Final Discrete Mathemetics Project

Assigned by Prof. Manish Gupta
Course : SC-105
DA-IICT, Gandhinagar

Project Title : Food Web

FOOD WEB

Isha Shah - 202001163,
Krunal Chhagani - 202001158,
Harshkumar Metkel - 202001154,
DA-IICT, GANDHINAGAR,
INDIA

July 5, 2021

Abstract

This model gives a brief introduction about the relationships between different organisms by graphically showing what eats what.

Contribution

Project Title and Content : 202001163, 202001158, 202001154.

Website : 202001163, 202001154.

Latex and Bibliography : 202001158.

Powerpoint Presentation : 202001163.

Youtube Video : 202001163 , 202001158 , 202001154.

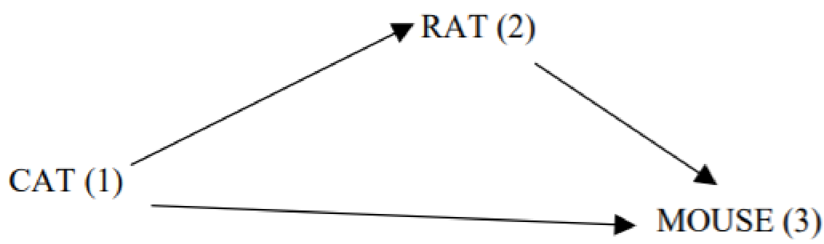
1 Introduction

A food web is a diagram for an ecosystem that shows the relationships between different organisms by graphically displaying what eats what.In this module, we will explore the complexity of food webs in mathematical terms, using a visual model (graph) name digraph which mainly helps in interaction among organisms.Mathematical modeling is a process by which a real-world situation is replaced with a mathematical representation. If both that real-world situation and the mathematical representation obtained by mathematical model are well matched, then whatever information is obtained from the mathematical representation is meaningful in the real-world setting.

We can use an adjacency matrix to hold the information from a food web and then use matrix algebra to help us learn more about the ecosystem. In an adjacency matrix for a food web, if an aij entry contains a 1, it means the organism at vertex i eats the organism at vertex j, and aij entry contains a 0, it means the organism at vertex i does not eat the organism at vertex j.

In this module,digraphs that consist of vertices and arcs are used to model the feeding relationships among species within a given area.

Let's understand with small example then consider the following digraph for a food web:



The above shown is the digraph of food web for 3 different organisms.

The adjacency matrix for above food web is:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

So, as explained above we can determine that the first organism i.e., cat eats rat so, A[1][2] = 1 and also, cat eats mouse so, A[1][3] = 1.

Similarly, rat eats mouse so, A[2][3] = 1.
And third row of adjacency matrix is fully 0 indicates that mouse in this food - web cannot eat anyone.
And first column in fully, zero which indicates that no organism in this food - web can eat cat.

2 History

2.1 Leonhard Euler

Leonhard Euler was a Swiss mathematician, physicist, astronomer, geographer, logician and engineer who founded the study of graph theory and topology and made influential discoveries in many other branches of mathematics.
In graph theory, Euler made a great contribution. The origin of graph theory is mainy arise from problem known as the Seven Bridges of Königsberg in Prussia.

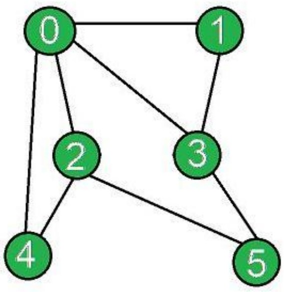
Leonhard Euler(Graph Theory)

In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A distinction is made between undirected graphs, where edges link two vertices symmetrically, and directed graphs, where edges link two vertices asymmetrically. Here, in our problem of food web solving directed graph is useful very much.

2.2 Adjacency Matrix

In graph theory and computer science, an adjacency matrix is a square matrix used to represent a finite graph.

The elements of the matrix indicate whether pairs of vertices are adjacent or not in the graph.
The adjacency matrix is simply a (0,1) matrix with zeros on its diagonal. If the graph is undirected, the adjacency matrix is symmetric.



	0	1	2	3	4	5
0	0	1	1	1	1	0
1	1	0	0	1	0	0
2	1	0	0	0	1	1
3	1	1	0	0	0	1
4	1	0	1	0	0	0
5	0	0	1	1	0	0

Adjacency Matrix

The adjacency matrix of a directed graph can be asymmetric.

- A non-zero element Aij indicates an edge from i to j
- It indicates an edge from j to i.

3 Demonstration

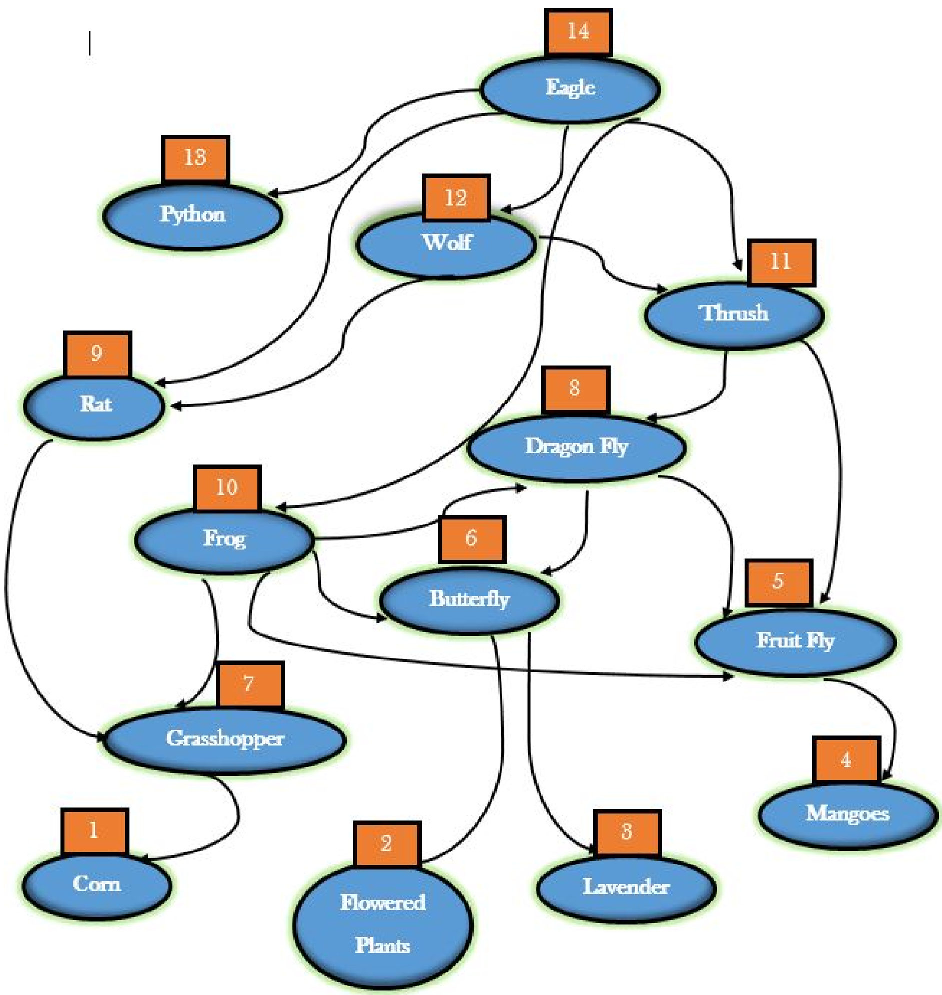
3.1 Problem Statement

As we all know that each living being on this planet is in some way dependent on any other animal or plant. But how this balance of dependency is balanced? And how can one perform systematic analysis of this dependencies? How do someone can predict if some of important animals are getting endangered just because some later animals on which former one is dependent?

3.2 Solution and Approach

We have seen Food-Web, Which is defined as A diagram for an ecosystem that shows the relationships between different organisms by graphically displaying what eats what.

Here is the example of the food web for small birds/insects/plants/sm
animals ecosystem.
The following digraph is :



Digraph of Food Web

Here one can see each creature has been given a certain number. This numbers will represent that animal in our analysis. Now see below a square matrix, which consists of only zeros and ones. This matrix is known as Adjacency matrix. To construct such Adjacency matrix, one first needs to know number of Creatures in any ecosystem which will represents the dimensions of Adjacency matrix. Then on looking on Food-Web graph one can easily identify, that which creature survives on which another creature. And looking on that one can assigns ones to the Adjacency matrix. If we consider rows of an Adjacency matrix as i and columns as j. Then by following a rule i eats j implies one.
So if in any cell there is one, this means that animal representing that row i, will eat animal representing columns j. And so based on this same principle we can construct a matrix as shown below, for the Food-Web like shown above.
Adjacency Matrix for above food web is :

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0
\end{bmatrix}
$$

Adjacency Matrix

Now on carefully observing this matrix one can say that Creatures belonging to indexes 1 to 4 these does not depend on any other creatures. This are some king of plants and creates their own food through photosynthesis. But as we go down no of one's in row increases. Which shows food dependencies. For example if see tow 10 (Frog) depends on columns 5 (Fruit-Fly), 6 (Butter-Fly), 7 (Grass-Hopper) and 8 (Dragon-Fly).

This way this kind of Adjacency matrices become very helpful in understanding the Food-Web inter species dependencies on Food. Also this method can be scaled to any size of Food-Web just the dimensions of matrix will increase the complexity won't increase to much extent because the basic principle remains the same.
Now we know that , Adjacency matrix gives the information about the direct sources of food. Like which creature is directly dependent on which other creature for food.

This way this kind of Adjacency matrices become very helpful in understanding the Food-Web inter species dependencies on Food. Also this method can be scaled to any size of Food-Web just the dimensions of matrix will increase the complexity won't increase to much extent because the basic principle remains the same.

Now we will square the adjacency matrix.
The square adjacency matrix for this food web is :

$$A.A = A^2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 2 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 0 & 0
\end{bmatrix}$$

Square Adjacency Matrix

Here we have Square of Adjacency matrix. This tells us about the indirect sources (means those sources of food, which the creature does not directly consumes, but its direct resource depends on that creature) of Food for any particular creature. Like if we take example of row 10 (Frog) it has ones in columns 1 (Corn) , 2 (Flowered Plants) , 3 (Lavenders), 5 (Fruit-Fly) and 6 (Butter-Fly).

This are all the creature/plants which are not directly consumed by the Frog but its direct sources are dependent on this things so even it matters for Frog that this resources sustain. Also observe that column 4 (Mangoes) is numbered two. Which means that Frog has some source that is directly dependent on Mangoes. On looking at the Graph, we found that creature is Fruit-Fly. So this we one can do a detailed analysis on indirect food resources of any creature by looking at its Square of Adjacency.

When we add above two matrices we get the Matrix that represents overall food sources of any creature. This gives complete information about the overall dependencies including direct and indirect Food sources. See below is depiction of that matrix.

Total Dependencies Matrix is :

$$A^2 + A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 1 & 0 \end{bmatrix}$$

Total Dependencies Matrix($\mathbf{A + A^2}$)

From this matrix , we can get total number of direct and indirect food sources.

3.3 Commercialisation

We have seen in Demonstration, that calculations are very simple yet very powerful. Not only that this can have wide variety of applications in multiple domains. Like one of the important sector in which this kind of Study helps is Wild-Life.
This study of digraph tells us about how certain things are dependent on others. So as we know that today because of many reasons say many animals are facing adverse effects. Many animals are on verge of extinction. And if we loose any one species it is not that we only looses that species, but what is the cost to be paid is too much. All the animals in any ecosystem are important. And extinction of any one species also causes huge imbalance to that whole ecosystem.

Let's take an example... What happens if tigers go extinct?

Tiger is very unique animal which plays a very important role in health and diversity of an ecosystem. It is top predator which is at the apex of Food-Chain, and keeps the population of wild animals in check. Now if someday tigers go extinct what happens is entire ecosystem will collapse. For some early days we see the sudden rise in population of animals which were food for tigers but soon there population would increase drastically that somehow they will generate food shortage for themselves. And eventually dies.

This ain't that simple thing but main idea is to understand how whole ecosystem is interrelated.

So we have seen there is a problem. This can be solved by creating a software which will do the similar analysis as we did in Demonstration page, but on a large scale for larger ecosystem, consisting of large number of animal species. Also software should be such that it should constantly monitor the change in population of any particular species, and should also immediately propose the necessary steps to be taken by responsible authorities. This way this kind of Software can be very useful in Wild-Life management

Who are targeted group of customers?

- Authorities of Forest department.
- Authorities related to Aquatic life management.
- Wild-Life Scientists.
- NGO-s who work for Wild-Life protection.

Alternate Application

The similar/same software can be used in many other applications as well. Like in many estimation problems where certain things are dependent on other things. because this software is purely application of Graph-Theory One of important domains is in Stock investment prediction. Like if one company say A, is having three different plans of investment. Each plan with three sections as - High risk, low risk and medium risk. Then the calculations of which plan would yield max return, which the least return can be done easily using similar matrix operations on Adjacency matrix. Of course this example is simple, but similar kind of larger problems can be solved using similar and optimized software.

4 Conclusion

We have seen Food Web with Mathematics and how it is useful to real life.In this module, we have used digraph , adjacency matrices to find various direct and indirect dependencies of the Food sources.We can also use Linear Algebra here.
We can take one column matrix v = [1 1 1 1 1 1 1 1 1 1 1 1 1]' (with same dimension of adjacency matrix).Then, we can calculate Av and A^2v with matrix product.It will give us column vectors whose elements will contain sum of rows respectively.Then , we can find Av + A^2v in similar manner and it will give the total number of direct and indirect sources of food.
This application can be extended to find what happens when certain organisms are removed from the ecosystem.For example, we wanted to know in our food web Butterfly suddenly become extinct, we would remove 6th row and 6th column from adjacency matrix.Then , we will perform same calculations and get the matrices and could compare the new results with our original ones to see the influence of the removal of Butterfly had on the food sources of the remaining organisms.

Another application would be to determine the importance of the organism in an ecosystem.An organism is important if other important organisms rely on it.So the spices would be ranked according to the number of important spices rely on it.

[4] [2] [1] [3] [5]

References

[1] kephakx17. *Food Webs*. 2019. URL: https://linearalgebraapplications19.wordpress.com/2019/04/29/food-webs/.

[2] David Poole. *Linear algebra: A modern introduction*. Cengage Learning, 2014.

[3] Wikipedia contributors. *Adjacency matrix — Wikipedia, The Free Encyclopedia*. [Online; accessed 5-July-2021]. 2021. URL: https://en.wikipedia.org/w/index.php?title=Adjacency_matrix&oldid=1032077200.

[4] Wikipedia contributors. *Leonhard Euler — Wikipedia, The Free Encyclopedia*. [Online; accessed 4-July-2021]. 2021. URL: https://en.wikipedia.org/w/index.php?title=Leonhard_Euler&oldid=1031823148.

[5] Wikipedia contributors. *Leonhard Euler — Wikipedia, The Free Encyclopedia*. [Online; accessed 5-July-2021]. 2021. URL: https://en.wikipedia.org/w/index.php?title=Leonhard_Euler&oldid=1031935336.