Harsh Mittal 2K19CSUN01082
CSE B

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T1 Examination
Probability and Statistics

Ans-1 (a). $E(x) = 10$ $VAR(x) = 25$
$E(y) = 0$ $VAR(y) = 1$

$$Y = ax + b$$
$$Var(Y) = Var(ax + b)$$
$$= a^2 \, Var(x)$$
$$1 = a^2 + 25$$
$$a = \frac{1}{5}$$
$$E(Y) = E(ax + b)$$
$$= a E(x) + b$$
$$0 = \frac{1}{5} \times 10 + b$$
$$b = -2$$

Ans-1 (b)

$$f(x) = \begin{cases} 0 & \text{if } 0 \le m < 2 \\ 2k & \text{if } 2 \le m < 4 \\ -kx + 6k & \text{if } 4 \le m < 6 \end{cases}$$

$$\int_0^6 f(x)\,dx = 1$$

$$\int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx + \int_4^6 f(x)\,dx = 1$$

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$$0 + \int_2^4 2k\,dx + \int_4^6 (1 - kx + 6k)\,dx = 1$$

$$2k\left[\frac{x^2}{2}\right]_2^4 + \left[-\frac{kx^2}{2} + 6kx\right]_4^6 = 1$$

$$2k(4-2) + (-18k + 36k) - (1 - 8k + 24k) = 1$$

$$4k + 18k - \cancel{16}\,k = 1$$

$$k = \frac{1}{6}$$

Ans 1 (C) mean = 3

Variable = 4

$$E(x) = np = 3$$

$$Var(x) = 4 = npq$$

$$\frac{Var(x)}{E(x)} = \frac{4}{3}\; q$$

Since q cannot be greater than 1 so binomial distribution not exist.

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Ans-1 (d). Y = Absolute difference of upturned faces

Y :	0	1	2	3	4	5
P(Y) :	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

$$E(Y) : 0 \times \frac{6}{36} + \frac{1 \times 10}{36} + 2 \times \frac{8}{36} + 3 \times \frac{6}{36} + 4 \times \frac{4}{36} + 5 \times \frac{2}{36}$$

$$E(Y) : 0 + \frac{10}{36} + \frac{16}{36} + \frac{18}{36} + \frac{16}{36} + \frac{10}{36}$$

$$E(Y) : \frac{70}{36}$$

$$E(Y) : \frac{35}{18}$$

Ans-1 (E) $P A = 0$ To Prove or disprove

$P(A \cap B) = 0$

if we consider tossing of face coins both biased such that toss always shows head (H) $\Rightarrow$ {HH, HT, TH, TT}

$P(HH) = 1$. $P(HT) = P(TH) = P(TT) = 0$

If A: Exactly one head

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$\therefore A = \{HT, TH\}$ is non empty but

$$P(A) = 0 + 0 = 0$$

$\therefore AB = A \cap B \subseteq A \quad \therefore P(A \cap B) \leq P(A) = 0$

$$\therefore P(A \cap B) = 0 \text{ if } P(A) = 0$$

~~Ans 2(a)~~ $\cancel{E(x) = \int_0^x \frac{100-x}{5000}\,dx}$

$$\cancel{= 100x - 0}$$

Ans- 3(B) (i) Probability of raining $= \frac{10}{30} = \frac{1}{3}$

Probability of not raining $= \frac{2}{3}$

The probability of rainfall on 3 or more days

$$\sum_{k=3}^{7} {}^{7}C_k \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{7-k}$$

$$= {}^{7}C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^4$$

$$= 35 \left(\frac{1}{27}\right)\left(\frac{16}{81}\right)$$

$$\approx 0.256$$

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Ans 13(A) To get a solution assume that the number of flaws per unit area is Poisson distributed.

Let X be the Poisson random variable with the mean equal to $E(X) = \lambda \frac{S}{S_0}$

$$P(X=k) = \left(\lambda \frac{S}{S_0}\right)^k \cdot e^{-\left(\lambda \frac{S}{S_0}\right)}$$

$$\lambda = 1,\ S_0 = 50\ ft^2,\ S = 4\ ft \cdot 8\ ft = 32\ ft^2$$

$$\lambda \frac{S}{S_0} = 1 \cdot \frac{32\ ft^2}{50\ ft^2}$$

$$= 0.64$$

$$P(X=0) = \frac{0.64^0 \cdot e^{-0.64}}{0!} = e^{-0.64}$$

$$\approx 0.5273$$

$$P(X \le 1) = P(X=0) + P(X=1) =$$

$$\frac{0.64^0 \cdot e^{-0.64}}{0!} + \frac{0.64^1 \cdot e^{-0.64}}{1!} = e^{-0.64} + 0.64 e^{-0.64}$$

$$\approx 0.8647$$

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Ans- 2 (A) $f(x) = \frac{100-x}{5000}$, $0 \le x \le 100$

(a) $\int_0^x \frac{100-x}{5000}$

$= \frac{1}{5000} \int_0^x (100-x)\,dx$

$= \frac{1}{5000} \left(100(15) - \frac{(15)^2}{2} \right)$

$= \frac{2775}{10000}$

$F(40) - F(0) = 0.27 - 0$

$= 0.27$

(b) By Bernoulli's distribution

$P(x=1) = (0.27)^1 (1-0.27)^{8-1}$

$= (0.27)(0.73)^7$

$= 0.0298$

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Ans 2(b) X = no. of head

$X = 0 \quad 1 \quad 2 \quad 3$

$P(x) = \frac{7}{24} \quad \frac{11}{24} \quad \frac{5}{24} \quad \frac{1}{24}$

$SL_1 = [H, T]$

$SL_2 = [HH, HT, TH, TT]$

$SL_3 = [HHH, HTH, HHT, THH, THT,$
$TTH, TTT]$

$$P(x=0) = P(x_1)\left(x = \frac{0}{x_1}\right) + P(x_2)\left(\frac{x=0}{x_2}\right)$$

$$+ P(x_3)\left(x = \frac{0}{x_3}\right)$$

$$= \frac{1}{6} + \frac{1}{12} + \frac{1}{24} = \frac{7}{24}$$

$$P(x=1) = \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{3}{8}$$

$$= \frac{11}{24}$$

$$P(x=2) = \frac{1}{3}\cdot 0 + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{3}{8}$$

$$= \frac{5}{24}$$

$$E(x) = 0\cdot\frac{1}{24} + 1\cdot\frac{11}{24} + 2\cdot\frac{5}{24} + 3\frac{1}{24}$$

$$= 1$$

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$P(x=2) = \frac{1}{3}\cdot 0 + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{3}{8}$

$= \frac{5}{24}$

$E(x) = 0\cdot\frac{1}{24} + 1\cdot\frac{11}{24} + 2\cdot\frac{5}{24} + \frac{3}{24}$

$= 1$

Ans- 3 (b)(ii)

for first three days will be fine

and rest 4 days no

$P = \left(\frac{2}{3}\right)$ as no rain

$\left(\frac{1}{3}\right)$ as rain

${}^{7}C_{3} \left(\frac{2}{3}\right)^{3} \left(\frac{1}{3}\right)^{4}$

$= 21 \left(\frac{2}{3}\right)^{3} \cdot \left(\frac{1}{3}\right)^{4}$

$= \frac{168}{2187}$