ME2220: Kinetics And Dynamics of Machinery

Kinematic analysis and synthesis of the toy-based mechanism

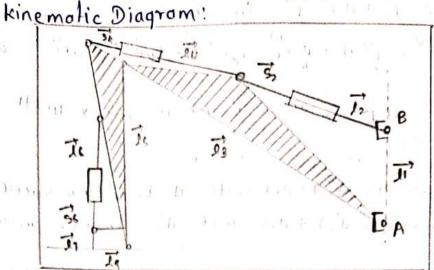
Description: A backhoe excavator toy is an excellent choice for studying mechanical linkages and movement. Its design includes articulated arms, hydraulic-like prismatic joints, and rotational motion, closely resembling real-world excavators. By analyzing its mechanism, We can understand kinematic relationships, such as how prismatic displacements affect angular velocities and link rotations. This makes it a great model for exploring excavator dynamics in a controlled and simplified manner.





Overview: The backhoe excavator toy's mechanism will be analyzed mathematically using loop closure equations to connect the movement of different points to the input crank angle. By calculating positions for different crank angles, we can simulate the motion using Python. Then, velocity and acceleration vector diagrams will be created. If these vectors form closed shapes, it confirms that their total sum is zero, verifying the accuracy of the mechanism's motion analysis.

Displacement Analysis:



Here, 1) 3, 54, 56 are prismatic lengths

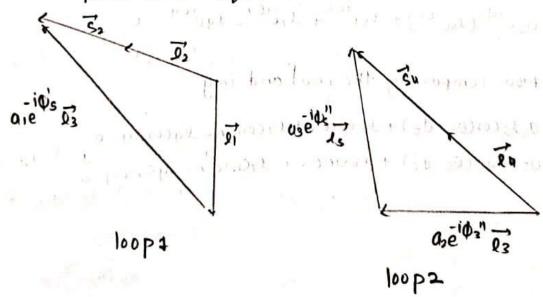
- 2) d', d', d', d' angles are also known
- 3) 1, 12, 15, 14, 15, 16, 17, 18, 19 all links are known.
- 4) 02,03,04,05,06,07,08,09 are unknowns

Loop 1: The loop closure equation is

$$\vec{l}_1 + \vec{a}_1 e^{-i\vec{Q}_1} \vec{l}_3 - \vec{S}_2 - \vec{l}_2 = 0$$

$$\vec{l}_1 e^{i\frac{3\pi}{2}} + \vec{a}_1 e^{i\vec{Q}_3} (l_3 e^{i63}) - S_2 e^{i62} - l_2 e^{i62} = 0$$

Now comparing the real and imaginary ports of this equation we get



Loop 2: The loop closure equation is:

$$0_{2}e^{-i\theta_{3}^{"}} + 0_{3}e^{-i\theta_{5}^{"}} - 1_{4}e^{-i\theta_{5}^{"}} - 1_{4}e^{i\theta_{4}} - 1_{4}e^{i\theta_{4}} = 0$$

$$0_{2}e^{-i\theta_{3}^{"}} + 0_{3}e^{-i\theta_{5}^{"}} + 0_{3}e^{-i\theta_{5}^{"}} + 0_{3}e^{-i\theta_{5}^{"}} - 1_{4}e^{i\theta_{4}} - 1_{4}e^{i\theta_{4}} = 0$$

Now comparing the real and imaginary parts of the equation.

02l3(0)(03-03)+03l5(0)(05-03)-l4(0)04-54(0)04=0
02l3sin(03-03)+03l5sin(05-03)-l4sinO4-54sinO4=0

Loop 3: loop closure equation is.

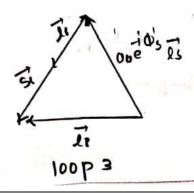
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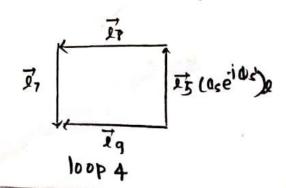
Loop 4: loop closure equation is

$$0_5 e^{-i\phi_5} + 1_8 + 1_7 - 1_9 = 0$$

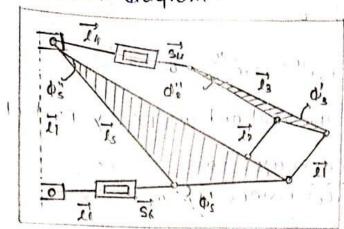
$$0_5 e^{-i\phi_5} (1_5 e^{i\theta_5}) + 1_8 e^{i\theta_5} + 1_7 e^{i\theta_7} - 1_9 e^{i\theta_9} = 0$$

Now, comparing the real and img.



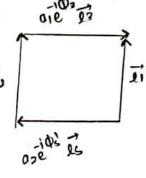


Displacement Analysis Kinematic diagram



Here, 1) II, Iz, Iz, Iu, Is, Io, Ii all links lengths are known 2) su, So prismatic length are also known.

4) 01,02,03,04,05,06 are unknowns.



100p1: Loop closure equation are

aie 12 +02e 10s 1 + 12 - 1 = 0

Now comparing the real and imaginary part of eq. $0.13\cos(\theta_3-\phi_3') + 0.21\sin(\theta_5-\phi_5') + 1.2\cos\theta_2 - 1.\cos\theta_1 = 0$ $0.13\sin(\theta_3-\phi_3') + 0.21\sin(\theta_5-\phi_5') + 1.2\sin\theta_3 - 1.\sin\theta_1 = 0$

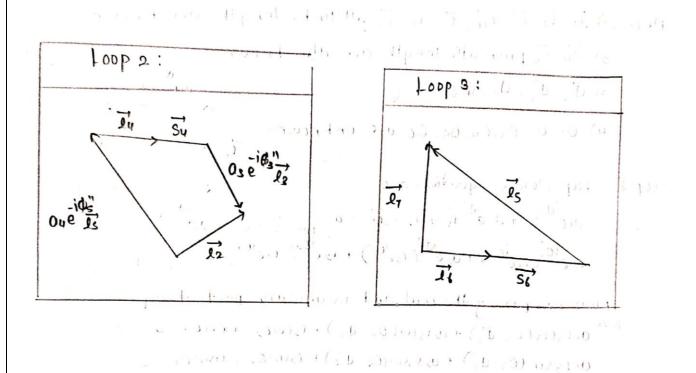
Loop2: $0_4 e^{i\phi_5^{"}} + l_4 + s_4 + s_9 e^{i\phi_5^{"}} + s_9 e^{i\phi_5^$

 $04l_{5}(0)(\theta_{5}-\phi_{5}^{"})+l_{4}(0)\theta_{4}+S_{4}(0)\theta_{4}+03l_{3}(0)(\theta_{3}-\phi_{5}^{"})-l_{2}(0)\theta_{2}=0$ $04l_{5}(0)(\theta_{5}-\phi_{5}^{"})+l_{4}(0)\theta_{4}+S_{4}(0)\theta_{4}+03l_{3}Sin(\theta_{3}-\phi_{5}^{"})-l_{2}Sin\theta_{2}=0$

100p 3: The loop closure equation is $\overrightarrow{l_6} + \overrightarrow{S_6} + \overrightarrow{l_5} - \overrightarrow{l_7} = 0$ $lee^{i\theta_6} + S_6e^{i\theta_6} + l_5e^{i\theta_5} - l_7e^{i\frac{71}{2}} = 0$

Now, composing the real and imaginary part of the eq. 100006 + 560006+ 250005 = 0

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Velocity Analysis : (FRONTSIDE-RIGHT)
             Differentiating loop closure equation:
            100p1:
                    ailywzsin(03-Φ'a) + V2(0)(Θ2) - S2ωz sinΘ2 - 22ωz sinΘ2 = 0
                   01/3W3(01(03-03) - V25InO2-52W2(01O2-22W2(01O2 =0
           Loop 2:
                  0, 13w3 sin (03-4") + 0, 15 ws sin (05-4") - (14+54) w4 sin 04 + V4(0)64= C
                  02 13 W3 (03 (03 - 0)) + 03 LSW5(03 (O5 - 0") - (14+54) W45 IN 04 - V4(0) 64 = 0
          L00 P3 !
                04/565 sin (05-05) + (16+56) w6 sin 06 - V6 (0166 - L8 W8 sin 08 = 0
                04 25 W5 CO) (O5-0/6) + (26+56) W6 CO) O6 + V6 SIN O6 - L8 W8 CO) O6 = 0
         Loop 4:
               Oslowesin (Os -d's) + lows sinos + lowsinos - lowg sinoq = 0
               05 15 W5 COS(86-0'5) + 18 W8 COS 68 + 27 W7 COS 67 - 29 W9 COS 69 =0
        Acceleration Analysis
Loop 1 1
   a113(k3 sin(03-43) + w3 (0)(03-43)) - (5+12)(k2 sin 62 + w2 cos 62) - V2 w2 sin 02 = 0
  01 /3( k3 C0) (03-013) - W3 sin(03-03)) - (52+22) (K2 C0162-W2 sin 02) - V2 W2(0162 = 0
Loop 2 !
  02 28 ( Ks sin (03 - 0") + 63 cos(03 - 0")) + 03 (Ks sin (05 - 0") + 45 cos (05 - 46"))
           - ( lu+ sy) (x4sin 64 + W42(0)64) + V4 W4 sin 64 = 0
  a= 13 (K3(0)(03-013)-w3sin(03-03")) + a3 15 (K5(0)(05-05"))-w2sin(05-05"))
           - (24+54) (x4cose4-w42sino4)- V4w4cose4 =0
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Loop 3:
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0425 (x5 sin(85-0/5) + w2 (0) (85-0/6)) + (16+56) (x6 sin 86 + w6 (0) 86) Vewering 6 - 28 (x8 sin 8 + w8 (0) 88) = 0

Que (0) θ6 - ds) - ws sin(05-ds)) + (le+si) (de (0) θ6-w62 sin 66) +

Loop4:

ds ls(ds sin(05-0's) + ws (0) (05-0's)) + ls(de sin 08 + wg² (0) 08) + lγ (dγ sin 07 + ωγ² (0) 07) - lg (dq sin 09 + wg² (0) 69) = 0

 $k_5 l_5(k_5(0)(\Theta_5 - \Phi_5') - \omega_5^2 sin(\Theta_5 - \Phi_5')) + l_8(k_8(0)\Theta_8 - \omega_8^2 sin\Theta_8) + l_7(k_7(0)\Theta_7 - \omega_7^2 sin\Theta_7) - l_9(k_9(0)\Theta_9 - \omega_9^2 sin\Theta_9) = 0$

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, + without (2)) - (2+12)(624/16) + 163 (016) + 164 (016)

Velocity Analysis. (Backside - LEFT)

loop1:

$$a_{1}l_{3}w_{3}sin(\theta_{3}-d_{3}')+a_{2}l_{5}w_{5}sin(\theta_{5}-\varphi_{5}')+l_{2}w_{2}sin\theta_{2}-l_{1}w_{1}sin\theta_{1}=0$$

$$a_{1}l_{3}w_{3}(\theta_{3}-\varphi_{3}')+a_{2}l_{5}w_{5}(\theta_{5}-\varphi_{5}')+l_{2}w_{2}cos\theta_{2}-l_{1}w_{1}cos\theta_{1}=0$$

Loop 2:

$$a_4 l_5 w_5 sin(\theta_5 - \theta_5'') + (l_4 + s_4) w_4 sin \theta_4 - V_4 (0) \theta_4 + 0_3 l_3 w_3 sin(\theta_3 - \theta_5'')$$

$$- l_2 w_2 sin \theta_2 = 0$$

$$O4lsw_{5}(0)(\Theta_{5}-\Phi_{5}'')+(14+54)w_{4}(0)\Theta_{4}+V_{4}sin\Theta_{4}+0_{3}l_{3}w_{3}(0)(\Theta_{3}-\Phi_{3}'')$$

$$-l_{2}w_{2}sin(0)(\Theta_{5})=0$$

Loop 3:

Acceleration Analysis

Loop 1:

$$0.11_{9} (\% \sin(03 - 0\frac{1}{3}) + W_{3}^{2} \cos(03 - 0\frac{1}{3})) + 0.21_{5} (\% \sin(05 - 0\frac{1}{5}) + W_{3}^{2} \cos(05 - 0\frac{1}{5}))$$

$$+ 1_{2} (\% \sin 02 + W_{2}^{2} \cos 02) - 1_{3} (\% \sin 01 + W_{3}^{2} \cos 01) = 0$$

$$\alpha_1 l_3 (k_3 cos (\theta_3 - \phi_3') - w_3^2 sin (\theta_3 - \phi_3')) + \alpha_2 l_3 (k_3 cos (\theta_3 - \phi_3') - w_3^2 sin (\theta_5 - \phi_3'))$$

+ $l_2 (k_2 cos \theta_2 - w_2^2 sin \theta_2) - l_1 (k_1 cos \theta_1 - w_1^2 sin \theta_1) = 0$

Loop 2:

Quls (xs sin (0s-4s") + ws (0) (0s-4s")) + (lu+su) (xusino 4 + wf (0) θ ψ)
- Vuwu sino 4 + 0sls (xssin (03-4;") + ws (0) (03+4;")) -lo (xssin 02 + ws (0)05)=0

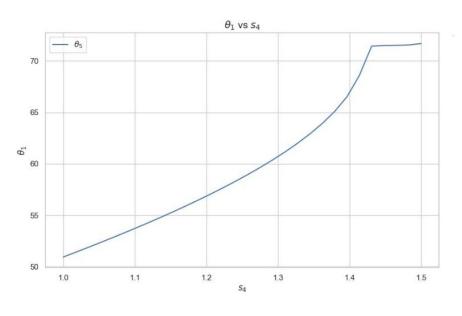
Quls (xs sin (0s-4s") - ws cin (0s-4s")) + (lu+su) (xu (0) 6 4 - wu cin 64) +

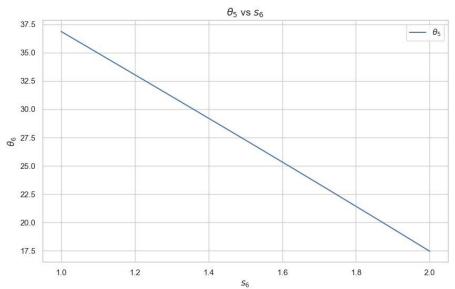
Vuwu (0) (6u) + 0sls (xs (0) (03-4;") - ws cin (03 - (4;")) -lo (xo (0) - ws cin 62) = 0

Loop 3:

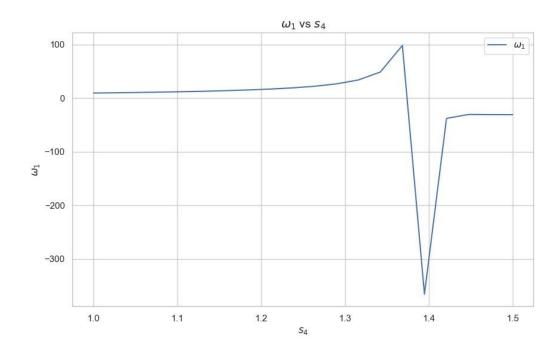
(15+56) (x651066 + w62 (0066) - V6 w651006 + 25(d551005 + w52(0)65) = 0 (15+56) (x661066 - w6251066) + V6 w660166 + 25(d5 (0)65 - w351065) = 0

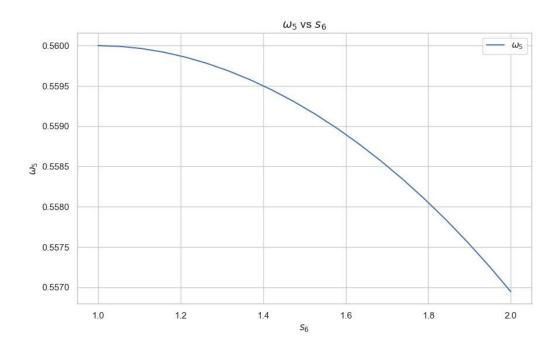
For Leftside arm displacement diagram



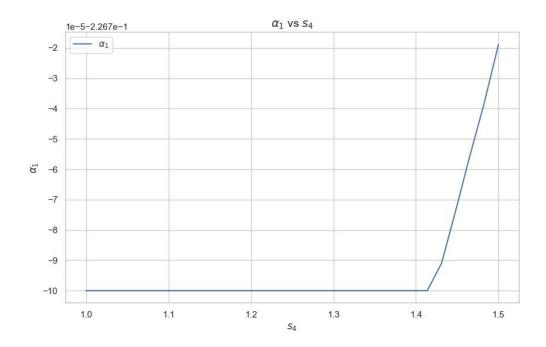


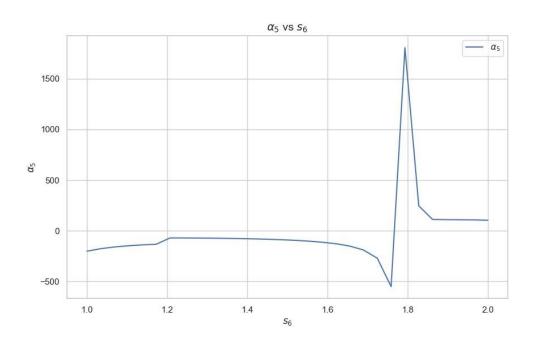
For Leftside arm velocity diagram



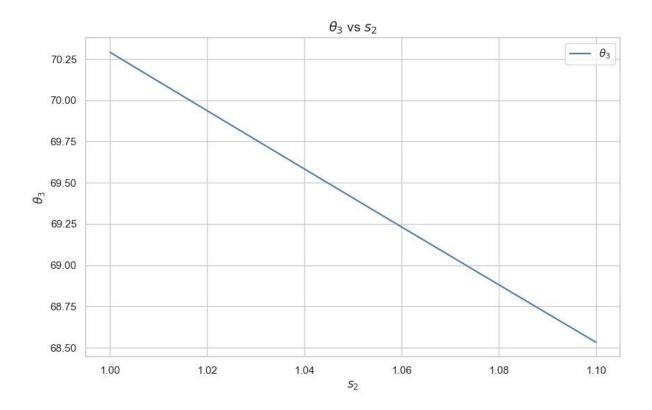


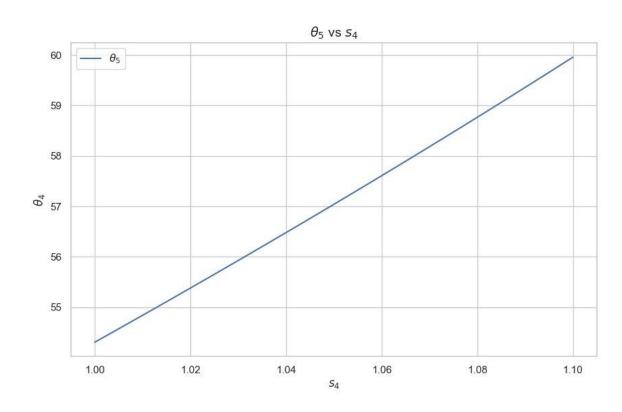
For Leftside arm Acceleration diagram





For Rightside arm displacement diagram





For Rightside arm velocity diagram

