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PAS

Tut-3

Q-1

Is the function below a density f^n:-

$$f(x) = \begin{cases} 0 & \text{for } x < 2 \\ \frac{3+2x}{18} & \text{for } 2 \le x \le 4 \\ 0 & \text{for } x > 4 \end{cases}$$

For density f^n

$\int_{-\infty}^{\infty} f(x)\,dx$ = should equal to 1

✓ ① Integrable
✓ ② more than or equal to 0.
③ Integrable from 2 to 4.

$$\Rightarrow \int_2^4 \frac{3+2x}{18}\,dx$$

$$\Rightarrow \frac{1}{18}\left[3x + \frac{\cancel{2}x^2}{\cancel{2}}\right]_2^4$$

$$\Rightarrow \frac{1}{18}\left[12+16-6-4\right] \Rightarrow \frac{1}{18}\left[18\right]$$

$$= 1$$

Therefore, it is an integrable function.

→ ~~As it~~ It is a density function.

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② If $f(x) = \begin{cases} \dfrac{x+1}{2} , & -1 < x < 1 \\ 0 , & \text{elsewhere} \end{cases}$

Represents the density function of a Random Variable (X). Find E(X) and Var (X).

$\Rightarrow$ Mean $= E(x) = \displaystyle\int_{-\infty}^{\infty} x . f(x)\, dx$

Variance $= V(x) = E(x^2) - (E(x))^2$

$\Rightarrow E(x) = \displaystyle\int_{-\infty}^{\infty} x \left(\frac{x+1}{2} \right) dx$

$= \dfrac{1}{2} \left[x^2 + x \right]_{-1}^{1}$

$= \dfrac{1}{2} \left[(1^2 + 1) - ((-1)^2 + (-1)) \right]$

$= \dfrac{1}{2} \left[2 - (1 - 1) \right]$

$= 1$

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$$= \frac{1}{2}\int_{-1}^{1} (x^2 + x)\, dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{2} - \left(\frac{-1}{3} + \frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{3} + \cancel{\frac{1}{2}} + \frac{1}{3} - \cancel{\frac{1}{2}}\right] \Rightarrow \frac{1}{3}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{-1}^{1} x^2 \frac{(x+1)}{2}\, dx$$

$$= \frac{1}{2}\int_{-1}^{1} x^3 + x^2\, dx$$

$$= \frac{1}{2}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \frac{1}{3}$$

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$$\text{Variance} = \frac{1}{3} - \left(\frac{1}{3}\right)^2 = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$$

Q-3 The frequency distribution of a Measurable characteristic varying between 0 and 2 is as under:

$$f(x) = \begin{cases} x^3, & 0 \le x \le 1 \\ (2-x)^3 & 1 \le x \le 2 \end{cases}$$

Calculate S.D. and Mean Deviation.

$\Rightarrow$

$$\text{Mean Deviation about Mean} = \int_{-\infty}^{\infty} |x-\mu|\, f(x)\, dx$$

$$\text{Mean} = E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^2 x\, f(x)\, dx \Rightarrow \int_0^1 x(x^3)\, dx + \int_1^2 x(2-x)^3\, dx$$

$$\Rightarrow \int_0^1 x^4\, dx + \int_1^2 x.\left(8 - x^3 - 3.4.x + 3.2.x^2\right) dx$$

$$\Rightarrow \left[\frac{x^5}{5}\right]_0^1 + \int_1^2 8x - x^4 - 12x^2 + 6x^3\, dx$$

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96 $\begin{array}{r} 16 \\ \times 6 \\ \hline 96 \end{array}$

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$$\Rightarrow \left[\frac{x^5}{5}\right]_0^1 + \int_1^2 8x - x^4 - 12x^2 + 6x^3 \; dx$$

$$\Rightarrow \left[\frac{1}{5} - 0\right] + \left[8.\frac{x^2}{2} - \frac{x^5}{5} - 12.\frac{x^3}{3} + 6\frac{x^4}{4}\right]_1^2$$

$$\Rightarrow \frac{1}{5} + \left[\frac{32}{2} - \frac{32}{5} - \frac{96}{3} + \frac{96}{4} - \frac{8}{2} + \frac{1}{5} + \frac{12}{3} - \frac{6}{4}\right]$$

$$\Rightarrow \frac{1}{5} + 0.3 = 0.5 \Rightarrow \frac{1}{2}$$

Mean deviation about Mean:-

$$= \int_0^2 |x - 0.5| \, f(x) \, dx$$

$$= \int_0^1 |x - 0.5| \, x^3 \, dx + \int_1^2 |x - 0.5| (2-x)^3 \, dx$$

$$\Rightarrow \int_0^{1/2} (0.5 - x) x^3 \, dx + \int_{1/2}^1 x^3 (x - 0.5) \, dx + \int_1^2 (x - 0.5)(2-x)^3 \, dx$$

$$\Rightarrow \int_0^{1/2} \frac{1}{2}.x^3 - x^4 \; dx + \int_{1/2}^1 x^4 - \frac{x^3}{2} \; dx + \int_1^2 (x-0.5)(2-x)^3 dx$$

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$\Rightarrow$ $0.0015625 + 0.0765625 + 0.175$

$\Rightarrow$ 0.253125

Ans-4 $f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$

Find $E(x)$

$$E(x) = \int_{-\infty}^{\infty} x_i f(x)\, dx = \int_0^1 x(3x^2)$$

$$\int_0^2 3x^3 \Rightarrow 3 \times \frac{x^4}{4} = \frac{3}{4} \int_0^1 x^4$$

$$\boxed{E(x) = \frac{3}{4}}$$

$$E(x^4) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$\int_0^1 x^2(3x^2)\, dx = 3 \times x^4 = \frac{3x^5}{5}$$

$$= \frac{3}{5}(1-0)$$

$$\boxed{E(x^2) = \frac{3}{5}}$$

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$E(3x-2)$

$3E(x)-2$

$\Rightarrow 3\times\frac{3}{4}-2$

$\Rightarrow \frac{9}{4}-2 \Rightarrow \boxed{\frac{1}{4}}$

Ans-5 $f(x) = y_0 e^{-|x|}, \quad -\infty < x < +\infty$

$\int_{-\infty}^{\infty} x\, f(x)\, dx = 1$ It is a probability density function.

$\int_{-\infty}^{\infty} y_0 x e^{-x}\, dx = 1$

$y_0 \int_{-\infty}^{\infty} x e^{-x}\, dx = 1$

Integrate By Parts:-

$\Rightarrow y_0 \int_{-\infty}^{\infty} x . -e^{-x} - [-e^{-x}]$

$\Rightarrow y_0 \int_{-\infty}^{\infty} -x e^{-x} + e^{-x}$

$y_0 [1+1] = 1 \Rightarrow \boxed{y_0 = \frac{1}{2}}$