

Joint Distribution. Discrete Case.

Hair Eye	Black	Brunette	Red	Blond	Marginal eye col.
Brown	0.11	0.20	0.04	0.01	0.36
Blue	0.03	0.14	0.03	0.16	0.36
Hazel	0.03	0.09	0.02	0.02	0.16
Green	0.01	0.05	0.03	0.03	0.12
Marginal hair col.	0.18	0.48	0.12	0.22	1

$$\rightarrow P\{\text{Eyes} = \text{Brown}, \text{Hair} = \text{Blond}\} = 0.01$$

$$\rightarrow P\{\text{Eyes} = \text{Blue}\} = 0.36$$

$$\rightarrow P\{\text{Hair} = \text{Blond}\} = 0.22$$

Eye colors: e_1, \dots, e_m

Hair colors: h_1, \dots, h_n

Joint Distribution

$$P\{E=e_i, H=h_j\} = P_{ij}$$

Marginal Distr.

$$P\{E=e_i\} = \sum_j P_{ij} = P_{i\cdot}$$

$$P\{H=h_j\} = \sum_i P_{ij} = P_{\cdot j}$$

$$\text{Independence: } P_{ij} = P_{i\cdot} \cdot P_{\cdot j}$$

Conditional Distribution. Discrete Case.

Hair Eye	Black	Brunette	Red	Blond	Marginal eye col.
Brown	0.11	0.20	0.04	0.01	0.36
Blue	0.03	0.14	0.03	0.16	0.36
Hazel	0.03	0.09	0.02	0.02	0.16
Green	0.01	0.05	0.03	0.03	0.12
Marginal hair col.	0.18	0.48	0.12	0.22	1

$$E = \text{Brown} | H = \text{Red}$$

$$\frac{0.04}{0.12} = 0.33$$

$$E = \text{Blue} | H = \text{Red}$$

$$\frac{0.03}{0.12} = 0.25$$

$$E = \text{Hazel} | H = \text{Red}$$

$$\frac{0.02}{0.12} = 0.17$$

$$E = \text{Green} | H = \text{Red}$$

$$\frac{0.03}{0.12} = 0.25$$

$$\frac{0.12}{0.12} = 1$$

What is the probability distribution of hair color of a person if her eyes are blue?

$$\frac{0.03}{0.36} = 0.08 \quad \frac{0.14}{0.36} = 0.39 \quad \frac{0.03}{0.36} = 0.08 \quad \frac{0.16}{0.36} = 0.45 \quad \frac{0.36}{0.36} = 1$$

$$\begin{aligned} P\{H = \text{Black} | E = \text{Blue}\} \\ P\{H = \text{Brunette} | E = \text{Blue}\} \\ P\{H = \text{Red} | E = \text{Blue}\} \\ P\{H = \text{Blond} | E = \text{Blue}\} \end{aligned}$$

$$P\{H|E\} = \frac{P\{H,E\}}{P\{E\}}$$

Row i : r_i . Column j : c_j .

$$P\{r_i | c_j\} = \frac{P\{r_i, c_j\}}{P\{c_j\}} = \frac{P\{r_i, c_j\}}{\sum_i P\{r_i, c_j\}}$$

$$P\{c_j | r_i\} = \frac{P\{c_j, r_i\}}{P\{r_i\}} = \frac{P\{c_j, r_i\}}{\sum_j P\{r_i, c_j\}}$$