For any inquiries contact the author, Yuri Balasanov, at ybalasan@uchicago.edu or yuri.balasanov@research-soft.com .

Probability Space.

Every random experiment is associated with the object $\{\Omega, \mathcal{F}, P\}$ called Probability Space.

Ω - all elementary outcomes.

$\mathcal{F}$ - all possible events related to elementary outcomes

P - probability measure that assigns probability number to each event.

Example. Tossing a fair coin once.

$\Omega = \{\text{"Heads"}, \text{"Tails"}\}$

$\mathcal{F} = \{\emptyset, \text{"Heads"}, \text{"Tails"}, \Omega\}$

$P = \{\emptyset\} \doteq 0;\ P\{\text{"Heads"}\} = P\{\text{"Tails"}\} = \frac{1}{2};\ P\{\Omega\} \doteq 1.$

Event $\emptyset$ includes all "impossible" events (the coin overcomes the force of gravity and goes into space).

Event Ω means that either "Heads" or "Tails" occurs.

Coin tossed 2 times. What is the probability space?

Let the outcomes of each toss be: "H"=0, "T"=1.

The probability space $\{\Omega, \mathcal{F}, P\}$ is:

$\Omega = \{00, 01, 10, 11\}$.

either both Heads or both Tails

$\mathcal{F} = \{ \emptyset, 00, 01, 10, 11, 00+01, 00+10, 00+11, 01+10, 01+11, 10+11,$

$00+01+10, 00+01+11, 01+10+11, 00+01+10+11 \}$

$P = \{ 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2},$

$\frac{3}{4}, \frac{3}{4}, \frac{3}{4}, 1 \}$

A coin is tossed 2 times. Define random variables associated with this experiment

$\Omega = \{HH, HT, TH, TT\}$.

1. R.V. $X: \Omega \to R$ is the number of Tails in the first toss.

$$X = \begin{cases} x_0 = 0 : HH, HT, \\ x_1 = 1 : TH, TT. \end{cases}$$

2. R.V. $Y: \Omega \to R$ is the total number of Tails.

$$Y = \begin{cases} y_0 = 0 : HH \\ y_1 = 1 : HT, TH \\ y_2 = 2 : TT \end{cases}$$

Marginal distributions:

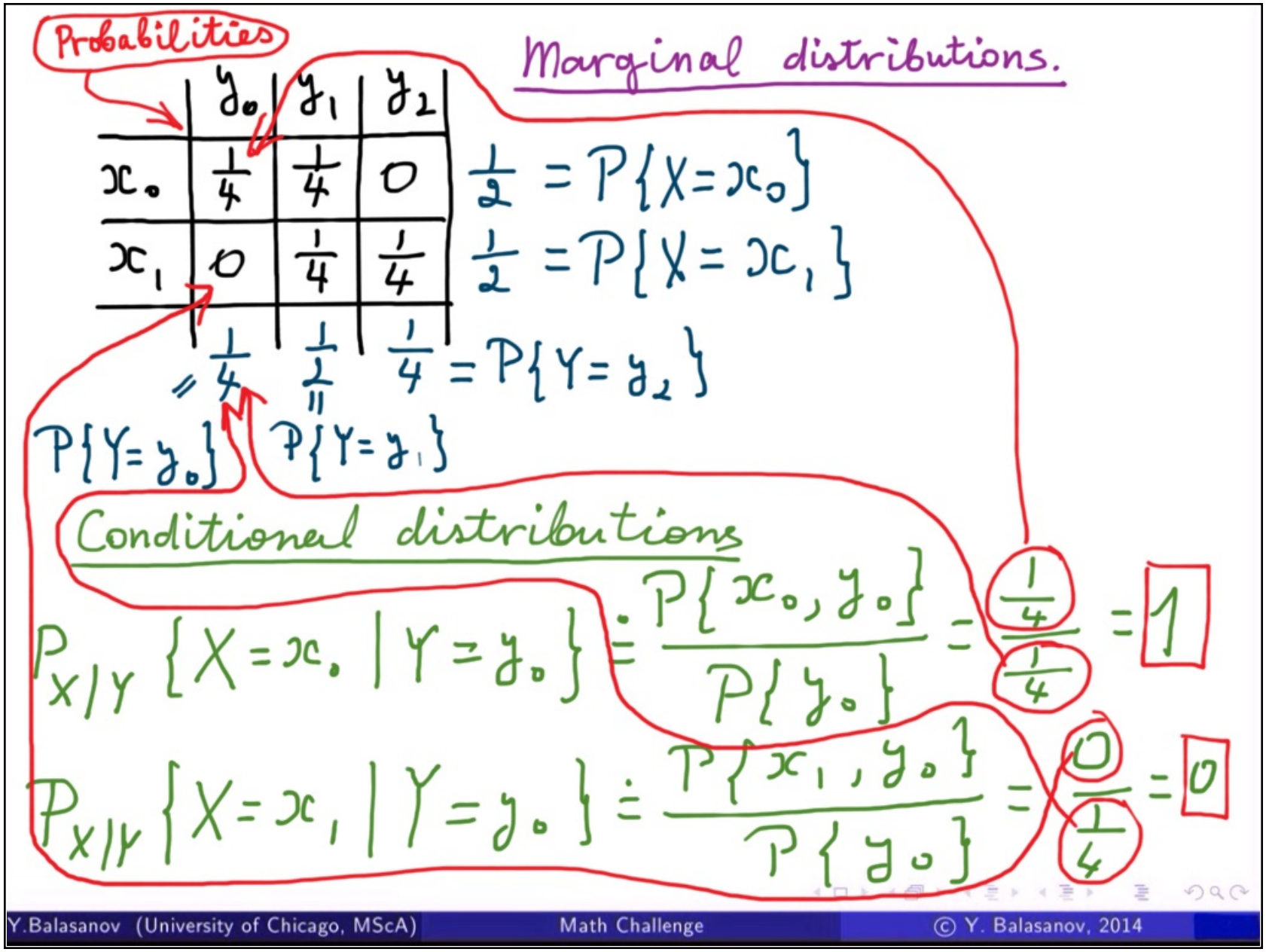
$P\{X=x_0\} = P\{X=x_1\} = \frac{1}{2}$; $P\{Y=y_0\} = P\{Y=y_2\} = \frac{1}{4}$; $P\{Y=y_1\} = \frac{1}{2}$.

Joint distribution: (Events)

	y_0	y_1	y_2
x_0	HH	HT	$\varnothing$
x_1	$\varnothing$	TH	TT

$P\{X=x_0, Y=y_0\} = P\{X=x_0, Y=y_1\} = \frac{1}{4}$,
$P\{X=x_1, Y=y_1\} = P\{X=x_1, Y=y_2\} = \frac{1}{4}$,
$P\{X=x_0, Y=y_2\} = P\{X=x_1, Y=y_0\} = 0$.

Y.Balasanov (University of Chicago, MScA) Math Challenge


Probabilities
Marginal distributions.
y_0 y_1 y_2
x_0 $\frac{1}{4}$ $\frac{1}{4}$ 0 $\frac{1}{2} = P\{X = x_0\}$
x_1 0 $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2} = P\{X = x_1\}$
$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{4} = P\{Y = y_2\}$
$= P\{Y = y_0\}$ $= P\{Y = y_1\}$
Conditional distributions
$P_{X|Y}\{X = x_0 \mid Y = y_0\} = \frac{P\{x_0, y_0\}}{P\{y_0\}} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$
$P_{X|Y}\{X = x_1 \mid Y = y_0\} = \frac{P\{x_1, y_0\}}{P\{y_0\}} = \frac{0}{\frac{1}{4}} = 0$
Y.Balasanov (University of Chicago, MScA)
Math Challenge
© Y. Balasanov, 2014