

EE5327 : Optimization

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Question

Plot the circles

$$f(\mathbf{x}) = (x_1 - 8)^2 + (x_2 - 6)^2 = r^2 \quad (1)$$

$\mathbf{x} = (x_1, x_2)^T$, for different values of r along with the line

$$g(\mathbf{x}) = x_1 + x_2 - 9 = 0 \quad (2)$$

From the graph, find

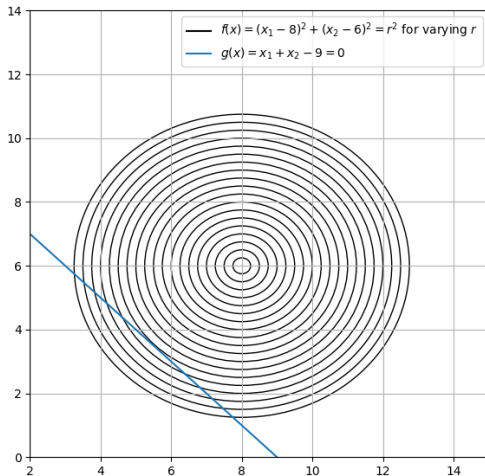
$$\begin{aligned} \min_{\mathbf{x}} f(\mathbf{x}) \quad \text{s.t.} \\ g(\mathbf{x}) = x_1 + x_2 - 9 = 0 \end{aligned}$$

Also obtain a theoretical solution for the problem above using coordinate geometry.

Graphical Approach

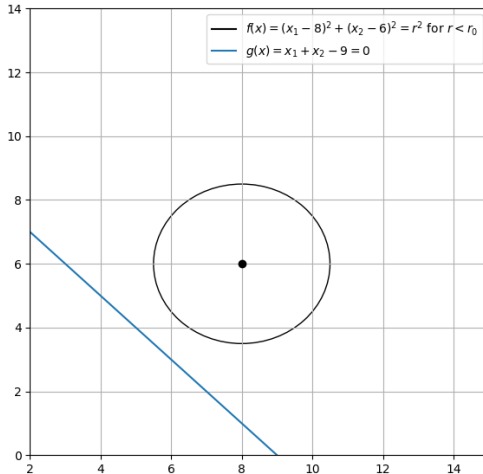
Graphical Approach

Plotting for different values of r



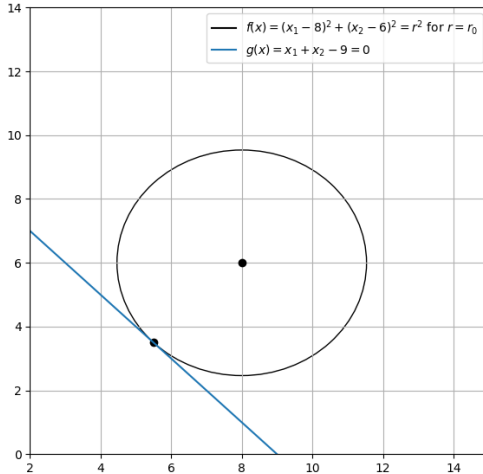
GRAPHICAL APPROACH

From the graph, find minimum value of $f(\mathbf{x})$ satisfying $g(\mathbf{x}) = 0$



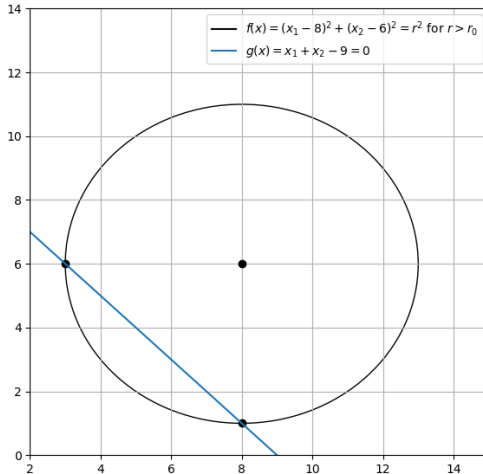
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From graph,

$$r^2 = (x_1 - 8)^2 + (3 - x_1)^2 \quad (3)$$

$$= 2x_1^2 - 22x_1 + 73 \quad (4)$$

$$\Rightarrow r^2 = \frac{(2x_1 - 11)^2 + 5^2}{2} \quad (5)$$

This is minimum at $x_1 = \frac{11}{2}, x_2 = \frac{7}{2}$.

$$f(\mathbf{x})_{\min} = 25\sqrt{2}, r_{\min} = \frac{5}{\sqrt{2}}.$$

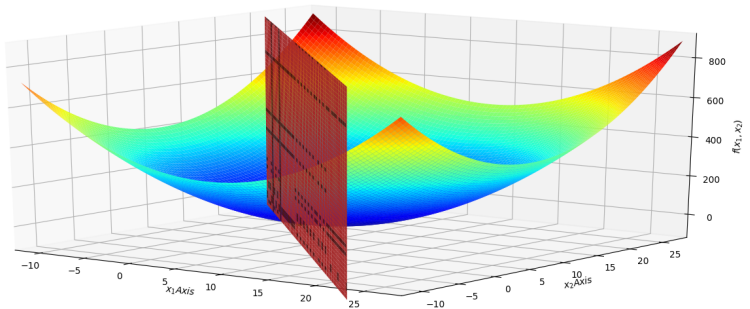
Another Graphical Approach

Another Graphical Approach

Plot Objective $z=f(x)$ and Feasible Region $g(x)=0$.

■ : Feasible Region $x_1 + x_2 = 9$

Objective $r^2 = f(x_1, x_2) = (x_1 - 8)^2 + (x_2 - 6)^2$

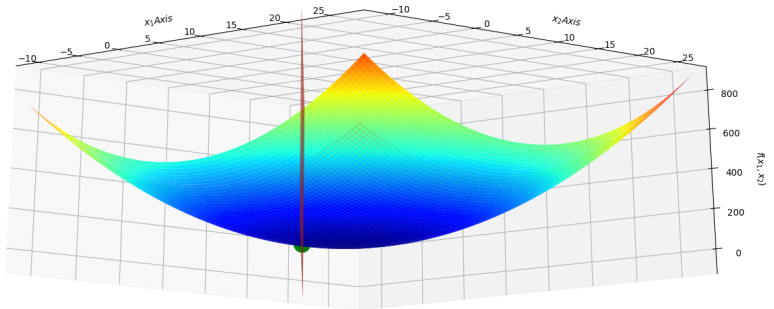


Another Graphical Approach

From graph, find minimum $z=f(x)$ such that point (x,z) lies on feasible region $g(x)=0$.

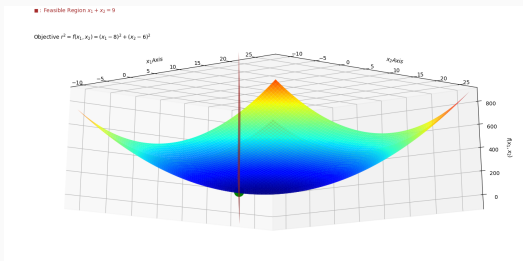
■ : Feasible Region $x_1 + x_2 = 9$

Objective $r^2 = f(x_1, x_2) = (x_1 - 8)^2 + (x_2 - 6)^2$



Another Graphical Approach

Finding Constraints for Solving :



As \mathbf{x} lies on $g(\mathbf{x}) = 0$, $x_1 + x_2 = 9$ -(i)

As $f(\mathbf{x})$ is decreasing along $-\nabla f(\mathbf{x})$, and $f(\mathbf{x})$ cannot decrease in any neighbourhood of (\mathbf{x}, z) in plane, $-\nabla f(\mathbf{x})$ should be perpendicular to plane $g(\mathbf{x}) = 0$.

$$-\nabla f(\mathbf{x})^T \cdot \begin{bmatrix} 9 \\ -9 \end{bmatrix} = 0 \text{ -(ii)}$$

Solve (i) and (ii) to get $x_1 = \frac{11}{2}, x_2 = \frac{7}{2}, f(\mathbf{x})_{\min} = \frac{25}{2}, r_{\min} = \frac{5}{\sqrt{2}}$