20/01/2021

$E(X) = 10, \quad var(X) = (25)$

(40)

10, 10, 20

$Y = aX - b$ $\quad \boxed{a, b = ?}$

$E(Y) = 0, \quad var(Y) = 1$

$E(Y) = E(aX - b)$

$= a\,E(X) - E(b)$

$0 = a \cdot 10 - b$

$\Rightarrow \boxed{10a - b = 0}$ — (1)

$25a^2 = 1$

$a^2 = 1/25$

$\boxed{a = 1/5}$

$b = 2$

$\therefore var(Y) = 1$

$\Rightarrow var(aX - b) = 1$

$\Rightarrow E[(aX-b)^2] - (E(aX-b))^2 = 1$

$\Rightarrow E(a^2X^2 + b^2 - 2aXb) - (aE(X) - b)^2 = 1$

$\Rightarrow a^2E(X^2) + b^2 - 2ab\,E(X) - a^2(E(X))^2 - b^2 + 2ab\,E(X) = 1$

$\Rightarrow a^2[E(X^2) - (E(X))^2] = 1$

$= a^2\, var(X) = 1 \Rightarrow a^2 \cdot 25 = 1$

500 solenoids → 25 all defective.

$P(0, 1, 2, 3)$ → lot of 20 solenoids.

$P(\text{defective solenoid}) = \frac{25}{500} = 0.05$

$\boxed{n = 20}$ ✓ $\boxed{p = 1/20 = 0.05}$ $q = 1 - p = 1 - 0.05$

$P(X=0) = P(\text{no defective}) = {}^{20}C_0\, p^0 q^{20}$ $\boxed{q = 0.95}$

$= (0.95)^{20}$

$P(X=1) = P(\text{one defective}) = {}^{20}C_1\, p^1 q^{19} = 20\,(0.5)^1 (0.95)^{19}$

$P(X=2) = P(\text{2 defective}) = {}^{20}C_2\, p^2 q^{18} = \frac{20 \cdot 19}{2 \cdot 1}(0.5)^2 (0.95)^{18}$

$P(X=3) = P(\text{3 defective}) = {}^{20}C_3\, p^3 q^{17} = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1}(0.5)^3 (0.95)^{17}$

POISSON DISTRIBUTION →

Limiting case of BINOMIAL distribution

where $n \to \infty$, $p \to 0$

$$P(X=r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

$\boxed{\lambda = np}$ = Mean of the distribution.

$\boxed{\text{Mean} = \text{Variance} = \lambda}$

$n = 2000$ people

$P(\text{disease}) = 0.002 = p$

$\lambda = np$

$= 2000 \times 0.002$

$\boxed{\lambda = 4}$

P(not more than 2 persons will be affected) = ??

$= P[X=1] + P[X=2] + P[X=0]$ ✓

$$= \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^0}{0!}$$

$$= e^{-4}\left[\frac{4}{1!} + \frac{4^2}{2!} + \frac{1}{0!}\right] = 13e^{-4}$$

Q2: A car hire firm has 2 cars.

no. of demands → 'X' $\boxed{\text{mean} = 1.5}$ λ

1. P(neither car is used) = ?
2. P(some demand is refused) = ?

1. $P(X = 0) = \dfrac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda} = e^{-1.5}$

2. P(demand is refused) = $P(X > 2)$

$$= 1 - P(X \leq 2)$$

$$= 1 - \left[P[X=0] + P[X=1] + P[X=2]\right]$$

$$= 1 - \left[\frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!}\right]$$