15/01/2021

Mean Deviation

about Mean $\qquad |x-\bar{x}|\, f(x)$

$$= \int_a^b |x-\bar{x}|\, f(x)\,dx$$

Ques.

Given $\boxed{f_X(x) = kx(2-x)}$, $0 \le x \le 2$

(1) k (2) Mean (3) Variance (4) M.D about Mean

$\because f_X(x)$ is a pdf $\therefore \int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$$\Rightarrow \int_0^2 k(x)(2-x)\,dx = 1 \Rightarrow k = 3/4$$

$$\text{Mean} = E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\,dx = \int_0^2 x\, f_X(x)\,dx$$

Mean $= \int_0^2 x\, \frac{3}{4} x(2-x)\,dx = 1$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_0^2 x^2\, \cancel{k}x(2-x)\,dx$$

$$= \frac{3}{4}\int_0^2 x^3(2-x)\,dx = \frac{3}{4}\int_0^2 2x^3 - x^4\,dx$$

$$= \frac{3}{4}\left(\frac{2x^4}{4} - \frac{x^5}{5}\right)_0^2 = \frac{6}{5}$$

$$\therefore \ Var(X) = E(X^2) - (E(X))^2 = \frac{6}{5} - (1)^2 = \frac{1}{5}$$

<u>Mean Deviation about Mean</u>: $\int_{-\infty}^{\infty} |x-\bar{x}|\, f_X(x)\,dx$

$$= \int_0^2 |x-1|\, f_X(x)\,dx =$$

$$= \int_0^2 |x-1|\, \frac{3}{4}\, x(2-x)\,dx$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \checkmark \\ -x & \text{if } x < 0 \checkmark \end{cases}$$

$$|x-1| = \begin{cases} x-1 & \text{if } x-1 \geq 0 \\ -(x-1) & \text{if } x-1 < 0 \end{cases} = \begin{cases} x-1 & \text{if } x \geq 1 \\ 1-x & \text{if } x < 1 \end{cases}$$

$\therefore$ M.D about Mean $= \int_0^2 |x-1| \, \frac{3}{4} x(2-x)\,dx$

$$= \int_0^1 (1-x)\frac{3}{4} x(2-x)\,dx + \int_1^2 (x-1)\, \frac{3}{4} x(2-x)\,dx$$

$$= \frac{3}{8} \checkmark$$

$$f(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$E(X)$, $E(3X-2)$, $E(X^2)$

$E(X) = \cancel{\tfrac{1}{4}}$

$\tfrac{3}{4}$

$E(3X-2)$

$= E(3X) - E(2)$

$= 3\,E(X) - 2$

$= \cancel{1}$

$= 3 \cdot \tfrac{3}{4} - 2$

$= \tfrac{1}{4}$

$E(X^2)$

$= \int_0^1 x^2 \cdot 3x^2 \, dx$

$= 3\int_0^1 x^4 \, dx$

$= \tfrac{3}{5}$

$E(X) = \int_0^1 x f_X(x) \, dx$

$= \int_0^1 x(3x^2) \, dx$

$= 3\int_0^1 x^3 \, dx$

$= 3 \left(x^4/4 \right) \Big|_0^1$

$= \tfrac{3}{4}$