# SSD Clustering

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#### Abstract

# 1 Introduction

The premise of this investigation is to use a form of clustering to drive the training of deep learning models. With clusters, we hope to promote small intracluster variability and reduce the need for large capacity models by training a model for each cluster. With that a corresponding reducing in the amount of training data.

# 2 Clustering

There are two distinct set of approaches to doing clustering without training:

- 1. cluster the feature vectors of a pre-trained model
- 2. cluster the input images using:
  - (a) "2nd-order" region statistics: e.g.
    - i. region covariance 2.1 (RC).
    - ii. Sigma-set 2.1.2 is a condensed RC. One of its attraction is being able to work with difference sized regions (think patches or SuperPixels).
  - (b) some form of non-Local Means (NLM)

There the algorithm to perform the clustering also deserve some investigation. K-means is certain a starting point but it is very sensitive to initialization. Next, the question of producing a quasi balanced cluster should also be considered

If we are using a NLM approach, naive approach will make k passes for k clusters while testing for nearest center. At least we need a fast lookup structure so that the nearest center can be retrieved in O(logn) or O(1) time. Or use the very nice SigmaSet 2.1.2 or [KH10]

#### 2.1 Region Covariance

The region covariance  $\mathbf{C}_R$  is a  $d \times d$  matrix of the feature points:

$$\mathbf{C}_R = \frac{1}{n-1} \sum_{k=1}^{n} (\mathbf{z}_k - \mu) (\mathbf{z}_k - \mu)^T$$
 (1)

where  $\mu$  is the mean.  $\mathbf{C}_R$  is  $d \times d$  instead of  $n \times d$  if we are using the raw features. Also, RC does not have any information regarding the ordering and the number of points. This implies a certain scale and rotation invariance.

The covariance matrix proposes a natural way of fusing multiple features which might be correlated. The diagonal entries of the covariance matrix represent the variance of each feature and the nondiagonal entries represent the correlations.

There are several advantages of using covariance matrices as region descriptors. A single covariance matrix extracted from a region is usually enough to match the region in different views and poses. In fact we assume that the covariance of a distribution is enough to discriminate it from other distributions.

Region covariance can be computed very efficiently using 'integral images/sumarea table' [PT06, TPM06]. Let  $F(x,y) = \phi(I,x,y)$  be the  $W \times H \times d$  dimensional feature image extracted from I, where  $\phi$  can be any mapping such as intensity, color, gradients, filter response etc.

See an early application of region covariance Using covariance improves computer detection and tracking of humans.



Figure 1: an image using  $4 \times 4$  covariance. "F.E." (feature extraction), "Stat" and "COV"

#### 2.1.1 Metrics for Covariance

Covariance matrices do not lie on Euclidean space. There are two commonly used metrics - 1) log-Euclidean 2) "affine-invariant" distance [FM99]. The affine-invariant" distance is computed using generalized eigenvalues which follows from the Lie group structure of positive definite matrices (PD):

$$\rho(\mathbf{C}_1, \mathbf{C}_2) = \sqrt{\sum_{k=1}^{n} \ln^2 \lambda_i(\mathbf{C}_1, \mathbf{C}_2)}$$
 (2)

where  $\lambda_i(\mathbf{C}_1, \mathbf{C}_2)$  are the generalized eigenvalues of  $\mathbf{C}_1$  and  $\mathbf{C}_2$  computed from  $\lambda_i \mathbf{C}_1 \mathbf{x}_i - \lambda_i \mathbf{C}_2 \mathbf{x}_i$  and  $\mathbf{x}_i \neq 0$  are the generalized eigenvectors.

#### 2.1.2 Sigma set

[HCS<sup>+</sup>09] proposes a novel 2nd-order statistics based region descriptor, named "Sigma Set", in the form of a small set of vectors, which can be uniquely constructed through Cholesky decomposition on the covariance matrix. This is basically an optimized form of region covariance using sparsity.

For any matrix A that satisfies  $C_R = AA^T$ , the set of columns of A has the same 2nd order statistics as R. More specifically, we can construct the Sigma Set descriptor for region R through Cholesky decomposition ( $\mathbf{C} = \mathbf{L}\mathbf{L}^T$ ) which is unique for SPD:

$$S = \{\mathbf{L}_1, \cdots, \mathbf{L}_d, -\mathbf{L}_1, \cdots, -\mathbf{L}_d\}$$
(3)

where  $\mathbf{L}_i$  is th *i*th column of  $\sqrt{d} \times \mathbf{L}$ .

An example application of sigma-set [UMM13].

#### 2.2 Kwatra2010

"Fast Covariance Computation and Dimensionality Reduction for Sub-Window Features in Images"

## $2.2.1 \quad Faulkner 2015$

 $[{\rm FSS}^+15]$  "A Study of the Region Covariance Descriptor: Impact of Feature Selection and Image Transformations"

# 3 Super-pixels

# 3.1 SLIC

 $[{\rm ASS}^+12]$  this is a more efficient form of Superpixel. I have used this in my previous work.

# 3.2 Superpixel and salient objects

 $[\mathrm{HLL}^+15]$  "SuperCNN: A Superpixelwise Convolutional Neural Network for Salient Object Detection"

# 4 Non-Local Means

# 4.1 Qian2013

[QY13] nonlocal similarity and spectral-spatial structure of hyperspectral imagery into sparse representation. Non-locality means the self-similarity of image, by which a whole image can be partitioned into some groups containing similar patches. The similar patches in each group are sparsely represented with a shared subset of atoms in a dictionary making true signal and noise more easily separated.

## 4.2 Fu2017

[FLSS17]

# 5 K-means

# 5.1 Balanced K-means

[MF14] in k-means assignment phase, the algorithm solves the assignment problem by Hungarian algorithm with time complexity  $O(n^3)$ .

# 5.2 Balanced K-means & min-cut

[CNMY14]

# 6 SSD

## 7 Conclusion

### 8 Remarks

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