

2/2/2024

$$E(X) = 1, \quad E(Y) = 1$$

↑ ↑

$$E(XY) = \frac{5}{4}$$

$$\text{Cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$$

$$= \frac{5}{4} - 1 \cdot 1 = \frac{5}{4} - 1 = \frac{1}{4}$$

$$\rho = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$$= \frac{1/4}{\sigma_X \sigma_Y} = \frac{1/4}{\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}} = \frac{1/4}{1/2} = \frac{1}{2}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\text{Var}(Y) = E(Y^2) - (E(Y))^2$$

$$E(X^2) = \sum x^2 f_X(x) \checkmark$$

$$E(Y^2) = \sum y^2 f_Y(y) \checkmark$$

$$\sigma_X = \sqrt{\text{Var}(X)}$$

$$\sigma_Y = \sqrt{\text{Var}(Y)}$$

$$E(X^2) = \sum_x \sum_y (x^2) f_{X,Y}(x,y) = \sum_x x^2 \sum_y f_{X,Y}(x,y)$$

$$= \sum_x x^2 f_X(x)$$

$$= 0^2 \cdot 2/8 + 1^2 \cdot 1/8 + 2^2 \cdot 2/8$$

$$= \frac{4}{8} + \frac{8}{8} = \left(\frac{12}{8}\right) \checkmark$$

$$E(Y^2) = \sum_y y^2 f_Y(y) = 0^2 \cdot 2/8 + 1^2 \cdot 1/8 + 2^2 \cdot 2/8$$

$$= \left(\frac{12}{8}\right) \checkmark$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 = \frac{12}{8} - 1^2 = 4/8 = 1/2$$

$$\text{Var}(Y) = 1/2$$

$$\therefore \text{S.D.}(X) = \sigma_X = \sqrt{1/2}$$

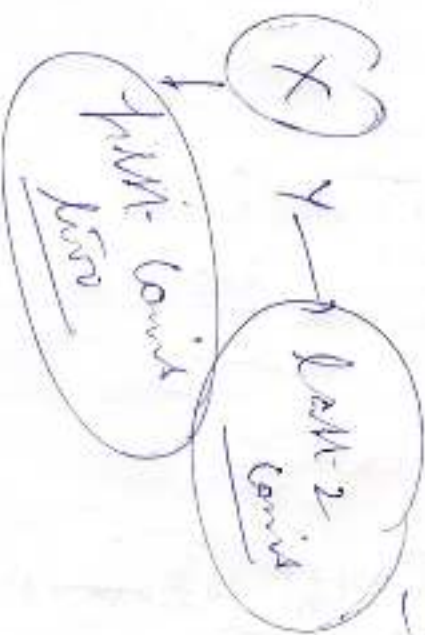
$$\sigma_Y = \frac{1}{\sqrt{2}}$$

Jo-hansen Conf. Exts

✓

λ

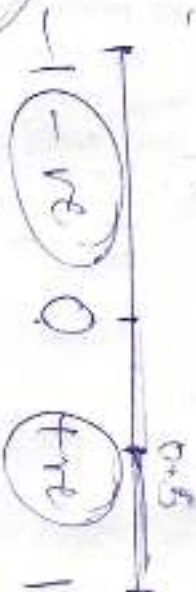
$$10 \quad -1 \leq \rho \leq 1$$



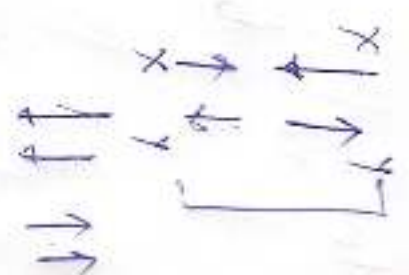
Any sample?

$$f=0$$

$$f=0.8$$



$$f(x_1) = 0.5$$



(x, y)

$$f(x, y) = \begin{cases} \frac{1}{8} (6 - x - y) & 0 \leq x \leq 2 \\ 0 & 2 \leq y \leq 4 \end{cases}$$

$$\boxed{0 \leq x \leq 2}$$
$$\boxed{2 \leq y \leq 4}$$

otherwise

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$$

$$\boxed{0 \leq x \leq 2}$$
$$\boxed{2 \leq y \leq 4}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$$

$$= \textcircled{1}$$

$\therefore f_{X,Y}(x, y)$ is a p.d.f.

03/03/2021

ap, mean

npq, var

$N(\mu, \sigma^2)$

$N=200$

$p=1/2$

$P(X > 150)$

P.D of

$X+Y$

$X-Y$

$= 1 - P(X \leq 150)$

gR, 2G, 4W

2 balls at

random

X: no. of Red Balls
Y: no. of Green Balls

$X=0,1,2$
 $Y=0,1,2$

(0,0)

(0,1)

Red Green Red

2

X Y
0 0
0 1
1 0
1 1
2 0
2 1
2 2

$\frac{4C_2}{9C_2}$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx dy = 1$$

$$F_X(x) = \int_{-\infty}^x \left(\int_{-\infty}^{\infty} f(x,y) dy \right) dx$$

$$= P[X \leq x]$$

$$F_Y(y) = \int_{-\infty}^y \left(\int_{-\infty}^{\infty} f(x,y) dx \right) dy$$

$$(0 < Z < 1) \quad (0,1)$$

0.9999

$$F_X(x) = P[X \leq x]$$

$$= P[X \leq x, Y < \infty]$$

$$F_Y(y) = P[Y \leq y]$$

$$= P[X < \infty, Y \leq y]$$

$$f_X(x) = P[X=x] \quad \checkmark$$

$$f_Y(y) = P[Y=y] \quad \checkmark$$

X \ Y	0	1	2	$f_X(x)$
0	0	1	2	3
1	0	1	2	3
2	0	1	2	3

Ques:

$$f_{X,Y}(x,y) = \begin{cases} 2 & 0 < x < 1, 0 < y < x \\ 0 & \text{elsewhere} \end{cases}$$

0 < x < 1, 0 < y < x
elsewhere

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy \quad \checkmark$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx$$

$$f_X(x) = \int_0^x 2 dy$$

$$= (2y)_0^x = 2x$$

$$f_X(x) = 2x, \quad 0 < x < 1$$

(TS) f is a p.d.f

$$\int_0^1 \int_0^x f(x,y) dy dx = 1$$

$$\text{LHS} = \int_0^1 \int_0^x 2 dy dx$$

$$= \int_0^1 (2y)_0^x dx$$

$$= \int_0^1 2y^2 dx$$

$$= 2(x^2/2)_0^1 = \textcircled{1}$$

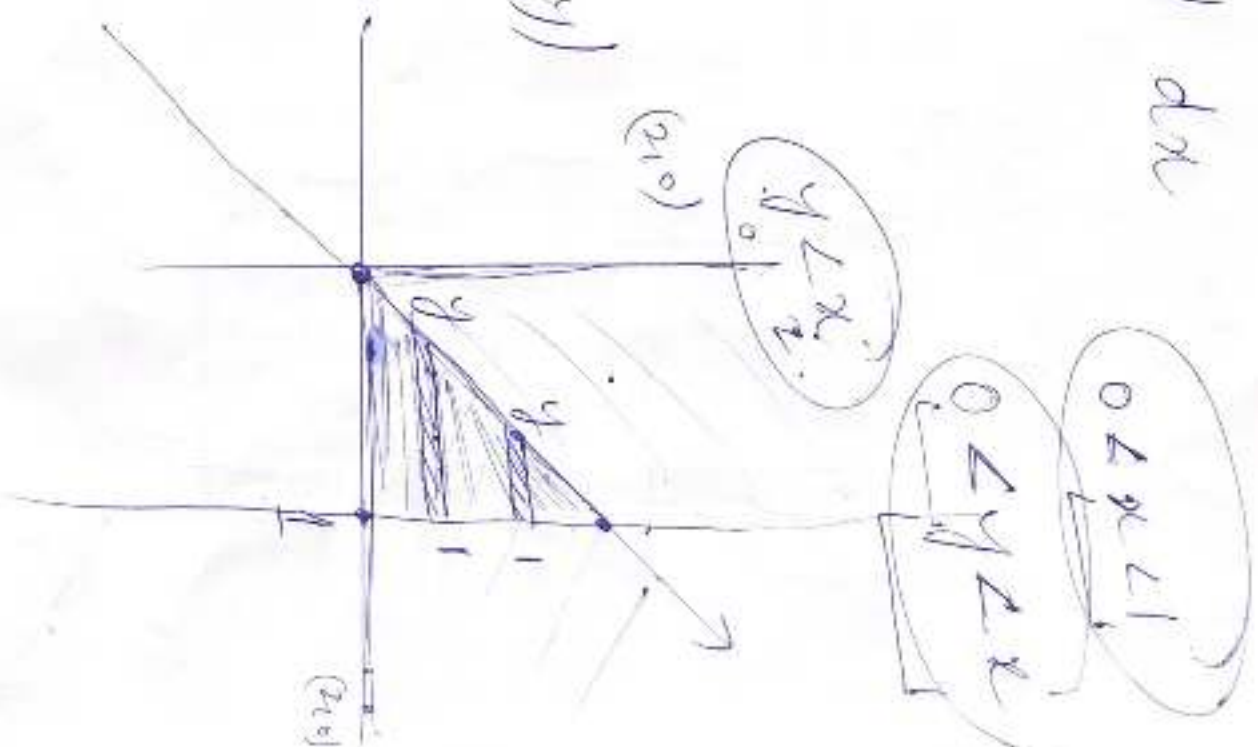
$$f_y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx$$

$$= \int_y^1 2 dx$$

$$= (2x)_y^1 = 2(1-y)$$

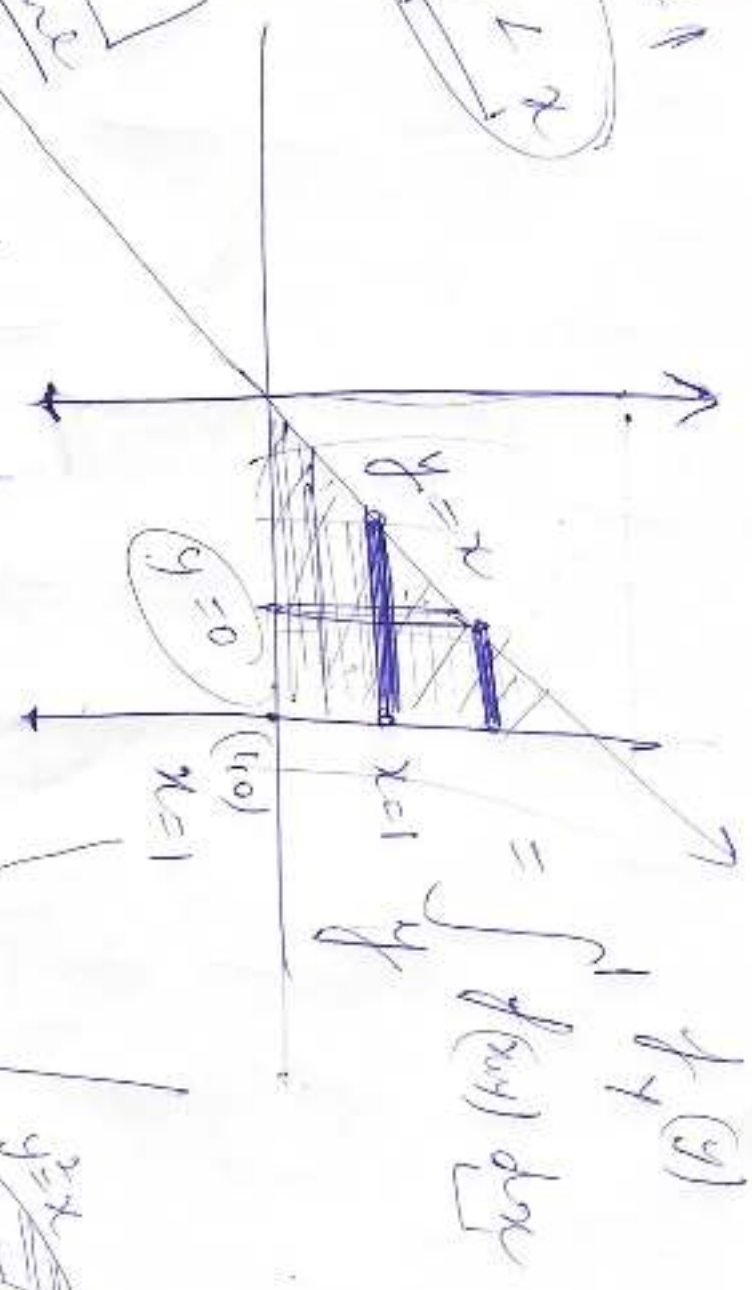
$$f_y(y) = 2(1-y)$$

$$0 < y < 1$$

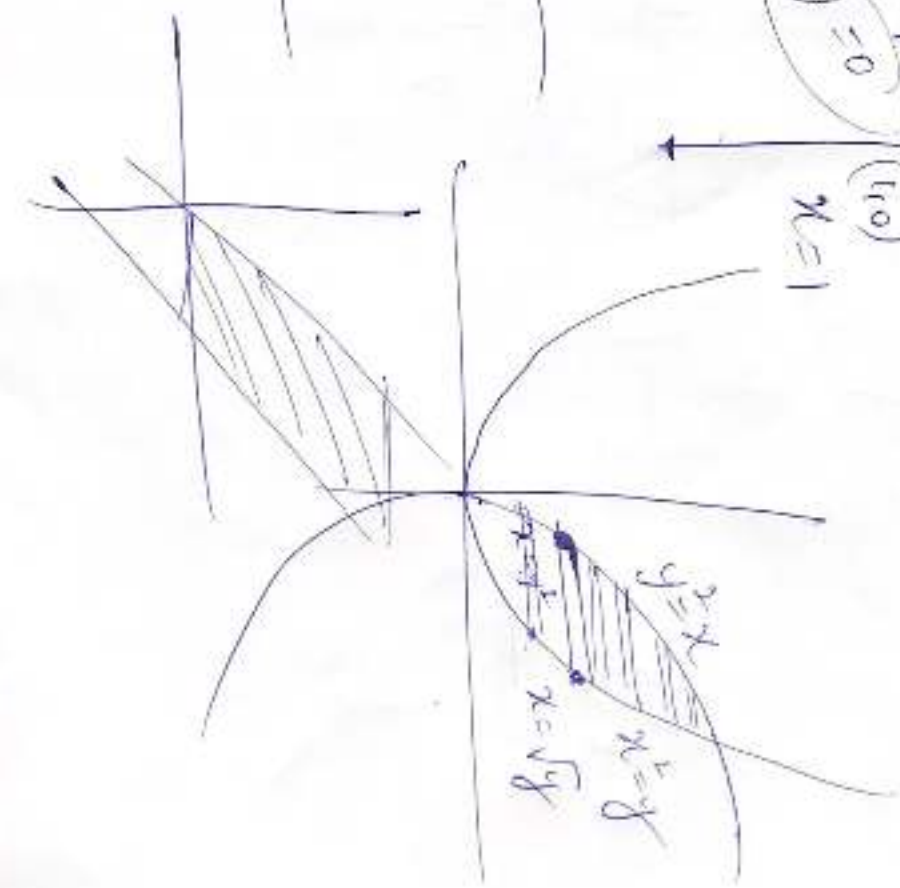
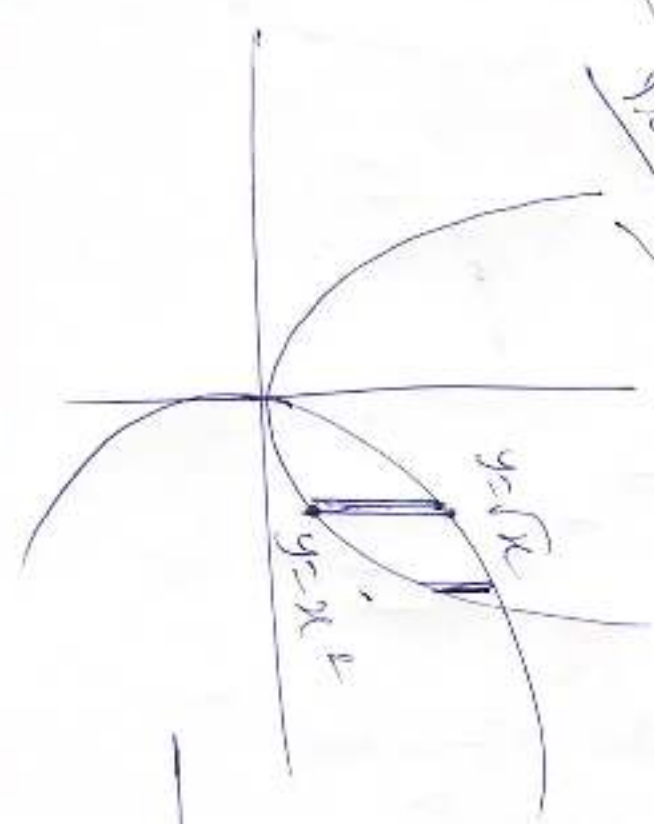


$0 < x < 1$
 $y < x$
 $0 < y < x$
 $0 < x < 1$

Time



$$\int_0^1 f(x) dx = \int_0^1 f(y) dy$$



05/02/2021

$$f(x, y) = \begin{cases} 1/8 (6 - x - y) & 0 \leq x \leq 2, \\ & 2 \leq y \leq 4 \end{cases}$$

0 otherwise

$$\textcircled{1} \quad P(X < 1 \cap Y < 3)$$

$$\begin{aligned} &= \int_{-\infty}^{-\infty} \int_3^1 f_{X,Y}(x, y) \cancel{dx dy} \\ &= \int_0^1 \int_2^3 \frac{1}{8} (6 - x - y) dy dx \quad \checkmark \end{aligned}$$

$$P\left(\sqrt{x+y} < 3\right) = ?$$

$$0 \leq x \leq 2$$

$$2 \leq y \leq 4$$

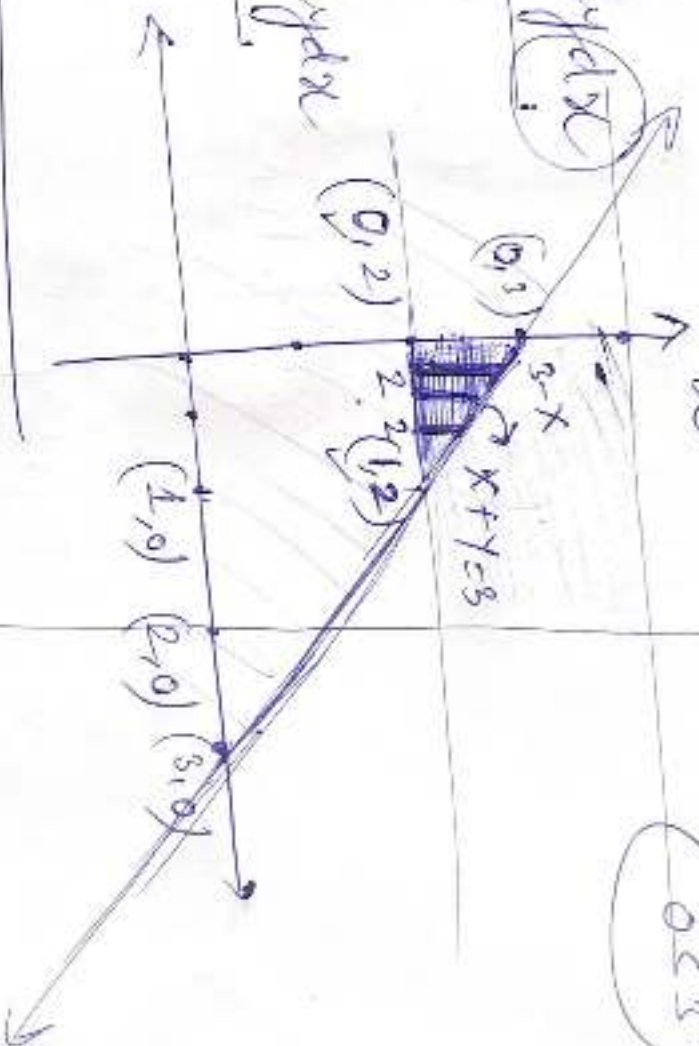
$$\boxed{x+y < 3}$$

$$0+0 < 3$$

$$0 < 3$$

$$= \int_0^2 \int_{2-x}^4 (x+y) dy dx$$

$$= \int_0^2 \int_2^{3-x} \frac{1}{8} (6-x-y) dy dx$$



$$\text{or } \int_2^4 \int_0^{3-y} \frac{1}{8} (6-x-y) dx dy$$

$$\textcircled{3} \quad P(X < 1 \mid Y < 3)$$

$$= \frac{P(X < 1 \cap Y < 3)}{P(Y < 3)} = \frac{P_{\text{part (i)}}}{P(Y < 3)}$$

$$P(Y < 3) = \int_0^2 \int_2^3 f(x,y) dx$$

$$f(x,y) = \begin{cases} 2 & 0 < x < 1, 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{cases} y < x \\ 0 < 1 \end{cases}$$

True

(i) Marginal density for X & Y

$$f_x(x) = P[X=x] = \int_{-\infty}^{\infty} f(x,y) dy = \int_0^x 2 dy$$

$$f_x(x) = 2x, 0 < x < 1$$

$$= (2y)'_0^x = 2x$$

$$f_y(y) = \int_{-\infty}^{\infty} f(x,y) dx = \int_0^1 2 dx$$

$$f_y(y) = (2x)'_y^1 = 2 - 2y \checkmark$$



$$f_y(y) = 2 - 2y \quad 0 < y < 1$$

Conditional density functions

$$f_{y|x}(y|x) = \frac{f_{xy}(x,y)}{f_x(x)} = \frac{2}{2x} \quad \boxed{0 < x < 1}$$

$$f_{x|y}(x|y) = \frac{f_{xy}(x,y)}{f_y(y)} = \frac{2}{2(1-y)} = \frac{1}{1-y}$$

Are (X,Y) independent?

$$f_{xy}(x,y) = f_x(x) \cdot f_y(y)$$

2

\neq

$2x(2-2y)$

$$P(A \cap B) = P(A) \cdot P(B)$$

not independent

(X, Y) cos R.V.

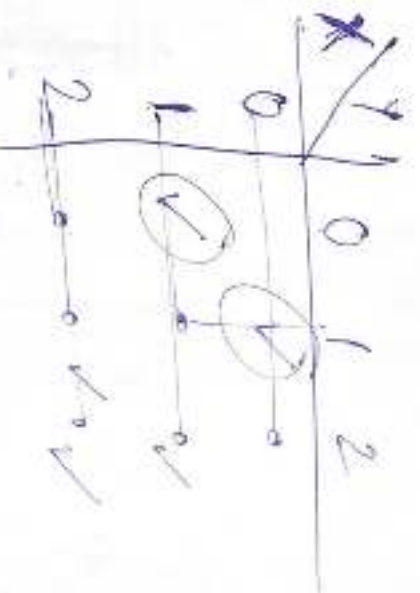
joint pdf

$$f_{X,Y}(x,y)$$

$$E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (g(x,y) f_{X,Y}(x,y)) \, dx \, dy$$

$$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_{X,Y}(x,y) \, dx \, dy$$

$$E(X) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f_{X,Y}(x,y) \, dx \, dy$$



$$Z = X + Y$$

$$= 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$= \int_{-\infty}^{\infty} x \left(\int_{-\infty}^{\infty} f_{X,Y}(x,y) \, dy \right) \, dx$$

$$= \int_{-\infty}^{\infty} x f_X(x) \, dx$$

Quest:

$$f(x,y) = \begin{cases} x+y & 0 \leq x \leq 1 \\ 0 & 0 \leq y \leq 1 \end{cases}$$

$$P(0 < x < 1/2, 0 < y < 1/4) = \int_0^{1/2} \int_0^{1/4} (x+y) dy dx$$

$$E(X) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f(x,y) dx dy$$

$$= \int_0^1 \int_0^1 x (x+y) dx dy$$

$$E(X) = \int_{-\infty}^{\infty} x f_X(x) dx$$



$$= \int_0^1 x \left(\int_0^1 y (x+y) dy \right) dx$$

$$= \int_0^1 \int_0^1 xy f(xy) dy dx$$

$$= \int_0^1 y \left(\int_0^1 (x+y) dx \right) dy$$

$$= \int_0^1 \int_0^1 y f(xy) dx dy$$

$\int_0^1 \int_0^1$

$$E(x+y) = \int_0^1 \int_0^1 (x+y) f_{x,y} dx dy$$

$$= \int_0^1 \int_0^1 (x+y) dx dy$$

$$E(x^2) = ?$$

$$E(y^2) = ?$$

$$\text{Var}(x) = E(x^2) - (E(x))^2 \quad \text{Cov}(x,y)$$

$$\text{Var}(y) = E(y^2) - (E(y))^2 = E(xy) - E(x) \cdot E(y)$$

$$\rho(x,y) = \frac{\text{Cov}(x,y)}{\sqrt{\text{Var}(x)} \sqrt{\text{Var}(y)}} \quad \checkmark$$

$$\text{Cov}(x,y) = \frac{-1/11}{\sqrt{\text{Var}(x)} \sqrt{\text{Var}(y)}}$$