Assignment - 1

Q Test for Consistency and solve

(i) $2x - 3y + 7z = 5$

$3x + y - 3z = 13$

$2x + 19y - 47z = 32$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$$[A:B] = \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 3 & 1 & -3 & : & 13 \\ 2 & 19 & -47 & : & 32 \end{bmatrix} \xrightarrow{R_1 \to \frac{1}{2}R_1} \begin{bmatrix} 1 & -3/2 & 7/2 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 3/2 & 7/2 & 5 \\ 0 & 11/2 & -27/2 & -2 \\ 0 & 22 & -54 & 22 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 4R_1} \begin{bmatrix} 1 & 7/2 & 7/2 & 5 \\ 0 & 11/2 & -27/2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A:B) = 3 \quad \rho(A) = 2 \neq$ No. of coloumn

$\rho(A) \neq \rho(A:B)$

Inconsistent (No soln)

(ii) $2x - y + 3z = 8$

$-x + 2y + z = 4$

$3x + y - 4z = 0$

: $AX = B$

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

$$(A:B) = \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix} \xrightarrow{R_1 \to 1/2 R_1} \begin{bmatrix} 1 & -1/2 & 3/2 & 4 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & 4 & 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$
$R_3 \to R_3 + 3R_1$

$$\begin{bmatrix} 1 & -1/2 & 3/2 & 4 \\ 0 & 3/2 & 1/2 & 8 \\ 0 & 0 & -4/3 & -25/2 \end{bmatrix} \xleftarrow{R_3 \to R_3 - \frac{5}{3}R_2} \begin{bmatrix} 1 & -1/2 & 3/2 & 4 \\ 0 & 3/2 & 1/2 & 8 \\ 0 & 5/2 & -1/2 & -12 \end{bmatrix}$$

Rank $\rho(A:B) = \rho(A) = 3$

System is consistent (unique solⁿ)

(c) find for what value of λ the given eqn. have a sol^n and solve them completely in each case

$x+y+z=1$

$x+2y+4z=\lambda$

$x+4y+10z=\lambda^2$

$AX=B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 0 & 3 & 9 & (\lambda^2-1) \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 0 & 0 & 0 & (\lambda^2-3\lambda+2) \end{bmatrix}$$

$\rho(A:B)=3,\ \rho(A)=2$

$\rho(A:B) \neq \rho(A)$

$\therefore$ Inconsistent

given eqⁿ has solⁿ, it should be consistent

$\rho(A:B) = \rho(A)$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda + \lambda + 2 = 0$

$\lambda(\lambda-2) - 1(\lambda-2)$

$\lambda = 1, 2$

for $\lambda = 1$.

$x + y + z = 1$

$y + 3z = 0$

No. of unknown (3) > Rank of A (2)

Eqⁿ has infinite solⁿ assign $3 - 2 = 1$

let $z = k$

$y + 3k = 0$

$y = -3k$

$x + y + z = 1$

$x - 3k + k = 1$

$x = 2k + 1$

$\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore \quad x + y + z = 1$

$y + 3z = 1$

assign to $3 - 2 = 1$ say k

let $z = k$

$y = 1 - 3k$

$x + y + z = 1$

$x + 1 - 3k + k = 1$

$\boxed{x = 2k}$

(d) find the soln of system of eqn

$x + 3y - 2z = 0, \quad 2x - y + 4z = 0, \quad x - 11y + 14z = 0$

$x + 3y - 2z = 0$

$2x - y + 4z = 0$

$x - 11y + 14z = 0$

$Ax = B$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$AX = 0$ $\therefore$ system is consistent (unique or infinite)

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_1 - 2R_1} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 8 \\ 0 & -14 & 16 \end{bmatrix}$$

$R_3 \to R_3 - \frac{14}{5} R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 8 \\ 0 & 0 & -32/5 \end{bmatrix}$$

$\rho(A) = 3$ = No. of unknowns

Consistent and has unique soln

(e) Find for what values of λ the given eqn $3x + y - \lambda z = 0$, $4x - 2y - 3z = 0$, $2\lambda x + 4y + \lambda z = 0$ may posses non-trivial soln & solve them completely in each case

eqn can be in form $AX = B$

$$\begin{bmatrix} 3 & -1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \xrightarrow{R_1 \to \frac{1}{3}R_1} \begin{bmatrix} 1 & -1/3 & -\lambda/3 \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$$

$$\Big\downarrow R_2 \to R_2 - 2R_3$$

$$\begin{bmatrix} 1 & -1/3 & -\lambda/3 \\ 0 & -10 & -3-2\lambda \\ 0 & 14/3 & 5\lambda/3 \end{bmatrix} \xleftarrow{R_3 \to R_3 - 2R_1} \begin{bmatrix} 1 & -1/3 & -\lambda/3 \\ 0 & -10 & -3-2\lambda \\ 2 & 4 & \lambda \end{bmatrix}$$

$$\Big\downarrow R_3 \to R_3 + \frac{7}{15} R_2$$

$$\begin{bmatrix} 1 & -1/3 & -\lambda/3 \\ 0 & -10 & -3-2\lambda \\ 0 & 0 & \left(\frac{11\lambda - 21}{15}\right) \end{bmatrix}$$

$\rho(A) =$ No. of unknown $= 3$

Consistent & unique

system to process non-trivial sol^n (infinite)

$\rho(A) <$ No. of unknown

$$\frac{11\lambda - 21}{15} = 0 \Rightarrow 11\lambda = 21$$

$$\lambda = \frac{21}{11}$$

Assignment - 2

(1) [1 0 0], [1 1 0], [1 1 1]

$a_1 = (1, 0, 0)$, $b_2 = (1, 1, 0)$, $c_2 = (1, 1, 1)$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$|A| = 1(1-0) - 0 + 0 = 1 \neq 0$

Linearly independent

(2) [7 -3 11 -6], [-56 24 -88 48]

$a[7 \; -3 \; 11 \; -6] + b[-56 \; 24 \; -88 \; 48] = 0$

$7a - 56b = 0$

$-3a + 24b = 0$

$11a - 88b = 0$

$-6a + 48b = 0 \rightarrow -a + 8b = 0$

$\boxed{a = 8b}$

$-3a + 24b = 0$

$-3(8b) + 24b = 0$

$0 = 0$ non-trivial sol^n

Eigenspace

$k \begin{bmatrix} 8 \\ 1 \end{bmatrix}$

(3) [-1 5 0], [16 8 -3], [-64 56 9]

$$A = \begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix}$$

$\det = -1(72+168) - 16(45-0) + (-64)(-15-0)$

$= 960 - 816$

$= 144 \neq 0$

Linearly Independent

(4) [2 -4], [1, 9], [3 5]

$a(2,-4) + b(1,9) + c(3,5) = 0$

$2a + b + 3c = 0 \quad \rightarrow \quad b = -3c - 2a$

$-4a + 9b + 5c = 0$

$-4a + 9b + 5c = 0$

$-4a + 9(-3c - 2a) + 5c = 0$

$-4a + (-27c - 18a) + 5c = 0$

$-4a - 27c - 18a + 5c = 0$

$-22a - 22c = 0$

$\boxed{a = -c}$

$b = -3a - 2a = -5a$

let $a = k$

$X = \begin{bmatrix} k \\ -5k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix}$ is a non-trivial sol^n

linearly independent

(5) $[3\ -2\ 0\ 4]$, $[5\ 0\ 0\ 1]$, $[-6\ 1\ 0\ 1]$, $[2\ 0\ 0\ 3]$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$c_1 \begin{bmatrix} 3 \\ 5 \\ -6 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_4 \begin{bmatrix} 4 \\ 1 \\ 1 \\ 3 \end{bmatrix} = 0$

$3c_1 - 2c_2 + 4c_4 = 0$

$5c_1 + c_4 = 0, \Rightarrow \boxed{c_4 = -5c_1}$

$-6c_1 + c_2 + c_4 = 0$

$2c_1 + 3c_4 = 0$

$\rightarrow 2c_1 + 3(-5c_1) = 0$

$2c_1 - 15c_1 = 0$

$c_1 = 0$

$$-c_2 + 5c_4 = 0$$

$$\boxed{c_2 = 5c_4}$$

$\therefore$ $c_4 = 0 \to c_2 = 0$ $\qquad c_1 = c_2 = c_3 = c_4 = 0$

linearly independent

(6) $[3\ 4\ 7],\ [2\ 0\ 3],\ [8\ 2\ 3],\ [5\ 5\ 6]$

$$A = \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 3 & 3 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 3 & 2 & 8 & 5 \\ 1 & -2 & -6 & 0 \\ 0 & 1 & -7 & -4 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 1 & -2 & -6 & 0 \\ 3 & 2 & 8 & 5 \\ 0 & 1 & -7 & -4 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$ $\begin{bmatrix} 1 & -2 & -6 & 0 \\ 0 & 8 & 26 & 5 \\ 0 & 1 & -7 & -4 \end{bmatrix}$ $R_2 \leftrightarrow R_3$ $\begin{bmatrix} 1 & -2 & -6 & 0 \\ 0 & 1 & -7 & -4 \\ 0 & 8 & 26 & 5 \end{bmatrix}$

$R_3 \to R_3 - 8R_2$

$$\begin{bmatrix} 1 & 2 & -6 & 0 \\ 0 & 1 & -7 & -4 \\ 0 & 0 & 82 & 37 \end{bmatrix}$$

$$\rho(A) + N = D$$

$$3 + N = 4$$

$$N = 1$$

$\rho(A) = 3 \neq N$ Infinite sol^n

linearly independent

Assignment - 3

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$[A][X] = \lambda [X]$

$[A][X] = \lambda [I][X]$

$[A - \lambda I)[X] = 0$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$-1(-12-(-3+3\lambda)) + 2(-6+(12+6\lambda)) - \lambda((-2-\lambda)(1-\lambda)-4)$

$\Rightarrow -1(-12+3-3\lambda) + 2($

$-\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$

$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

$(\lambda-3)(\lambda^2 - 2\lambda - 15) = 0$

$(\lambda+3)(\lambda+3)(\lambda-5) = 0$

$\lambda = -3, -3, 5$

$\boxed{\lambda = 3}$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 + 2x_2 - 3x_3 = 0$$

$$x_1 = -2x_2 + 3x_3$$

Eigen vector $x_1 = \begin{bmatrix} -2k_1 + 3k_2 \\ k_1 \\ k_2 \end{bmatrix}$

$$x_1 = k_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = 5}$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = k$

$$-x_1 - 2x_2 - 5x_3 = 0$$

$$-8x_2 - 16x_3 = 0$$

$$x_2 = -2k$$

$-x_1 = k$

$x_1 = -k$

Eigin vector

$$X = \begin{bmatrix} -k \\ -2k \\ k \end{bmatrix} = k\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

(2) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$[A - \lambda I][x] = 0$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$$

$-2(-1+\lambda) + (1-\lambda)\left((4-\lambda)(1-\lambda)\right) = 0$

$(1-\lambda)\left(2 + (4-\lambda)(1-\lambda)\right) = 0$

$(\lambda-1)(\lambda^2 - 5\lambda + 6) = 0$

$(\lambda-1)(\lambda-3)(\lambda-2) = 0$

$\lambda_1 = 1, \lambda_2 = 3, \lambda_3 = 2$

$\lambda = 1$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 = k$

$3x_1 + x_3 = 0$

$2x_3 = 0$

$2x_3 = 0$

$x_1 = 0, \quad x_2 = k, \quad x_3 = 0$

$$x = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix} \quad k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$X = k\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$x_1 + x_3 = 0$ $\qquad x_3 = k$

$-2x_2 + 2x_3 = 0$ $\qquad x_1 = -k$

$x_2 = k$

$\lambda_3 = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 + x_3 = 0$

$-x_2 + x_3 = 0$

$x_3 = k$

$x_2 = k$

$x_1 = -k/2$

$$X = k \begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$$

③ $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 3 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & 0 & -2 \\ 0 & -5 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 - 2x_3 = 0$

$-5x_2 + 3x_3 = 0$

$x_3 = k$

$x_1 = -2k$

$x_2 = 3/5\, k$

$X = k\begin{bmatrix} -2 \\ 3/5 \\ 1 \end{bmatrix}$

$\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 = 0$

$-x_1 + 3x_3 = 0$

$X = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$x_2 = k$

$x_1 = 0$ & $x_3 = 0$

$\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 = 0$

$-3x_2 = 0$

$x_3 = k$

$x_1 = 0$ & $x_2 = 0$

$$X = k\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(4)
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

$\boxed{\lambda = 0}$

$$\begin{bmatrix} 0 & 0 & -2 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 = 0$

$3x_2 + 4x_3 = 0$

let $x_1 = k$

$x_2 = 0$ & $x_3 = 0$

$$X = k\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 = 0$ $\qquad$ $x_2 = k$

$4x_2 = 0$ $\qquad$ $x_1 = 0$

$-5x_3 = 0$ $\qquad$ $x_3 = 0$

$$X = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda = -2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 = 0$

$5x_2 + 4x_3 = 0$

$x_3 = k$

$x_1 = 0$ & $x_2 = -\frac{4}{5}k$

$$X = k \begin{bmatrix} 0 \\ 4/5 \\ 1 \end{bmatrix}$$

Assignment - 4

① find the rank of the matrix A by using Row reduced echelon form

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow[R_4 \to R_4 - 2R_3]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix}$$

$$\downarrow R_3 \to R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix} \xleftarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix}$$

$$\downarrow R_4 \to R_4 - R_2 \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of A = 3

(2) let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} =$ $(a-b) + (b-c)x + (c-a)x^2$. find the rank and nullity of T

: A/p

every matrix and vector space W is of the form $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

linear transformation, $T: W \to P_2$

$$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

$1, x, x^2$ (R^3) three cofficients

Dimension $= 2+1 = 3$

The standard Basis for W is

$$\left\{\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

$$T \quad T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = \begin{bmatrix} a-b \\ b-c \\ c-a \end{bmatrix}$$

T represent in terms of standard Basis element

$$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

these column vector

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

Convert into row-echlon form

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

linearly independent

Rank = 3

Nullity = 0

$\rho(A) + N = n$

$3 + N = 3$

$N = 0$

③ let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ find the eigen values and eigen vector of A^{-1} and $A + 4I$

→ characteristic eqn

$\det(A - \lambda I) = 0$

$$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$4 + \lambda^2 - 4\lambda - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda^2 - 3\lambda - \lambda + 3 = 0$$

$$\lambda(\lambda - 3) - 1(\lambda - 3) = 0$$

$\lambda = 1, 3$ → eigen values

$\underline{\lambda = 1}$ $[A - \lambda I]\, x = [0]$

$$\left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

$$\left(\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

$x - y = 0$

$x = y$

$-x + y = 0$

if $x = k$, then $x = y = k$

∴ eigen space

$\begin{bmatrix} k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\underline{\lambda = 3}$

$[A - 3I][X] = [0]$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x - y = 0 \Rightarrow \boxed{-x = y}$

if $x = k$, then $x = -y = -k$

Eigen space $= \begin{bmatrix} k \\ -k \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Eigen values for $A + 4I$

$\Rightarrow \lambda_1 + c,\ \lambda_2 + c$

$1 + 4,\ 3 + 4$

$\boxed{5, 7}$

(4) Solve by Gauss-Siedel method

$3x + 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x + 0.2y + 10z = 71.4$

With initial values $x(0)=0$, $y(0)=0$, $z(0)=0$

⇒ Now initial $k = 0$

$$x^{(k+1)} = \frac{1}{a_{11}}\left(b_1 - a_{12}y^{(k)} - a_{13}z^{(k)}\right)$$

$$y^{(k+1)} = \frac{1}{a_{22}}\left(b_2 - a_{21}x^{(k+1)} - a_{23}z^{(k)}\right)$$

$$z^{(k+1)} = \frac{1}{a_{33}}\left(b_3 - a_{31}x^{(k+1)} - a_{23}y^{(k+1)}\right)$$

1st iteration

$$x^{(1)} = \frac{1}{3}\left(7.85 + 0.1\cdot 0 + 0.2\cdot 0\right) = \frac{7.85}{3} = 2.6167$$

$$y^{(1)} = \frac{1}{7}\left(-19.3 - 0.1x^{(1)} + 0.3\times 0\right) = \frac{1}{7}\left(-19.3 - \frac{0.785}{3}\right)$$

$$= -2.7571$$

$$z^{(1)} = \frac{1}{10}\left(-71.4 - 0.1\cdot x^{(1)} + 0.3y^{(1)}\right) : \frac{1}{10}\left(-71.4 - \frac{0.705}{3} + 10\right)$$

$$= \frac{+198.1505}{30} = 6.7752$$

2nd.

$$x^{(2)} = \frac{1}{3}(7.85 + 0.1 \cdot y^{(1)} + 0.2\, z^{(1)})$$

$$= \frac{1}{3}\left(7.85 + 0.1\left(\frac{56.115}{21}\right) + 0.2\left(\frac{-198.1505}{30}\right)\right)$$

$$= 1.7908$$

$$y^{(2)} = \frac{1}{7}(-19.3 - 0.1(-2.6167) + 0.3(6.77571)) = -3.3377$$

$$z^{(2)} = \frac{1}{10}(71.4 - 0.3(1.7908) + 0.2(-3.7777)) = 7.2991$$

3rd iteration

$$x^{(3)} = \frac{1}{3}(7.85 + 0.1(-3.3377) + 0.2(7.2991)) = 1.6792$$

$$y^{(3)} = \frac{1}{7}(-19.3 - 0.1(1.6792) + 0.3(7.2991)) = -3.2744$$

$$z^{(3)} = \frac{1}{10}(71.4 - 0.3(1.6792) + 0.2(-3.2744)) = 7.3561$$

(5) Define Consistent & inconsistent system of eqn
Hence the solve following system of eqn
if Consistent

$x+3y+2z=0$, $2x-y+3z=0$, $3x-5y+4z=0$
$x+17y+4z=0$

: $Ax=B$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Since $B=0$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1}]{R_4 \to R_4 - R_1} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix} \xrightarrow{R_4 \to R_4 + R_1} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$(\rho(A) = 2) < (n = 3)$

infinite soln

$1x + 3y + 2z = 0$

$-7y - z = 0$

$\boxed{z = 7y}$

$x + 3y + 14y = 0$

$x + 17y = 0$

$x = -17y$

$$X = \begin{bmatrix} -17k \\ k \\ 7k \end{bmatrix} = k \begin{bmatrix} -17 \\ 1 \\ 7 \end{bmatrix}$$

$y = k$, $x = -17k$, $z = 7k$

(6) Determine the function $T : P_2 \to P_2$ is linear transformation or not when

$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

Check linear Transformation

(1) Additivity $T(u+v) = T(u) + T(v)$

(2) Homogenity $T(cu) = \cancel{cT(u)}\ cT(u)$

(1) ~~Additivity~~ $T(a_1 + b_1 x + c_1 x^2) + (a_2 + b_2 x + c_2 x^2))$

$T((a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2)$

$(a_1 + a_2 + 1) + (b_2 + b_1 + 1) + (c_1 + c_2 + 1)x^2$

$T(a_1 + b_1 x + b_2 x^2) + T(a_2 + b_2 x + c_2 x^2)$

$(a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$

$(a_1 + a_2 + 2) + (b_1 + b_2 + 2)x + (c_2 + c_1 + 2)x^2 = R.H.S$

L.H.S $\neq$ R.H.S

T is not additive

② Homogenity

$T(cu) = T(c(a+bx+cx^2)) =$

$(ca+1) + (cb+1) + (cc+1)x^2$

$cT(u) = cT(a+bx+cx^2) = c((a+1)+(b+1)x+(c+1)x^2)$

$(ca+c) + (cb+c)x + (cc+c)x^2$

Not equal

T is not Homogenous

Therefore T is not linear Transformation

1. Determine whether the set $S = \{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis determine the dimension and the basis of the subspace spanned by S

determine the set S is a basis for $V_3(R)$

we need 2 things

1. linear independence
2. spanning

→ form a ~~vector~~ matrix with vector of S as its column

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 9/5 R_2} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank = 2 < n = 3

sol is infinite

linearly independent

Hence don't span $V_3(R)$ and not basis

(9.) Explain one application of matrix operation in image processing with example

→ one common application of matrix operation in image processing is image rotation

Image rotation is a geometric transformation that maps the position of every object in the image to a new location. This transformation is often represented as a matrix. for a 2D image, the transformation matrix for a rotation by an angle θ is

$$\begin{bmatrix} \cos\theta & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

let's consider a simple eg where we have 2D image represented as a matrix. we want to rotate this image by 90 deg counter clockwise. The transformation matrix for a 90 deg rotation

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

If our image is represented by the matrix 'A' the rotated image 'B' can be obtained by the matrix multiplication $B = K$ k is the rotation matrix.

(10) Give a brief description of linear transformation for Computer vision for rotating 2D image

→ linear transformation in computer vision, particularly in the content of rotating 2D images involving applying transformation matrices to the images coordinates to achieve rotation

In a 2D space, rotation can be described using rotation matrix for example, to rotate a point (x, y) Counter clockwise by an angle θ.

To rotate an entire image, we apply this rotation matrix to each pixel coordinates

Here a simple algorithm to rotate an image

(1.) apply the inverse transformation to bring the pixel back to the original image coordinates center

- apply the rotation matrix R to the pixel coordin
- apply the forward transformation to bring the pixel back to the image coordinates system.

(2.) Repeat this process for all pixel in the original image

~~(3.)~~ ~~Hsox~~