1

1 (a): $E(X) = 10$, $VAR(X) = 25$

$\boxed{Y = aX + b}$ $E(Y) = 0$, $Var(Y) = 1$

$E(Y) = E(aX) + E(b)$

$\Rightarrow E(Y) = a\,E(X) + b$

$\Rightarrow 0 = a \cdot 10 + b$

$\Rightarrow \boxed{10a + b = 0}$

$Var(Y) = 1$

$\Rightarrow Var(aX + b) = 1$

$\Rightarrow a^2\, Var(X) = 1$

$\Rightarrow a^2 \cdot 25 = 1$

$\Rightarrow a^2 = 1/25$

$a = \pm 1/5$ (discarding $a = -1/5$)

if $a = -1/5$; $b = 2$

$\because \boxed{a = 1/5}$

$10(1/5) + b = 0$

$\boxed{b = -2}$

(b) $\because f(x)$ is a pdf $\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_2^4 f(x)\,dx + \int_4^6 f(x)\,dx = 1$

$\Rightarrow \int_2^4 2k\,dx + \int_4^6 (-kx + 6k)\,dx = 1$

$\Rightarrow 2k(x)\Big|_2^4 + (-k)\left(\frac{x^2}{2}\right)\Big|_4^6 + 6k(x)\Big|_4^6 = 1$

$\Rightarrow 2k(2) + (-k)/_2\,(6^2 - 4^2) + 6k \cdot 2 = 1$

$\Rightarrow 4k - \frac{k}{2} \cdot 20 + 12k = 1$

$\Rightarrow 16k - 10k = 1 \Rightarrow 6k = 1 \Rightarrow \boxed{k = 1/6}$

(c) $\left.\begin{array}{l} \text{Mean} = np = 3 \\ \text{Variance} = npq = 4 \end{array}\right] \Rightarrow 3q = 4$

$\boxed{q = 4/3} > 1$

which is not possible.

$\therefore$ A binomial distribution cannot have mean 3 and Variance 4.

(d) Y: absolute difference of the upturned faces.

$Y = 0, 1, 2, 3, 4, 5$

$Y=0$: $\{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$Y=1$: $\{(1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (4,5), (5,4), (5,6), (6,5)\}$

$Y=2$: $\{(1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,6), (6,4)\}$

$\boxed{Y=3}$: $\{(1,4), (2,5), (3,6), (4,1), (5,2), (6,3)\}$

$Y=4$: $\{(1,5), (2,6), (5,1), (6,2)\}$

$Y=5$: $\{(1,6), (6,1)\}$

$P[Y=0] = 6/36$

$P(Y=1) = 10/36$

$P(Y=2) = 8/36$

$P(Y=3) = 6/36$

$P(Y=4) = 4/36$

$P(Y=5) = 2/36$

$\therefore$ $Y = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$P(Y) = \frac{6}{36} \quad \frac{10}{36} \quad 8/36 \quad 6/36 \quad 4/36 \quad 2/36$

$$E(Y) = \frac{0}{36} + \frac{10}{36} + \frac{16}{36} + \frac{18}{36} + \frac{16}{36} + \frac{10}{36}$$

$$= \frac{70}{36} = 35/18$$

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1 e Given $P(A) = 0$; $P(A \cap B) = 0$

$\because A \cap B \subseteq A$

$\Rightarrow P(A \cap B) \leq P(A)$

$\Rightarrow P(A \cap B) \leq 0$

$\Rightarrow \boxed{P(A \cap B) = 0}$ Proved.

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2 a :
$$f_X(x) = \frac{100 - x}{5000} \qquad 0 \leq x \leq 100$$

(i) Probability that a small bomb will disrupt the traffic

$= P(|X| \leq 15)$

$= P(-15 \leq X \leq 15)$

$= P(0 \leq X \leq 15)$

$$= \int_0^{15} f_X(x)\, dx$$

$$= \int_0^{15} \left(\frac{100 - x}{5000}\right) dx$$

$(\because 0 \leq x \leq 100)$

$$= \frac{1}{5000}\left(1500 - \frac{225}{2}\right) = \frac{2775}{10000} = 111/400$$

Ans.

(ii) The probability for a small bomb not to disrupt the traffic (or not to fall within 15 feet of the track $= 1 - 111/400 = 289/400$

$\therefore$ probability that the traffic will be disrupted when the plane uses all the eight bombs,

$$= 1 - \left(\frac{289}{400}\right)^8$$

2 (B): Let X: no of Heads

$\therefore X = 0, 1, 2, 3$

$$\therefore E(X) = 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2 \cdot P(X=2) + 3 \cdot P(X=3) \quad \text{— (1)}$$

Also; $\boxed{P(B_1) = P(B_2) = P(B_3) = 1/3}$

$\Omega_1 = \{H, T\}, \quad \Omega_2 = \{HH, HT, TH, TT\}$

$\Omega_3 = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$$\therefore P(X=0) = P(B_1) \cdot P(X=0 \mid B_1) + P(B_2) \cdot P(X=0 \mid B_2) + P(B_3) \cdot P(X=0 \mid B_3)$$

$$= 1/3 \cdot 1/2 + 1/3 \cdot 1/4 + 1/3 \cdot 1/8 = 7/24$$

$$P(X=1) = P(B_1) \cdot P(X=1 \mid B_1) + P(B_2) P(X=1 \mid B_2) + P(B_3) \cdot P(X=1 \mid B_3)$$

$$= 1/3 \cdot 1/2 + 1/3 \cdot 2/4 + 1/3 \cdot 3/8 = \frac{11}{24}$$

$$P(X=2) = P(B_1)\cdot P(X=2|B_1) + P(B_2)\, P(X=2|B_2)$$
$$+ P(B_3)\cdot P(X=2|B_3)$$
$$= \tfrac{1}{3}\left[\tfrac{0}{2} + \tfrac{1}{4} + \tfrac{3}{8}\right) = \tfrac{5}{24}$$

$$P(X=3) = P(B_1)\cdot P(X=3|B_1) + P(B_2)\cdot P(X=3|B_2)$$
$$+ P(B_3)\cdot P(X=3|B_3)$$
$$= \tfrac{1}{3}\left(0 + 0 + \tfrac{1}{8}\right) = \tfrac{1}{24}$$

$$\therefore E(X) = 0\cdot \tfrac{7}{24} + 1\cdot \tfrac{11}{24} + 2\cdot \tfrac{5}{24}$$
$$+ 3\cdot \tfrac{1}{24} = 1$$

3(a): $\cancel{\lambda_1\ \lambda = 1/50}$ (flaws on an average per square foot)

$$\lambda_1 = 1/50$$

Flaws per 32 square feet sheet

$$= 32 \times 1/50 = 0.64$$

$$\therefore \boxed{\lambda = 0.64}$$

$$\therefore P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-0.64}\cdot 1}{1} = 0.527$$

$$P(X \le 1) = P(X=0) + P(X=1)$$

$$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!}$$

$$= e^{-\lambda} + e^{-\lambda}\cdot\lambda$$

$$= e^{-\lambda}(1+\lambda)$$

$$= e^{-0.64}(1+0.64)$$

$$= 0.865 \checkmark$$

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3 (b): Probability of rain fall $= 10/30$

$\boxed{q = 2/3}$ $\boxed{n = 7}$ $\boxed{p = 1/3}$ ✓

(i) Probability (rain will fall at least 3 days of a given week)

$P(X \geq 3)$

$$= P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7)$$

OR

$$1 - P(X < 3)$$

$$= 1 - \left[P(X=0) + P(X=1) + P(X=2) \cancel{+ P(X=3)}\right]$$

$$= 1 - \left[{}^7C_0\, p^0 q^7 + {}^7C_1\, p^1 q^6 + {}^7C_2\, p^2 q^5 \right]$$

$$= 1 - \left[q^7 + 7 p^1 q^6 + \frac{7 \cdot 6}{2 \cdot 1} p^2 q^5 \right]$$

$$= 1 - \left[\left(\frac{2}{3}\right)^7 + 7 \frac{1}{3} \left(\frac{2}{3}\right)^6 + 21 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^5 \right]$$

$$= 1 - \left(\frac{2}{3}\right)^5 \left[\left(\frac{2}{3}\right)^2 + \frac{7}{3} \left(\frac{2}{3}\right)^1 + \frac{21}{9} \right]$$

$$= 1 - \frac{32}{243} \left[\frac{4}{9} + \frac{14}{9} + \frac{21}{9} \right]$$

$$= 1 - \frac{32 \times 39}{9 \times 243} = 1 - \frac{1248}{2187}$$

$$= \frac{939}{2187} = 0.429$$

(ii) P(first three days fine & remaing 4 days wet)

$$= \cancel{P(X=} \; P(qqqpppp)$$

$$= \frac{2}{3} \frac{2}{3} \frac{2}{3} \left(\frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \right)$$

$$= \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^4 = \frac{8}{3^7}$$

$$= \frac{8}{2187} = 0.0037$$