Motion profile planning for reduced jerk and vibration residuals

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Motion profile planning for reduced jerk and vibration residuals

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Abstract – Jerk limitation for a motion stage is important to suppress transient vibration and to reduce the settling time. This paper presents a novel motion control method which aims to reduce jerk and transient vibration for linear motion stages. In this approach, the acceleration profile is designed to obtain a smoother movement without any abrupt corner. The velocity and the displacement profiles are calculated by integration. To implement the proposed motion profile on a DSP-based motion controller, a table look-up and linear interpolation method is used to relieve the calculation burden to the DSP controller in real-time. A voice coil motor driving motion stage with a fixed-point DSP controller is used as the test-bed, where a point-to-point linear motion task is designed and implemented to test the proposed motion profile. Experimental results have shown that by using the new profile, the motion induced vibration on the stage frame can be greatly reduced.

Keywords: Vibration control, Jerk, Motion control, DSP controllers

1 BACKGROUND

High-performance motion control is widely needed in modern electronics manufacturing industry. Due to the global competency, there is an increasing demand for high-precision and high-speed automatic assembly equipment.

An important issue for such equipment is the residual vibration caused by the inappropriate acceleration or deceleration motion, which results in decreased accuracy and increased settling time to the positioning table.

To solve such problems, the new-generation motion controller must have the capability to plan trajectory and generate motion profile in such a way that the accelerating and decelerating phases become much smoother to reduce inertia force and residual vibration.

A typical point-to-point motion profile can be divided into three phases: accelerating, constant speed, and decelerating. Traditional motion control schemes use a simple trapezoidal velocity profile. The machine is accelerated at a constant acceleration until it reaches a maximum velocity, then keeps this constant velocity for a specified time, and is decelerated to rest at a constant deceleration.

One of the main problems with this trapezoidal velocity profile is the induced vibration. The sudden change of acceleration will result in a large jerk force to the structure of the motion stage, and the higher the acceleration, the higher the force to excite vibration. Hence it has the disadvantages of longer settling time and limited acceleration, thus lower speed and accuracy.

To improve the performance, *s-curve* velocity profiles have been used to reduce the jerk force and the tendency to excite structural vibration. In the *s-curve* motion profile, the velocity profile is modified to have an *s-shape* during the acceleration and deceleration periods to reduce the residual vibrations caused in a mechanical system by a moving mass. It means that the trajectory ramps up to peak acceleration and ramps down to constant speed. Compared with the trapezoidal velocity profile, the resulting motion can be both faster and more precise. However, it is noted that the *s-curve* profile still exhibit a sudden change of jerk profile, and the finite jerk spreads out over a period of time [1].

In recent years, methods using input design or input shaping have been developed to reduce the unwanted oscillations caused by poorly damped poles of the system, by specific design of the motion trajectory to minimise energy in the input signal at the frequencies corresponding to the poles [2-5].

Meckl et al [2] proposed an approach to optimise the selection of the ramp-up and ramp-down time for *s-curve* motion profiles to reduce the residual vibration. This method was based on a frequency analysis that minimises the excitation energy of the input forcing function at the system natural frequency. However, one problem with these methods is that knowledge of the system natural frequencies are needed.

Higher order trajectory planning approaches have also been used to smooth the trajectory and bound jerk. For example, Lambrechts et al [6] proposed design method using fourth-order feed-forward with fourth-order trajectories for single-axis motion control. Their results showed superior effectiveness of fourth-order feed-forward in comparison with lower-order feed-forward.

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Macfarlane and Croft [1] presented a method using a concatenation of fifth-order polynomial to provide a smooth trajectory for point-to-point motion with jerk limits. They used a sine wave template to calculate the end conditions for ramps from zero acceleration to non-zero acceleration. It is also noted that for higher order trajectories, the burden for the controller can be quite high due to the complexity in computation. In high-speed cam design, there exist some sophisticated approaches to determine a cam contour so as to deliver a specific smooth motion with constraint jerk [7,8], however, few of these techniques has been applied to a linear motion stage.

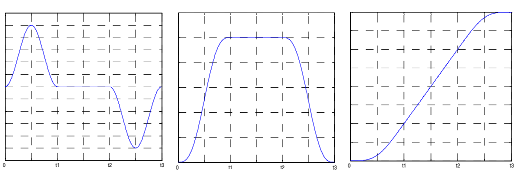
2 OBJECTIVE

This study aims to develop a novel motion control method to minimise jerk and to reduce the residual vibration. It employs a new motion profile generation approach to smooth the acceleration profile which results in a smoother jerk profile. To simplify the computing complexity and to relief the burden of the motion controller, a look-up table is used in an embedded controller for real-time motion profile generation and control. This approach is implemented in a voice-coil driven linear stage with a TI DSP controller. This paper presents the methodology, implementation and experimental results.

3 MOTION PROFILE PLANNING

The purpose of this method is to minimise the high acceleration induced jerk in high-speed, high-precision motion control. As jerk is proportional to the derivative of acceleration, so the basic idea is to reduce the change rate of the acceleration.

A schematic view of the developed acceleration, velocity, and displacement profiles is shown in Fig. 1. The new motion profile also comprises the three parts, i.e., accelerating phase where $t \in [0, t_1]$, constant velocity phase where $t \in [t_1, t_2]$, and decelerating phase where $t \in [t_2, t_3]$. In the accelerating and decelerating phases, the acceleration profile is derived from a level-shifted cosinoidal function to obtain a smooth movement. The velocity and the final displacement profile can be calculated by integration.



(a) (b) (c)

Fig. 1. The proposed motion profile in the format of: (a) acceleration, (b) velocity, and (c) displacement.

At an arbitrary time instant, t, the acceleration can be expressed as:

$$a = \frac{A}{2}\left(1 - \cos\left(\frac{2\pi}{T_1}t\right)\right) \quad t \in [0, t_1] \tag{1}$$

$$a = 0 \quad t \in [t_1, t_2] \tag{2}$$

$$a = -\frac{A}{2}\left(1 - \cos\left(\frac{2\pi}{T_1}(t - t_2)\right)\right) \quad t \in [t_2, t_3] \tag{3}$$

where A is a factor determining the maximum acceleration, T_1, T_2, and T_3 are time intervals for acceleration, constant velocity, and deceleration phase respectively. T_1, T_2, and T_3 can be written as:

$$T_1 = t_1 - t_0 = t_1 \quad (let\, t_0 = 0) \tag{4}$$

$$T_2 = t_2 - t_1 \tag{5}$$

$$T_3 = t_3 - t_2 \tag{6}$$

The time interval for the first and third part, T_1 and T_3 respectively, are set equal, i.e., $T_1 = T_3$. The time interval for the constant velocity, T_2, can be freely altered based on the moving distance. Then the time for the whole movement is:

$$T = 2T_1 + T_2 \tag{7}$$

The acceleration profile is given in Fig. 1(a), which shows a smooth transition of the acceleration. The velocity profile can be developed by integration of the acceleration profile. Integration of Eq. (1) leads to the velocity, v, at the acceleration phase as:

$$v = \frac{A}{2}\left(\frac{T_1}{2\pi}\right)\left(\frac{2\pi}{T_1}t - \sin\left(\frac{2\pi}{T_1}t\right)\right) \quad t \in [0, t_1] \tag{8}$$

After t_1, the motion reaches a constant velocity, v_c, as:

$$v = v_c = \frac{1}{2}AT_1 \quad t \in [t_1, t_2] \tag{9}$$

The velocity at the deceleration phase is developed by integration of Eq. (3) and using the constant velocity as its initial condition. It leads to:

$$v = \frac{A}{2}\left(\frac{T_1}{2\pi}\right)\left(\frac{2\pi}{T_1}\left(1 - \frac{t - t_2}{T_1}\right) + \sin\left(\frac{2\pi}{T_1}(t - t_2)\right)\right) \quad t \in [t_2, t_3] \tag{10}$$

A plot of the velocity profile is given in Fig. 1(b). The start value and stop value for the velocity profile must be zero.

Integrating Eqs. (8), (9), and (10) one more time gives the final formulas for the displacement, d, as:

$$d = \frac{A}{2}\left(\frac{T_1}{2\pi}\right)^2 \left(\frac{1}{2}\left(\frac{2\pi}{T_1}t\right)^2 - \left(1-\cos\left(\frac{2\pi}{T_1}t\right)\right)\right) \quad t \in [0, t_1] \tag{11}$$

$$d = \frac{1}{4}AT_1^{\,2} + \frac{1}{2}AT_1(t - t_1) \quad t \in [t_1, t_2] \tag{12}$$

$$d = \frac{1}{4}AT_1^2 + \frac{1}{2}AT_1T_2 + \frac{A}{2}\left(\frac{T_1}{2\pi}\right)^2 \cdot \left(\frac{(2\pi)^2(t-t_2)}{T_1} - \frac{1}{2}\left(\frac{2\pi(t-t_2)}{T_1}\right)^2 + \left(1-\cos\left(\frac{2\pi}{T_1}(t-t_2)\right)\right)\right)$$
$$t \in [t_2, t_3] \tag{13}$$

It is easy to show that the moving distances at the acceleration phase and deceleration phase are equal, which is:

$$l_1 = l_3 = \frac{1}{4}AT_1^{\,2} \tag{14}$$

The moving distance at the constant velocity phase is:

$$l_2 = \frac{1}{2}AT_1T_2 \tag{15}$$

The total distance moved is:

$$l_{total} = l_1 + l_2 + l_3 = \frac{1}{2}AT_1^{\,2} + \frac{1}{2}AT_1T_2 \tag{16}$$

The final movement profile is shown in Fig. 1(c).

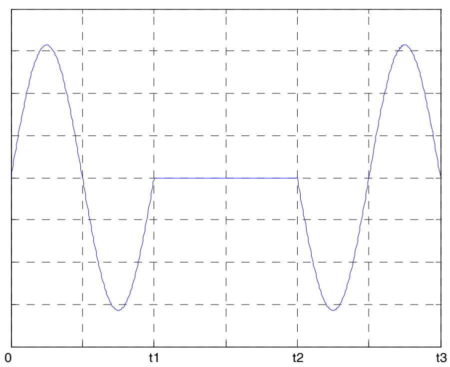


Fig. 2. Jerk profile based on the proposed motion profile.

The resulted jerk profile, as shown in Fig. 2, can be calculated by differentiating Eqs. (1) – (3), which is:

$$jerk = \frac{A\pi}{T_1}\sin\left(\frac{2\pi}{T_1}t\right) \quad t \in [0, t_1] \tag{17}$$

$$jerk = 0 \quad t \in [t_1, t_2] \tag{18}$$

$$jerk = -\frac{A\pi}{T_1}\sin\left(\frac{2\pi}{T_1}(t - t_2)\right) \quad t \in [t_2, t_3] \tag{19}$$

The maximum acceleration, A'_{max}, and maximum speed, v'_{max}, that the motion stage can reach are determined by the capacity of the motor and the load. If a specified maximum acceleration, A_{max}, and maximum speed, v_{max}, are used in the motion scheduling, then the maximum time interval for both the acceleration phase and the deceleration phase is:

$$T_{1\max} = T_{3\max} = \frac{2v_{\max}}{A_{\max}} \tag{20}$$

The sum of the moving distances at the acceleration phase and the deceleration phase is:

$$l_{1+3} = \frac{1}{2}A_{\max}T_{1\max}^2 = \frac{2v_{\max}^2}{A_{\max}} \tag{21}$$

Assume the stage is required to move a distance of l. If $l>l_{1+3}$, then there is a need for the constant velocity phase which takes a time interval of:

$$T_2 = \frac{2l_2}{A_{\max}T_{1\max}} = \frac{2(l - l_{1+3})}{A_{\max}T_{1\max}} = \frac{2\left(l - \frac{2v_{\max}^2}{A_{\max}}\right)}{A_{\max}T_{1\max}} \tag{22}$$

If $l<l_{1+3}$, then there is no need for the constant velocity phase, and the time intervals for the acceleration phase and the deceleration phase should be reduced based on the needed moving distance, the total moving distance is divided into two equal parts for acceleration and the deceleration, which takes a time interval of:

$$T_1 = T_3 = \sqrt{\frac{4l_1}{A}} = \sqrt{\frac{2l}{A}}, \quad A \le A_{\max} \tag{23}$$

4 CONTROLLER IMPLEMENTATION

The developed motion profile is implemented on a fixed-point DSP controller. The DSP controller performs the servo motion control by providing motion profile generation, position determination, position error calculation, and motion commands creation in real-time. To release the calculation burden of the DSP controller, the new motion profile generation algorithm is implemented by using table look-up and linear interpolation method.

In this approach, the profile data is first calculated using a Matlab script, which generates a header-file containing the look-up table and other data such as the start and stop positions. The look-up table only holds the information for the acceleration part of the profile (T_1). The values for the constant velocity and deceleration parts are calculated by the DSP. For the constant velocity segment the start and end points are calculated and the values between are interpolated by the DSP. For the deceleration phase the look-up table is read in reverse. The Matlab script allows users to set the resolution of the look-up table. The values between the look-up table steps are linearly interpolated. This interpolation is also done by the DSP.

After power on, the controller system first initialises the main DSP registers and the peripherals. The profile generation is integrated into the main interrupt routine. The program knows from the header file how many interrupt steps the whole motion takes and uses a counter to track which part of the profile has to be calculated. After reaching the end position, the program rewrites the table so that it can be used for the motion back to the start position.

5 EXPERIMENTAL RESULTS

A testbed has been developed as shown in Fig. 3 to implement the proposed motion control scheme. It comprises a BEI integrated linear voice coil motion module sitting on a flexible aluminium structure, and a TI DSP controller. The motion module assembly consists of a voice coil actuator integrated with a linear guidance for linear motion, and a linear encoder system as the linear feedback device. The motor is attached to the top of an aluminium frame so that the vibrations generated by its movement can be easily measured for evaluation.

Voice coil actuators are a kind of non-commutated DC linear motors that operate at high speeds without cogging or ripple and with infinite resolution. They are widely used in many industry applications where high speed, short stroke and high accuracy are needed. In this set-up, a BEI LA13-12-000 linear voice coil motor is used as the motion actuator. The moving coil actuator produces a force proportional to the applied current. The direction of travel can be reversed by reversing the polarity of the applied current. The linear stage has a stroke of 11.1 mm. A BTA-28V-6A amplifier is used to drive the motor. A MicroE Mercury 2000 linear encoder system is attached to the linear stage as a position feedback device. The linear encoder provides a resolution up to 1 μm.

The digital controller is implemented on a TI TMS320LF2407A evaluation module (EVM). The LF2407A DSP is a 16-bit fixed-point digital signal processor. It offers 40 MIPS performance and a combination of standard on-chip peripherals such as communication interfaces, ultra-fast A/D converters, PWM generation and QEP/CAP modules. The EVM also provides four 12-bit digital to analogue converter (DAC) channels.

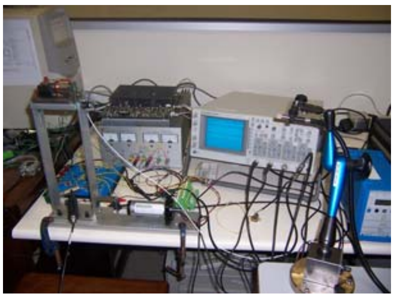



Fig. 3. The motion stage and testbed setup.

To achieve the desired motion profile, the control system regulates the motor in order to allow the position output to behave as expected.

In the close loop servo system, the controller gets the position information from the encoder using its QEP (quadrature encoded pulses) interface, and outputs the motor-driving command to the motor amplifier using one external DAC channel, and compared with the target position generated from the ideal motion profile. The position error is then used in a proportional-integral-derivative (PID) compensator to calculate the next required motor-driving command value so as to control the motion of the stage and let it move along the given profile. The analogue input and output of the DSP are from 0 to 3.3 V DC.

Signal conditioning and level shifting circuits are designed to interface the controller with the motor drive. The system sample-rate is set to 2.5 kHz. The controller software is developed using TI's Code Composer Studio. Mixed C and assembly languages are used for optimised code execution efficiency. Q-format is employed in the fixed-point DSP programming for a better data dynamic range and to avoid multiplication that can cause data overflow.

In the experiment, a point-to-point linear motion task was designed on the motion stage to move a distance of 8 mm in 200 ms, stop for a period and move back to the starting position. The time for acceleration and deceleration was set to 70 ms, the time for the constant velocity to 60 ms.

Two different motion profiles were implemented for comparison, where one was the proposed new motion profile, and the other was a simple linear motion profile. Both profiles were implemented on the fixed-point DSP controller. The measured stage position through the encoder was output by the digital to analogue converter of the DSP Board for monitoring and debugging purpose.

To investigate the performance of the new motor profile, a Polytec laser Doppler vibrometer (LDV) is used to monitor the induced vibration on the frame at the motion direction. The LDV was set to a measurement range with a sensitivity of 2 mm/s/V. A Fluke Scopemeter is used to display both the position signal and the vibration velocity of the frame. Channel A was connected to the LDV output for the vibration, and Channel B was connected to the DAC output of the DSP board for the measured motor position.

When the motor was set to the stop status, the background vibration disturbance was recorded as shown in Fig. 4. The measured vibration signal had a root-mean-square (RMS) value of 32 mV, corresponding to a RMS velocity of 0.064 mm/s, and a peak-to-peak (PTP) value of 168 mV, corresponding to a PTP velocity of 0.336 mm/s.

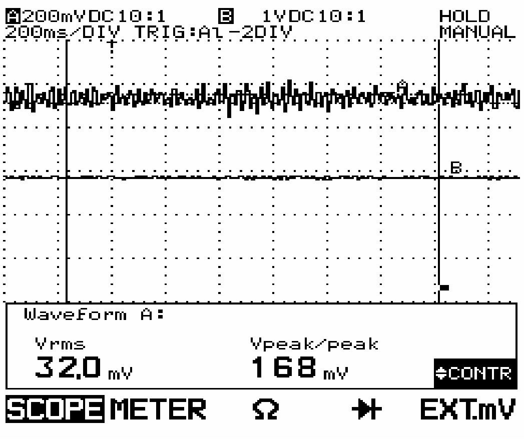


Fig. 4. The measured background vibration when the stage was in stop mode.

When the stage was executing the abovementioned motion using the new motion profile generation strategy, the frame vibration and stage motion profile were recorded as shown in Fig. 5. In this screenshot, the vibration signal had a RMS value of 160 mV, corresponding to 0.32 mm/s for the RMS velocity, and a PTP value of 1.52 V, corresponding to 3.04 mm/s for the PTP velocity.

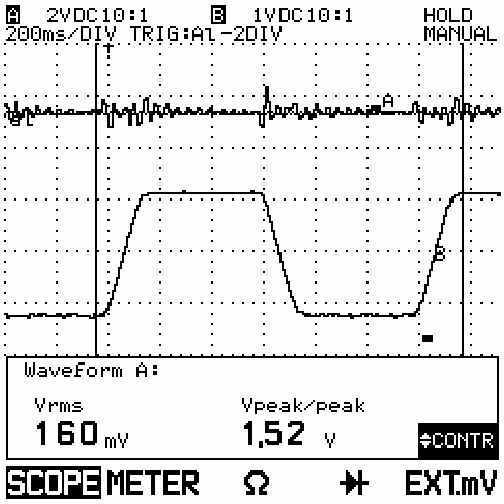


Fig. 5. The measured vibration profile and motor position profile using the developed motion profile generation method.

For comparison, the stage was set to run the same distance using a linear motion profile generation method. The recorded vibration and motion profile were shown in Fig. 6. The measured vibration signal value was 400 mV in RMS, corresponding to a RMS velocity of 0.80 mm/s. The measured PTP value was 2.32 V, corresponding to a PTP velocity of 4.64 mm/s. Comparing the motion profile and vibration profile as shown in Figs. 5 and 6, it is evident that peak vibration usually occurs at the beginning and ending phase of a point-to-point motion, which can be mainly attributed to the jerk and sudden change of acceleration. It can be seen that the new motion profile generation strategy resulted in lower vibration and quicker settling time.

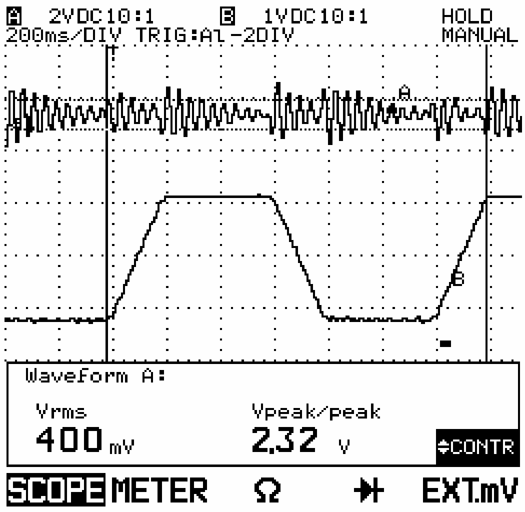


Fig. 6. Measured vibration profile and motor position profile using a linear motion profile generation method.

Another set of experiment was done with an increased motion speed. The motion stage was set to move the same distance of 8 mm in 150 ms. The measured vibration profiles and motor position profiles are shown in Figs. 7 and 8, where Fig. 7 corresponds to using the developed motion profile generation strategy, and Fig. 8 corresponds to using the linear motion profile scheme. It is clear that higher running speed can cause higher stage vibration. Comparing Figs. 7 and 8, the vibration resulted from the linear motion profile generation method was almost double of that using the proposed method. It can be seen from the results that the new profile greatly reduced the vibration to the plant.

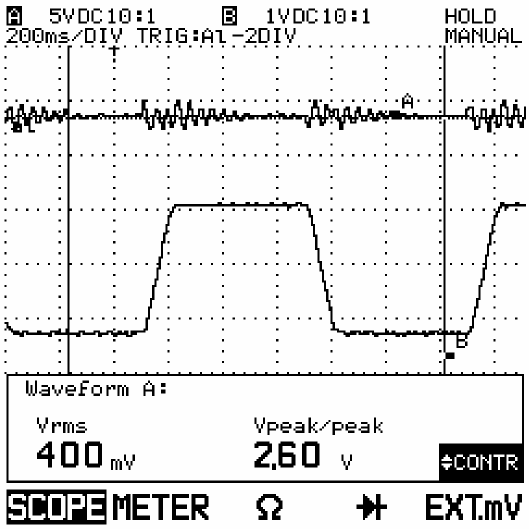


Fig. 7.The measured vibration profile and motor position profile at higher speed using the developed motion profile generation method.

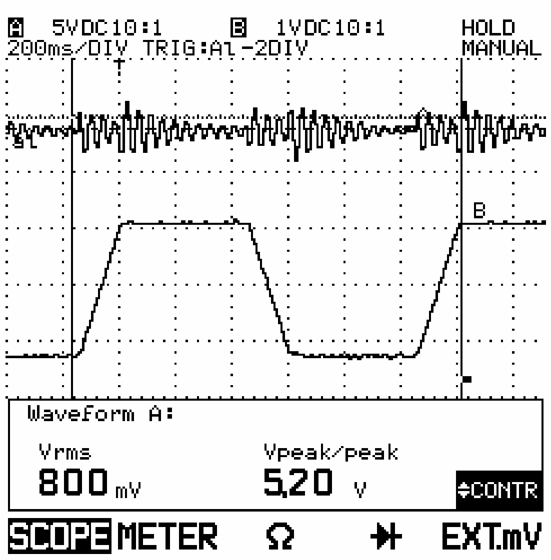


Fig. 8. The measured vibration profile and motor position profile at higher speed using a linear motion profile generation method.

The results showed that it is possible to reduce the motion-induced transient vibration of a structure by using the proposed new motion profile generation strategy. In comparison to the simple linear motion profile, the vibration of the structure is greatly reduced by using the new method. The cost of this reduction lies in the greater need of computation power and the more difficult implementation. Further research work is still needed to explore if it is possible to further reduce the vibration by altering the time constants T_1 and T_2, and the performance of the profile at higher speeds and accelerations.

6 CONCLUSION

This paper introduced the development of a new motion control approach for linear motion stages where a novel motion profile generation strategy was proposed to reduce the motion-induced jerk and vibration. In this method, the acceleration profile is designed based on a level-shifted cosinoidal function to obtain a smoother movement without any corner. The velocity and the displacement profiles are calculated by integration. To implement the proposed motion profile on a DSP-based motion controller, a table look-up and linear interpolation method is used to relieve the calculation burden to the DSP controller in real-time.

The profile data is first calculated using a Matlab script, which generates a header-file containing the look-up table and other variables. On the DSP controller side, the profile generation is integrated into the main interrupt routine. The program knows from the header file how many interrupt steps the whole motion takes and uses a counter to track which part of the profile has to be calculated. The values between the look-up table steps are linearly interpolated. In the experiment, a voice coil motor driving motion stage with a fixed-point DSP controller was used as the testbed, where a point-to-point linear motion task was designed. The results showed that the new profile dramatically reduced the vibration to the stage frame.

7 INDUSTRIAL SIGNIFICANCE

The proposed motion control method can be applied to a wide range of precision machines in electronics manufacturing industry which uses high-speed high-precision motion stages, such as pick-and-place manipulator, industrial robots, positioning systems, wafer inspection machine, and so on.

REFERENCES

[1] S. Macfarlane, and E.A. Croft, "Jerk-bounded manipulator trajectory planning: design for real-time applications", *IEEE Trans. Robotics and Automation*, vol. 19(1), pp. 42-52, 2003.

[2] P.H. Meckl, P.B. Arestides, and M.C. Woods, "Optimized S-Curve Motion Profiles for Minimum Residual Vibration", *Proc. American Control Conf.*, Pennsylvania, USA, June 1998, pp. 2627-2631.

[3] N.C. Singer, and W.P. Seering "Preshaping command inputs to reduce systems vibration", *Trans. ASME, J. Dynamic Systems, Measurement and Control*, vol. 112, pp. 76-82, 1990.

[4] L.Y. Pao, T.N. Chang, and E. Hou, "Input shaper designs for minimizing the expected level of residual vibration in flexible structures", *Proc. American Control Conf.*, 1997, pp. 3542-3546.

[5] B.G. Dijkstra, N.J. Rambaratsingh, C.W. Scherer, O.H. Bosgra, M. Steinbuch, and S. Kerssemakers, "Input design for optimal discrete time point-to-point motion of an industrial xy-positioning table", in *Signals, Systems and Control*, Edited by O.H. Bosgra, P.M.J. Van den Hof and C.W. Scherer, Delft: Delft University Press, pp. 9-14, 2001.

[6] P. Lambrechts, M. Boerlage, and M. Steinbuch, "Trajectory planning and feedforward design for electromechanical motion systems," Control Engineering Practice, vol. 13, pp. 145-157, 2005.

[7] A.G. Erdman, and G.N. Sandor, *Mechanism Design: Analysis and Synthesis*, Prentice-Hall, Inc., 1984.

[8] J.E. Shigley, and J.J. Uicker Jr., Theory of Machines and Mechanisms, 2nd Ed., McGraw-Hill, Inc., 1995.