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Department of Physics

Summer Internship Report

Study of Michelson Interferometer to Determine Spectral Purity of a Source

Submitted by

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ABSTRACT

This report presents a detailed study on the modeling of a Michelson Interferometer, with a particular emphasis on how real-world light sources—characterized by finite spectral bandwidth—affect the resulting interference pattern. The report begins by detailing the setup and working of the Michelson Interferometer, followed by study of the concept of coherence, both temporal and spatial, which plays a critical role in the formation and visibility of interference fringes. The concept of partial coherence is discussed in the context of realistic optical sources. The third section of the report focuses entirely on the algorithm of the interferometer model using Python. This simulation provides valuable insights into the relationship between coherence properties and interference behavior in practical optical systems. In the final section of the report Coherence length and hence the spectral purity of a give source is determined.

Chapter 1

Michelson Interferometer

Michelson interferometer is based on interference by the division of amplitude where a light beam is split by a beam splitter and recombined to observe the interference pattern. The key lies in the fact that the light beams emerging from same source combine at same point but coming from different paths.

1.1 Setup of Michelson Interferometer

Michelson interferometer includes a source E_s whose beam of light is split into two parts when it passes through a beam splitter (50:50), E_1 reflected goes to Mirror M_1 and E_2 gets transmitted and goes to Mirror M_2. E_1 also undergoes a phase shift of π due to reflection from rarer to dense medium. Both E_1 and E_2 reflect from mirror M_1 and M_2 and then recombine at the beam splitter, where E_1 is transmitted and E_2 is reflected (note that E_2 does not go any phase change because it is reflecting from dense to rarer medium). Both E_1 and E_2 recombine to form the interference pattern observed on screen.

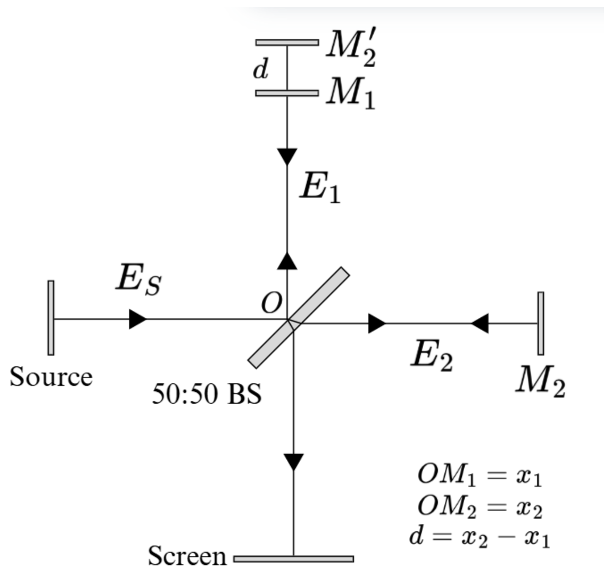
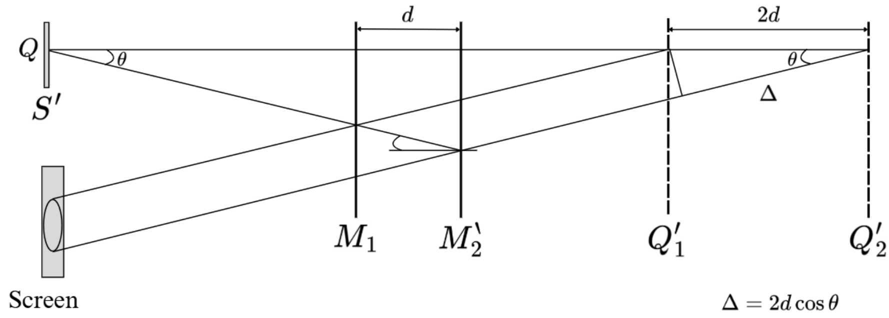


Figure 1.1: Setup of a Michelson interferometer

One of the Mirrors (say M_2) is mounted on a precision translation stage with a micrometer screw which can be used to change the optical path difference.

1.2 Interference Condition

For the interference condition we first need to calculate the effective path difference. The actual interferometer in Figure above possesses two optical axes at right angles to one another. A simpler but equivalent optical system, having a single optical axis, can be drawn by working with virtual images of the source S and mirror M_2 via reflection in the BS mirror. E_S and E_2 line are rotated counterclockwise 90° about the point of intersection of beams with the BS mirror. The resulting geometery is shown below.



Now if the distance of the BS from M_1 is x_1 and from M_2 is x_2 then the resulting difference between the two is $d = x_2 - x_1$. The new position of the source plane is S', and position of M_2 is M_2'. Light from a point Q on the source plane S' is reflected from both the mirrors M_1 and M_2', shown parallel with optical path difference d. The two reflected beams appear to come from the two virtual images Q_1' and Q_2' is $2d$, and the optical path difference Δ between the two beams emerging from the interferometer is

$$\Delta = 2d\cos\theta \tag{1.1}$$

Where θ is the inclination of the beams relative to the optical axis. For a normal beam $\theta = 0$ and $\Delta = 2d$. Now we can calculate the interference condition.

$$\text{For Bright fringe}: \; 2d\cos\theta = \left(n + \frac{1}{2}\right)\lambda \tag{1.2}$$

$$\text{For Dark fringe}: \; 2d\cos\theta = n\lambda \tag{1.3}$$

where $n = 0, 1, 2, ...$ The $(n + 1/2)\lambda$ for bright fringe and $n\lambda$ for dark fringe comes from fact that there is an addition phase of π accumulated by the beam. Mirror M_1 is mounted on a precision translation stage with a micrometer screw

1.3 Image Formation

Assume $E_P = E_1 + E_2$ both E_1 and E_2 emerge from the same source path, superpose at the same point P but traveling different paths accumulating individual phases. Let E_1 and E_2 be-

$$E_1 = Ae^{-iwt+\phi_1} \text{ and } E_2 = Ae^{-iwt+\phi_2}$$

We assume here that both the field have the same amplitudes, then-

$$E_P = E_1 + E_2$$

$$E_P = Ae^{-iwt+\phi_1} + Ae^{-iwt+\phi_2}$$

Intensity or irradiance is equal to-

$$I = |E_P|^2 = |Ae^{-iwt+\phi_1} + Ae^{-iwt+\phi_2}|^2$$

$$I = |A|^2|e^{iwt+\phi_1} + e^{iwt+\phi_2}|^2$$

$$I = 2I_0(1 + \cos\phi) \tag{1.4}$$

where $\phi = \phi_2 - \phi_1$ this ϕ is calculated by-

$$\phi = \frac{2\pi}{\lambda}(2d\cos\theta)$$

Here we can see that if $\cos\phi = 1$ we get $I = 4I_0$ (Bright Fringe) and $\cos\phi = -1$ we get $I = 0$ (Dark Fringe).

Some Observations

1. For Dark fringe we have $2d\cos\theta = n\lambda$,
   If we fix $n = 998$, $\lambda = 6000$ and vary d then-

   (a) For $d = 0.29$mm $\theta = 1.03°$

   (b) For $d = 0.3$mm $\theta = 3.62°$

   (c) For $d = 0.31$mm $\theta = 15.026°$

   Conclusion: The 998th dark fringe appears at the above angles when d varies. Meaning when d increases, fringes move outward and when d decreases, fringes move inward.

2. Again for Dark fringe we have $2d\cos\theta = n\lambda$,
   If we fix $\theta = 2.562°$, $\lambda = 6000$ and vary d then-

   (a) For $d = 0.29$mm $n = 966$

   (b) For $d = 0.3$mm $n = 999$

   (c) For $d = 0.31$mm $n = 1032$

   Conclusion: On that angle if d decreases a lower order fringe appears (pattern expands) and d increases a higher order fringe appears (pattern contracts)

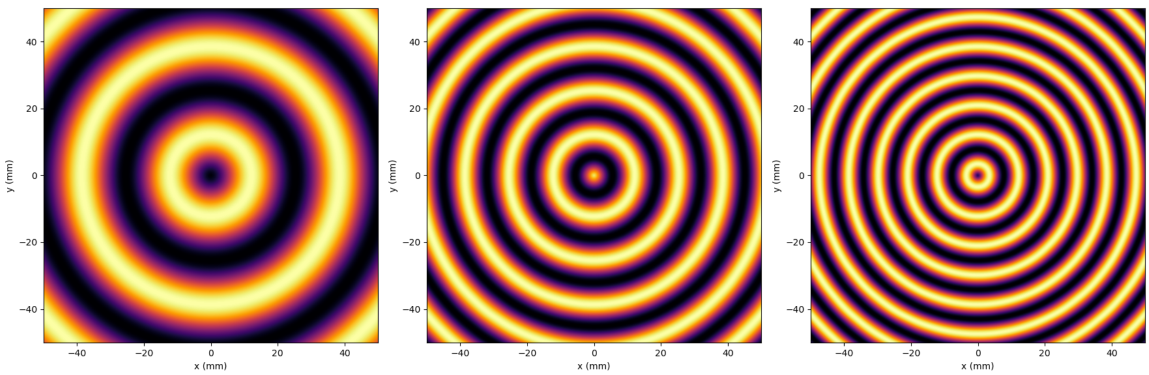


Figure 1.2: Fringe pattern for $d = 3\mu m$, $d = 6\mu m$, $d = 9\mu m$. Notice how the fringe pattern contracts as d increases.

3. The fringe pattern can be observed as curved lines or straight lines if one of the mirrors is tilted.

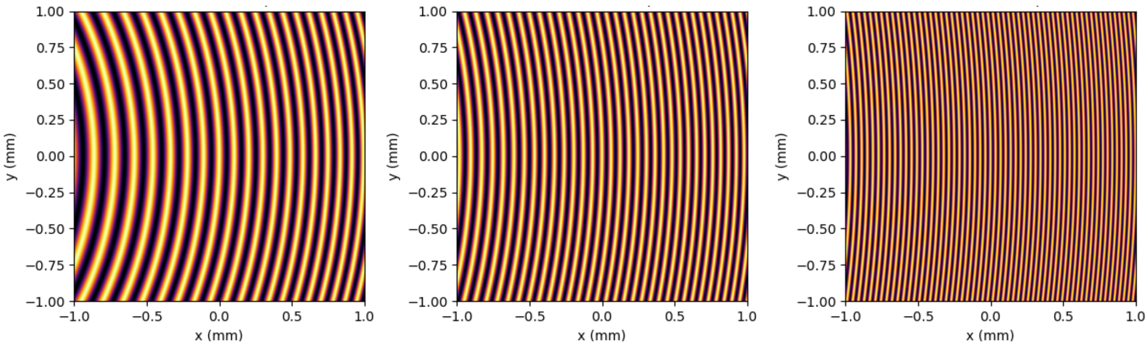


Figure 1.3: Fringe pattern when one mirror is tilted at $\alpha = 0.17°$, $\alpha = 0.29°$ and $\alpha = 0.52°$. Notice how the lines go from curved to straight as α increases

4. At central fringe m fringes collapse when mirror moves distance d_0 then-

$$2d = n\lambda,\ 2d_0 = m\lambda$$

$$2(d - d_0) = (n - m)\lambda$$

$$\lambda = 2\frac{\Delta d}{\Delta N} \tag{1.5}$$

The above expression can be used to determine wavelength of light.

Chapter 2

Coherence

2.1 Introduction

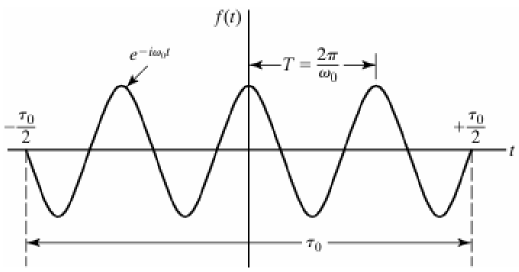
Coherence refers to correlation between the phases of monochromatic light waves. When beams of light maintain a stable phase relationship, they are said to be coherent. On the other hand, if the phase relationship fluctuates randomly, the beams are incoherent. The presence of coherence is essential for the formation of visible interference fringes. In the case of coherent beams, their amplitudes combine during superposition, whereas for incoherent beams, it is the intensities (irradiances) that add up. In this chapter we go through the concept of coherence, with a particular emphasis on Temporal coherence, which is closely related to the spectral purity of the light source. We also introduce a quantitative way to describe partial coherence, which is used to model the interferometer.

2.2 Temporal Coherence and Spatial Coherence

Temporal Coherence: Fields from the same source arrive at the same point via differenent paths. E.g- Michelson Interferometer
Spatial Coherence: Fields from a finite source arrive at a same point. E.g YDSE. In further section we will only be focussing on the temporal coherence and its role in the Michelson interferometer for determining the spectral purity of a source. First we start by fourier analysis of a wave.

2.3 Fourier Analysis of a finite harmonic wave

The spectral analysis of an infinitely extended sinusoidal wave is straightforward—it consists of a single term in the Fourier series that corresponds to the wave's frequency, with all other terms being zero. However, such infinitely long sinusoidal waves are purely mathematical idealizations. In practice, the wave is turned on and off at finite times. The result is a wave train of finite length, such as the one pictured below-



For the Fourier analysis of such finite wave we regard it as a non periodic fuction. Unlike an infinite sinusoidal wave represented by a single frequency this cannot be represented as such. Instead, many sine waves of different frequencies must be combined in just the right way so that they recreate the wave train during its lifetime and cancel out completely outside of that period. The turning of wave on and off introduces additional frequency components beyond the main one. When we apply the Fourier transform to such a signal, it results in a continuous spectrum of frequencies. This same idea applies to any isolated pulse, no matter its shape. For simplicity, we consider a pulse that behaves like a sine wave while it exists. To analyze its spectrum properly, we use the Fourier integral. In our case, we set the time origin so the pulse is centered around zero. The wave train lasts for a time τ_0 and oscillates at a frequency ω_0, and is described as:

$$f(t) = \begin{cases} e^{-i\omega_0 t} & \text{for} \quad \frac{-\tau_0}{2} < t < \frac{\tau_0}{2} \\ 0 & \text{for} \quad \text{elsewhere} \end{cases}$$

The frequency $g(\omega)$ is calculated as-

$$g(\omega) = \frac{1}{2\pi} \int_{-\inf}^{+\inf} f(t) e^{i\omega t} dt = \frac{1}{2\pi} \int_{-\tau_0/2}^{+\tau_0/2} f(t) e^{i(\omega-\omega_0)t} dt$$

Integrating it we have,

$$g(\omega) = \left[\frac{e^{i(\omega-\omega_0)t}}{2\pi i(\omega-\omega_0)} \right]_{-\tau_0/2}^{+\tau_0/2}$$

$$g(\omega) = \frac{\tau_0}{2\pi} \left\{ \frac{\sin\left((\tau_0/2(\omega-\omega_0)\right)}{(\tau_0/2)(\omega-\omega_0)} \right\} = \frac{\tau_0}{2\pi} \frac{\sin x}{x}$$

Where $x = (\tau_0/2)(\omega - \omega_0)$, and $\sin x/x$ peaks to 1 at $x \to 0$ i.e when $\omega \to \omega_0$, then

$$g(\omega) = \frac{\tau_0}{2\pi}$$

Further more the $sinc(x) \to 0$ whenever $x = \pm n\pi$, when $n = 1, 2, 3, ...$, then we can say-

$$g(\omega) = 0, \text{ when } \omega = \omega_0 \pm \frac{2n\pi}{\tau_0} \tag{2.1}$$

As ω increases (or decreases) from ω_0, then $g(\omega)$ periodically passes through zero. Below is the plot of the function- The square of the plot of $g(\omega)$ is called power spectrum, although frequencies far from ω_0 contribute to power spectrum, but the bulk of the energy lies in the frequencies present in the central maximum whose width is $4\pi/\tau_0$.
For the first zeros $x = \pm\pi$ we calculate the width of the central maximum where most of the frequencies are present-

$$(\omega - \omega_0) = \pm \frac{2\pi}{\tau_0}$$

$$\Delta\omega = \frac{4\pi}{\tau_0}, \text{ Central Maximum width}$$

To calculate the most dominant frequencies and preserve the frequency time inverse relationship, we calculate the FWHM of the central maximum width-

$$\text{FWHM } \Delta\omega = \frac{2\pi}{\tau_0}$$

$$\Delta\nu = \frac{1}{\tau_0} \tag{2.2}$$

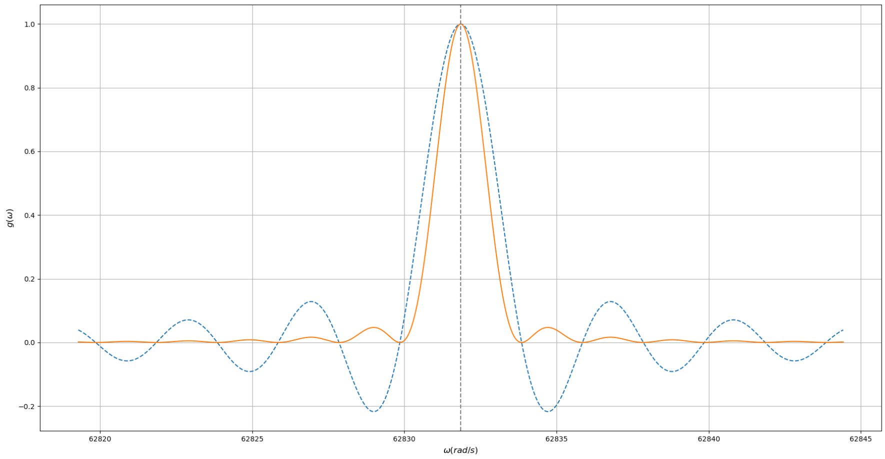


Figure 2.1: Fourier transform of the finite harmonic wave train. The dashed line gives the amplitude of the frequency spectrum $g(\omega)$ and the solid line gives its square, the power spectrum $|g(\omega)|^2$

The above relation $\Delta\nu = 1/\tau_0$ shows that if-

1. Case 1: $\tau_0 \to \infty$ then $\Delta\nu \to 0$
   This case represents a wave train of infinite length and only a single frequency is required to represent the whole wave train.

2. Case 2: $\tau_0 \to 0$ then $\Delta\nu \to \infty$
   Narrower the pulse, meaning a very small wave train more is the number of frequencies required to represent it.

In colclusion it can be said that larger the τ_0 smaller is the number frequencies (Bandwidth) required to represent it or Vice versa.

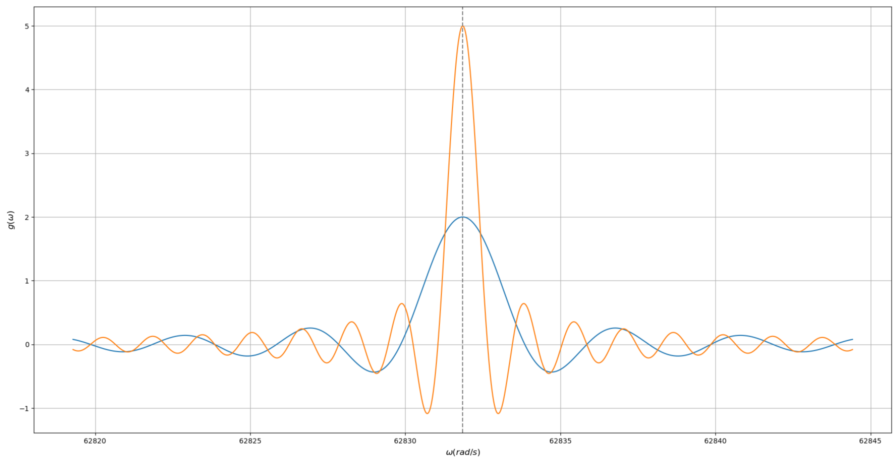


Figure 2.2: Shorter the τ_b (Blue line), broader the central maximum is. Larger the τ_o (Orange line) narrower the central maximum.

2.4 Temporal Coherence and Linewidth of source

For the purpose of spectral purity we need to understand what causes the dissappear of the fringes. The decrease in the contrast of the fringes can also be interpreted as being due to the

fact that the source is not emitting a single frequency but a band of frequencies (as discussed in previous section). When the path difference between the two interfering beams is zero or very small, the different wavelength components produce fringes superimposed on one another and the fringe contrast is good. On the other hand, when the path difference is increased, different wavelength components produce fringe patterns which are slightly displaced with respect to one another, and the fringe contrast becomes poorer. One can equally well say that the poor fringe visibility for a large optical path difference is due to the nonmonochromaticity of the light source.
The equivalence can be understood if we consider Michaelson interferometer experiment using two closely space wavelengths λ_1 and λ_2. The interference pattern will disappear once the bright fringe of λ_1 starts to overlap dark fringe of λ_2 or vice versa in this case-

$$2d = n\lambda_1, \; 2d = \left(n + \frac{1}{2}\right)\lambda_2$$

It can be calculated that-

$$2d = \frac{\lambda_1\lambda_2}{2(\lambda_1 - \lambda_2)}$$

where $2d$ is the path difference between two beams. Instead of two discrete wavelengths we consider that the beam consists of all wavelengths lying in between λ and $\lambda + \Delta\lambda$, the resultant interference pattern for increasing path difference will loose constrast once maximas start to overlap minimas. Then we can write-

$$2d \approx \frac{\lambda^2}{\Delta\lambda} \tag{2.3}$$

Thus for $2d \geq \lambda^2/\Delta\lambda$ the contrast of the interference fringes will be very poor, and we can rewrite it again as-

$$\Delta\lambda \sim \frac{\lambda^2}{L_c}, \text{ Where } 2d = L_c \tag{2.4}$$

L_c is called the Coherence length and is defined as the distance over which fringes can be observed, once the path difference exceeds $2d \geq L_c$ the fringes disappear. Also $L_c = c\tau_0$, the coherence time can be directly related to the bandwidth of the source.

2.5 Partial Coherence

The source field can be written as-

$$E_s(t) = \frac{1}{2}(E(t) + E^*(t))$$

where,

$$E(t) = E_0 e^{-iwt + i\phi(t)}$$

Here $\phi(t)$ models the departure from the monochromaticity of the source field. meaning its a time dependent phase fluctuation. Similarly we can write the superposed field at P-

$$E_{1P}(t) = \frac{1}{2}(E_1(t) + E_1^*(t))$$

$$E_{2P}(t) = \frac{1}{2}(E_2(t) + E_2^*(t))$$

Both E_1 and E_2 are related to source field via relations-

$$E_1(t) = \beta_1 E(t - T_1) = \beta_1 E_0 e^{-i\omega(t - T_1)} e^{i\phi(t - T_1)}$$

$$E_2(t) = \beta_2 E(t - T_2) = \beta_2 E_0 e^{-i\omega(t-T_2)} e^{i\phi(t-T_2)}$$

where β_1 and β_2 are the multiplicative factors resulting from the splitting of the source field amplitude due to reflection and transmission in the propogation of the fields from S to P. Further, T_1 and T_2 are the times of flight for the field propogating along path 1 and 2. The irradiance at point P will be given by-

$$I_P = \epsilon_0 c \langle (E_{1P} + E_{2P})^2 \rangle = \epsilon_0 c \left(\langle E_{1P}^2 \rangle + \langle E_{2P}^2 \rangle + 2\langle E_{1P} E_{2P} \rangle \right) \tag{2.5}$$

Substituting the values of E_{1P} and E_{2P} we have-

$$I_P = I_{1P} + I_{2P} + \frac{\epsilon_0 c}{2} \langle E_1 E_2 + E_1^* E_2^* + E_1 E_2^* + E_1^* E_2 \rangle$$

where $I_{1P} = \epsilon_0 c \langle E_{1P}^2 \rangle$ and $I_{2P} = \epsilon_0 c \langle E_{2P}^2 \rangle$ irradiance due to individual fields. The last term in the expression above is the interference term determines whether the resultant irradiance at the Point P is more or less than their individual sums. Also-

$$\langle E_1 E_2 \rangle = 0, \ \langle E_1^* E_2^* \rangle = 0$$

because these have terms oscillating at 2ω. Then we can write-

$$I_P = I_{1P} + I_{2P} + \frac{\epsilon_0 c}{2} \langle E_1 E_2^* + E_1^* E_2 \rangle$$

$$I_P = I_{1P} + I_{2P} + \epsilon_0 c \beta_1 \beta_2 \mathrm{Re} \langle E(t - T_1) E^*(t - T_2) \rangle$$

We later shift the time origin by T_1 and define the time difference in times of flight for the two paths as $\tau = T_1 - T_2$, then the irradiance at point P is given by-

$$I_P = I_{1P} + I_{2P} + \epsilon_0 c \beta_1 \beta_2 \mathrm{Re} \langle E(t) E^*(t + \tau) \rangle$$

The remaining time average has a form of correlation function. We define $\Gamma(\tau)$ i.e-

$$\Gamma(\tau) = \langle E(t) E^*(t + \tau) \rangle \tag{2.6}$$

This correlation function determines the size of the interference term and depends on the amount of correlation that exists in the values of the source fields at two different times. For convenience normalized correlation function,

$$\gamma(\tau) = \frac{\epsilon_0 c \beta_1 \beta_2}{2} \frac{\Gamma(\tau)}{\sqrt{I_{1P} I_{2P}}} \tag{2.7}$$

Then the irradiance at point P is-

$$I_P = I_{1P} + I_{2P} + 2\sqrt{I_{1P} I_{2P}} \mathrm{Re}[\gamma(\tau)] \tag{2.8}$$

The function $\gamma(\tau)$ we need to calculate now is function of τ and depends therefore on the location of point P. Now we know if $\tau > \tau_0$ the coherence between the two beams is lost. The dependence of $\gamma(\tau)$ on τ_0 is derived based on the assumption that τ_0 is constant in time rather than being average. This Kind of wavetrain is shown in the figure below with regular discontinuities in the phase seperated by τ_0. $\gamma(\tau)$ is also called degree of coherence and it further can be simplified by substituting the values of E, I_{1P} and I_{2P} i.e-

$$I_{1P} = \frac{\epsilon_0 c}{2} (\beta_1 E_0)^2, \ I_{2P} = \frac{\epsilon_0 c}{2} (\beta_2 E_0)^2$$

Using them the degree of coherence is simplified as-

$$\gamma(\tau) = \frac{\epsilon_0 c}{2}\beta_1\beta_2 \frac{\langle E_0 e^{-i\omega t} e^{i\phi(t)} E_0 e^{-i\omega(t+\tau)} e^{i\phi(t+\tau)} \rangle}{\sqrt{(\epsilon_0 c/2)^2 (\beta_1\beta_2 E_0)^2}}$$

$$\gamma(\tau) = e^{iw\tau} \langle e^{i(\phi(t)-\phi(t+\tau))} \rangle \tag{2.9}$$

We next need to calculate the time average $\langle\rangle$ i.e-

$$\langle e^{i(\phi(t)-\phi(t+\tau))} \rangle = \frac{1}{T}\int_0^T e^{i(\phi(t)-\phi(t+\tau))} dt \tag{2.10}$$

where T is sufficiently long time. $e^{i(\phi(t)-\phi(t+\tau))}$ in the exponent is pictured in the figure and can be seen as a series of regularly spaced rectangular pulses with random magnitude falling between $-\pi$ and $+\pi$.

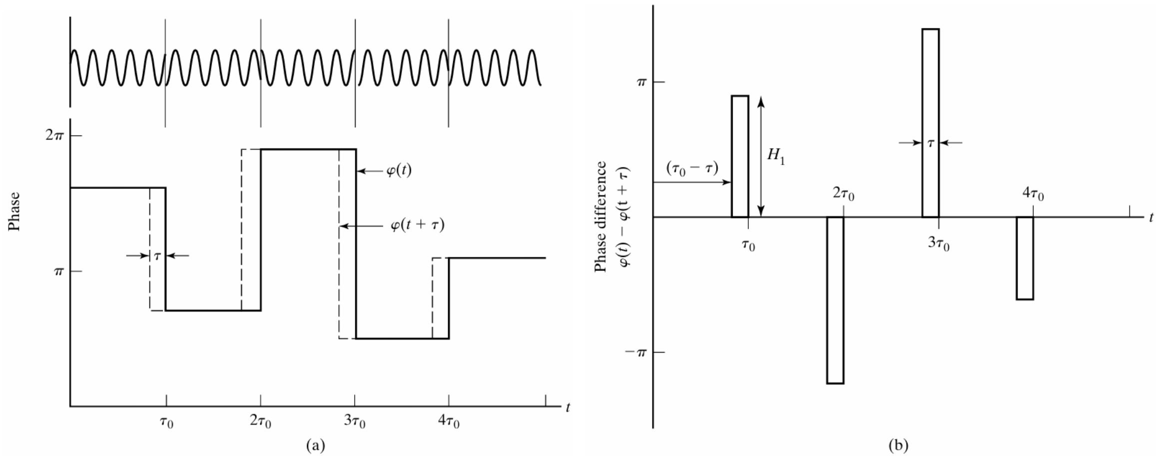


Figure 2.3: (a) Random phase Fluctuations $\phi(t)$ every τ_0 of a wave (Solid line) and the same phase fluctuations $\phi(t+\tau)$ of the wave (dashed line) at a time τ earlier. (b) Difference in the phase between the two waves described in figure on left.

Consider the first coherence time interval τ_0 in which pulse function is-

$$\phi(t)-\phi(t+\tau) = \begin{cases} 0 & \text{for} \quad 0 < t < (\tau_0-\tau) \\ H_1 & \text{for} \quad (\tau_0-\tau) < t < \tau_0 \end{cases}$$

Now in the succesive intervals the expressions are similar except the value of H_1. Then the degree of Coherence γ is written as-

$$\gamma = e^{i\omega t}\frac{1}{N\tau_0}\Bigg[\underbrace{\int_0^{\tau_0-\tau} e^{i\cdot 0}\, dt + \int_{\tau_0-\tau}^{\tau_0} e^{iH_1 t}\, dt}_{\text{Interval } N=1} \\ + \text{ similar terms for } (N-1) \text{ successive intervals}\Bigg]$$

Integrating over N terms-

$$\gamma = \frac{e^{i\omega\tau}}{N\tau_0}\left\{(\tau_0-\tau+\tau e^{iH_1}) + (\tau_0-\tau+\tau e^{iH_2}) + ...\right\}$$

$$\gamma = \frac{e^{i\omega\tau}}{N\tau_0}\left\{N(\tau_0-\tau)+\tau\sum_{j=1}^{N}e^{iH_j}\right\}$$

Because of the random nature of H_j, the terms in the summation average to zero when N is sufficiently large. Thus only the times when the waves coincide are when $\phi(t) = \phi(t+\tau)$ contribute to the integral, therefore at end we have degree of coherence as-

$$\gamma(\tau) = \left(1-\frac{\tau}{\tau_0}\right)e^{i\omega t} \tag{2.11}$$

We require the real part that is given by-

$$\mathrm{Re}[\gamma(\tau)] = \left(1-\frac{\tau}{\tau_0}\right)\cos\omega\tau \tag{2.12}$$

The degree of coherence takes values 0 to 1 when-

$$\mathrm{Re}\,\gamma(\tau) = \begin{cases} 1 & \text{for} \quad \tau = 0 \\ \left(1-\frac{\tau}{\tau_0}\right)\cos\omega\tau & \text{for} \quad 0<\tau<\tau_0 \\ 0 & \text{for} \quad \tau=\tau_0 \end{cases}$$

Where $\tau = 0$ denotes the case where the path difference between two paths is equal and $\tau = \tau_0$ is the case where the path difference is equal to coherence length. The amplitude of the cosine term $(1-\tau/\tau_0)$ can be written as $|\gamma|$, i.e is the magnitude of the degree of cosine term going from $1 \to 0$.
Visibility is defined as

$$V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}} \tag{2.13}$$

Now we can look upon the cases below, it is convenient to write τ as Δ and τ_0 as L_c as both are related.

1. Complete incoherence: $\Delta \to L_c$ then $|\gamma| = 0$

$$I_P = I_1 + I_2 = 2I_0, \text{ for equal beams}$$
$$V = \frac{2I_0 - 2I_0}{4I_0} = 0$$

2. Complete Coherence: $\Delta = 0$ and $|\gamma| = 1$

$$I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\omega\tau$$
$$I_P = 2I_0(1+\cos\omega\tau) \text{ for equal beams}$$
$$I_{\max} = 4I_0,\ I_{\min} = 0$$
$$V = \frac{4I_0 - 0}{4I_0} = 1$$

3. Partial Coherence: $0 < \Delta < L_c$ and $1 > |\gamma| > 0$

$$I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}(\gamma)$$
$$I_P = 2I_0(1+\mathrm{Re}(\gamma)) \text{ for equal beams}$$
$$I_{\max} = 2I_0(1+|\gamma|),\ I_{\min} = 2I_0(1-|\gamma|)$$
$$V = \frac{4I_0|\gamma|}{4I_0} = |\gamma|$$

This case is particularly important for modeling a Michelson interferometer which measures the spectral purity of a given Laser.

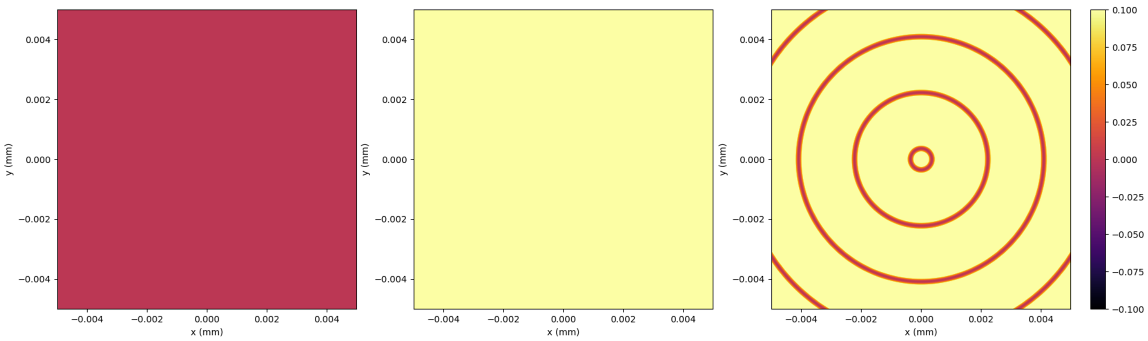


Figure 2.4: Visibility for (a) Complete Incoherence case (V=0) (b) Complete Coherence (V=1) (c) Partial Coherence (V=0.79) for d=64mm, and coherence length $L_c = 300$mm

Conclusion: In all the cases of equal beams, the fringe visibility is equal to the magnitude of the correlation function $|\gamma|$, either one is a measure of the degree of coherence

Chapter 3

Modelling the Michelson Interferometer in Python

In this Chapter I used the concepts in previous chapters to model a Michelson interferometer. First step is to get the ideal intensity for a single wavelength. Second is to model the visibility into the intensity plot. The end result is an interferometer whose fringes disappear once the path difference reaches the coherence length.

3.1 Algorithm

1. Define constants and Grid Size
   (a) λ: Wavelength
   (b) D: Focal Distance
   (c) d: initial Path difference
   (d) L_c: Coherence length of the source
   (e) Define a spatial grid x and y in the transverse plane.
   (f) Use np.meshgrid() to create a 2D coordinate system X, Y
2. Calculate the intensity for a given d path difference
   (a) Optical path difference $\Delta = 2d\cos\theta$ where $\cos\theta$ is expanded to account for small angle approximation then Δ becomes-

   $$\Delta = 2d\left(1 - \frac{\sqrt{X^2 + Y^2}}{D^2}\right)$$

   (b) Calculate the phase difference ie

   $$\phi = \frac{2\pi}{\lambda}\Delta$$

   (c) Using ϕ calculate the interference intensity $I = 2(1 + \cos\phi)$
   (d) Calculate the visibility because of Partial Coherence:

   $$V = 1 - \frac{\Delta}{L_c}$$

   Using np.clip V is clipped between 0 and 1.

(e) Final observed intensity is given by $I = V.I_0$

3. To account for the change in path difference to see the visibility dropping once path difference approaches the coherence length L_c I added buttons using matplotlib.widgets.Button

   (a) $+d$ button increases the path difference

   (b) $-d$ button decreases the path difference

   (c) Bind buttons to update the intensity plot

4. Plot the initial interference pattern and allow the user to interact using buttons

3.2 Simulation Result and Plots

For a wavelength of $\lambda = 600$ nm, the following results and intensity plots have been obtained through simulation using a Michelson interferometer setup. The source is assumed to have a finite coherence length of $L_c = 0.3$ m. This coherence length represents the maximum optical path difference over which interference fringes can still be observed. As expected from theory, the visibility of the interference fringes gradually diminishes as the optical path difference between the two arms of the interferometer approaches the coherence length. Once the path difference exceeds L_c, the phase relationship between the two beams becomes completely random, resulting in the disappearance of the interference pattern. This directly illustrates the role of temporal coherence in interference phenomena.

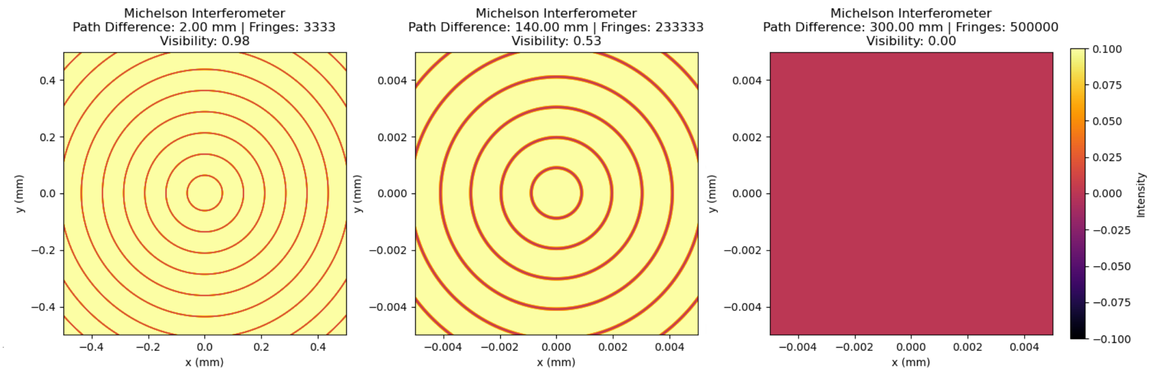


Figure 3.1: Formation of fringes for path difference $2d = 2$mm, $2d = 0.14$m and $2d = 0.30$m

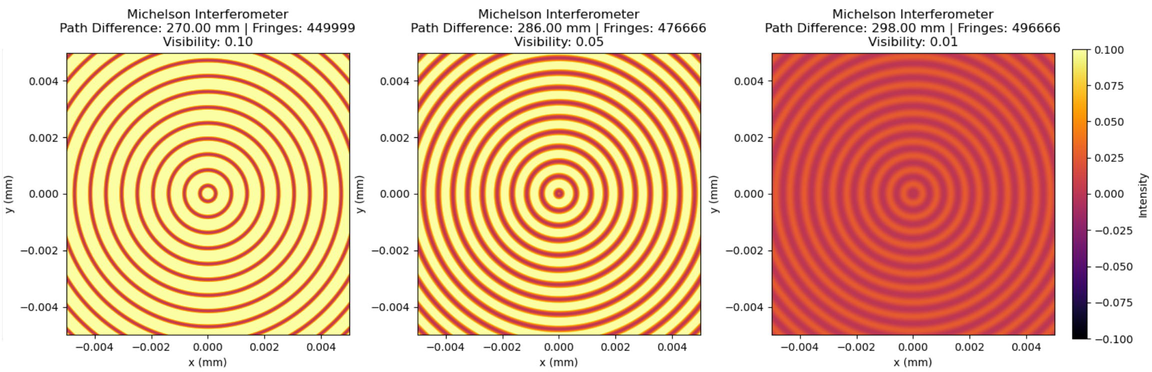


Figure 3.2: As $2d$ approaches L_c the fringes start to loose contrast

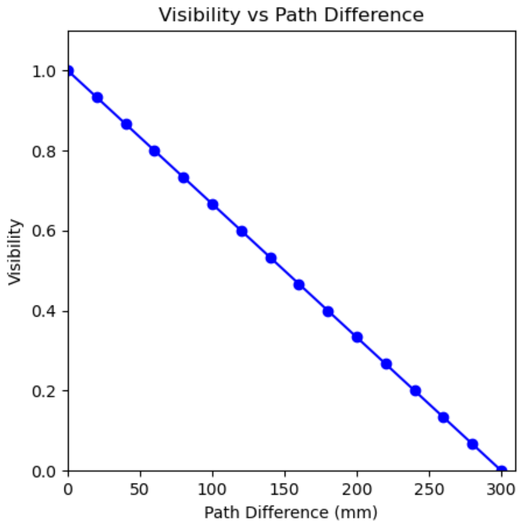


Figure 3.3: Fringe Visibility as a function of the path difference

3.3 Result

The above results confirm that the Michelson interferometer can be effectively used to estimate the spectral purity of a light source. The sharper and more sustained the fringe pattern, the longer the coherence length—and hence the higher the spectral purity.

Chapter 4

Determining the Coherence length using Michelson Interferometer

In this section we go through the experimental procedure to determine the coherence length using a diode LASER. To start with I setup a Michelson Interferometer to get the circular fringe on the screen. Then setting up a translation stage on the optical table with a pinhole detector to measure the visibility.

4.1 Setup

In this Section I discuss the assembly to measure the coherence length. The approach I used rather than moving the mirror to the point to see when the fringe contrast drops, instead measuring the visibility of the interference pattern on two different path differences and calculating the coherence length.

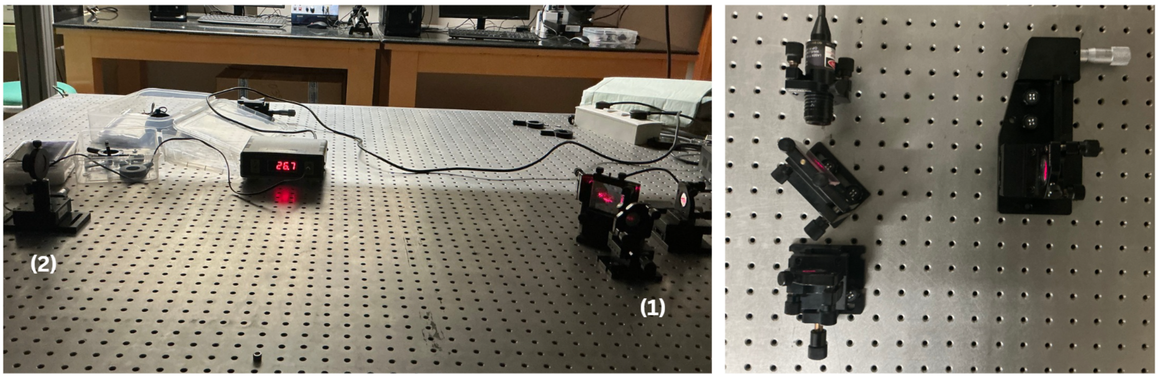


Figure 4.1: The setup is on the Left (a) is the Michelson Interferometer setup also shown on right image. (b) is the Pinhole detector used to measure the intensity

4.2 Calculations and Plots

The following values were taken for a Holmarc Red 655nm Laser for Path difference of 20.10mm and 20.15mm repectively to ensure correct intensity readings are obtained each value has been averaged.

Distance (mm)	Intensity at 20.10 mm	Intensity at 20.15 mm
0	11.7706	4.0765
0.25	19.0029	3.3824
0.5	23.7441	3.6500
0.75	25.7706	5.0882
1	27.2471	10.6971
1.25	25.5206	8.9441
1.5	21.4088	6.8500
1.75	15.7088	6.1000
2	13.1735	7.0294
2.25	12.7706	5.7471
2.5	11.9882	8.5676
2.75	7.9206	13.0500
3	9.5765	16.1088
3.25	8.5706	14.5706
3.5	4.5765	15.1000
3.75	3.7441	12.5176
4	4.2206	16.9559
4.25	4.6765	18.2000
4.5	5.8412	17.6147
4.75	7.6882	21.7412
5	8.7971	27.5529
5.25	9.8382	29.1735
5.5	16.1588	28.9529
5.75	14.2735	26.6735
6	17.7588	27.4824
6.25	18.7412	27.0353
6.5	20.2206	26.9588
6.75	22.8735	28.7029
7	30.3206	30.6647
7.25	28.4735	28.9824
7.5	31.7206	30.1853
7.75	30.4588	31.1088
8	26.5353	27.0294
8.25	22.8500	18.7941
8.5	19.1529	12.4412
8.75	20.3118	11.6059
9	16.5059	9.8765
9.25	17.2235	11.1294
9.5	11.5324	12.0412
9.75	12.0118	11.6441
10	7.6441	9.6000

1. For 20.10mm $I_{max} = 31.72\mu$A and $I_{min} = 3.74\mu$A then Visibility is calculated as $V = 0.788$
   Hence coherence length $L_c = 95.19$mm

2. For 20.15mm $I_{max} = 31.11\mu$A and $I_{min} = 3.38\mu$A then Visibility is calculated as $V = 0.803$
   Hence coherence length $L_c = 102.73$mm

The mean Coherence Length is 98.96mm. Using this the spectral purity of the laser is calculated as-

$$\Delta\lambda = \frac{\lambda^2}{2L_c}$$

$$\Delta\lambda = 0.0268\,\text{Å}$$

4.3 Result

Over the 9.86cm the given Laser goes out of phase and the interference pattern vanishes. Near this distance the fringes are very small and have very low intensities.

4.3.1 Experimental Limitations and Suggestions

The intensity pattern was Measured using pinhole detector. The values were fluctuating so to mitigate that the detector was exposed to a single point in pattern for say 20s and the average of those values was taken. So If a 10mm of the pattern is recorded via that detector in the increments of 0.25mm that would be around 40 readings for a particular path difference. It is way too time consuming. If a ccd would have been used instead of the pinhole detector than the pattern could have been scanned more quickly and visibility could have been calculated for number of path differences.
To confirm the calculated coherence length, one of the Mirror had to be moved to a position corresponding to that length. However, as the path difference approaches the coherence length, the fringe pattern becomes faint and loses contrast. Observing this transition requires very slow and precise movement of the mirror, but even a slight shift often causes the pattern to vanish entirely. If optical rails had been available on the optical table, it would have been much easier to control and confirm the coherence length through smooth and stable mirror translation.

.1 Appendix: Python Simulation of Michelson Interferometer

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.widgets import Button

# Constants
wavelength = 4790e-10          # Wavelength in meters
N = 500                        # Grid size
D = 0.5                        # Distance to screen (focal length)
d = 0.15                       # Initial path difference in meters
L_c = 0.3                      # Coherence length in meters

# Grid
x = np.linspace(-5e-6, 5e-6, N)
y = np.linspace(-5e-6, 5e-6, N)
X, Y = np.meshgrid(x, y)

# Figure and axis
fig, ax = plt.subplots(figsize=(8, 8))
plt.subplots_adjust(bottom=0.3)

# Initialize image
I = np.zeros_like(X)
image = ax.imshow(I, cmap='inferno',
                  extent=[x[0]*1e3, x[-1]*1e3, y[0]*1e3, y[-1]*1e3],
                  origin='lower')
plt.colorbar(image, ax=ax, label="Intensity")
ax.set_xlabel("x (mm)")
ax.set_ylabel("y (mm)")

#   Intensity Calculation
def calculate_intensity(d):
    delta = 2 * d * (1 - (np.sqrt(X**2 + Y**2) / D**2))
    delta_phi = (2 * np.pi / wavelength) * delta
    I_0 = 2 * (1 + np.cos(delta_phi))
    V = 1 - (np.abs(delta) / L_c)
    V = np.clip(V, 0, 1)
    I = V * I_0
    return I, V

#   Update plot and title
def update_plot(_=None):
    global d
    I, V = calculate_intensity(d)
    image.set_data(I)
    fringe_counter = int((2 * d) / wavelength)
    avg_visibility = np.mean(V)
    ax.set_title(f"Michelson Interferometer\nPath Difference: {2*d*1e6:.2f} µm | Fringe
```

```
    fig.canvas.draw_idle()

#   Button Callbacks
def increase_path(event):
    global d
    d += 0.1e-2  # increase by 0.1 mm
    update_plot()

def decrease_path(event):
    global d
    d = max(0, d - 0.1e-2)  # decrease by 0.1 mm
    update_plot()

# Buttons
ax_dec = plt.axes([0.44, 0.1, 0.1, 0.075])
ax_inc = plt.axes([0.64, 0.1, 0.1, 0.075])
b_dec = Button(ax_dec, '- d')
b_inc = Button(ax_inc, '+ d')
b_dec.on_clicked(decrease_path)
b_inc.on_clicked(increase_path)

# Initial plot
update_plot()
plt.show()
```

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- M. Born and E. Wolf, Principles of Optics, Pergamon Press, Oxford, 1975.