15. The probability that a man aged 50 years will die within a year is 0.01125. What is the probability that out of 12 such men, at least 11 will reach their fifty first birthday?

(Given $e^{-0.135} = 0.87371$)

Answers

1. (*i*) 0.1804 (*ii*) 0.3235 (*iii*) 0.1353 (*iv*) 0.5941

2. 1 **3.** (*i*) 2 (*ii*) $\dfrac{2}{3e^2}$ **4.** (*i*) e^{-4} (*ii*) $4e^{-4}$ **5.** $\dfrac{(10)^{15}\,e^{-10}}{(15)!} = 0.035$

6. 0.3235 **7.** $320 \times \dfrac{e^{0.503}\,(9.503)^r}{r!}$

8. (*a*) $P(r) = 121.36 \times \dfrac{(0.5)^{r!}}{r!}$, where $r = 0, 1, 2, 3, 4$

Theoretical frequencies are 121, 61, 15, 3, 0 respectively

(*b*) $P(r) = 43.964 \times \dfrac{(0.97)^r}{r!}$, where $r = 0, 1, 2, 3, 4$

Theoretical frequencies are 44, 43, 14, 3, 1 respectively

9. Theoretical frequencies are 109, 142, 92, 40, 13, 3, 1, 0, 0, 0, 0

11. (*a*) 9802, 196, 2 (*b*) 19604, 392, 4, 0 **12.** 0.01936 **13.** 0.4795

14. (*i*) 0.1784 (*ii*) 0.7150 **15.** 0.99166.

NORMAL DISTRIBUTION

5.26. NORMAL DISTRIBUTION

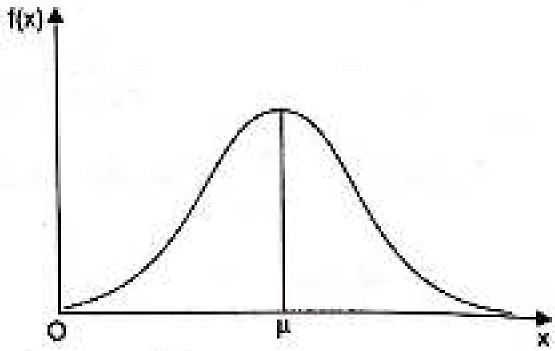
(*K.U.K. 2009, Dec. 2015*)

The normal distribution is a continuous distribution. It can be derived from the Binomial distribution in the limiting case when n, the number of trials is very large and p, the probability of a success, is close to $\frac{1}{2}$. The general equation of the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where the variable x can assume all values from $-\infty$ to $+\infty$. μ and σ, called the parameters of the distribution, are respectively the mean and the standard deviation of the distribution and $-\infty < \mu < \infty$, $\sigma > 0$. x is called the normal variate and $f(x)$ is called probability density function of the normal distribution.

If a variable x has the normal distribution with mean μ and standard deviation σ, we briefly write x: $N(\mu, \sigma^2)$.



The graph of the normal distribution is called the *normal curve*. It is bell-shapped and symmetrical about the mean μ. The two tails of the curve extend to $+\infty$ and $-\infty$ towards the positive and negative directions of the x-axis respectively and gradually approach the x-axis without ever metting it. The curve is unimodal and the mode of the normal distribution coincides with its mean μ. The line $x = \mu$ divides the area under the normal curve above x-axis into two equal parts. Thus, the median of the distribution also coincides with its mean and mode. The area under the normal curve between any two given ordinates $x = x_1$ and $x = x_2$ represents the probability of values falling into the given interval. The total area under the normal curve above the x-axis is 1.

5.27. BASIC PROPERTIES OF THE NORMAL DISTRIBUTION

The probability density function of the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

1. ***The total area under normal probability curve is unity.***

Normal probability curve is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Area under this curve

$$= \int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

Put $\quad \dfrac{x-\mu}{\sigma\sqrt{2}} = t, \quad dx = \sigma\sqrt{2}\,dt$

Required area $= \dfrac{1}{\sigma\sqrt{2\pi}} \displaystyle\int_{-\infty}^{\infty} e^{-t^2} \sigma\sqrt{2}\,dt = \dfrac{1}{\sqrt{\pi}} \displaystyle\int_{-\infty}^{\infty} e^{-t^2}\,dt = \dfrac{1}{\sqrt{\pi}}\sqrt{\pi} = 1.$

2. ***The mean of the normal distribution.***

The general form of the normal curve is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Mean $= \displaystyle\int_{-\infty}^{\infty} x f(x)\,dx$ [See Art. 5.18(*b*)]

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

Putting $\dfrac{x-\mu}{\sigma\sqrt{2}} = t, \; dx = \sigma\sqrt{2}\,dt$

$\therefore$ Mean $= \dfrac{1}{\sigma\sqrt{2\pi}} \displaystyle\int_{-\infty}^{\infty} (\mu + \sigma\sqrt{2}t) e^{-t^2} \sigma\sqrt{2}\,dt$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} (\mu + \sigma\sqrt{2}t) e^{-t^2} dt = \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} dt + \frac{\sigma\sqrt{2}}{\sqrt{\pi}} \int_{-\infty}^{\infty} te^{-t^2} dt$$

$$= \frac{\mu}{\sqrt{\pi}} \sqrt{\pi} + \frac{\sigma\sqrt{2}}{\sqrt{\pi}} (0) = \mu \qquad | \because \ te^{-t^2} \text{ is an odd function of } t$$

3. *For a normal curve, the ordinate at the mean is the maximum ordinate.*

The equation of the normal curve is

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad \text{Mean} = \mu$$

$$\frac{dy}{dx} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \cdot \frac{-2(x-\mu)}{2\sigma^2} = -\frac{(x-\mu)}{\sigma^3\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{\sigma^3\sqrt{2\pi}} \left[1 . e^{-\frac{(x-\mu)^2}{2\sigma^2}} + (x-\mu) . e^{-\frac{(x-\mu)^2}{2\sigma^2}} . \frac{-2(x-\mu)}{2\sigma^2}\right]$$

$$= -\frac{1}{\sigma^3\sqrt{2\pi}} . e^{-\frac{(x-\mu)^2}{2\sigma^2}} \left[1 - \frac{(x-\mu)^2}{\sigma^2}\right]$$

Now $\qquad \frac{dy}{dx} = 0 \quad \text{when } x = \mu$

and $\qquad \left[\frac{d^2y}{dx^2}\right]_{x=\mu} = -\frac{1}{\sigma^3\sqrt{2\pi}} < 0.$

Hence y, the ordinate is maximum when $x = \mu$ *i.e.*, the ordinate at the mean is the maximum ordinate.

4. *The mode of the normal distribution.*

The equation of the normal curve is

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Mode is the value of x corresponding to $y = y_0$, where y_0 is the maximum frequency.

Proceeding as in Example 3, y is maximum when $x = \mu$.

Hence **the mode = the mean** $= \mu$.

5. *The median of the normal distribution.*

If M is the median of the normal distribution, we have

$$\int_{-\infty}^{M} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \frac{1}{2}$$

$$\Rightarrow \qquad \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{M} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx = \frac{1}{2}$$

$$\Rightarrow \qquad \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\mu} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx + \frac{1}{\sigma\sqrt{2\pi}}\int_{\mu}^{M} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx = \frac{1}{2} \qquad \ldots(1)$$

Now $\quad \dfrac{1}{\sigma\sqrt{2\pi}}\displaystyle\int_{-\infty}^{\mu} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$

$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{\infty}^{0} e^{-t^2}(-\sigma\sqrt{2})\,dt, \quad \text{where} \quad -\frac{x-\mu}{\sigma\sqrt{2}} = t$$

$$= \frac{1}{\sqrt{\pi}}\int_{0}^{\infty} e^{-t^2}\,dt = \frac{1}{\sqrt{\pi}}\cdot\frac{1}{2}\sqrt{\pi} = \frac{1}{2}$$

$\therefore$ From (1)

$$\frac{1}{2} + \frac{1}{\sigma\sqrt{2\pi}}\int_{\mu}^{M} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx = \frac{1}{2}$$

$$\Rightarrow \int_{m}^{M} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx = 0 \quad \Rightarrow \quad M = \mu$$

Hence, for the normal distribution, mean, median and mode coincide.

6. *The variance and standard deviation of a normal distribution.*

The equation of the normal curve is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \text{ Mean} = \mu$$

$$\text{Variance} = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} (x-\mu)^2 . e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

Putting $\quad \dfrac{x-\mu}{\sigma\sqrt{2}} = t, \quad dx = \sigma\sqrt{2}\,dt$

$$\text{Variance} = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} 2\sigma^2 t^2 . e^{-t^2} . \sigma\sqrt{2}\,dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}}\int_{-\infty}^{\infty} t^2 e^{-t^2}\,dt = \frac{4\sigma^2}{\sqrt{\pi}}\int_{0}^{\infty} t^2 e^{-t^2}\,dt$$

Putting $t^2 = z$, $2t\,dt = dz$ or $dt = \dfrac{dz}{2\sqrt{z}}$

$$\therefore \qquad \text{Variance} = \frac{4\sigma^2}{\sqrt{\pi}}\int_{0}^{\infty} z e^{-z} . \frac{dz}{2\sqrt{z}} = \frac{2\sigma^2}{\sqrt{\pi}}\int_{0}^{\infty} z^{1/2} e^{-z}\,dz$$

$$= \frac{2\sigma^2}{\sqrt{\pi}}\int_{0}^{\infty} z^{3/2-1} e^{-z}\,dz = \frac{2\sigma^2}{\sqrt{\pi}}\Gamma\left(\tfrac{3}{2}\right) = \frac{2\sigma^2}{\sqrt{\pi}}.\frac{1}{2}\sqrt{\pi} = \sigma^2.$$

Standard deviation $= \sqrt{\text{Variance}} = \sigma.$

7. *The points of inflexion of the normal curve.*

Let the equation of the normal curve be

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Taking logarithms

$$\log y = \log \frac{1}{\sigma\sqrt{2\pi}} - \frac{(x-\mu)^2}{\sigma^2}$$

Differentiating w.r.t. x

$$\frac{1}{y} \cdot \frac{dy}{dx} = -\frac{x-\mu}{\sigma^2}$$

Differentiating again

$$\frac{1}{y} \cdot \frac{d^2y}{dx^2} - \frac{1}{y^2} \cdot \left(\frac{dy}{dx}\right)^2 = -\frac{1}{\sigma^2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{y} \cdot \left(\frac{dy}{dx}\right)^2 - \frac{y}{\sigma^2} = \frac{1}{y} \cdot \frac{y^2(x-\mu)^2}{\sigma^4} - \frac{y}{\sigma^2}$$

$$= \frac{y}{\sigma^4}\,[(x-\mu)^2 - \sigma^2]$$

At a point of inflexion,

$$\frac{d^2y}{dx^2} = 0 \qquad \left(\text{and } \frac{d^3y}{dx^3} \neq 0 \text{ which can be shown}\right)$$

$$\therefore \qquad (x-\mu)^2 = \sigma^2$$

or $$x - \mu = \pm\,\sigma \quad \text{or} \quad x = \mu \pm \sigma.$$

Thus, the curve has two points of inflexion, one at $\mu - \sigma$ and the other at $\mu + \sigma$, *i.e.*, at a distance from the mean, equal to the standard deviation.

8. *The mean deviation from the mean of the normal distribution is about $\frac{4}{5}$ of its standard deviation.* (M.D.U. 2011)

Let the equation of the normal curve be

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Standard deviation $= \sigma$.

Mean deviation from the mean

$$= \int_{-\infty}^{\infty} y\,|x-\mu|\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} |x-\mu| \,.\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$

$$= \frac{\sigma^2}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} |z|\, e^{-\frac{1}{2}z^2}\, dz, \text{ where } z = \frac{x-\mu}{\sigma}$$

$$= \frac{\sigma}{\sqrt{2\pi}} \left[\int_{-\infty}^{0} -z \,.\, e^{-\frac{1}{2}z^2}\, dz + \int_{0}^{\infty} z e^{-\frac{1}{2}z^2}\, dz \right] \qquad \left|\because\ |z| = \begin{cases} -z & \text{if } z<0 \\ z & \text{if } z \geq 0 \end{cases}\right.$$

$$= \frac{\sigma}{\sqrt{2\pi}} \left[-\int_{\infty}^{0} t e^{-\frac{1}{2}t^2}\, dt + \int_{0}^{\infty} z\, e^{-\frac{1}{2}z^2}\, dz \right]$$

where $t = -z$

$$= \frac{\sigma}{\sqrt{2\pi}} \left[\int_{0}^{\infty} z e^{-\frac{1}{2}z^2}\, dz + \int_{0}^{\infty} z\, e^{-\frac{1}{2}z^2}\, dz \right]$$

$$\left(\because\ \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \text{ and } \int_a^b f(x)\,dx = \int_a^b f(z)\,dz\right)$$

$$= \sigma \sqrt{\frac{2}{\pi}} \int_0^{\infty} z e^{-\frac{1}{2}z^2}\, dz = \sigma \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{1}{2} e^{-\frac{1}{2}t}\, dt, \text{ where } t = z^2$$

$$= \sigma \sqrt{\frac{2}{\pi}} \left[-e^{-\frac{1}{2}t} \right]_0^{\infty} = -\sigma \sqrt{\frac{2}{\pi}}\, [0-1] = \sqrt{\frac{2}{\pi}} \,.\, \sigma = 0.7979\sigma = \tfrac{4}{5}\sigma \text{ (approx.)}$$

$$= \frac{4}{5} \times \text{standard deviation (approx.)}.$$

9. *Standard deviation for a normal distribution is approximately 25% more than the mean deviation.*

(K.U.K. Dec. 2015)

Standard deviation – Mean deviation $= \sigma - \dfrac{4}{5}\sigma$ (approx.)

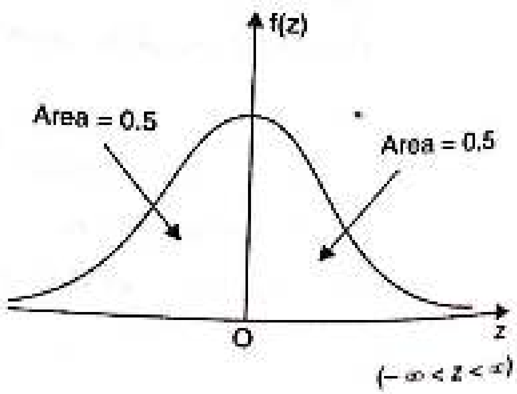
$$= \frac{1}{5}\sigma = \frac{1}{4}\left(\frac{4}{5}\sigma\right) = \frac{1}{4} \text{ (Mean deviation)}$$

= 25% of Mean Deviation.

5.28. STANDARD FORM OF THE NORMAL DISTRIBUTION

If X is a normal random variable with mean μ and standard deviation σ, then the random variable $Z = \dfrac{X-\mu}{\sigma}$ has the normal distribution with mean 0 and standard deviation 1. The random variable Z is called the *standardized* (or *standard*) normal random variable.

The probability density function for the normal distribution in standard form is given by



$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

It is free from any parameter. This helps us to compute areas under the normal probability curve by making use of standard tables.

Note 1. If $f(z)$ is the probability density function for the normal distribution, then

$$P(z_1 \le Z \le z_2) = \int_{z_1}^{z_2} f(z)\, dz = F(z_2) - F(z_1), \quad \text{where} \quad F(z) = \int_{-\infty}^{z} f(z)\, dz = P(Z \le z)$$

The function $F(z)$ defined above is called the *distribution function* for the normal distribution.

Note 2. The probabilities $P(z_1 \le Z \le z_2)$, $P(z_1 < Z \le z_2)$, $P(z_1 \le Z < z_2)$ and $P(z_1 < Z < z_2)$ are all regarded to be the same.

Note 3. $F(-z_1) = 1 - F(z_1)$.

ILLUSTRATIVE EXAMPLES

Example 1. *A sample of 100 dry battery cells tested to find the length of life produced the following results:*

$$\bar{x} = 12 \text{ hours}, \sigma = 3 \text{ hours}.$$

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life

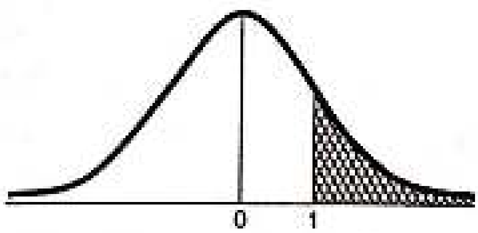
(*i*) *more than 15 hours* (*ii*) *less than 6 hours*

(*iii*) *between 10 and 14 hours?* (Cochin 2011)

Sol. Here x denotes the length of life of dry battery cells.

Also $z = \dfrac{x - \bar{x}}{\sigma} = \dfrac{x - 12}{3}$.

(*i*) When $x = 15$, $z = 1$

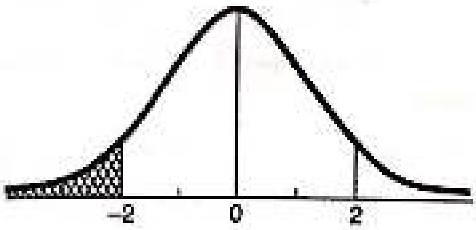


$\therefore \quad P(x > 15) = P(z > 1)$

$= P(0 < z < \infty) - P(0 < z < 1)$

$= 0.5 - 0.3413 = 0.1587 = 15.87\%$.

(*ii*) When $x = 6$, $z = -2$



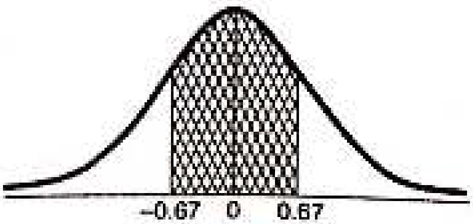
$\therefore \quad P(x < 6) = P(z < -2)$

$= P(z > 2) = P(0 < z < \infty) - P(0 < z < 2)$

$= 0.5 - 0.4772 = 0.0228 = 2.28\%$.

(*iii*) When $x = 10$, $z = -\dfrac{2}{3} = -0.67$

When $x = 14$, $z = \dfrac{2}{3} = 0.67$



$P(10 < x < 14)$

$= P(-0.67 < z < 0.67)$

$= 2P(0 < z < 0.67) = 2 \times 0.2485$

$= 0.4970 = 49.70\%$.

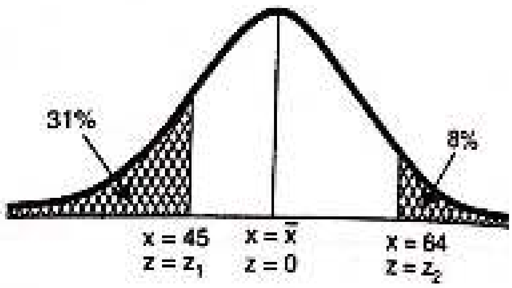
Example 2. *In a normal distribution, 31% of the items are under 45 and 8% are over 6 Find the mean and standard deviation of the distribution.*

(Cochin 2010; Calicut 2011; K.U.K., 2010, Jan. 2014; M.D.U. 2012, May 201

Sol. Let $\bar{x}$ and σ be the mean and S.D. respectively.

31% of the items are under 45.

$\Rightarrow$ Area to the left of the ordinate $x = 45$ is 0.31



When $x = 45$, let $z = z_1$

$$P(z_1 < z < 0) = .5 - .31 = .19$$

From the tables, the value of z corresponding to this area is 0.5

$$\therefore \qquad z_1 = -0.5 \qquad [z_1 < 0]$$

When $x = 64$, let $z = z_2$

$$P(0 < z < z_2) = 0.5 - 0.08 = 0.42$$

From the tables, the value of z corresponding to this area is 1.4.

$$z_2 = 1.4$$

Since $$z = \frac{x - \bar{x}}{\sigma}$$

$$-0.5 = \frac{45 - \bar{x}}{\sigma} \quad \text{and} \quad 1.4 = \frac{64 - \bar{x}}{\sigma}$$

$$\Rightarrow \qquad 45 - \bar{x} = -0.5\sigma \qquad ...(1)$$

and $$64 - \bar{x} = 1.4\sigma \qquad ...(2)$$

Subtracting $\quad -19 = -1.9\sigma \quad \therefore \quad \sigma = 10$

From (1), $\quad 45 - \bar{x} = -0.5 \times 10 = -5 \quad \therefore \quad \bar{x} = 50.$

Example 3. *Obtain the equation of the normal curve that may be fitted to the following data:*

Class:	*60–65*	*65–70*	*70–75*	*75–80*	*80–85*	*85–90*	*90–95*	*95–100*
Frequency:	*3*	*21*	*150*	*335*	*326*	*135*	*26*	*4*

Sol. The equation of the normal curve is

$$y = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Now, from the given data, we shall find $N = \Sigma f$; μ, the mean and the S.D. σ.

Class	Mid-values (x)	f	$u = \frac{x - 77.5}{5}$	fu	fu^2
60–65	62.5	3	−3	−9	27
65–70	67.5	21	−2	−42	84
70–75	72.5	150	−1	−150	150
75–80	77.5	335	0	0	0
80–85	82.5	326	1	326	326
85–90	87.5	135	2	270	540
90–95	92.5	26	3	78	234
95–100	97.5	4	4	16	64
		N = 1000		489	1425

$$\text{Mean } \mu = A + \frac{h}{N}\sum fu = 77.5 + \frac{5}{1000} \times 489$$

$$= 77.5 + 2.445 = 79.945$$

$$\text{S.D. } \sigma = h\sqrt{\frac{1}{N}\sum fu^2 - \left(\frac{\sum fu}{N}\right)^2} = 5\sqrt{\frac{1425}{1000} - \left(\frac{489}{1000}\right)^2}$$

$$= \frac{5}{1000}\sqrt{1425000 - 239121} = \frac{5}{1000}\sqrt{1185879}$$

$$= \frac{5}{1000} \times 1088.98 = 5.445$$

Hence the equation of the normal curve fitted to the given data is

$$y = \frac{1000}{5.445\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x - 79.945}{5.445}\right)^2}$$

EXERCISE 5.6

1. If z is the standard normal variate, then find the following probabilities:
   (i) $P(1 \le z \le 2)$
   (ii) $P(-2.3 \le z \le -1.5)$
   (iii) $P(-0.42 \le z \le 2)$
   (iv) $P(z \le -0.56)$
   (v) $P(|z| \le 1)$
   (vi) $P(|z| \ge 1)$
2. (a) Let X be a random variable having a normal distribution with mean 30 and standard deviation 5. Find the probability that
   (i) $20 \le x \le 40$
   (ii) $|x - 30| > 5$
   (iii) $|x - 24| < 8$
   (iv) $x \ge 45$
   (v) $x \le 45$
   (M.D.U. 2010, May 2013)

(*b*) For a normally distributed variate with mean 1 and S.D. 3, find the probabilities that:

(*i*) $3.43 \le x \le 6.19$ (*ii*) $-1.42 \le x \le 6.18$ (K.U.K. Jan. 2013)

3. The mean height of 500 male students in a certain college is 151 cm and the standard deviation is 15 cm. Assuming the heights are normally distributed, find how many students have heights between 120 and 155 cm?

4. An aptitude test for selecting officers in a bank is conducted on 1000 candidates. The average score is 42 and the standard deviation of score is 24. Assuming normal distribution for the scores, find

(*i*) the number of candidates whose scores exceed 60

(*ii*) the number of candidates whose scores lie between 30 and 60.

5. If the mean height of an Indian police inspector be 170 cm with variance 25 cm^2, how many inspectors out of 1000 would you expect

(*i*) between 170 cm and 180 cm (*ii*) less than 160 cm?

6. The marks obtained by a large group of students in a final examination in statistics have a mean of 58 and a standard deviation of 8.5. Assuming that these marks are approximately normally distributed, what percentage of the students can be expected to have obtained marks from 60 to 69?

7. The income of a group of 10,000 persons was found to be normally distributed with mean ₹ 750 p.m. and standard deviation ₹ 50. Show that, of this group, about 95% had income exceeding ₹ 668 and only 5% had income exceeding ₹ 832. What was the lowest income among the richest 100?

8. (*a*) In a normal distribution, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution?

(*b*) In a normal distribution 17% of the items are below 30 and 17% of the items are above 60. Find the mean and standard deviation.

(*c*) In a normal distribution 10% of the items are below 55 and 20% are above 59. Find the mean and variance of the normal distribution.

9. Let X denote the number of scores on a test. If X is normally distributed with mean 100 and standard deviation 15, find the probability that X does not exceed 130.

10. It is known from the past experience that the number of telephone calls made daily in a certain community between 3 p.m. and 4 p.m. have a mean of 352 and a standard deviation of 31. What percentage of the time will there be more than 400 telephone calls made in this community between 3 p.m. and 4 p.m.?

11. Students of a class were given a mechanical aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What per cent of students scored

(*i*) more than 60 marks? (*ii*) less than 56 marks?

(*iii*) between 45 and 65 marks?

12. In an examination taken by 500 candidates, the average and the standard deviation of marks obtained (normally distributed) are 40% and 10%. Find approximately

(*i*) how many will pass, if 50% is fixed as a minimum?

(*ii*) what should be the minimum if 350 candidates are to pass?

(*iii*) how many have scored mark...

13. In a distribution, exactly normal, 9.85% of the items are under 40 and 89.97% are under 60. What are the mean and standard deviation of the distribution?

14. The income distribution of workers in a certain factory was found to be normal with mean of ₹ 500 and standard deviation of ₹ 50. There were 228 workers getting above ₹ 600. How many workers were there in all?

15. The mean inside diameter of a sample of 500 washers produced by a machine is 5.02 mm and the standard deviation is 0.05 mm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 4.96 to 5.08 mm, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed.

16. Assuming that the diameters of 1000 brass plugs taken consecutively from a machine, form a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm. How many of the plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm?

(DCRUST Murthal, May 2014)

17. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for

(*i*) more than 2150 hours

(*ii*) less than 1950 hours

(*iii*) more than 1920 hours but less than 2160 hours

(*iv*) not less than 2000 hours. *(Calicut 2011)*

18. Obtain the equation of the normal curve that may be fitted to the following data:

7. ₹ 866.50 **8.** (*a*) $\mu = 50.29$, $\sigma = 10.33$ (*b*) $\mu = 45$, $\sigma = 15.8$

(*c*) $\mu = 57.41$, $\sigma^2 = 3.469$ **9.** 0.9772 **10.** 6.06%

11. (*i*) 50% (*ii*) 21.2% (*iii*) 84%

12. (*i*) 79 (*ii*) 35% (*iii*) 11

13. $\mu = 50.04$, $\sigma = 7.78$ **14.** 10,000 **15.** 23 **16.** 52

17. (*i*) 72 (*ii*) 134 (*iii*) 1909

(*iv*) 14.97

18. $y = \dfrac{60}{1.45\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-1.93}{2.09}\right)^2}$ **19.** $y = \dfrac{1}{2\sqrt{2\pi}} e^{-\frac{(x-6)^2}{8}}$ **20.** 97.

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