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Probability Space.

Every random experiment is associated with the object $\{\Omega, \mathcal{F}, P\}$ called **Probability Space**.

Ω - all elementary outcomes.

\mathcal{F} - all possible events related to elementary outcomes

P - probability measure that assigns probability number to each event.

Example. Tossing a fair coin once.

$$\Omega = \{\text{"Heads"}, \text{"Tails"}\}$$

$$\mathcal{F} = \{\emptyset, \text{"Heads"}, \text{"Tails"}, \Omega\}$$

$$P = \{\emptyset\} \doteq 0; P\{\text{"Heads"}\} = P\{\text{"Tails"}\} = \frac{1}{2}; P\{\Omega\} \doteq 1.$$

Event \emptyset includes all "impossible" events (the coin overcomes the force of gravity and goes into space).

Event Ω means that either "Heads" or "Tails" occurs.

Coin tossed 2 times. What is the probability space?

Let the outcomes of each toss be: "H"=0, "T"=1.

The probability space $\{\Omega, \mathcal{F}, P\}$ is:

$$\Omega = \{00, 01, 10, 11\}.$$

either both
Heads or
both Tails

$$\mathcal{F} = \{ \emptyset, 00, 01, 10, 11, 00+01, 00+10, 00+11, 01+10, 01+11, 10+11, 00+01+10, 00+01+11, 01+10+11, 00+01+10+11 \}$$

$$P = \{ 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, 1 \}$$

$$00+01+10, 00+01+11, 01+10+11, 00+01+10+11 \}$$

$$\frac{3}{4}, \frac{3}{4}, \frac{3}{4}, 1 \}$$

A coin is tossed 2 times. Define random variables $\Omega = \{HH, HT, TH, TT\}$. associated with this experiment

1. R.V. $X: \Omega \rightarrow \mathbb{R}$ is the number of Tails in the first toss.

$$X = \begin{cases} x_0 = 0 : HH, HT, \\ x_1 = 1 : TH, TT. \end{cases}$$

2. R.V. $Y: \Omega \rightarrow \mathbb{R}$ is the total number of Tails.

$$Y = \begin{cases} y_0 = 0 : HH \\ y_1 = 1 : HT, TH \\ y_2 = 2 : TT \end{cases}$$

marginal distributions:

$$P\{X=x_0\}=P\{X=x_1\}=\frac{1}{2}; \quad P\{Y=y_0\}=P\{Y=y_2\}=\frac{1}{4}; \quad P\{Y=y_1\}=\frac{1}{2}.$$

Joint distribution: Events

	y_0	y_1	y_2
x_0	HH	HT	\emptyset
x_1	\emptyset	TH	TT

$$\begin{aligned} P\{X=x_0, Y=y_0\} &= P\{X=x_0, Y=y_1\} = \frac{1}{4}, \\ P\{X=x_1, Y=y_1\} &= P\{X=x_1, Y=y_2\} = \frac{1}{4}, \\ P\{X=x_0, Y=y_2\} &= P\{X=x_1, Y=y_0\} = 0. \end{aligned}$$

Probabilities

	y_0	y_1	y_2
x_0	$\frac{1}{4}$	$\frac{1}{4}$	0
x_1	0	$\frac{1}{4}$	$\frac{1}{4}$
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Marginal distributions.

$$\frac{1}{2} = P\{X=x_0\}$$

$$\frac{1}{2} = P\{X=x_1\}$$

$$\frac{1}{4} = P\{Y=y_0\}$$

$$\frac{1}{2} = P\{Y=y_1\}$$

$$\frac{1}{4} = P\{Y=y_2\}$$

Conditional distributions

$$P_{X/Y}\{X=x_0 \mid Y=y_0\} = \frac{P\{x_0, y_0\}}{P\{y_0\}} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$$

$$P_{X/Y}\{X=x_1 \mid Y=y_0\} = \frac{P\{x_1, y_0\}}{P\{y_0\}} = \frac{0}{\frac{1}{4}} = 0$$

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