04/02/21 **NORMAL DISTRIBUTION**

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty$$

$$f'(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \cdot \left(-\frac{1}{2}\cdot 2\left(\frac{x-\mu}{\sigma}\right)\frac{1}{\sigma}\right) \Rightarrow \boxed{f'(x) = 0} \text{ to get stationary points}$$

$$\boxed{f''(x) < 0}$$

$$\Rightarrow f'(x) = \frac{-1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \left(\frac{x-\mu}{\sigma^2}\right)$$

$$f'(x) = \frac{-1}{\sqrt{2\pi}\,\sigma^3}(x-\mu)\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\Rightarrow \boxed{f'(x) = 0} \Rightarrow \boxed{x = \mu}$$

$$\Rightarrow f''(x) = \frac{-1}{\sqrt{2\pi}\,\sigma^3}\left[e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \cdot 1 + (x-\mu)\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\left(-\frac{1}{2}\, 2\left(\frac{x-\mu}{\sigma}\right)\frac{1}{\sigma}\right)\right]$$

$$\Rightarrow f''(x) = \frac{-1}{\sqrt{2\pi}\,\sigma^3} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\left[1 - \left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$\because f''(x) < 0$ at $x = \mu$

$\therefore f(x)$ has MAXIMUM value at $x = \mu$

$\therefore \boxed{\text{MODE} = \mu}$

(4) MEDIAN: if M is the Median

$$\therefore \underbrace{\int_{-\infty}^{M} f(x)\,dx} = \int_{M}^{\infty} f(x)\,dx = 1/2$$

mean = Median = Mode = μ

Consider;

$$\int_{-\infty}^{M} f(x)\,dx = 1/2$$

$$\Rightarrow \int_{-\infty}^{M} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{1}{2}$$

$$\Rightarrow \boxed{\int_{-\infty}^{\mu} f(x)\,dx} + \int_{\mu}^{M} f(x)\,dx = 1/2$$

Consider $\int_{-\infty}^{\mu} f_X(x)\,dx = \int_{-\infty}^{\mu} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$

put $\frac{x-\mu}{\sqrt{2}(\sigma)} = t$

$\Rightarrow \boxed{dx = \sqrt{2}\,\sigma\,dt}$

$$= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-t^2} (\sigma\sqrt{2}\,dt)$$

$$= \frac{1(\sigma\sqrt{2})}{\sigma\sqrt{2\pi}} \int_{-\infty}^{0} e^{-t^2} dt$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{0} e^{-t^2} dt = \frac{1}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2} dt$$

$$= \frac{1}{\sqrt{\pi}} \left(\sqrt{\pi}/2\right) = 1/2$$

$$\therefore \int_{-\infty}^{M} f(x)\,dx = \int_{-\infty}^{\mu} f(x)\,dx + \int_{\mu}^{M} f(x)\,dx$$

$$\Rightarrow 1/2 = 1/2 + \int_{\mu}^{M} f(x)\,dx$$

$$\therefore \int_{\mu}^{M} f_X(x)\,dx = 0 \Rightarrow \boxed{M = \mu}$$

$\therefore$ Median $= M$.

VARIANCE & STANDARD DEVIATION

$$\boxed{Var(X) = E(X^2) - (E(X))^2}$$

$$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

$$= \int_{-\infty}^{\infty} (x^2 + \mu^2 - 2x\mu) f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx + \mu^2 \int_{-\infty}^{\infty} f(x)\,dx - 2\mu \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= E(X^2) + \mu^2 \cdot 1 - 2\mu\, E(X)$$

$$= E(X^2) + (E(X))^2 - 2(E(X))^2$$

$$= E(X^2) - (E(X))^2$$

$$\therefore \quad Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

put $\boxed{\frac{x-\mu}{\sigma\sqrt{2}} = t}$

$\Rightarrow dx = \sigma\sqrt{2}\, dt$

$$\therefore \quad Var(X) = \int_{-\infty}^{\infty} 2t^2\sigma^2 \frac{1}{\sqrt{2\pi}\,\sigma} e^{-t^2} \sigma\sqrt{2}\, dt$$

$$= \cancel{4}\ \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t^2 e^{-t^2} dt$$

$$= \frac{4\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} t^2 e^{-t^2} dt$$

$t^2 = z$

$Var(X) = \sigma^2$ ✓

$S.D = \sqrt{\sigma^2}$

$= \sigma$ ✓

05/02

$$Var(x) = \frac{4\sigma^2}{\sqrt{\pi}} \int_0^\infty t^2 e^{-t^2} dt ,$$

$$= \frac{4\sigma^2}{\sqrt{\pi}} \int_0^\infty z e^{-z} \frac{1}{2} z^{-1/2} dz$$

$$= \frac{4\sigma^2}{2\sqrt{\pi}} \int_0^\infty z^{1/2} e^{-z} dz$$

$$= \frac{4\sigma^2}{2\sqrt{\pi}} \int_0^\infty z^{3/2-1} e^{-z} dz$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \left[\Gamma(3/2) \right] = \frac{2\sigma^2}{\sqrt{\pi}} \frac{1}{2}\sqrt{\pi}$$

$$= \sigma^2$$

$$Var(x) = \sigma^2 \Rightarrow \boxed{S.D = \sigma}$$

$t^2 = z$

$2t\,dt = dz$

$dt = \frac{dz}{2t}$

$= \frac{dz}{2z^{1/2}}$

$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$

$\Gamma(n+1) = n\Gamma(n)$

$\Gamma 3/2 = 1/2\, \Gamma(1/2)$

$= \frac{1}{2}\sqrt{\pi}$

$X \sim N(\mu, \sigma^2)$

$X \rightarrow$ height ✓ CSE4

μ, σ^2

$P(176 \le X \le 184) = ?$

$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$

Standard Normal

$Z = \frac{X-\mu}{\sigma}$ ✓

Ques: $X \sim N(\mu, \sigma^2)$ $\boxed{Z = \frac{X-\mu}{\sigma}}$ ✓

Mean = 30, S.D = 5

$\boxed{\begin{array}{l} |x| < a \\ -a < x < a \end{array}}$

$P[|X-24|) < 8]$

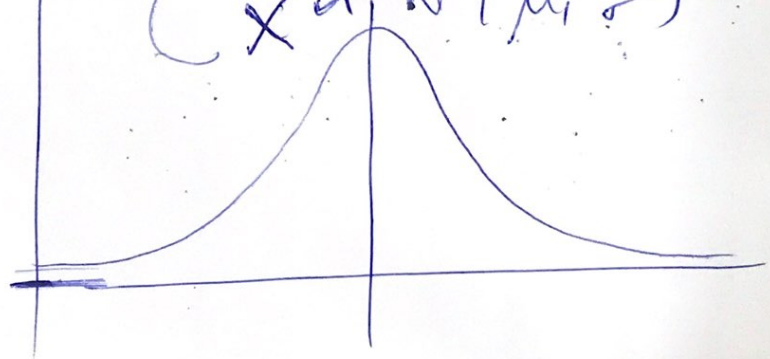
$\Rightarrow P(-8 < X-24 < 8)$

$\Rightarrow P(24-8 < X < 24+8)$

$\Rightarrow P(16 < X < 32)$

$Z \sim N(0,1)$

$X \sim N(\mu, \sigma^2)$



$= P(16 < X < 32)$

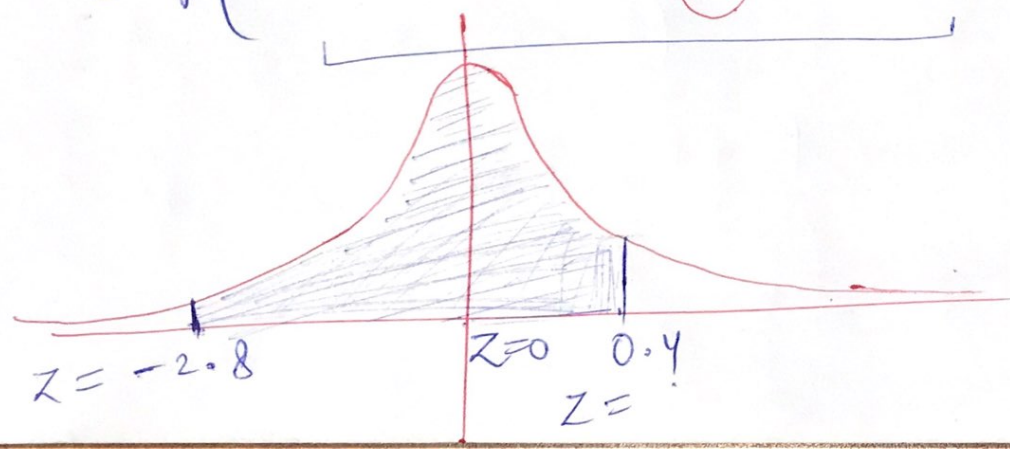
$= P(16-\mu < X-\mu < 32-\mu)$

$= P\left(\frac{16-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{32-\mu}{\sigma}\right)$

$= P\left(\frac{16-30}{5} < Z < \frac{32-30}{5}\right)$

$= P\left(-\frac{14}{5} < Z < \frac{2}{5}\right)$

$= P(-2.8 < Z < 0.4)$



$= P(-2.8 < Z < 0) + P(0 < Z < 0.4)$

$= P(0 < Z < 2.8) + P(0 < Z < 0.4)$

$= 0.4974 + 0.1554$

$= 0.6528$ ✓

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X ~ Marks of the students

$X \sim N(\mu, \sigma^2)$

$X \sim N(60, 5^2)$

$P(X > 60) = ?$

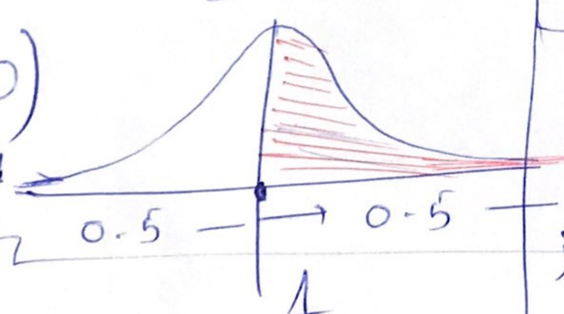
$P(X - \mu > 60 - \mu)$

$= P\left(\frac{X-\mu}{\sigma} > \frac{60-\mu}{\sigma}\right)$

$= P\left(\frac{X-60}{5} > \frac{60-60}{5}\right)$

$= P(Z > 0)$

$= 0.5$



Mean

Median

Mode

~~1000~~

50%

$\sigma =$

$\bar{X} = 15000$ | 3000
A | B
n_1 12 | n_2 15
25000 | 15000 K
20 K | 16000 K
14 K | 15500 K
— | 16 K
40 K | $\bar{X} = 15500$
35 K |
12000 |
8000 |

A | B
15 Lacs | 13 lacs
35 L | 10 ± 20
10 ± 15 |
5 ± 10 L | 10 – 20

3000

S.D: 10 hrs

$\bar{X} = 130$ hrs

$\bar{X} = 140$ hrs, 25 hrs