C15Q22

$Y'_j = \mu + \alpha_j B_j$

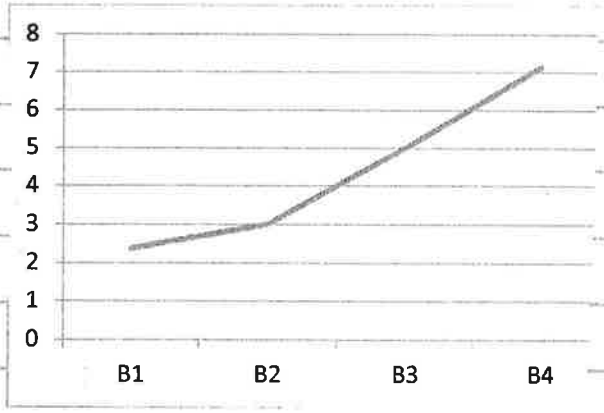
$\sum \alpha = 0$

H0: all means are equal

omnibus test (1)

B1	B2	B2	B4
1	2	3	4
3	2	4	5
4	4	2	6
5	5	6	8
1	2	4	8
1	3	6	9
2	2	7	9
2	4	8	8

$n = 8$ $N = 32$



means

2.375 3 5 7.125

$4.375 = \bar{X}_{..}$

$$SST = \sum_j^K \sum_i^n (X_{ij} - \bar{X}_{..})^2 = (1-4.375)^2 + (3-4.375)^2 \ldots (8-4.375)^2 = 191.5$$

$$SSB = n \sum_{j=1}^K (\bar{X}_{.j} - \bar{X}_{..})^2 = 8\left((2.375-4.375)^2 + (3-4.375)^2 \ldots (7.125-4.375)\right)$$

$= 110.75$

$SST = SSB + SSE$

$SSE = 191.5 - 110.75 = 80.75$

$$SE = \sum\sum (X_{ij} - \bar{X}_{.j})^2 = 80.75$$

Source	df	SS	MS	F	p
B	3	110.75	36.92	12.82	0.000
Error	28	80.75	2.88		
Total	31				

C1522

Multiple Comparisons.

Family wise error rate.

α : α x # of comparisons :

$$\binom{4}{2} = \frac{4!}{2!\,2!} = 6$$

$$\alpha = 0.05 \times 6 = .30$$

Bonferroni → reject H_i if $p_i \leq \frac{\alpha}{m}$

$$0.05 = \alpha \cdot 6$$

$$\alpha = \frac{.05}{6} = 0.008$$

for two tailed = .004 df : 28

$$SE = \sqrt{\frac{MSE}{n}} = \sqrt{\frac{2.88}{8}} = 0.6$$

Critical $t_{(.004,\,28)} = -2.85$

$t_{(.004;28)} \times SE = 1.71$

1 and 2 ~~3 - 2.375~~

2 and 1 → 2.375 − 3 = 0.625 → [−1.085, 2.33]

3 and 2 → 5 − 3 = 2 → [0.29, 3.71]

4 and 2 → 7.125 − 3 = 4.125 → [2.415, 5.835]

3 and 1 → 5 − 2.375 = 2.625 → [0.915, 4.33]

4 and 3 → 7.125 − 5 = 2.125

4 and 1 → 7.125 − 2.375 = 4.75

C15Q17 - Reading Comprehension

	12-year old	16-year old	
30 min	66	74	
	68	71	
	59	67	68.3
	72	82	
	46	76	
60 min	69	95	
	61	92	
	69	95	74.5
	73	98	
	61	94	
	64.6	84.4	74.5
	B (j)		

$n = 5$

$N = 20$

Y: Reading comprehension

A(k)			
	62.6	74	68.3
	66.6	84.4	80.7
	64.6	84.4	74.5

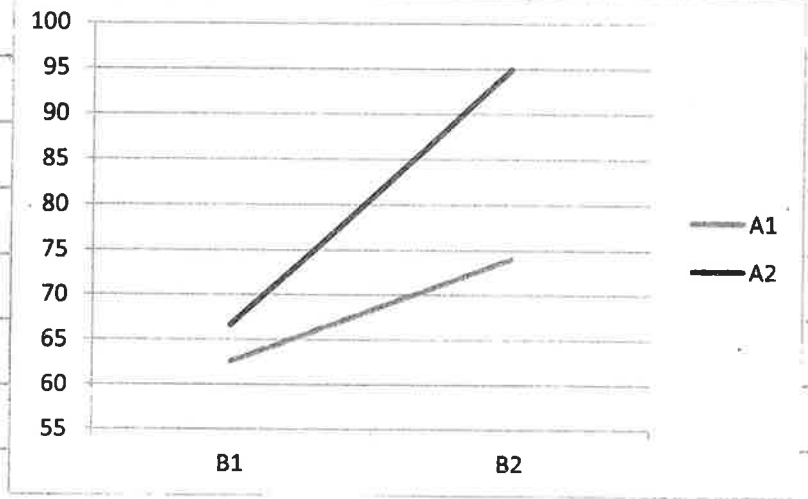
$$Y'_{jk} = \mu + \alpha_k A_k + \beta_j B_j + \alpha\beta_{jk}(A_k B_j)$$

$\Sigma \alpha_k = 0 \quad \Sigma A_j = 0 \quad \Sigma\Sigma$

$H_{0_1}: A_1 - A_2 = 0$ A main effect

$H_{0_2}: B_1 - B_2 = 0$ B main effect

H_{0_3}: Interaction effect is zero



C1517

$$SST = \sum_i \sum_j \sum_k (x_{ijk} - \bar{x}_{..})^2 = (66 - 74.5)^2 + (68 - 74.5)^2 \ldots (94 - 74.5)^2$$
$$= 3697$$

$$SSA = 10 \times \sum_{k=1}^{2} (\bar{x}_{.k} - \bar{x}_{..})^2 = 10 \times \left[(68.3 - 74.5)^2 + (80.7 - 74.5)^2\right]$$
$$= 768.8$$

$$SSB = 10 \times \sum_{j=1}^{2} (\bar{x}_{j.} - \bar{x}_{..})^2 = 10 \times \left[(64.6 - 74.5)^2 + (84.4 - 74.5)^2\right]$$
$$= 1960.2$$

$$SSE = \sum_i \sum_J \sum_K (x_{ijk} - \bar{x}_{jk})^2 = (66 - 62.6)^2 + (68 - 62.6)^2 \ldots (94 - 94.8)^2$$
$$= 615.2$$

$$SSAB = SST - (SSE + SSA + SSB) = 352.8$$

$$= \left[\sum_j \sum_k (\bar{x}_{jk} - \bar{x}_{.k} - \bar{x}_{j.} + \bar{x}_{..})\right] \times 5$$

$$MSA = \frac{SSA}{K-1} = \frac{768.8}{1} = 768.8$$

$$MSB = \frac{SSB}{J-1} = \frac{1960.2}{1} = 1960.2$$

$$MSAB = \frac{SSAB}{(K-1)(J-1)} = \frac{352.8}{1} = 352.8$$

$$MSE = \frac{SSE}{N - K - J - (J-1)(K-1) + 1} = \frac{615.2}{16} = 38.45$$

C15Q17

$$F = \frac{MSA}{MSE}$$

Source	df	SS	MS	F	P
Age	1	1960.2	1960.2	50.98	< .001
Time	1	768.8	768.8	19.99	< .001
Age x Time	1	352.8	352.8	9.17	0.03
Error	16	615.2	38.45		
Total	19	3697			

C15Q25

A

$Y_{ij} = \mu + \alpha_j + \beta_i$

Subjects	T1	T2	T3	
1	4	6	7	5.67
2	3	7	7	5.67
3	2	8	5	5
4	1	4	7	4
5	4	6	9	6.3
	2.8	6.2	7.00	5.33

$n = 5$

$N = 5$

H0: None of the pairs are different from each other

$$SST = \sum_i \sum_j (x_{ij} - \bar{x}_{..})^2 = (4-5.33)^2 + (3-5.33)^2 \dots (9-5.33^2)$$

$$= 73.33$$

$$SSA = 5 \times \sum_{j=1}^{3} (\bar{x}_{.j} - \bar{x}_{..})^2 = (2.8-5.33)^2 + (6.2-5.33)^2 + (7-5.33)^2$$

$$= 49.73$$

$$SS_{Subject} = 3 \times \sum_{i=1}^{5} (\bar{x}_{i.} - \bar{x}_{..})^2 = (5.67-5.33)^2 + (5.67-5.33)^2 + \dots (6.3-5.3$$

$$= 6.33$$

$$SSW = \sum_j \sum_i (x_{ij} - \bar{x}_{.j})^2 = (4-2.8)^2 + (3-2.8)^2 + \dots + (9-7)^2$$

$$= 23.6$$

$$SSW = SS_{subject} + SS_{error}$$

$$SS_{error} = SSW - SS_{subject} = 23.6 - 6.33 = 17.27$$

Source		SS	MS	F	
Trial	2	49.73	24.86	11.51	0.01 = F(2,8)
SS	4	6.33			
within	12	23.6			
Error	8	17.27	2.16		

C15Q25

$$MS_{error} = \frac{SS_{error}}{(n-1)(J-1)} = \frac{17.27}{8} = 2.16$$

b) $\alpha = .01/3 = .003$

$$SE = \sqrt{\frac{2\,MSE}{n}} = \frac{2 \times 2.16}{5} = 0.93$$

$T1 - T2 = -3.4$ $\quad t = \frac{-3.4}{0.93} = -3.56$ $\quad p = 0.004$

$T1 - T3 = -4.2$ $\quad = -4.5$ $\quad \boxed{p = 0.001}$

$T2 - T3 = -0.8$ $\quad = -0.86$ $\quad p = 0.21$

c)

1	2	3	
-1	0	1	Linear
1	-2	1	Quadratic

$L1 = -1\bar{x}1 + 0\bar{x}2 + 1\bar{x}3 = -2.8 + 7 = 4.2$

$L2 = 1\bar{x}1 - 2\bar{x}2 + 1\bar{x}3 = 2.8 - 2 \times 6.2 + 7 = -2.6$

$$SS_{L1} = \frac{n\left(\sum_{j=1}^{3} c_j \bar{x}_j\right)^2}{\sum c_j^2} = \frac{5 \times (4.2)^2}{2} = 44.1 = MS_{L1}$$

$p < .001$ $\quad F_{(1,8)} = \frac{44.1}{MSE} = \frac{44.1}{2.16} = 20.42$

$$SS_{L2} = \frac{n\left(\sum c_j \bar{x}_j\right)^2}{\sum c_j^2} = \frac{5 \times (-2.6)^2}{6} = 5.63$$

$F = 2.6$ $\quad p = .15$