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ME2220 : Kinetics And Dynamics of Machinery

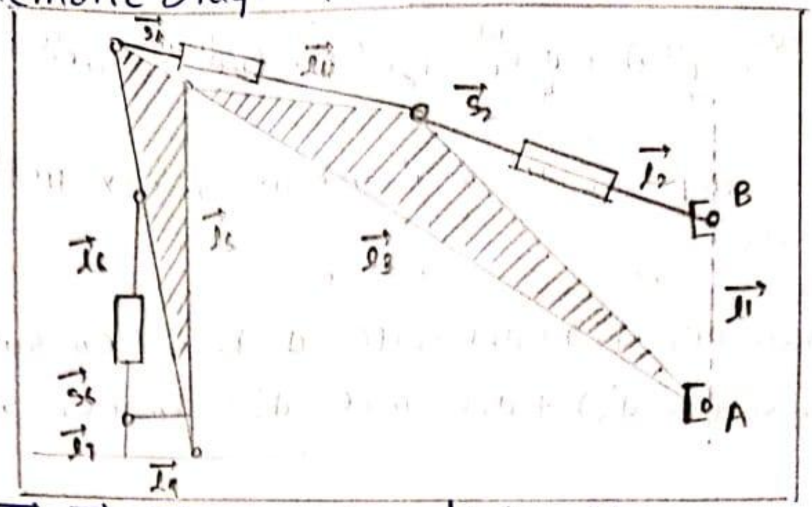
Kinematic analysis and synthesis of the toy-based mechanism

Description: A backhoe excavator toy is an excellent choice for studying mechanical linkages and movement. Its design includes articulated arms, hydraulic-like prismatic joints, and rotational motion, closely resembling real-world excavators. By analyzing its mechanism, We can understand kinematic relationships, such as how prismatic displacements affect angular velocities and link rotations. This makes it a great model for exploring excavator dynamics in a controlled and simplified manner.

Overview: The backhoe excavator toy's mechanism will be analyzed mathematically using loop closure equations to connect the movement of different points to the input crank angle. By calculating positions for different crank angles, we can simulate the motion using Python. Then, velocity and acceleration vector diagrams will be created. If these vectors form closed shapes, it confirms that their total sum is zero, verifying the accuracy of the mechanism's motion analysis.

Displacement Analysis:

Kinematic Diagram:



Here, 1) $\vec{s_2}, \vec{s_4}, \vec{s_6}$ are prismatic lengths

2) $\phi_3', \phi_5', \phi_3'', \phi_5''$ angles are also known

3) $\vec{l_1}, \vec{l_2}, \vec{l_3}, \vec{l_4}, \vec{l_5}, \vec{l_6}, \vec{l_7}, \vec{l_8}, \vec{l_9}$ all links are known.

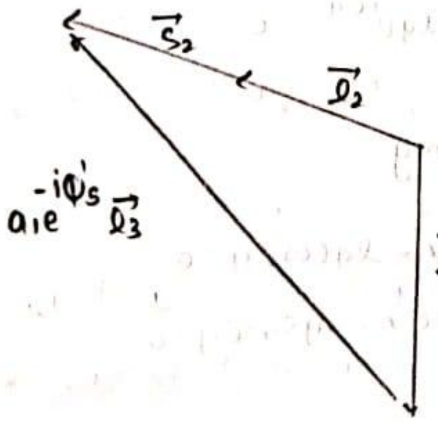
4) $\theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9$ are unknowns

Loop 1: The loop closure equation is

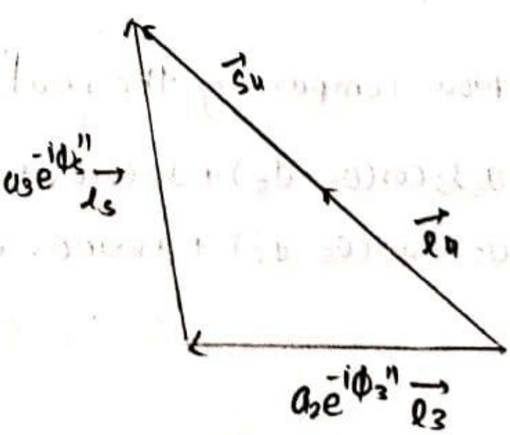
$$\vec{l_1} + a_1 e^{-i\phi_3'} \vec{l_3} - \vec{s_2} - \vec{l_2} = 0$$

$$\vec{l_1} e^{i\frac{3\pi}{2}} + a_1 e^{-i\phi_3'} (l_3 e^{i\theta_3}) - s_2 e^{i\theta_2} - l_2 e^{i\theta_2} = 0$$

Now comparing the real and imaginary parts of this equation we get



loop 1



loop 2

Loop 2: The loop closure equation is:

$$a_2 e^{-i\phi_3''}\vec{l_3} + a_3 e^{-i\phi_5'}\vec{l_5} - \vec{l_4} - \vec{S_4} = 0$$

$$a_2 e^{-i\phi_3''}(l_3 e^{i\theta_3}) + a_3 e^{-i\phi_5''}(l_5 e^{i\theta_5}) - l_4 e^{i\theta_4} - S_4 e^{i\theta_4} = 0$$

Now comparing the real and imaginary parts of the equation.

$$a_2 l_3 \cos(\theta_3 - \phi_3'') + a_3 l_5 \cos(\theta_5 - \phi_5'') - l_4 \cos\theta_4 - S_4 \cos\theta_4 = 0$$

$$a_2 l_3 \sin(\theta_3 - \phi_3'') + a_3 l_5 \sin(\theta_5 - \phi_5'') - l_4 \sin\theta_4 - S_4 \sin\theta_4 = 0$$

Loop 3: loop closure equation is,

$$a_4 e^{-i\phi_5'}\vec{l_5} + \vec{l_6} + \vec{S_6} - \vec{l_8} = 0$$

$$a_4 e^{-i\phi_5'}(l_5 e^{i\theta_5}) + \vec{l_6} e^{i\theta_6} + S_6 e^{i\theta_6} - \vec{l_8} e^{i\theta_8} = 0$$

Now comparing.

$$a_4 l_5 \cos(\theta_5 - \phi_5') + l_6 \cos\theta_6 + S_6 \cos\theta_6 - l_8 \cos\theta_8 = 0$$

$$a_4 l_5 \sin(\theta_5 - \phi_5') + l_6 \sin\theta_6 + S_6 \sin\theta_6 - l_8 \sin\theta_8 = 0$$

Loop 4: loop closure equation is

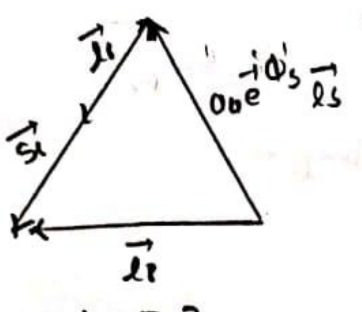
$$a_5 e^{-i\phi_5'}\vec{l_5} + \vec{l_8} + \vec{l_7} - \vec{l_9} = 0$$

$$a_5 e^{-i\phi_5'}(l_5 e^{i\theta_5}) + l_8 e^{i\theta_8} + l_7 e^{i\theta_7} - l_9 e^{i\theta_9} = 0$$

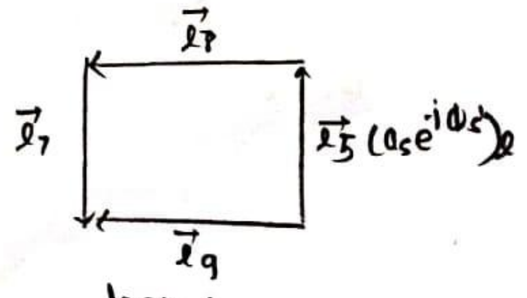
Now, comparing the real and img.

$$a_5 l_5 \cos(\theta_5 - \phi_5') + l_8 \cos\theta_8 + l_7 \cos\theta_7 - l_9 \cos\theta_9 = 0$$

$$a_5 l_5 \sin(\theta_5 - \phi_5') + l_8 \sin\theta_8 + l_7 \sin\theta_7 - l_9 \sin\theta_9 = 0$$



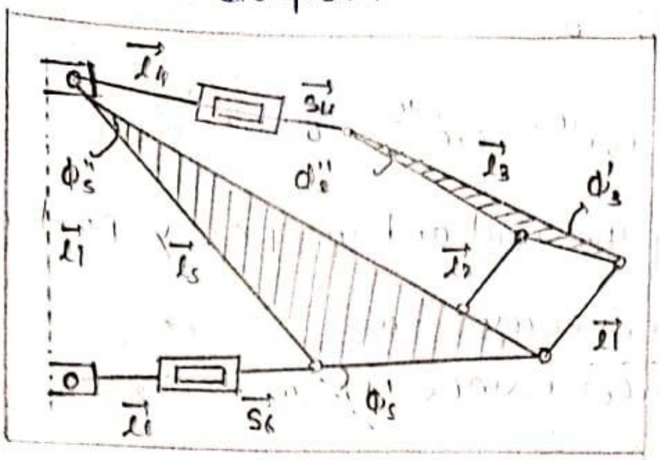
loop 3



loop 4

Displacement Analysis

Kinematic diagram:

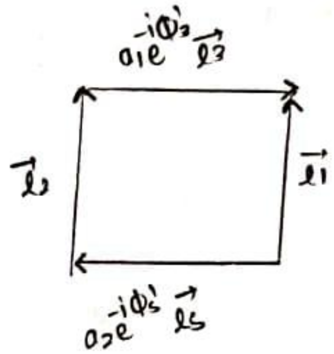


Here, 1) $\vec{l_1}, \vec{l_2}, \vec{l_3}, \vec{l_4}, \vec{l_5}, \vec{l_6}, \vec{l_7}$ all links lengths are known

2) $\vec{s_4}, \vec{s_6}$ prismatic length are also known.

3) $\phi_3', \phi_3'', \phi_5', \phi_5''$

4) $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ are unknowns.



Loop 1: Loop closure equation are

$$a_1 e^{-i\phi_3'} \vec{l_3} + a_2 e^{i\phi_5'} \vec{l_5} + \vec{l_2} - \vec{l_1} = 0$$

$$a_1 e^{-i\phi_3'} (l_3 e^{i\theta_3}) + a_2 e^{-i\phi_5'} (l_5 e^{i\theta_5}) + l_2 e^{i\theta_2} - l_1 e^{i\theta_1} = 0$$

Now comparing the real and imgginary part of eq.

$$a_1 l_3 \cos(\theta_3 - \phi_3') + a_2 l_5 \cos(\theta_5 - \phi_5') + l_2 \cos\theta_2 - l_1 \cos\theta_1 = 0$$

$$a_1 l_3 \sin(\theta_3 - \phi_3') + a_2 l_5 \sin(\theta_5 - \phi_5') + l_2 \sin\theta_2 - l_1 \sin\theta_1 = 0$$

Loop 2:

$$a_4 e^{-i\phi_5''} \vec{l_5} + \vec{l_4} + \vec{s_4} + a_3 e^{-i\phi_3''} \vec{l_3} - \vec{l_2} = 0$$

$$a_4 e^{-i\phi_5''} (l_5 e^{i\theta_5}) + l_4 e^{i\theta_4} + s_4 e^{i\theta_4} + a_3 e^{-i\phi_3''} (l_3 e^{i\theta_3}) - l_2 e^{i\theta_2} = 0$$

Now comparing the real and imaginary part.

$$a_4 l_5 \cos(\theta_5 - \phi_5'') + l_4 \cos\theta_4 + s_4 \cos\theta_4 + a_3 l_3 \cos(\theta_3 - \phi_3'') - l_2 \cos\theta_2 = 0$$

$$a_4 l_5 \sin(\theta_5 - \phi_5'') + (l_4 + s_4) \sin\theta_4 + a_3 l_3 \sin(\theta_3 - \phi_3'') - l_2 \sin\theta_2 = 0$$

Loop 3: The loop closure equation is

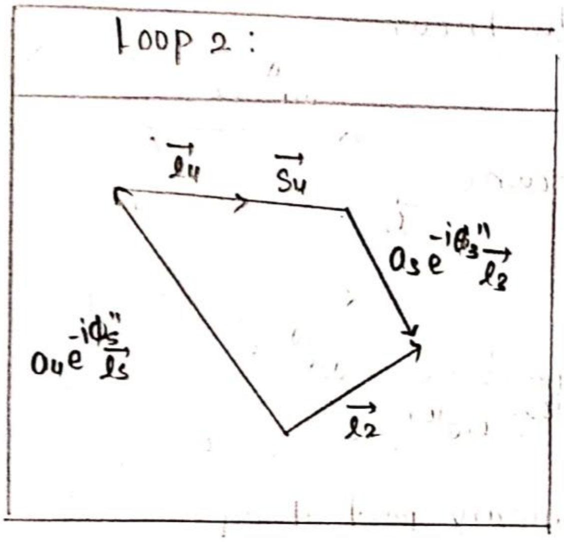
$$\vec{l_6} + \vec{S_6} + \vec{l_5} - \vec{l_7} = 0$$

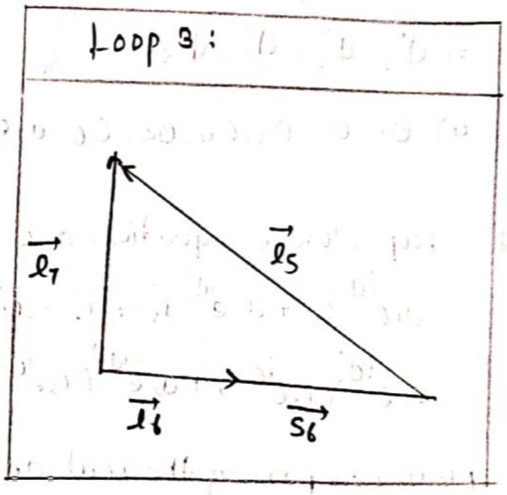
$$l_6 e^{i\theta_6} + S_6 e^{i\theta_6} + l_5 e^{i\theta_5} - l_7 e^{i\pi/2} = 0$$

Now, comparing the real and imaginary part of the eq.

$$l_6\cos\theta_6 + S_6\cos\theta_6 + l_5\cos\theta_5 = 0$$

$$l_6\sin\theta_6 + S_6\sin\theta_6 + l_5\sin\theta_5 - l_7 = 0$$





Velocity Analysis: (FRONTSIDE - RIGHT)

Differentiating loop closure equation:

Loop 1:

$$a_1 l_3 \omega_3 \sin(\theta_3 - \phi'_3) + V_2 \cos(\theta_2) - S_2 \omega_2 \sin\theta_2 - l_2 \omega_2 \sin\theta_2 = 0$$

$$a_1 l_3 \omega_3 \cos(\theta_3 - \phi'_3) - V_2 \sin\theta_2 - S_2 \omega_2 \cos\theta_2 - l_2 \omega_2 \cos\theta_2 = 0$$

Loop 2:

$$a_2 l_3 \omega_3 \sin(\theta_3 - \phi''_3) + a_3 l_5 \omega_5 \sin(\theta_5 - \phi''_5) - (l_4 + s_4)\omega_4 \sin\theta_4 + V_4 \cos\theta_4 = 0$$

$$a_2 l_3 \omega_3 \cos(\theta_3 - \phi''_3) + a_3 l_5 \omega_5 \cos(\theta_5 - \phi''_5) - (l_4 + s_4)\omega_4 \sin\theta_4 - V_4 \cos\theta_4 = 0$$

Loop 3:

$$a_4 l_5 \omega_5 \sin(\theta_5 - \phi'_5) + (l_6 + s_6)\omega_6 \sin\theta_6 - V_6 \cos\theta_6 - l_8 \omega_8 \sin\theta_8 = 0$$

$$a_4 l_5 \omega_5 \cos(\theta_5 - \phi'_5) + (l_6 + s_6)\omega_6 \cos\theta_6 + V_6 \sin\theta_6 - l_8 \omega_8 \cos\theta_8 = 0$$

Loop 4:

$$a_5 l_5 \omega_6 \sin(\theta_5 - \phi'_5) + l_8 \omega_8 \sin\theta_8 + l_7 \omega_7 \sin\theta_7 - l_9 \omega_9 \sin\theta_9 = 0$$

$$a_5 l_5 \omega_5 \cos(\theta_6 - \phi'_5) + l_8 \omega_8 \cos\theta_8 + l_7 \omega_7 \cos\theta_7 - l_9 \omega_9 \cos\theta_9 = 0$$

Acceleration Analysis

Loop 1:

$$a_1 l_3 (\alpha_3 \sin(\theta_3 - \phi'_3) + \omega_3^2 \cos(\theta_3 - \phi'_3)) - (S_2 + l_2)(\alpha_2 \sin\theta_2 + \omega_2^2 \cos\theta_2) - V_2 \omega_2 \sin\theta_2 = 0$$

$$a_1 l_3 (\alpha_3 \cos(\theta_3 - \phi'_3) - \omega_3^2 \sin(\theta_3 - \phi'_3)) - (S_2 + l_2)(\alpha_2 \cos\theta_2 - \omega_2^2 \sin\theta_2) - V_2 \omega_2 \cos\theta_2 = 0$$

Loop 2:

$$a_2 l_3 (\alpha_3 \sin(\theta_3 - \phi''_3) + \omega_3^2 \cos(\theta_3 - \phi''_3)) + a_3 l_5 (\alpha_5 \sin(\theta_5 - \phi''_5) + \omega_5^2 \cos(\theta_5 - \phi''_5))$$
$$- (l_4 + s_4)(\alpha_4 \sin\theta_4 + \omega_4^2 \cos\theta_4) + V_4 \omega_4 \sin\theta_4 = 0$$

$$a_2 l_3 (\alpha_3 \cos(\theta_3 - \phi''_3) - \omega_3^2 \sin(\theta_3 - \phi''_3)) + a_3 l_5 (\alpha_5 \cos(\theta_5 - \phi''_5)) - \omega_5^2 \sin(\theta_5 - \phi''_5))$$
$$- (l_4 + s_4)(\alpha_4 \cos\theta_4 - \omega_4^2 \sin\theta_4) - V_4 \omega_4 \cos\theta_4 = 0$$

Loop 3:

$$a_4 l_5 (\alpha_5 \sin(\theta_5 - \phi_5') + \omega_5^2 \cos(\theta_5 - \phi_5')) + (l_6 + s_6)(\alpha_6 \sin\theta_6 + \omega_6^2 \cos\theta_6) - v_6 \omega_6 \sin\theta_6 - l_8(\alpha_8 \sin\theta_8 + \omega_8^2 \cos\theta_8) = 0$$

$$a_4 l_5 (\alpha_5 \cos(\theta_5 - \phi_5') - \omega_5^2 \sin(\theta_5 - \phi_5')) + (l_6 + s_6)(\alpha_6 \cos\theta_6 - \omega_6^2 \sin\theta_6) + v_6 \omega_6 \cos\theta_6 - l_8(\alpha_8 \cos\theta_8 - \omega_8^2 \sin\theta_8) = 0$$

Loop 4:

$$\alpha_5 l_5 (\alpha_5 \sin(\theta_5 - \phi_5') + \omega_5^2 \cos(\theta_5 - \phi_5')) + l_8(\alpha_8 \sin\theta_8 + \omega_8^2 \cos\theta_8) + l_7(\alpha_7 \sin\theta_7 + \omega_7^2 \cos\theta_7) - l_9(\alpha_9 \sin\theta_9 + \omega_9^2 \cos\theta_9) = 0$$

$$\alpha_5 l_5 (\alpha_5 \cos(\theta_5 - \phi_5') - \omega_5^2 \sin(\theta_5 - \phi_5')) + l_8(\alpha_8 \cos\theta_8 - \omega_8^2 \sin\theta_8) + l_7(\alpha_7 \cos\theta_7 - \omega_7^2 \sin\theta_7) - l_9(\alpha_9 \cos\theta_9 - \omega_9^2 \sin\theta_9) = 0$$

Velocity Analysis. (Backside. LEFT)

loop1:

$$a_1 l_3 \omega_3 \sin(\theta_3 - \phi_3') + a_2 l_5 \omega_5 \sin(\theta_5 - \phi_5') + l_2 \omega_2 \sin\theta_2 - l_1 \omega_1 \sin\theta_1 = 0$$

$$a_1 l_3 \omega_3 \cos(\theta_3 - \phi_3') + a_2 l_5 \omega_5 \cos(\theta_5 - \phi_5') + l_2 \omega_2 \cos\theta_2 - l_1 \omega_1 \cos\theta_1 = 0$$

Loop 2:

$$a_4 l_5 \omega_5 \sin(\theta_5 - \phi_5'') + (l_4 + s_4)\omega_4 \sin\theta_4 - V_4 \cos\theta_4 + a_3 l_3 \omega_3 \sin(\theta_3 - \phi_3'') - l_2 \omega_2 \sin\theta_2 = 0$$

$$a_4 l_5 \omega_5 \cos(\theta_5 - \phi_5'') + (l_4 + s_4)\omega_4 \cos\theta_4 + V_4 \sin\theta_4 + a_3 l_3 \omega_3 \cos(\theta_3 - \phi_3'') - l_2 \omega_2 \cancel{\sin} \cos(\theta_2) = 0$$

Loop 3:

$$(l_6 + s_6)\omega_6 \sin\theta_6 + V_6 \cos\theta_6 + l_5 \omega_5 \sin\theta_5 = 0$$

$$(l_6 + s_6)\omega_6 \cos(\theta_6) + V_6 \sin\theta_6 + l_5 \omega_5 \cos\theta_5 = 0$$

Acceleration Analysis

Loop 1:

$$a_1 l_3 (\alpha_3 \sin(\theta_3 - \phi_3') + \omega_3^2 \cos(\theta_3 - \phi_3')) + a_2 l_5 (\alpha_5 \sin(\theta_5 - \phi_5') + \omega_5^2 \cos(\theta_5 - \phi_5')) + l_2 (\alpha_2 \sin\theta_2 + \omega_2^2 \cos\theta_2) - l_1 (\alpha_1 \sin\theta_1 + \omega_1^2 \cos\theta_1) = 0$$

$$a_1 l_3 (\alpha_3 \cos(\theta_3 - \phi_3') - \omega_3^2 \sin(\theta_3 - \phi_3')) + a_2 l_5 (\alpha_5 \cos(\theta_5 - \phi_5') - \omega_5^2 \sin(\theta_5 - \phi_5')) + l_2 (\alpha_2 \cos\theta_2 - \omega_2^2 \sin\theta_2) - l_1 (\alpha_1 \cos\theta_1 - \omega_1^2 \sin\theta_1) = 0$$

Loop 2:

$$a_4 l_5\left(\alpha_5 \sin(\theta_5-\phi_5'') + \omega_5^2\cos(\theta_5-\phi_5'')\right) + (l_4+s_4)(\alpha_4\sin\theta_4 + \omega_4^2\cos\theta_4)$$
$$- v_4\omega_4\sin\theta_4 + a_3 l_3\left(\alpha_3\sin(\theta_3-\phi_3'') + \omega_3^2\cos(\theta_3-\phi_3'')\right) - l_2(\alpha_2\sin\theta_2 + \omega_2^2\cos\theta_2) = 0$$

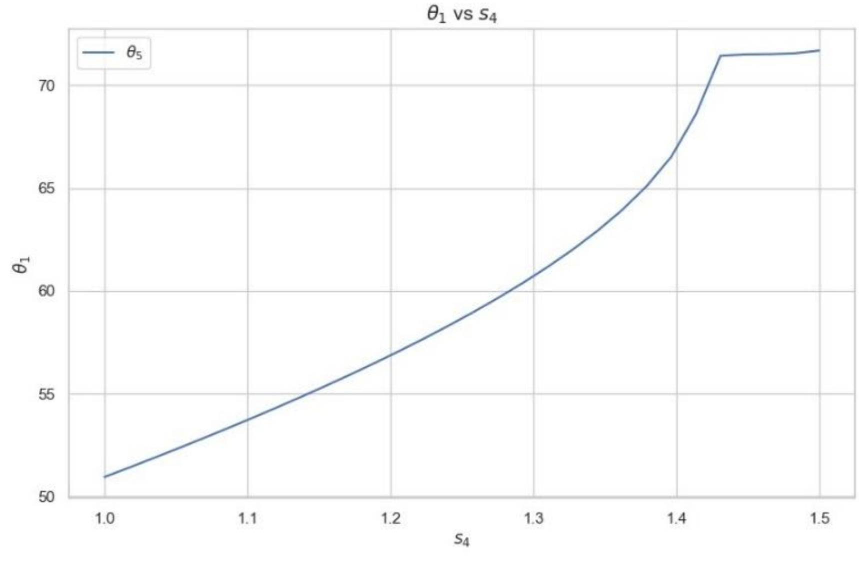
$$a_4 l_5\left(\alpha_5\cos(\theta_5-\phi_5'') - \omega_5^2\sin(\theta_5-\phi_5'')\right) + (l_4+s_4)(\alpha_4\cos\theta_4 - \omega_4^2\sin\theta_4) +$$
$$v_4\omega_4\cos(\theta_4) + a_3 l_3\left(\alpha_3\cos(\theta_3-\phi_3'') - \omega_3^2\sin(\theta_3-\phi_3'')\right) - l_2(\alpha_2\cos\theta_2 - \omega_2^2\sin\theta_2) = 0$$

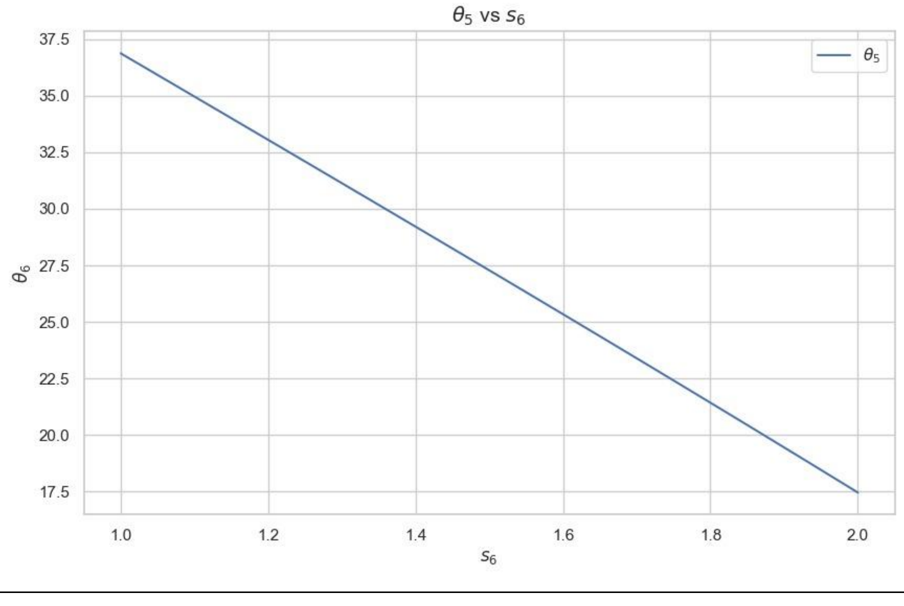
Loop 3:

$$(l_6+s_6)(\alpha_6\sin\theta_6 + \omega_6^2\cos\theta_6) - v_6\omega_6\sin\theta_6 + l_5(\alpha_5\sin\theta_5 + \omega_5^2\cos\theta_5) = 0$$

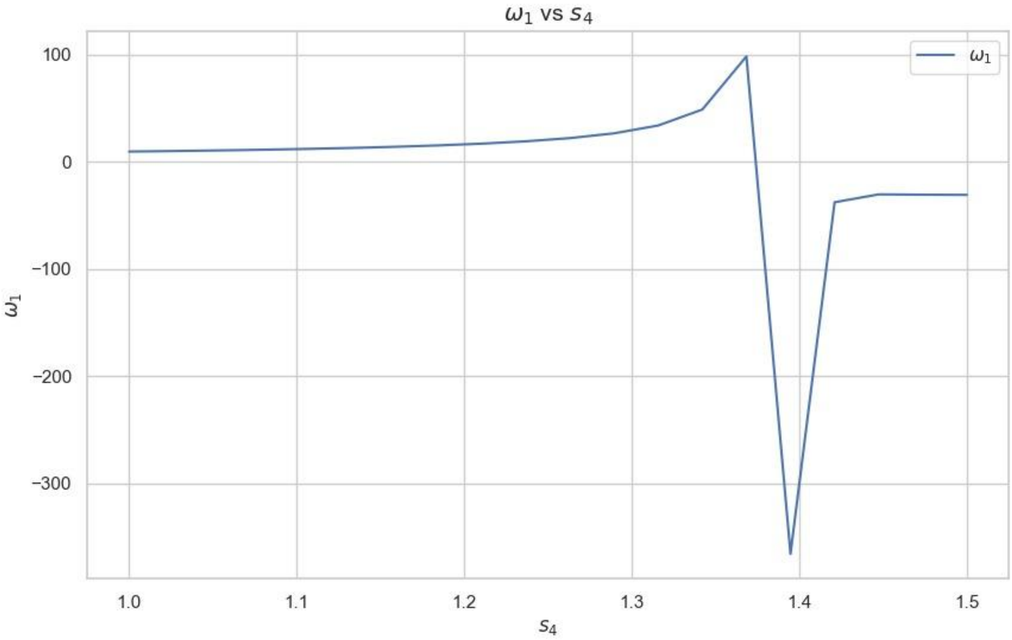
$$(l_6+s_6)(\alpha_6\cos\theta_6 - \omega_6^2\sin\theta_6) + v_6\omega_6\cos\theta_6 + l_5(\alpha_5\cos\theta_5 - \omega_5^2\sin\theta_5) = 0$$

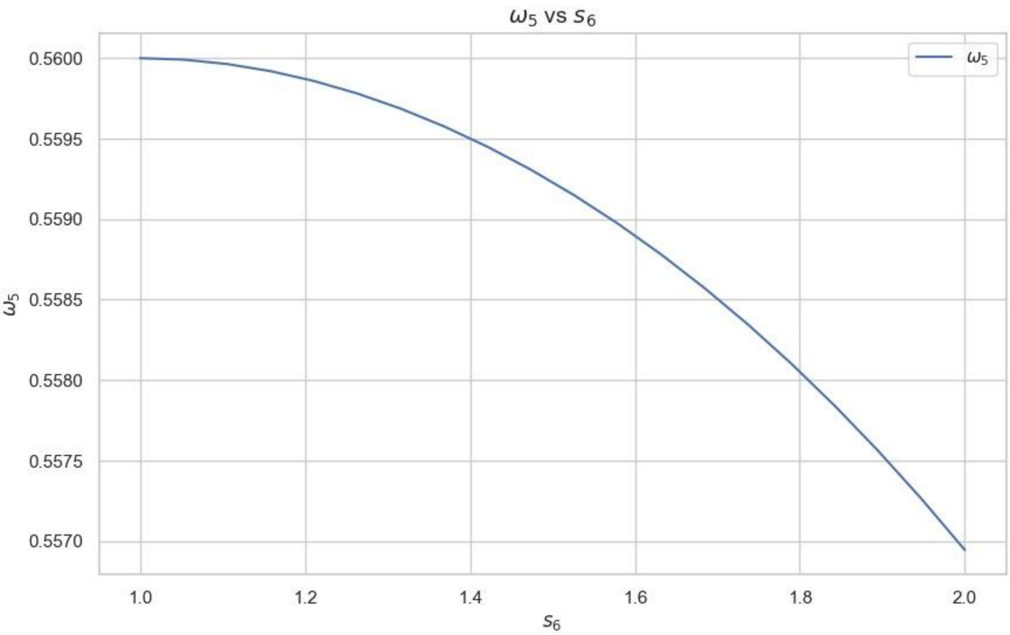
For Leftside arm displacement diagram



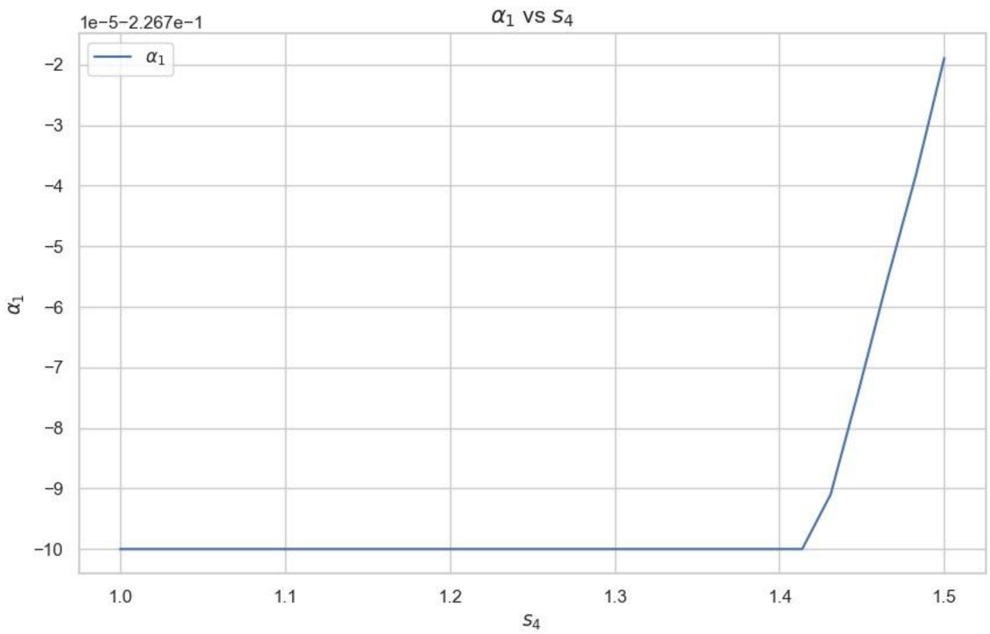


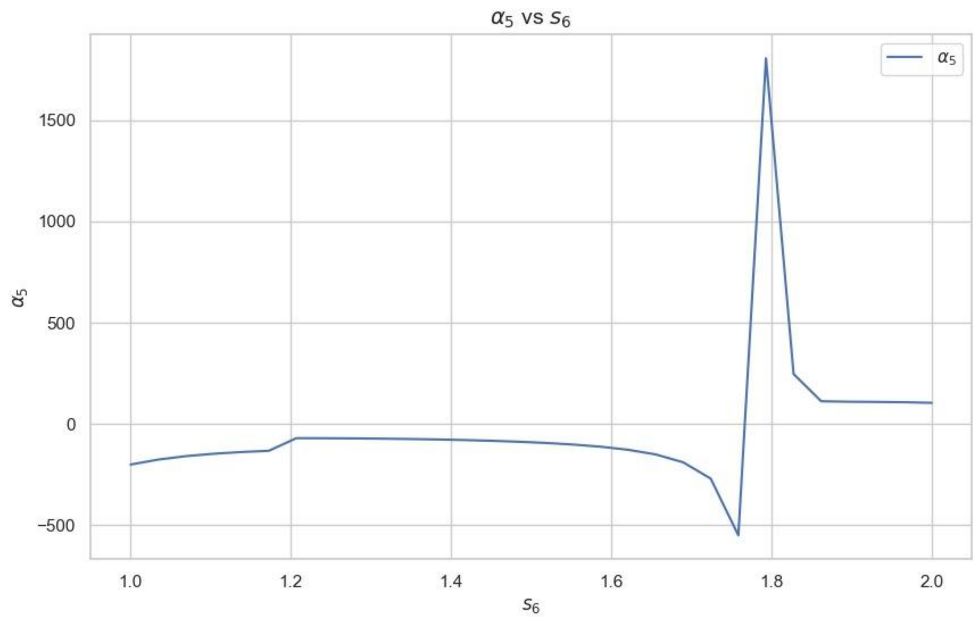
For Leftside arm velocity diagram



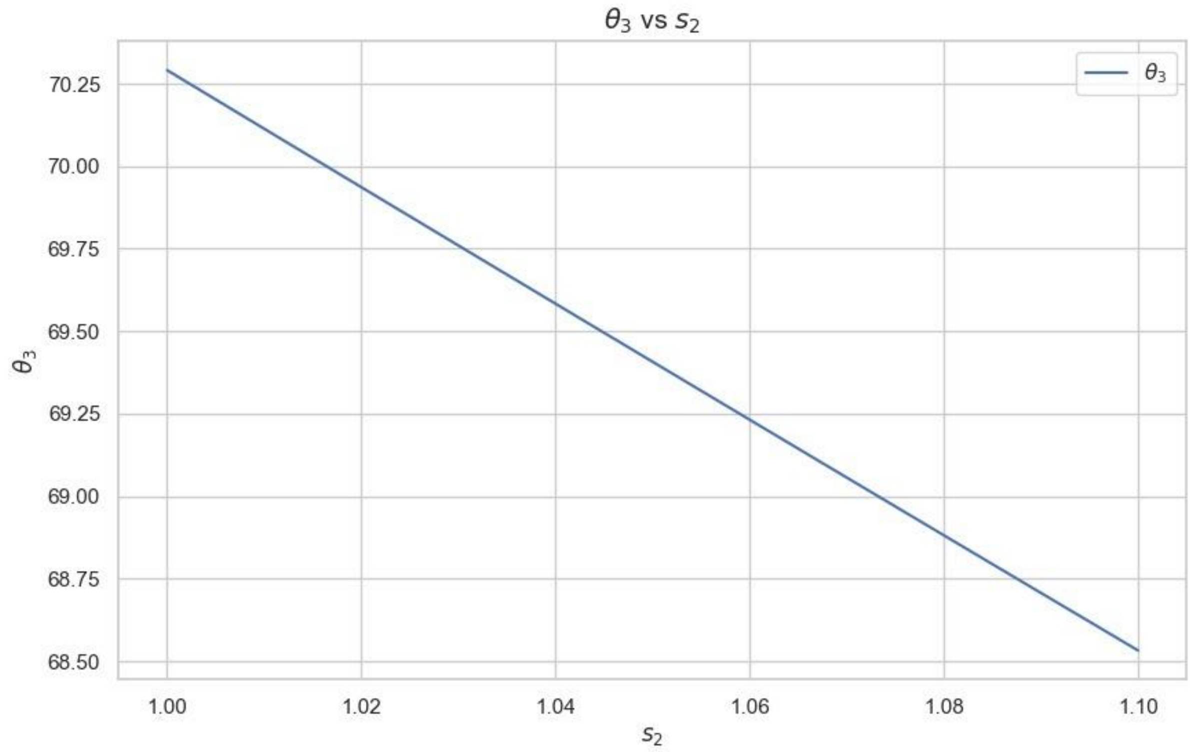


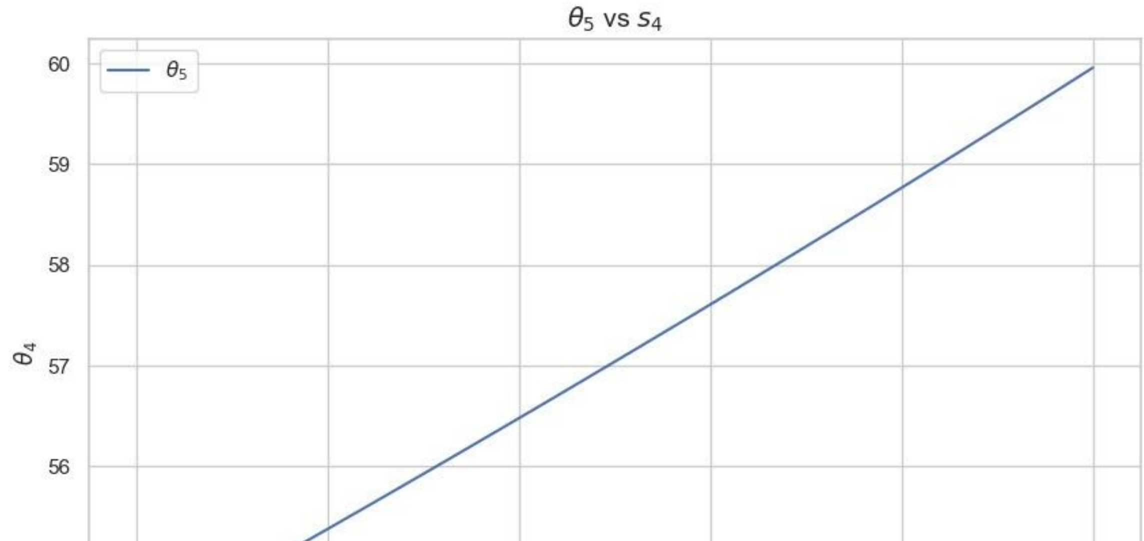
For Leftside arm Acceleration diagram





For Rightside arm displacement diagram





For Rightside arm velocity diagram