2/3/2021

$E(X) = 1, \quad E(Y) = 1 \qquad \uparrow\uparrow \quad \downarrow\downarrow$

$E(XY) = 5/4$

$\mathrm{Cov}(X, Y) = E(X \cdot Y) - E(X) \cdot E(Y)$

$= 5/4 - 1 \cdot 1 = 5/4 - 1 = 1/4$

$$\boxed{\rho = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \, \sigma_Y}} = \frac{1/4}{\sigma_X \, \sigma_Y} = \frac{1/4}{1/\sqrt{2} \; 1/\sqrt{2}} = \frac{1/4}{1/2} = 1/2$$

0.5

$\sigma_X = \sqrt{\mathrm{Var}(X)}$

$\sigma_Y = \sqrt{\mathrm{Var}(Y)}$

$\mathrm{Var}(X) = E(X^2) - (E(X))^2$

$\mathrm{Var}(Y) = E(Y^2) - (E(Y))^2$

$E(X^2) = \sum x^2 f_X(x)$ ✓

$E(Y^2) = \sum y^2 f_Y(y)$ ✓

$$E(X^2) = \sum_x \sum_y x^2 f_{X,Y}(x,y) = \sum_x x^2 \sum_y f_{X,Y}(x,y)$$

$$= \sum_x x^2 f_X(x)$$

$$= 0^2 \cdot 2/8 + 1^2 \cdot 4/8 + 2^2 \cdot 2/8$$

$$= \frac{4}{8} + \frac{8}{8} = \frac{12}{8} \checkmark$$

$$E(Y^2) = \sum_y y^2 f_Y(y) = 0^2 \cdot 2/8 + 1^2 \cdot 4/8 + 2^2 \cdot 2/8$$

$$= 12/8 \checkmark$$

$$Var(X) = E(X^2) - (E(X))^2 = \frac{12}{8} - 1^2 = 4/8 = 1/2$$

$$\boxed{Var(Y) = 1/2}$$

$$\therefore S.D(X) = \sigma_X = 1/\sqrt{2}, \quad \sigma_Y = \frac{1}{\sqrt{2}}$$

Tobacco Comp. Cate

1

2

3

r

r

10

$\boxed{-1 \le \rho \le 1}$

X Y → Latt-2 Cons

First Cons line

lung cancer

$\boxed{\rho = 0}$

$\rho = 0.8$

$x \downarrow \uparrow y$

$\uparrow \downarrow$

$x \quad y$

$\downarrow\downarrow \quad \uparrow\uparrow$

0.5

−1 (−ve) 0 (+ve) 1

$\rho(X, Y) = 0.5$

(X, Y) $\quad f(x,y) = \begin{cases} \frac{1}{8}(6-x-y) & 0 \le x < 2, \ 2 \le y < 4 \\ 0 & \text{otherwise} \end{cases}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\Rightarrow \int_{2}^{4}\int_{0}^{2} \frac{1}{8}(6-x-y)\,dx\,dy \qquad \boxed{\begin{matrix} 0 < x < 2 \\ 2 < y < 4 \end{matrix}}$$

$$= \int_{2}^{4} \frac{1}{8}\left(6x - \frac{x^2}{2} - xy\right)\Big|_{0}^{2} dy$$

$$= 1 \qquad \therefore f_{X,Y}(x,y) \text{ is a p.d.f.}$$

03/03/2021

np, npq $\rightarrow N(\mu, \sigma^2)$

Mean var

$N = 200$

$p = 1/2$

$P(X > 150)$

$= 1 - P(X \leq 150)$

P.D of $X+Y$ $X-Y$

3R, 2G, 4W 2 Balls at random

X: no of Red Balls $X = 0, 1, 2$

Y: no of Green Balls $Y = 0, 1, 2$

X \ Y (Green)	0	1	2
0	$\frac{^4C_2}{^9C_2}$		
1			
2			

(0, 0)

(0, 1)

Red green Red

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy = 1$$

$$F_X(x) = \int_{-\infty}^{x}\left(\int_{-\infty}^{\infty} f(x,y)\,dy\right)dx \quad \left(= P[X \le x]\right)$$

$$F_Y(y) = \int_{-\infty}^{y}\left(\int_{-\infty}^{\infty} f(x,y)\,dx\right)dy$$

$(0 < Z < Z_1)$

$(0, 1)$

0.9999

$$F_X(x) = P[X \le x]$$
$$= P[X \le x, Y < \infty]$$

$$F_Y(y) = P[Y \le y]$$
$$= P[X < \infty, Y \le y]$$

$$f_X(x) = P[X = x]$$
$$f_Y(y) = P[Y = y]$$

$X \backslash Y$	0	1	2	$f_X(x)$
0				
1				
2				

Ques: $f_{X,Y}(x,y) = \begin{cases} 2 & 0<x<1,\ 0<y<x \\ 0 & \text{elsewhere} \end{cases}$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$$

$$f_X(x) = \int_0^x 2\,dy$$

$$= \left(2y\right)_0^x = 2x$$

$$\boxed{f_X(x) = 2x,\ 0<x<1}$$

TS: f is a p.d.f

$$\int_0^1\int_0^x f(x,y)\,dy\,dx = 1$$

LHS

$$= \int_0^1\int_0^x 2\,dy\,dx$$

$$= \int_0^1 \left(2y\right)_0^x dx$$

$$= \int_0^1 2x\,dx$$

$$= 2\left(x^2/2\right)_0^1 = 1$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx$$

$$= \int_y^1 2\, dx$$

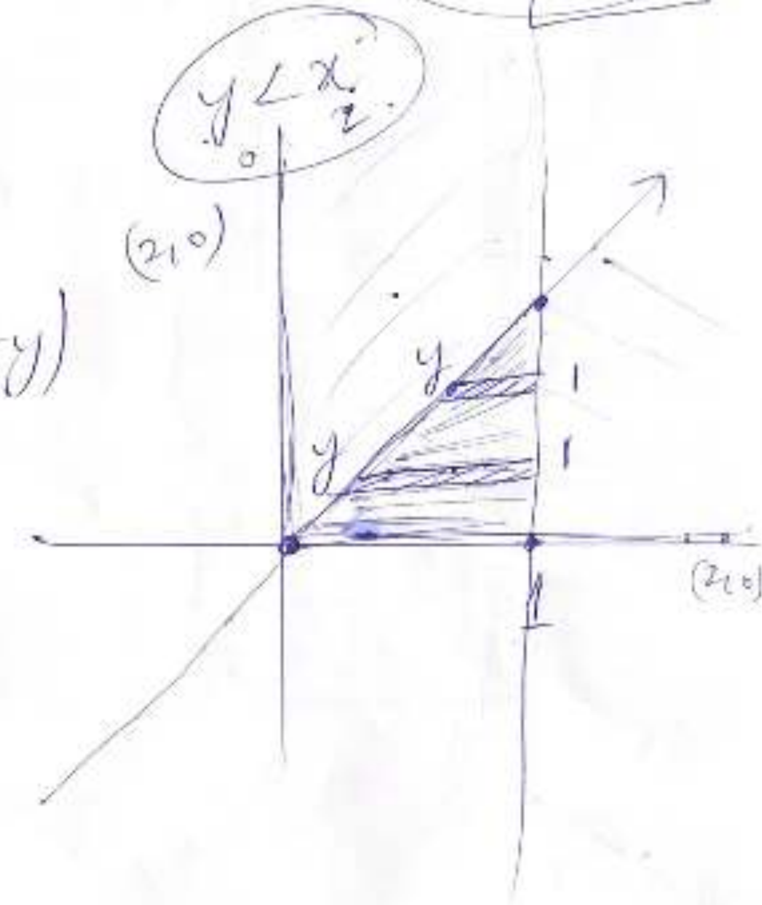
$$= \left(2x\right)_y^1 = 2(1-y)$$

$$\boxed{f_Y(y) = 2(1-y)}$$

$$0 < y < 1$$

$0 < x < 1$

$0 < y < x$

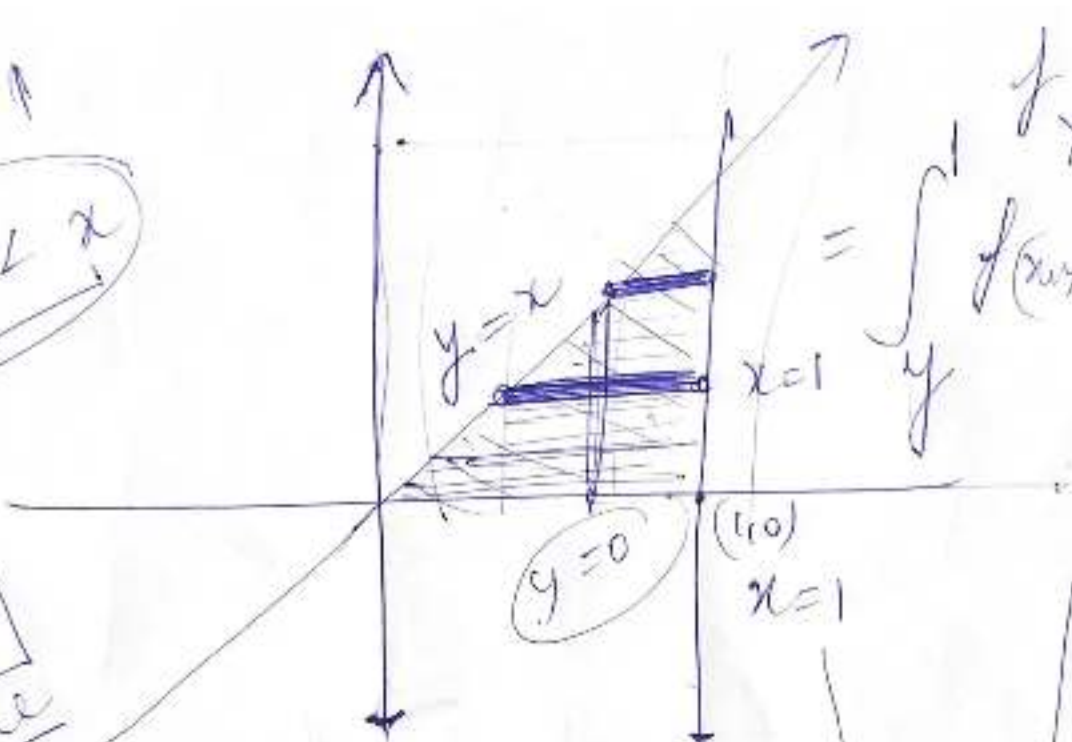


$0 < x < 1$

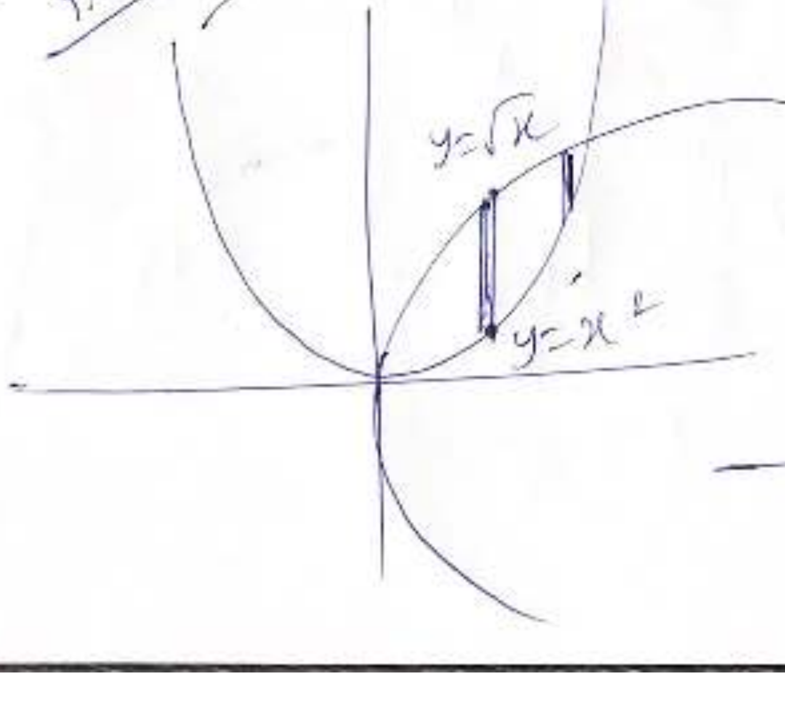
$0 < y < x$

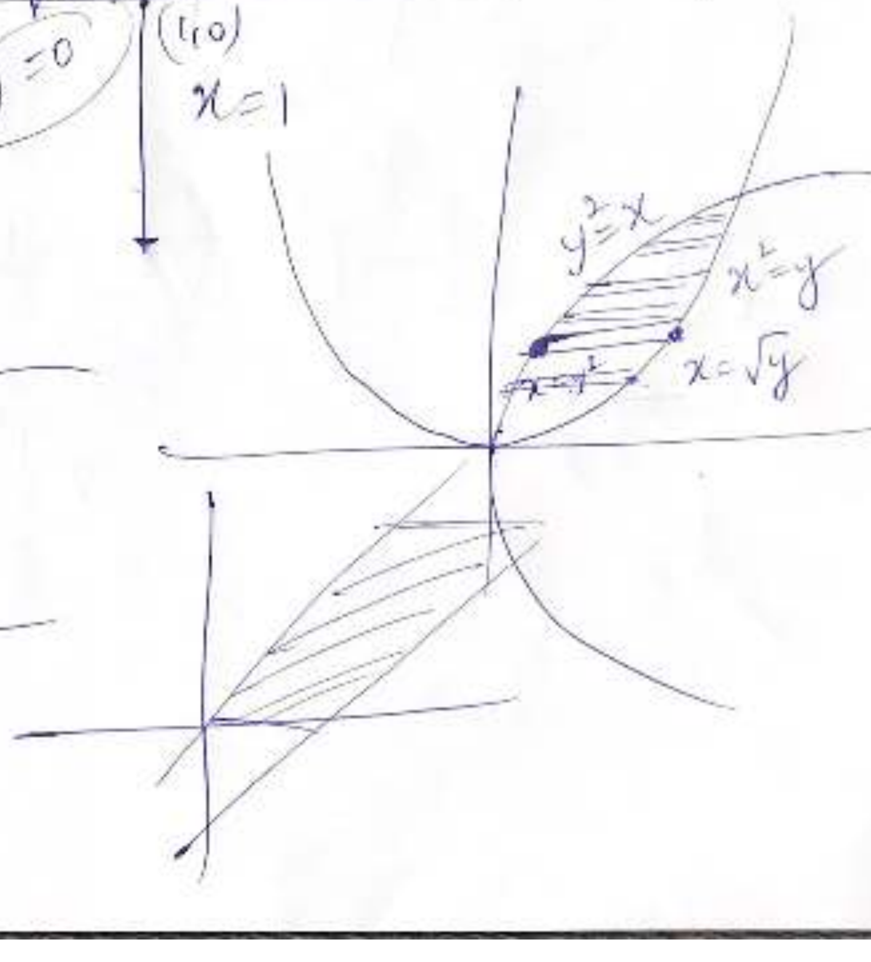
$y < x$

$\boxed{0 < 1}$ True



$f_Y(y) = \int_y^1 f(x,y)\, dx$





05/03/2021

$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y) & 0 \le x \le 2,\ 2 \le y \le 4 \\ 0 & \text{otherwise} \end{cases}$$

① $P(X<1 \cap Y<3)$

$$= \int_{-\infty}^{1}\int_{-\infty}^{3} f_{X,Y}(x,y)\, \cancel{dx\,dy}$$

$$dy\,dx$$

$$= \int_{0}^{1}\int_{2}^{3} \frac{1}{8}(6-x-y)\, dy\, dx$$

$$P[(X+Y) < 3] = ?$$

$0 \le x \le 2$

$2 \le y \le 4$

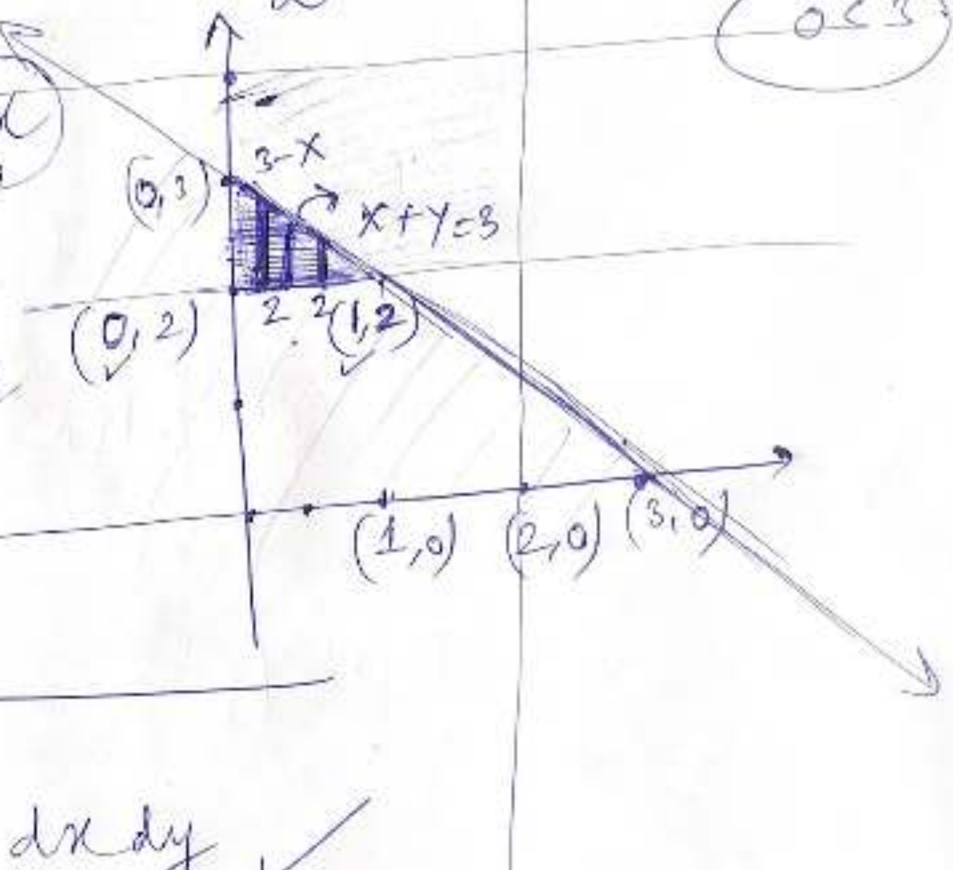
$$\boxed{X+Y<3}$$

$0+0<3$

$0<3$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy\, dx$$

$$= \int_{0}^{1}\int_{2}^{3-x} \frac{1}{8}(6-x-y)\, dy\, dx$$



OR

$$\int_{2}^{3}\int_{0}^{3-y} \frac{1}{8}(6-x-y)\, dx\, dy$$

(3) $P(X<1 \mid Y<3)$

$$= \frac{P(X<1 \cap Y<3)}{P(Y<3)} = \frac{\text{Part(i)}}{P(Y<3)}$$

$$P(Y<3) = \int_0^2 \int_2^3 f(x,y)\,dy\,dx$$

$$f(x,y) = \begin{cases} 2 & 0<x<1,\ 0<y<x \\ 0 & \text{elsewhere} \end{cases}$$

$y<x$
$0<1$
True

(i) Marginal density fn of X & Y

$$f_X(x) = P[X=x] = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^x 2\,dy$$

$$= (2y)_0^x$$

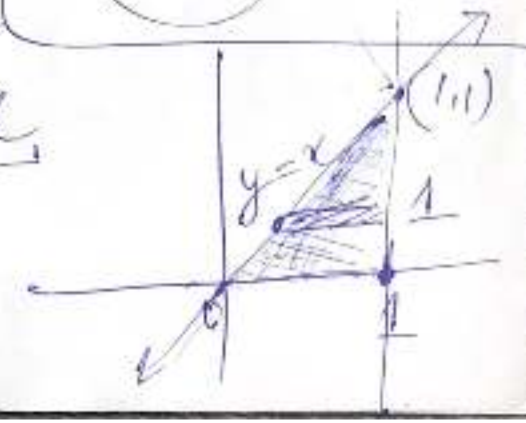
$$= 2x$$

$$\boxed{\therefore f_X(x) = 2x,\ 0<x<1}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_y^1 2\,dx$$

$P(Y=y)$

$$= (2x)_y^1 = 2-2y$$ ✓



$$\boxed{f_Y(y) = 2 - 2y} \quad 0 < y < 1$$

Conditional density functions

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{2}{2x} = 1/x \quad \boxed{0 < x < 1}$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{2}{2(1-y)} = \frac{1}{1-y} \quad \boxed{0 < y < 1}$$

Are (X,Y) Independent?

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$2 \neq 2x(2-2y)$$

not Independent

(X, Y) cos r.v.

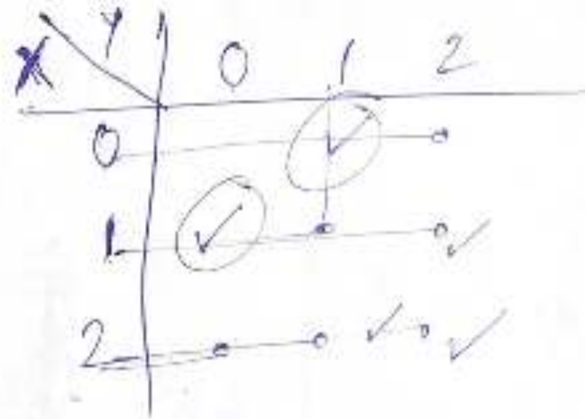
joint pdf

$$\boxed{f_{X,Y}(x,y)}$$

$$E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, f_{X,Y}(x,y)\, dx\,dy$$

$$E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{X,Y}(x,y)\, dx\,dy$$

$$E(X) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f_{X,Y}(x,y)\, dx\,dy$$



$Z = X + Y$

$= 0, 1, 2, 3, 4$

$$= \int_{-\infty}^{\infty} x \left(\int_{-\infty}^{\infty} f\, dy \right) dx$$

$$= \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

Ques:

$$f(x,y) = \begin{cases} x+y & \left.\begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 1 \end{matrix}\right] \\ 0 & \text{otherwise} \end{cases}$$

$$P(0 < X < 1/2, 0 < y < 1/4) = \int_0^{1/2}\int_0^{1/4} (x+y)\, dy\, dx$$

$$E(\underline{X}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f_{X,Y}(x,y)\, dx\, dy$$

$$= \int_0^1\int_0^1 x\,(x+y)\, \underline{dx}\, \underline{dy}$$

$$E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

$$E(Y) = \int_0^1 \int_0^1 y \, f_{X,Y}(x,y) \, dx \, dy \quad \begin{bmatrix} 0 \le x \le 1 \\ 0 \le y \le 1 \end{bmatrix}$$

$$= \int_0^1 y \left(\int_0^1 (x+y) \, dx \right) dy$$

$$E(XY) = \int_0^1 \int_0^1 xy \, f(x,y) \, dy \, dx$$

$$= \int_0^1 x \left(\int_0^1 y(x+y) \, dy \right) dx$$

$$E(X+Y) = \int_0^1 \int_0^1 (x+y)\, f_{X,Y}(x,y)\, dx\, dy$$

$$= \int_0^1 \int_0^1 (x+y)(x+y)\, dx\, dy$$

$E(X^2) = ?$ $\qquad$ $E(Y^2) = ?$

$$\left.\begin{aligned} \mathrm{Var}(X) &= E(X^2) - (E(X))^2 \\ \mathrm{Var}(Y) &= E(Y^2) - (E(Y))^2 \end{aligned}\right]$$

$\mathrm{Cov}(X,Y) = E(XY) - E(X) \cdot E(Y)$

$$\underbrace{\rho(X,Y)}_{-1/11} = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\,\sqrt{\mathrm{Var}(Y)}}$$