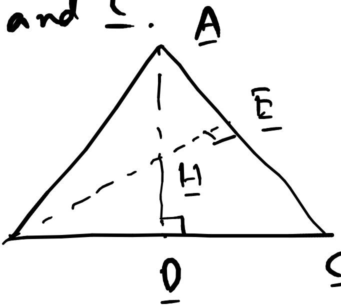
Problem: To find the orthocentre of ΔABC given $\underline{A}$, $\underline{B}$ and $\underline{C}$.



Solution

Let the altitudes AD and BE meet at $\underline{H}$. We need to find $\underline{H}$.

The direction vector of BC $= \underline{m}_{BC}$

$$= \underline{B} - \underline{C}.$$

The normal vector of AD $= \underline{n}_{AD} = \underline{m}_{BC}$

since $AD \perp BC$.

Hence, the equation of AD is

$$\underline{n}_{AD}^{t} (\underline{x} - \underline{A}) = 0$$

$$\Rightarrow (\underline{B} - \underline{C})^{t} (\underline{x} - \underline{A}) = 0$$

$$\Rightarrow (\underline{B} - \underline{C})^{t} \underline{x} = (\underline{B} - \underline{C})^{t} \underline{A}. \quad \text{--- } (1)$$

Similarly, the equation of BE is

$$(\underline{C} - \underline{A})^{t} \underline{x} = (\underline{C} - \underline{A})^{t} \underline{B} \quad \text{--- } (2)$$

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① and ② can be written as

$$\begin{pmatrix} \underline{B}-\underline{C} & \underline{C}-\underline{A} \end{pmatrix}^t \underline{x} = \begin{bmatrix} (\underline{B}-\underline{C})^t \underline{A} \\ (\underline{C}-\underline{A})^t \underline{B} \end{bmatrix}$$

$$\Rightarrow \underline{x} = \begin{pmatrix} \underline{B}-\underline{C} & \underline{C}-\underline{A} \end{pmatrix}^{-t} \begin{bmatrix} (\underline{B}-\underline{C})^t \underline{A} \\ (\underline{C}-\underline{A})^t \underline{B} \end{bmatrix}$$

$= \underline{H}$ - orthocentre

Now, you can do the numerical computations to obtain $\underline{H}$.

Note that we are using only matrix algebra here.

This is the approach that should be used to solve every problem.

Use python for computing the theoretical solution and verifying it by plotting the figures.