①

MSUA3100 Reading Notes - week 3

03/15/19 **Chapter 10: estimation:**

$$s^2 = \frac{\Sigma(x-m)^2}{N-1}$$

degrees of freedom

confidence interval computations:

Lower limit = $M - z_{.95}\sigma_m$

upper limit = $M - z_{0.05}\sigma_m$

→ # of SDs extending from the mean of a normal dist. required to contain 0.95 of the area

03/16/19 **Chapter 11: logic of hypothesis testing**

Chapter 12: Tests of means

A) Testing A single mean

- eg. subliminal messages
- $H_0: \mu = 50$
- significance test → computing P(x) that $M \neq \mu$ by 1 or more

① determine sampling distribution of the mean

$\mu_m = \mu \longrightarrow \mu_m = 50$

$\sigma_M = \frac{\sigma}{\sqrt{N}}$ ← can also estimate this with s

if H_0 = true, then, based on Binomial distribution: find var.

$\sigma^2 = N\pi(1-\pi)$

② Assume normal distribution, then compute probability for sample mean

$$z = \frac{M-\mu}{\sigma_m}$$

statistic − hypothesized value / estimated std error of statistic

t =

m = sample mean → use as an estimate then find 5th sem

- eg. ADHD & delay

estimation demo

$\boxed{t = \frac{\text{static} - \text{hypothesized value}}{\text{estimated standard error of the statistic}}}$

$$t = \frac{M-\mu}{s_m}$$

TA session 2 - confidence intervals

03/16/19

Difference Between the means - sample $\longrightarrow$ estimating the σ^2 when the population var is unknown

1. $M_1 - M_2 = 5.35 - 3.88$
$= 1.47$

$\mu_{M_1-M_2} = 1.47$

(harmonic mean)

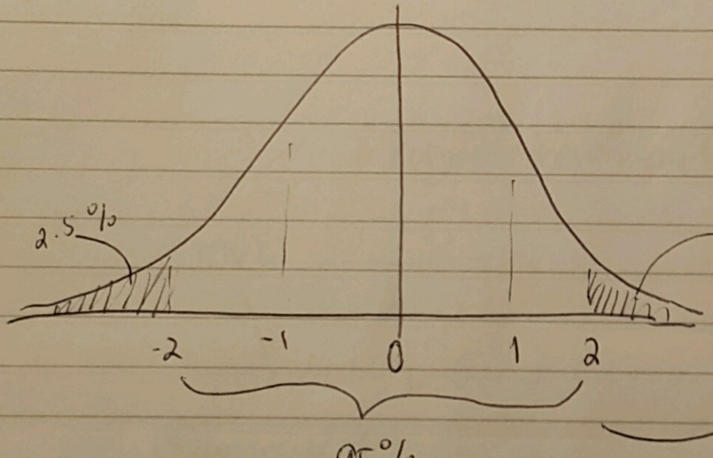
$$MSE = \frac{2.743 + 2.985}{2} = 2.864$$

$$s_{M_1-M_2} = \sqrt{\frac{2(MSE)}{n}} = \sqrt{\frac{2(2.864)}{17}} = .5805$$

- difference b/n knowing SE & estimating it

$$\sqrt{\frac{2.743}{17} + \frac{2.985}{17}} \longrightarrow \sigma_{M_1-M_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

④ t-distribution vs normal



$z = 1.96$

$z = 2.03$
$p = 0.041$

$$t = \frac{1.47}{0.5805} = 2.53$$

$0.0164 \longrightarrow$ under 5%

$df = (n_1 - 1) + (n_2 - 1)$
$= 16 + 16$

$df = 32$

(2)

R functions:

pnorm ⟶ HW 2A
qnorm
1-pnorm =

} opposites of each other

dbinom (0.2049)
qnorm

pnorm (.75)
  [out] 0.7733726
} similar functions to the Book

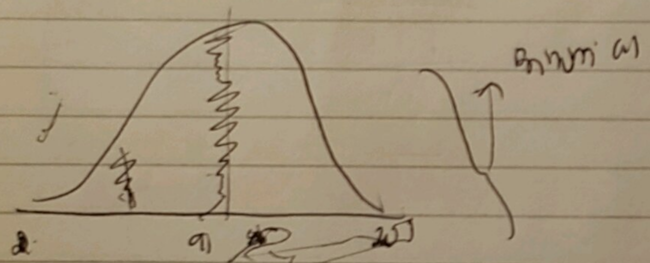
rnorm[ ] ( )
pbinom(lower tail)

@ harmonic mean
= need to know length.

N(75,8)

mc.prob ( )
replicate
fun

- P(<65) or P(≤65) - point difference



M ~ 112

M : 5.20 = 10

- p ~ V

M : 3.52 ~ 0

- percentile

Quantile function ⟶ type6

#22C ⟶ moods ← angry - moods

h ← quantile(moods$Angry-Exprimm, 6)
assum ← summary (moods$Angry-Exprimm)
assum [2] #27
assum [4] #37
assum[5] #44.5

summary(subset(moods, moods$Gender == 1))

B) Difference between 2 means

° eg. "Animal Research" case study
- diff. b/n population means rather than sample means — !
- Assumption:
  1) Both pop have the same var ⟶ homogeneity of variance
  2) Populations are normally distributed
  3) Each value sampled independently from each other value.
- Statistic = diff b/n means

① $t = \dfrac{\text{statistic} - \text{hypothesized value}}{\text{estimated standard error of the statistic}}$ ② null hypothesis that the diff b/n population means is 0.

③ $S_{m_1 - m_2} \longrightarrow \sigma_{m_1 - m_2} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

$= \sqrt{\dfrac{2\sigma^2}{n}}$

- estimate the variance by averaging 2 sample variances

$MSE = \dfrac{S_1^2 + S_2^2}{2}$

estimate of σ^2

④ compute t:

$t = \dfrac{1.4705}{0.5805}$

$= 2.533$

⑤ compute the probability of getting $t \geq 2.533$
$t \leq -2.533$

$df = (n_1 - 1) + (n_2 - 1)$

⑥ t distribution calculator

(3)

- computations for unequal sample sizes
  - MSE counts the group w/ the larger sample size more than the group w/ the smaller sample size

= SSE

$$\boxed{SSE = \sum (X - M_1)^2 + \sum (X - M_2)^2}$$

M for group 1

$$\boxed{MSE = \frac{SSE}{df}} \rightarrow df = (n_1 - 1) + (n_2 - 1)$$

$$S_{M_1 - M_2} = \sqrt{\frac{2MSE}{n_h}}$$

↳ harmonic mean of sample sizes

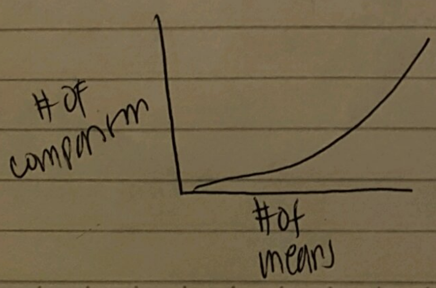
$$n_h = \frac{2}{\frac{1}{n_1} + \frac{1}{n_2}}$$

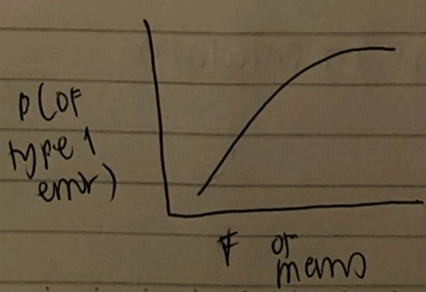
⊗ Robustness simulation

⊗ t distribution demo

c) Pairwise comparisons

- eg. Smiles & Leniency
- t-test for each group mean
- chances to make a type I error
- pairwise comparisons — between pairs of means
  ↳ function of the # of means





- Type I error can be controlled using Tukey Honestly Significant Difference Test or Tukey HSD → Based on a random · of the & distribution · that takes into account the # of means being compared
→ studentized range distribution
- steps:
  1. compare the means & var of each group
  2. compute the MSE
  3. compute

$$Q = \frac{M_i - M_j}{\sqrt{\frac{MSE}{n}}}$$

  4. compute each p for each comparison using the Studentized Range calculator
- Assumption:
  1. Normality
  2. Homogeneity of variance
  3. independent observations
- ANOVA → analysis of variance
  - computes MSE used for calculation of t-test
- MS = mean squared
- Tukey's test need not be a follow up to ANOVA

◦ computations for unequal sample sizes
  1. Find SSE

$$SSE = \sum (X - M_1)^2 + \sum (X - M_2)^2 + \cdots \sum (X - M_k)^2$$

  2. compute dfe

$$dfe = N - k$$

  3. $MSE = \frac{SSE}{dfe}$
  4. For each comparison of means, use harmonic mean. (n_h)

(4)

D) specific comparisons (independent groups)

* linear combination
- more complex comparison of means
- "planned comparisons"
- eg. self esteem
- Doing a significance test for diff in means:
  (1) Express diff in linear combination.
  - multiply each by 0.5 then add

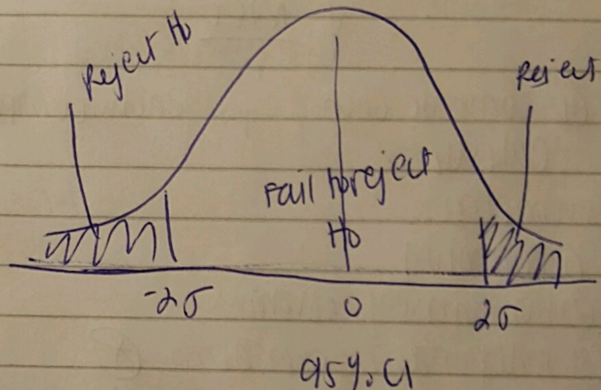
$$\mathcal{L} = 3.667 + 2.750 - 2.417 - 3.917$$

- if H_0 is rejected in favor of H_A, then the result is statistically significant

- p (parameter for the population proportion)

$$\mathcal{L} = \sum c_i M_i$$

↑ coefficient ↖ mean



Testing L significance:

$$t = \frac{\mathcal{L}}{\sqrt{\frac{\sum c_i^2 MS_E}{n}}}$$

$df = N - k$

- difference b/n differences ⟶ testing interaction

• multiple comparison
  - more comparisons = more chance of Type I error
  (1) per comparison error rate ⟶ α, c # of comparisons
  (2) family wise error rate ⟶ $\underbrace{FW \leq c\alpha}_{\text{Bonferroni inequality}}$

  • $\frac{\alpha}{c}$ → Bonferroni correction

• ORTHOGONAL COMPARISONS
  - "independent comparisons" → sum of the products of coefficients = 0

E) CORRELATED PAIRS

- ADHD Treatment case study
  - counterbalancing → method of avoiding confounding among variables
    eg. order of presentation
  - delay of gratification task
  - we don't have independent groups → correlated
- → correlated t test | related-pair t test
  ① compute diff b/n the 2 scores,
  ② test of a single mean
- each subject is their "own control" → keeps differences b/n subjects from entering into the analysis
- Details about the SE of the Difference Between Means
  - variance sum law:

$$s^2_{x \pm y} = s^2_x + s_y^2 \pm 2 r s_x s_y$$

※ correlated t demo.

F) SPECIFIC COMPARISONS (CORRELATED OBSERVATIONS)

- Weapons & Aggression case study
  - is the mean value significantly different?

$$t = \frac{M - \mu}{s_M}$$

  - priming effect
- multiple comparisons
- orthogonal comparisons
  - correlate them directly

G) PAIRWISE CORRELATED OBSERVATIONS

- one group w/ several scores from several subjects → Tukey makes an assumption that's unlikely to hold: the var of diff of scores is the same for all pairwise diff b/n means

H) ~~STATISTICAL LITERACY~~

Chapter 13: Power

® Part Demo 1 & 2

A) Introduction

- Power: the probability of correctly rejecting a false null hypothesis

power = $1 - \beta$

↑

Probability of failing to reject a null hypothesis

B) Example Calculations

- significance test based on the Binomial distribution

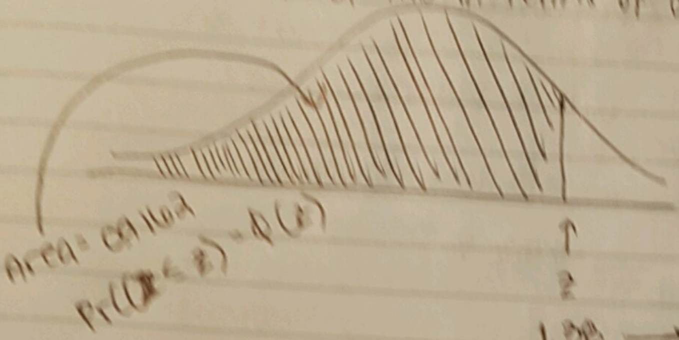
C) Factors Affecting Power

- sample size ⟶ ↑N = ↑P
- SD ⟶ ↓SD = ↑P
- Diff b/n hypothesized & true mean
  ↑effect = ↑P
- Significance level
  ↓ sig = ↓ power
- one vs 2 tailed tests
  P ↑ one tailed than 2 tailed as long as H is right

D) Statistical Literacy

The Standard Normal Distribution (PROBABILITY)

- normal distribution w/ $\mu = 0, \sigma = 1$ 03/19/19
- z used to denote random variable that has a standard normal distribution
  - ↳ think of this in terms of σ rather than "units"



- standard normal probability table gives the probability that a standard normal random variable Z is less than any given number z

1.88 → look for this value in the standard normal probability table to find the area

- word problems
  Given μ, σ, and a value x / probability to find
  ① standardize the #'s
  $$z = \frac{X - \mu}{\sigma}$$
  eg. $z = -1.67$

  z = the # of standard deviations below the mean
  ② check the z table to find z
  $\Phi(z) = \%$
  $\Phi(-1.67) = 0.0475$ → there is about a 5% probability that Z will be less than -1.67.

- types of probabilities:
  - less than z — $\Phi(z)$ → Φ = the standard normal cumulative dist function.
  - greater than z — $1 - \Phi(z)$
  - between z_1 & z_2 — $\Phi z_1 - \Phi z_2$
  - not b/n z_1 & z_2 — $1 - (\Phi z_1 - z_2)$

→ the # of times that something happens out of n independent trials

→ The Normal Approximation to the Binomial

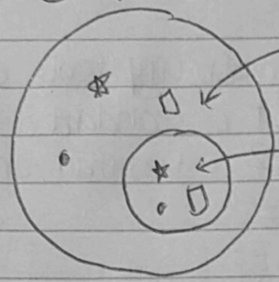
- normal can closely approx binom. ↳ not always symmetric (eg. $\pi \neq 0.5$)
  whenever binomial n is large and prob π not too close to 0 or 1
- this is useful bc we can use the simpler z formula w/ binom.

Binomial Prob	Approximation on normal
- exactly a	b/n a−0.5 and a+0.5
- b/n a and b	b/n a−0.5 & b+0.5

$\mu = n\pi$, $\sigma = \sqrt{n\pi(1-\pi)}$

Random Sampling (statistical inference)

- Benefit:
  1. will ensure a well-represented & unbiased sample
  2. will characterize diff b/n sample & population



POPULATION
N

SAMPLE n — with Replacement / without Replacement

※ ensure this is representative → "same %"

- "sample statistic" — any # computed from sample data
  — random variable (bec obtained thru random sampl)
  — "known & random"
- "population parameter" — fixed # (no randomness involved)
  — "unknown & fixed"

"estimator" of the population parameter, while "estimate" is the actual # computed from the data

"unbiased estimator" = neither systematically too high or too low

- simple Random sample

→ The sampling Distribution & Central Limit theorem

- sampling distribution of the statistic
  ↳ probability distribution
  - info about sample (have) → info about population (want)
- individual measurement vs average/sum of measurements
- central Limit theorem:
  - distributions become more and more normal as n gets large for both the average & the sum
  - means & sd:

	Ave	sum total
mean	$\mu_{\bar{x}} = \mu$	$\mu_{sum} = n\mu$
sd	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$\sigma_{sum} = \sigma\sqrt{n}$

- The standard error
  - → "estimated standard deviation"
  - the standard error of the statistic → how far the observed value of the statistic is from the mean
    - ↳ approximates the σ of taking a large # of samples, finding μ for each, and working w/ the sample averages as a dataset.
  - "amt of uncertainty in a summary number (statistic) representing the entire sample
    - ↳ vs standard deviation → amt of variability among individuals
  - comparing the sample average to the population mean:

$\bar{X}$ → sample average

S = standard dev of sample → uncertainty in $\bar{X}$

$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ } standard deviation of the average → we don't know this in statistics, but we do in probability

$S_{\bar{X}} = \frac{S}{\sqrt{n}}$ } SE of ave, so we use this
— how far sample ave $\bar{X}$ is from pop mean μ

S → individuals, $S_{\bar{X}}$ → averages

	POPULATION	SAMPLE
variability of individuals	σ	S
var of $\bar{X}$, the ave of n	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$	$S_{\bar{X}} = \frac{S}{\sqrt{n}}$

⊛ SMALL POPULATIONS — Adjusted SE:
- Apply the finite-population correction factor

$$\sqrt{\frac{(N-n)}{N}}$$

- so adjusted SE $= \sqrt{\frac{N-n}{N}} \times \frac{S}{\sqrt{n}}$

→ Standard error for Binomial Proportion:

	X (number of occurences)	$p = \frac{X}{n}$ (proportion %)
SD (pop)	$\sigma_X = \sqrt{n\pi(1-\pi)}$	$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$
SE (estimated from sample)	$S_X = \sqrt{np(1-p)}$	$S_p = \sqrt{\frac{p(1-p)}{n}}$

Confidence Intervals (statistical inference)

- tradeoff for higher confidence is wider confidence interval, which might not be as useful
- "95% sure that the population parameter is somewhere between the estimator minus 2 SE, and the estimator + 2 SE"

→ The confidence interval for a population mean or a population %

- "the probability that the ~~sample average~~ population mean is within 1.960 standard deviations of the sample average from the sample average is 0.95"
  - "the P(x) that the pop. mean is within 1.96 SE from the sample ave is approximately 0.95"

→ "We are 95% sure that the population mean μ is somewhere between $\bar{X} - tS_{\bar{X}}$ and $\bar{X} + tS_{\bar{X}}$," where t is taken from the t-table for 2-sided 95% confidence.

- degrees of freedom: the # of independent pieces of information in your standard error

→ Interpreting a confidence interval

- can't have a probability without (random) experiment
- one-sided confidence interval
- 2-sided confidence interval

① compute t

② find t value from ~~con~~ confidence level

Hypothesis Testing

- uses data to decide b/n 2 possibilities (hypotheses) → confidence or not?
- way of using statistics to make decisions
- hypothesis → statement about the population
  - NULL HYPOTHESIS
    H_0: default possibility that you will accept unless you have convincing evidence to the contrary
  - RESEARCH HYPOTHESIS
    H_1: accepted only if there's convincing statistical evidence that would rule out the null hypothesis at a reasonable prob.
    "alternative hypothesis"
- Results of a hypothesis test:
  1. Accept null H_0 → weak conclusion → innocent
  2. Reject null H_0 → strong conclusion. → guilty

- Testing the population mean against a known Reference value
  - Reference value: μ_0 that doesn't come from the sample data

$$\left.\begin{array}{l} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{array}\right\} \text{2-sided test}$$

- Another way to carry out 2-sided test of population mean is to compute the t-statistic $\left(\frac{\bar{x} - \mu_0}{S_{\bar{x}}}\right)$, and then use the t-table to decide which hypothesis to accept.