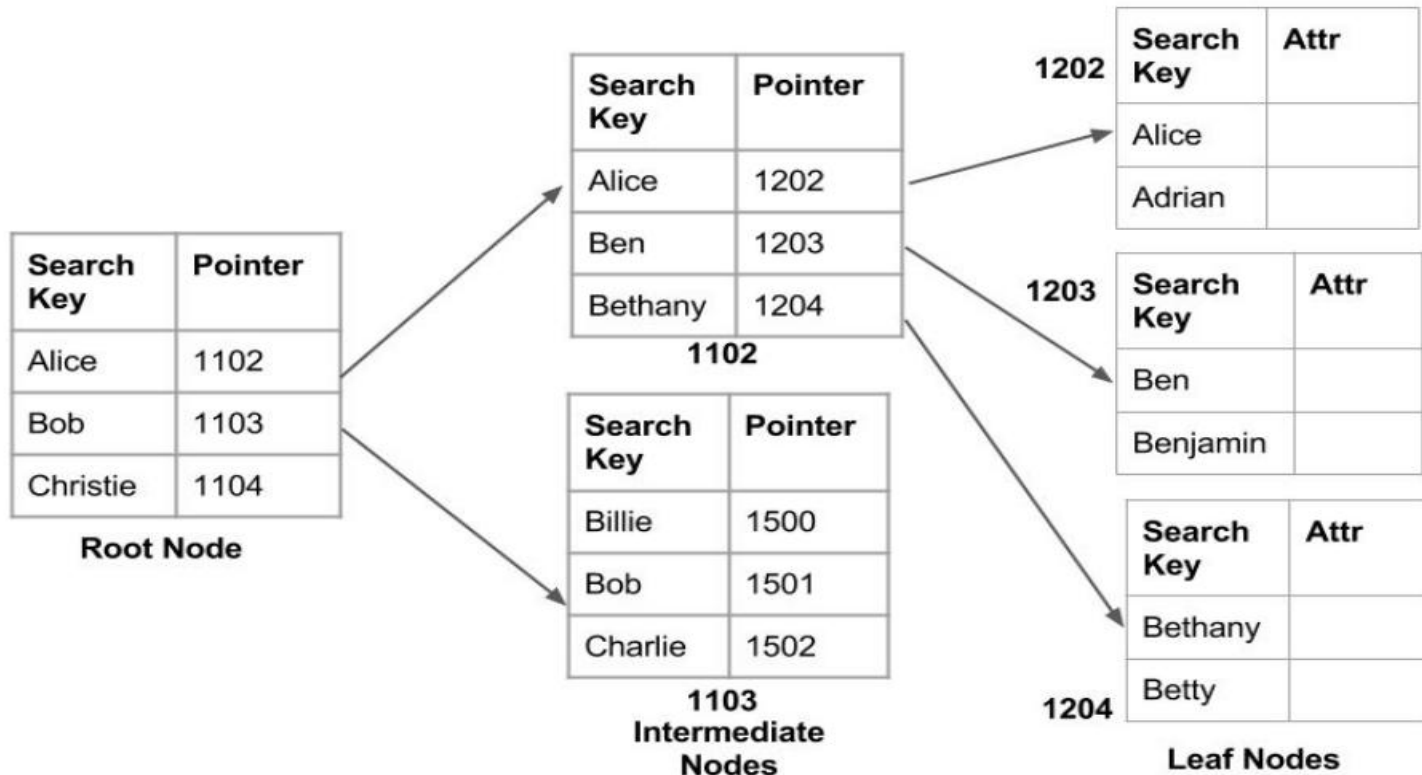


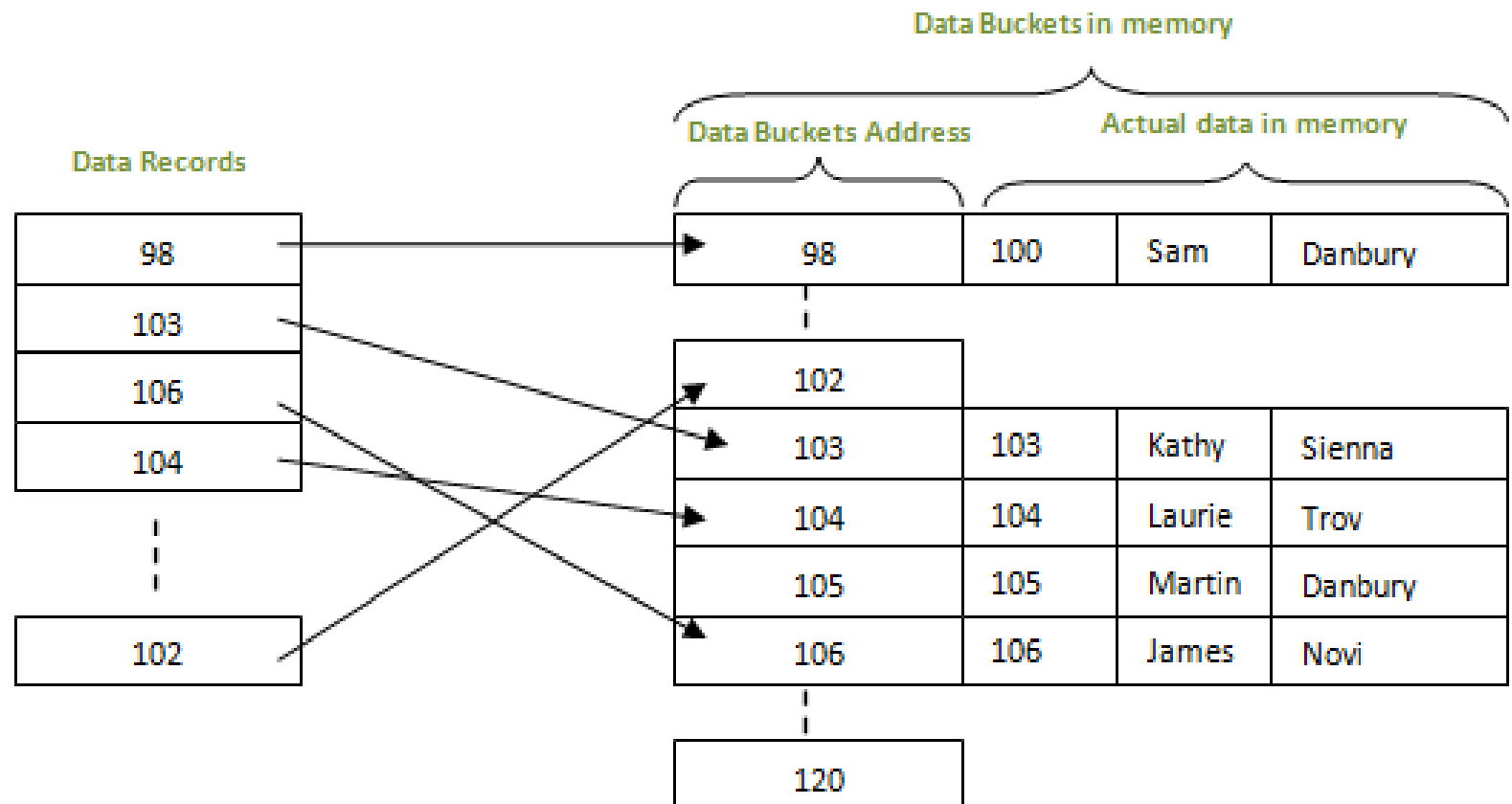
What is Hashing?

Difference between Indexing and Hashing

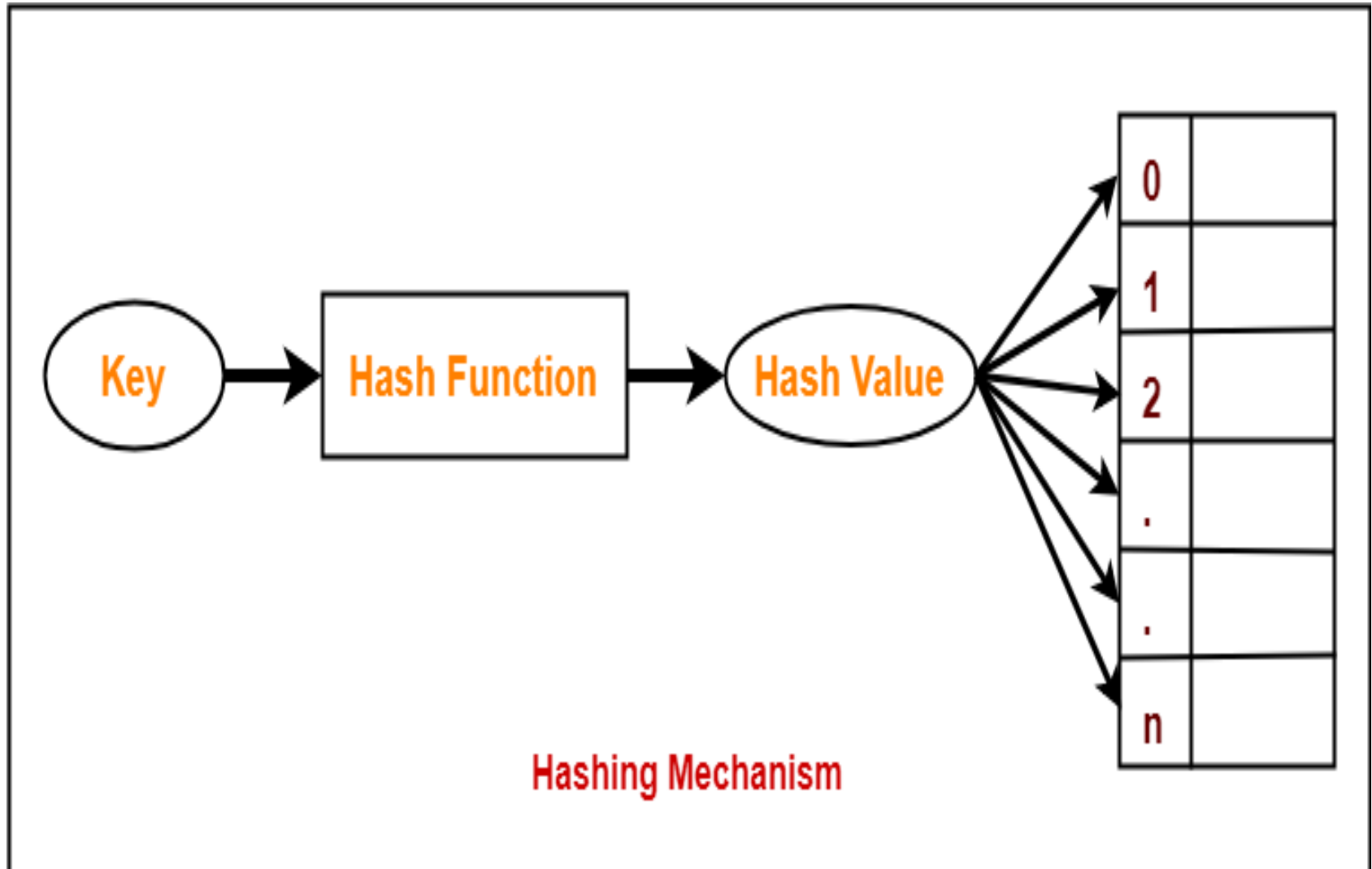


Non clustered index

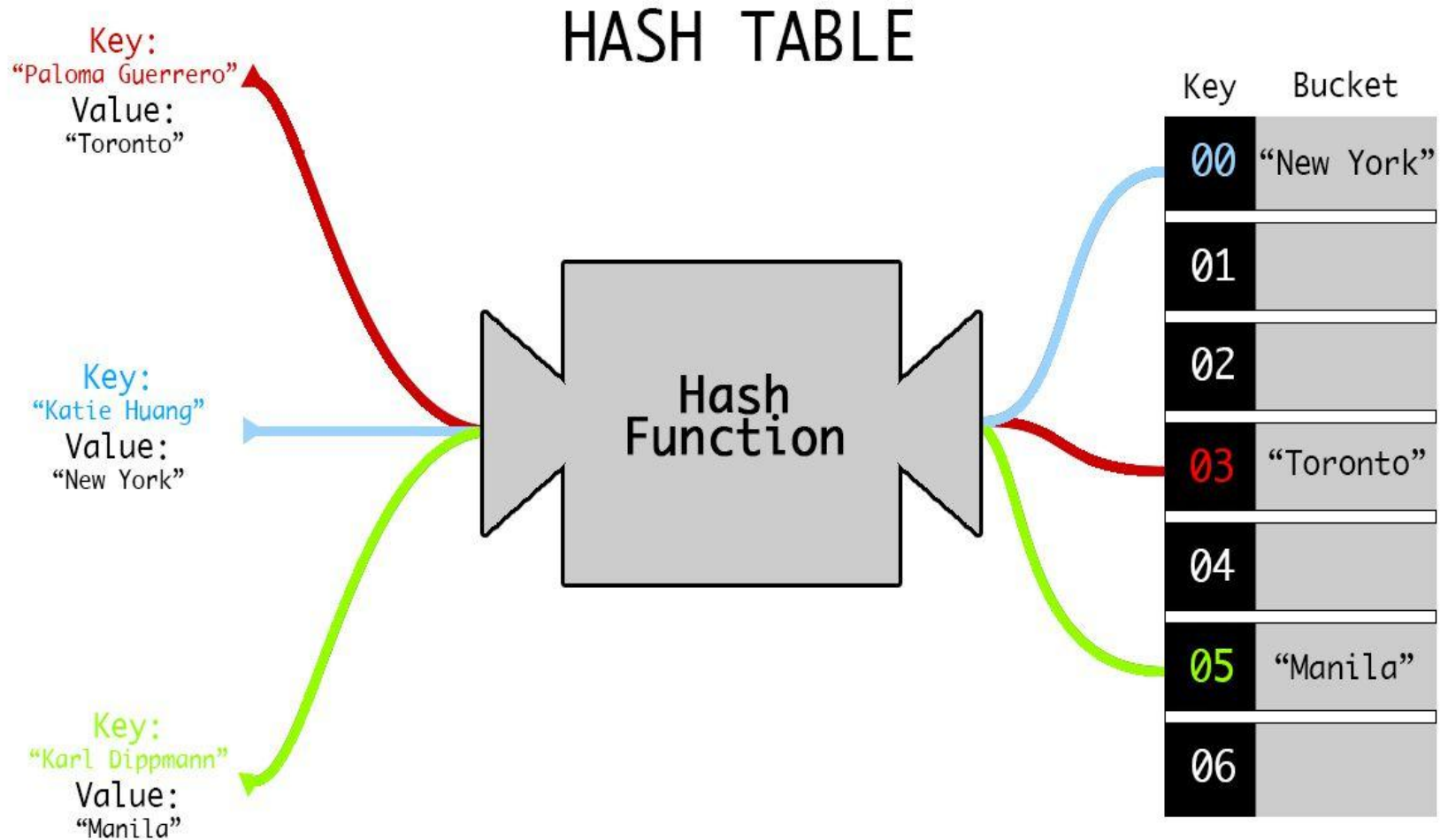
Difference between Indexing and Hashing



What is Hashing ?



Hash Table



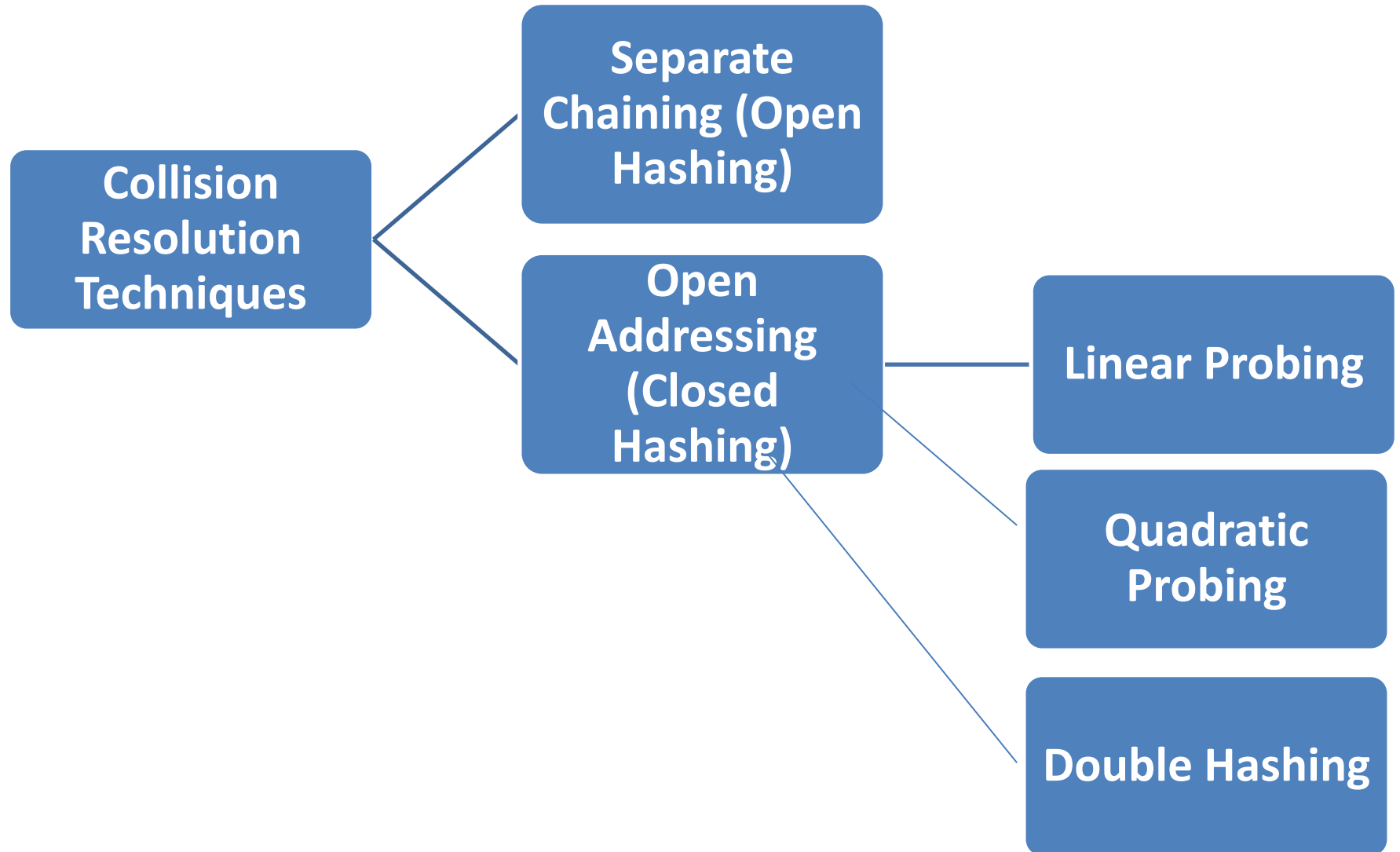


Collision

- If the hash function returns a slot that is already occupied then there is a collision.



| | Key | Value | |
|-----------|--------|---------|---|
| H(129007) | 129007 | Fred | 0 |
| H(926647) | | | 1 |
| H(975378) | 975378 | Richard | 2 |
| | | | 3 |
| H(269908) | 269908 | Robert | 4 |

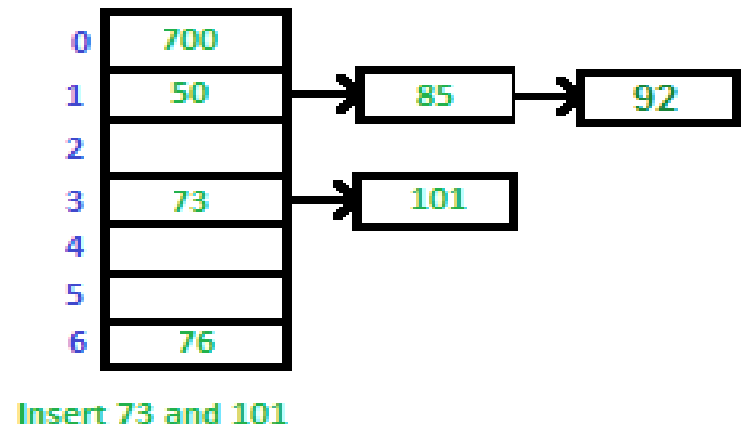
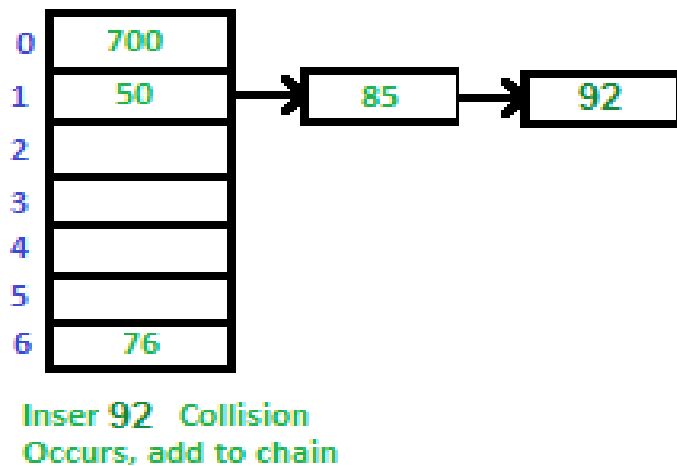
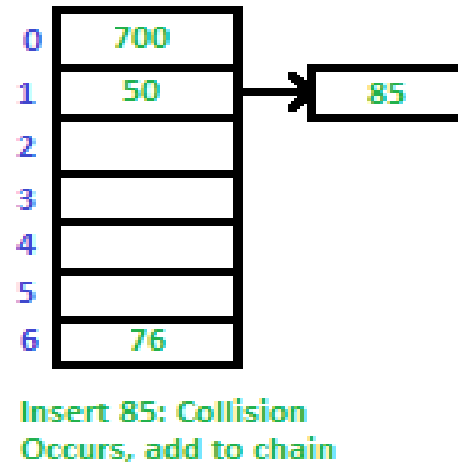
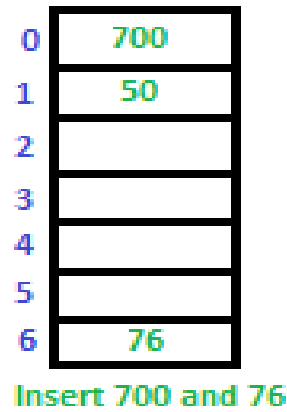
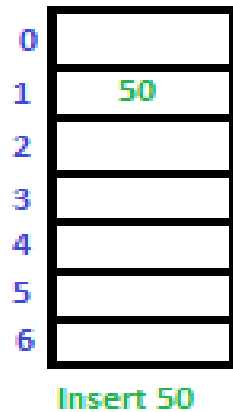
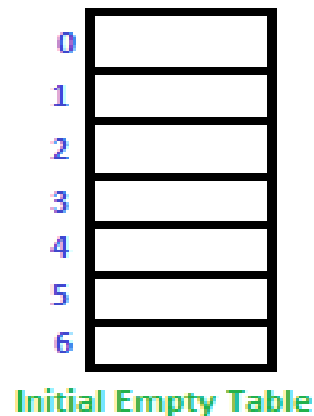


Separate Chaining

The idea is to make each cell of hash table point to a linked list of records that have same hash function value.

Let us consider a simple hash function as “**key mod 7**” and sequence of keys as 50, 700, 76, 85, 92, 73, 101.

Separate Chaining



Performance of Chaining

m = Number of slots in hash table

n = Number of keys to be inserted in hash table

Load factor $\alpha = n/m$

Expected time to search = $O(1 + \alpha)$

Expected time to insert/delete = $O(1 + \alpha)$

Time complexity of search insert and delete is $O(1)$ if α is $O(1)$

Open Addressing

All elements are stored in the hash table itself. So at any point, size of the table must be greater than or equal to the total number of keys.

Open Addressing is done following ways:

- Linear Probing
- Quadratic Probing
- Double Hashing

Linear Probing

- Linearly probe for next slot.
- Let **hash(x)** be the slot index computed using hash function and **S** be the table size
If slot $\text{hash}(x) \% S$ is full, then we try $(\text{hash}(x) + 1) \% S$

Let us consider a simple hash function as “key mod 7”
and sequence of keys as 50, 700, 76, 85, 92, 73, 101.

Linear Probing

| | |
|---|--|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

Initial Empty Table

| | |
|---|----|
| 0 | |
| 1 | 50 |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

Insert 50

| | |
|---|-----|
| 0 | 700 |
| 1 | 50 |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | 76 |

Insert 700 and 76

| | |
|---|-----|
| 0 | 700 |
| 1 | 50 |
| 2 | 85 |
| 3 | |
| 4 | |
| 5 | |
| 6 | 76 |

Insert 85: Collision Occurs, insert 85 at next free slot.

| | |
|---|-----|
| 0 | 700 |
| 1 | 50 |
| 2 | 85 |
| 3 | 92 |
| 4 | |
| 5 | |
| 6 | 76 |

Insert 92, collision occurs as 50 is there at index 1. Insert at next free slot

| | |
|---|-----|
| 0 | 700 |
| 1 | 50 |
| 2 | 85 |
| 3 | 92 |
| 4 | 73 |
| 5 | 101 |
| 6 | 76 |

Insert 73 and 101

Quadratic Probing

- We look for i^2 'th slot in i 'th iteration.
- Let $\text{hash}(x)$ be the slot index computed using hash function.

If slot $\text{hash}(x) \% S$ is full, then we try $(\text{hash}(x) + 1*1) \% S$

Double Hashing

- We use another hash function $\text{hash2}(x)$ and look for $i * \text{hash2}(x)$ slot in i 'th rotation.
- Let $\text{hash}(x)$ be the slot index computed using hash function.

If slot $\text{hash}(x) \% S$ is full, then we try $(\text{hash}(x) + 1 * \text{hash2}(x)) \% S$

Comparison

Linear Probing

- Best cache performance but suffers from clustering.
- Easy to compute.

Quadratic Probing

- Lies between the two in terms of cache performance and clustering.

Double Hashing

- Poor cache performance but no clustering.
- More computation time as two hash functions need to be computed.

Performance of Open Addressing

m = Number of slots in the hash table

n = Number of keys to be inserted in the hash table

Load factor $\alpha = n/m$ (< 1)

Expected time to search/insert/delete $< 1/(1 - \alpha)$

So Search, Insert and Delete take $(1/(1 - \alpha))$ time

Rehashing

- Hash Table may get too full.
- Solution: Rehash
 - Build another table that is twice as big and has a new hash function.
 - Move all the element from smaller to bigger table.
- Cost of Rehashing = $O(N)$

Rehashing Example

Original Hash Table

| | |
|---|----|
| 0 | 6 |
| 1 | 15 |
| 2 | |
| 3 | 24 |
| 4 | |
| 5 | |
| 6 | 13 |

After Inserting 23

| | |
|---|----|
| 0 | 6 |
| 1 | 15 |
| 2 | 23 |
| 3 | 24 |
| 4 | |
| 5 | |
| 6 | 13 |

After Rehashing

| | |
|----|----|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | 6 |
| 7 | 23 |
| 8 | 24 |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | 13 |
| 14 | |
| 15 | 15 |
| 16 | |

QUESTIONS

Given the following input (4322, 1334, 1471, 9679, 1989, 6171, 6173, 4199) and the hash function $x \bmod 10$, which of the following statements are true?

- i. 9679, 1989, 4199 hash to the same value
- ii. 1471, 6171 has to the same value
- iii. All elements hash to the same value
- iv. Each element hashes to a different value

- (A) i only
- (B) ii only
- (C) i and ii only
- (D) iii or iv

QUESTIONS

Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?

- (A) $(97 \times 97 \times 97)/100^3$
- (B) $(99 \times 98 \times 97)/100^3$
- (C) $(97 \times 96 \times 95)/100^3$
- (D) $(97 \times 96 \times 95)/(3! \times 100^3)$

QUESTIONS

Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for i ranging from 0 to 2020?

(A) $h(i) = i^2 \bmod 10$

(B) $h(i) = i^3 \bmod 10$

(C) $h(i) = (11 * i^2) \bmod 10$

(D) $h(i) = (12 * i) \bmod 10$

QUESTIONS

How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table given in Question 3 above?

- (A) 10
- (B) 20
- (C) 30
- (D) 40

QUESTIONS

A hash table of length 10 uses open addressing with hash function $h(k)=k \bmod 10$, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below.

| | |
|---|----|
| 0 | |
| 1 | |
| 2 | 42 |
| 3 | 23 |
| 4 | 34 |
| 5 | 52 |
| 6 | 46 |
| 7 | 33 |
| 8 | |
| 9 | |

Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

- (A) 46, 42, 34, 52, 23, 33
- (B) 34, 42, 23, 52, 33, 46
- (C) 46, 34, 42, 23, 52, 33
- (D) 42, 46, 33, 23, 34, 52