S.T $(Z^+, \times)$ is a Monoid.

Group codes

Ex: The word C: 1010110 is transmitted through a channel.
If e = 0101101 is the error pattern, find the word r received.
If P = 0.05 is the probability that a signal is incorrectly received. Find the probability with r is received.

(i) c = 1010110
e = 0101101
r = c+e → arithmatic mod 2 addition
r = 1111011

2^{nd} 4^{th} 5^{th} 7^{th}.

(ii) probability with which r is received with $P = 0.05 = \frac{5}{100}$.

$$P = P^4 (1-P)^{7-4}$$

~~5.8575×10^-6~~.

$$= 5.35 85 \times 10^{-6}$$

1) The word C = 1010110 is sent through a binary channel. If P = 0.02 is a probability of incorrect receipt of a signal. Find the probability that c is received as r = 1011111.
(Also determine error pattern.)
c = 1010110.
r = 1011111. change is 4^{th} & 7^{th}

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① probability with which r is received with $p = 0.02$

$$P = p^2 (1-p)^{7-2}$$

$$= 3.61568 \times 10^{-4}$$

② error pattern $(e = r - c)$

$$\begin{bmatrix} 1011111 \\ 1010110 \end{bmatrix} \times$$

r	=	c	+	e
1	=	1	+	0
0	=	0	+	0
1	=	1	+	0
1	=	0	+	1
1	=	1	+	0
1	=	1	+	0
1	=	0	+	1

$\therefore$ $e = 0001001$

Encoding function

(*) Let $E: Z_2^m \to Z_2^{m+1}$ be an encoding function defined as follows

$$W_{m+1} = \begin{cases} 0 & \text{if } W \text{ contains even no. of 1's} \\ 1 & \text{if } \text{" " odd no " "} \end{cases}$$

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Ex: Find the code words assigned by
(i) $E: Z^3 \to Z^4$, for 000, 001, 011, 100, 110, 101, 111, 010.
(ii) $E: Z^4 \to Z^5$ for 0000, 0001, 0101, 1111, 1010, 1100, 1101, 1001

Sol^n (i) $E: Z^3 \to Z^4$

$E(000) = 0000$
$E(001) = 0011$
$E(011) = 0110$
$E(100) = 1001$
$E(110) = 1100$
$E(101) = 1010$
$E(111) = 1111$
$E(010) = 0101$

(ii) $E: Z^4 \to Z^5$

$E(0000) = 00000$
$E(0001) = 00011$
$E(0101) = 01010$
$E(1111) = 11110$
$E(1010) = 10100$
$E(1100) = 11000$
$E(1101) = 11011$
$E(1001) = 10010$

(*) $(m, 3m)$ Encoding function

The encoding function is $E: Z^m \to Z^{3m}$

Ex: Find the code word assigned by encoding function $E: Z^m \to Z^{3m}$ for 000, 001, 010, 100, 011, 101, 110, 111

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Sol^n

size $3 \to 9$

E(000) = 000000000

E(011) = 011011011

E(010) = 010010010

E(100) = 100100100

E(001) = 001 001 001

E(101) = 101101 101

E(110) = 110110110

E(111) = 111 111111

Decoding function

(*) $(3m, m)$ decoding function

Let $D : Z^{3m} \to Z^m$ be a decoding funct^n defined by

$D(r) = S_1, S_2, S_3 \ldots S_m$

where $S_i = \begin{cases} 1 & \text{if } r_i\, r_{i+m}\, r_{i+2m} \text{ has majority 1's} \\ 0 & \text{if } r_i\, r_{i+m}\, r_{i+2m} \text{ has majority 0's} \end{cases}$

Ex: Find the decoded word assigned by $D : Z^6 \to Z^2$ for

111111, 101010, 010101, 100100, 010011, 110110, 010111, 000111

Sol^n

$D(r) = S_1 S_2$

$S_1 = \begin{cases} 1 & \text{if } R_1 R_3 R_5 \text{ has maj 1's} \\ 0 & \text{" " " " 0's} \end{cases}$

$D(111111) = 11$

$S_2 = \begin{cases} 1 & \text{if } R_2 R_4 R_6 \text{ has maj 1's} \\ 0 & \text{" " " " " " 0's} \end{cases}$

$D(101010) = 10$

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$\mathcal{D}(010101) = 01$

$\mathcal{D}(100100) = 00$

$\mathcal{D}(010011) = 01$

$\mathcal{D}(110110) = 11$

$\mathcal{D}(010111) = 01$

$\mathcal{D}(000111) = 01$

Ex: $\mathcal{D}$ $f_D : Z^9 \to Z^3$ for the following

$\mathcal{D}(r) = S_1 S_2 S_3$

$S_1 = \begin{cases} 1 \to R_1 R_4 R_7 \\ 0 \to R_1 R_4 R_7 \end{cases}$

$S_2 = \begin{cases} 1 \to R_2, R_5, R_8 \\ 0 \to \text{" " "} \end{cases}$

$S_3 = \begin{cases} 1 \to R_3 R_6 R_9 \\ 0 \to R_3 R_6 R_9 \end{cases}$

$\mathcal{D}(000000000) = 000$

$\mathcal{D}(011011011) = 011$

$\mathcal{D}(010010010) = 010$

$\mathcal{D}(100100100) = 100$

$\mathcal{D}(001001001) = 001$

$\mathcal{D}(101101101) = 101$

$\mathcal{D}(110110110) = 110$

$\mathcal{D}(111\ 111\ 111) = 111$

* **Hamming distance**

Let x and y be words in B^m where
$x = x_1 x_2 x_3 \dots x_m$ and $y = y_1 y_2 \dots y_m$
and $x_i \neq y_i$
The hamming distance between x and y is the weight of ~~x+x~~ $x+y$
It is denoted by $d(x,y) = H(x,y) = \text{weight}(x+y)$

Ex:

Find distance between x and y when
$x = 110110$ $y = 000101$

Sol:

$$\begin{array}{r} 110110 \\ 000101 \\ \hline 110011 \end{array}$$

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$H(x,y) =$ weight of $(x+y)$
$=$ weight of (110011)
$= 4$ (no. of 1's)

(ii) $x = 001100$
$y = 010110$
$\overline{\quad 011010}$

$H(x,y) =$ weight of $(x+y)$
$=$ weight of (011010)
$= 3$

Minimum distance

Minimum distance of an encoding function is $E: Z^m \to Z^n$ is minimum of the distances between all distinct pairs of codewords

Min dist $= \min(d(E(x), E(y)) \mid x, y \in B^m)$

Ex: Find min distance of $E: Z^2 \to Z^5$ encoding function $E(00) = 00000$
$E(10) = 00111$ $E(01) = 01110$
$E(11) = 11111$

Sol^n
$d(E(00), E(10)) = d(00000 + 00111) = wt(00111) = 3$
$d(E(00), E(01)) = d(00000 + 01110) = wt(01110) = 3$
$d(E(00), E(11)) = d(00000 + 11111) = wt(11111) = 5$
$d(E(10), E(01)) = d(00111 + 01110) = wt(01001) = 2$
$d(E(10), E(11)) = d(00111 + 11111) = wt(11000) = 2$
$d(E(01), E(11)) = d(01110 + 11111) = wt(10001) = 2$

Min. distance $= \min\{3, 3, 5, 2, 2, 2\}$
$= 2$

$H(x,y)$ = weight of $(x+y)$
= weight of (110011)
= 4 (no. of 1's)

(ii) x = 001100
y = 010110
011010

$H(x,y)$ = weight of $(x+y)$
= weight of (011010)
= 3

Minimum distance

Minimum distance of an encoding function is $E: Z^m \to Z^n$ is minimum of the distances between all distinct pairs of codewords

Min dist = $\min(d(E(x), E(y)) \mid x, y \in B^m)$

Ex: Find min distance of $E: Z^2 \to Z^5$ encoding function $E(00) = 00000$
$E(10) = 00111$ $E(01) = 01110$
$E(11) = 11111$

Sol^n $d(E(00), E(10)) = d(00000 + 00111) = wt(00111) = 3$
$d(E(00), E(01)) = d(00000 + 01110) = wt(01110) = 3$
$d(E(00), E(11)) = d(00000 + 11111) = wt(11111) = 5$
$d(E(10), E(01)) = d(00111 + 01110) = wt(01001) = 2$
$d(E(10), E(11)) = d(00111 + 11111) = wt(11000) = 2$
$d(E(01), E(11)) = d(01110 + 11111) = wt(10001) = 2$

Min. distance = min {3, 3, 5, 2, 2, 2}
= 2

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Q) Let $E: Z^3 \to Z^6$ defined by $E(000) = 000111$
$E(001) = 001001$ $E(010) = 010010$ $E(110) = 110001$
$E(111) = 111000$

* Parity check matrix

Generating matrix G

$$G = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mr} \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

An encoding function of G is denoted as
$E: Z^m \to Z^n$ such that
$E(b) = x$
where $b = b_1 b_2 \ldots b_m$
$x = b_1 b_2 \ldots b_m x_1 x_2 \ldots x_r$

To determine $x_1\ x_2\ x_3 \ldots$ $r + m = n$
following relations

$x_1 = b_1 a_{11} + b_2 a_{21} + \ldots + b_m a_{m1}$

$x_2 = b_1 a_{12} + b_2 a_{22} + \ldots + b_m a_{m2}$

$x_3 = b_1 a_{13} + b_2 a_{23} + \ldots + b_m a_{m3}$

$\vdots$

$x_r = b_1 a_{1r} + b_2 a_{2r} + \ldots + b_m a_{mr}$

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Ex: An encoding function $E: Z^2 \to Z^5$ is given by the generator matrix

$$G = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determine all codewords of Z^2

Soln

$Z^2 = \{00, 10, 01, 11\}$

$e(00) = 00x_1x_2x_3$

$e(10) = 10x_1x_2x_3$

$e(01) = 01x_1x_2x_3$

$e(11) = 11x_1x_2x_3$

(i) $e(00)$

$x_1 = 0.1 + 0.1 = 0$

$x_2 = 0.0 + 0.1 = 0$

$x_3 = 01 + 00 = 0$

(ii) $e(10)$

$x_1 = 1.1 + 0\times 1 = 1$

$x_2 = 1\times 1 + 0\times 1 = 1$

$x_3 = 1\times 0 + 0\times 0 = 0$

(iii) $e(01)$

$x_1 = 0\times 1 + 1\times 0 = 0$

$x_2 = 0\times 1 + 1\times 1 = 1$

$x_3 = 0\times 0 + 1\times 1 = 1$

(iv) $e(11)$

$x_1 = 1\times 1 + 1\times 0 = 1$

$x_2 = 1\times 1 + 1\times 1 = 0$

$x_3 = 1\times 0 + 1\times 1 = 1$

$e(00) = 00000$

$e(01) = 01011$

$e(10) = 10110$

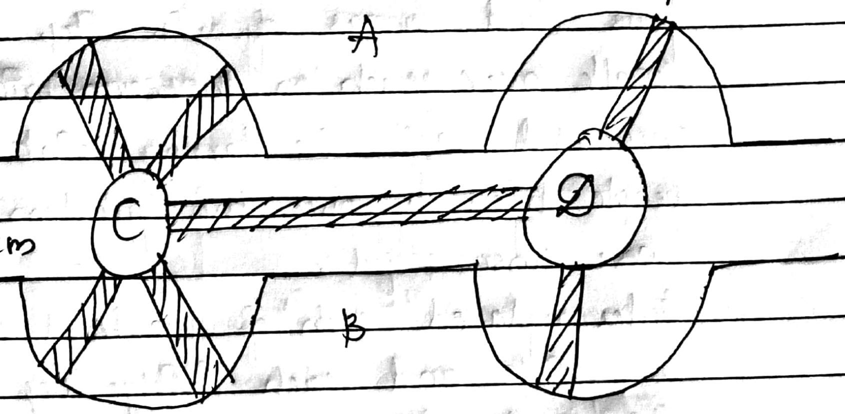
$e(11) = 11101$

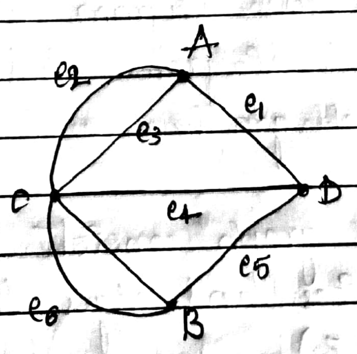
Unit 5

Graph theory

Konigsberg bridge problem (Seven bridge problem)

pregel problem





$V(G) = \{A, B, C, D, E\}$

$= \{v_1, v_2, v_3, v_4, \cancel{v_5} \ldots v_n\}$

$E(G) = \{e_1\ e_2\ e_3\ e_4\ e_5\ e_6\ e_7\}$

$= \{e_1\ e_2\ e_3 \ldots\ e_n\}$

$G(4, 7)$

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Graph theory was first introduced by Euler in 1736, through his research paper on Konigsberg bridge problem

Konigsberg bridge problem (Seven bridge problem)

Two islands C and D formed by the pregel river is connected to each other and the river lands A and B with seven bridges as shown in the figure. The problem was to start at any of the four land areas of the city A, B, C, D walk over each on the seven bridges exactly once and reach the starting point. Many people tried to solve this problem but were unsuccessful.

Euler replaced each of the land areas by a point and each bridge by a line. This produced diagram as shown. Through this diagram Euler gave the concept of graph.

Graph: A graph consists of objects called vertices and edges. It is denoted by

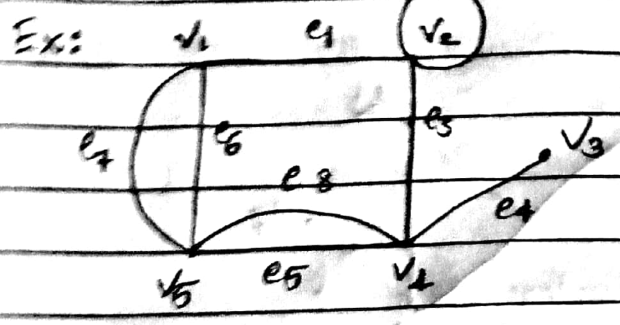
$G = G(V, E)$

where $V(G) = \{v_1\ v_2\ v_3 \ldots v_n\}$

$E(G) = \{e_1, e_2, e_3 \ldots e_n\}$

A graph G is also called as linear graph

Ex:



Vertices are called points, nodes, dots.
A vertex set of a graph G is denoted by V(G). For the graph $V(G) = \{v_1\ v_2\ v_3\ v_4\ v_5\}$

Edges: A line joining 2 vertices is called an edge. An edge can be a straight line or an arc, or a curved line or a loop.
The edge set is denoted by E(G). For this graph $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

Self-loop: An edge having same vertex as both its end vertices is called a self loop or loop.
In the graph edge e_2 is a self loop.

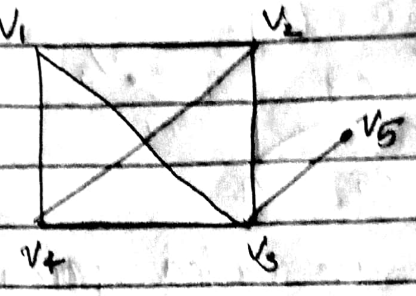
parallel edges: Given a pair of vertices, may have more than one edges. Such edges are called parallel edges.
In the graph $e_6\ e_7$ and $e_5\ e_8$ are called parallel edges.

Simple graph:

A graph without self loop and parallel edges is called a simple graph.

Ex:

G_1 :

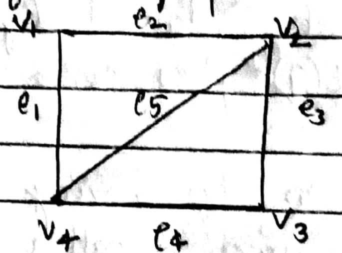


Finite graph :

Graph G having finite no of vertices and edges is called a finite graph.

G_1 is a finite graph

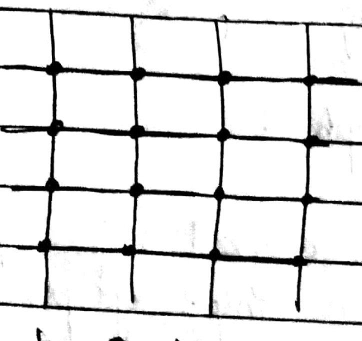
G_1 :



Infinite graph :

A graph G having infinite no. of vertices and edges is called infinite graph.

Ex:



non terminating

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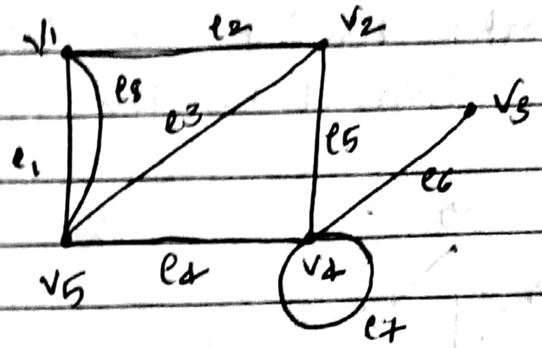
* ~~Component~~ Connected graph

A graph G is said to be a connected graph if there is atleast one path between every pair of vertices. Otherwise G is called a disconnected graph

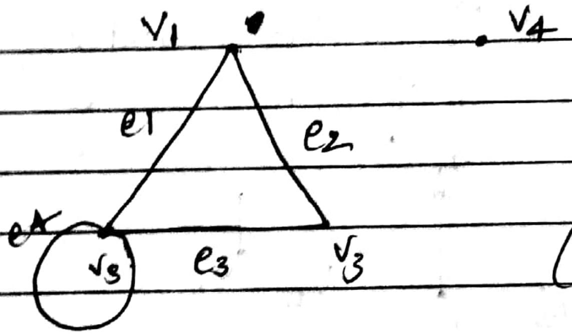
Ex: G_1:



$G_1 (5, 8)$

G_1 is called a connected graph

G_2:



$G_2 (4, 4)$

(2components disconnected graph).

G_2 is a disconnected graph

* Components of a graph

A disconnected graph has 2 or more connected subgraphs. Each of these connected subgraphs is called a component

In the above example G_2 is a 2-component disconnected graph.

* Order and size of a graph

The no. of vertices in graph G is called its order & no. of edges is called its size.

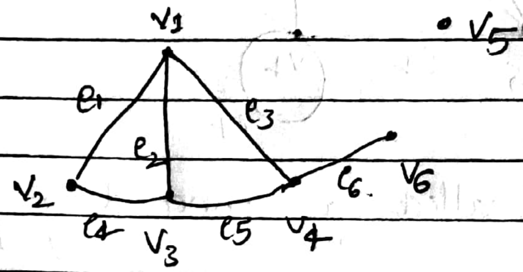
In the above example for graph G_1
order $(G_1) = O(G_1) = 5$
size $(G_1) = 6$

* Isolated vertex

A vertex having no incident edge is called isolated vertex

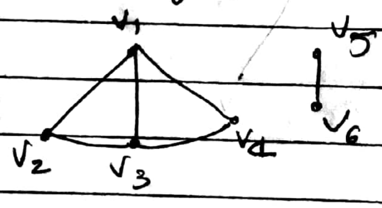
Ex :-

G_3 :



Here v_5 is isolated vertex.

(if insted $v_4 \xrightarrow{e_6} v_6$)



no isolated vertex.

* Pendant vertex (End vertex)

A vertex of degree 1 is called a pendant vertex.

(Self loop contributes 2 to same vertex).

In G_1 vertex V_3 has degree 1
$\therefore$ v_3 is pendant vertex.

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* Adjacent edges
2 non parallel edges are said to be adjacent if they incident on a common vertex.

In G_1, e_2 & e_3 are adjacent edges because they incident on same vertex v_2.
(don't consider self loop).

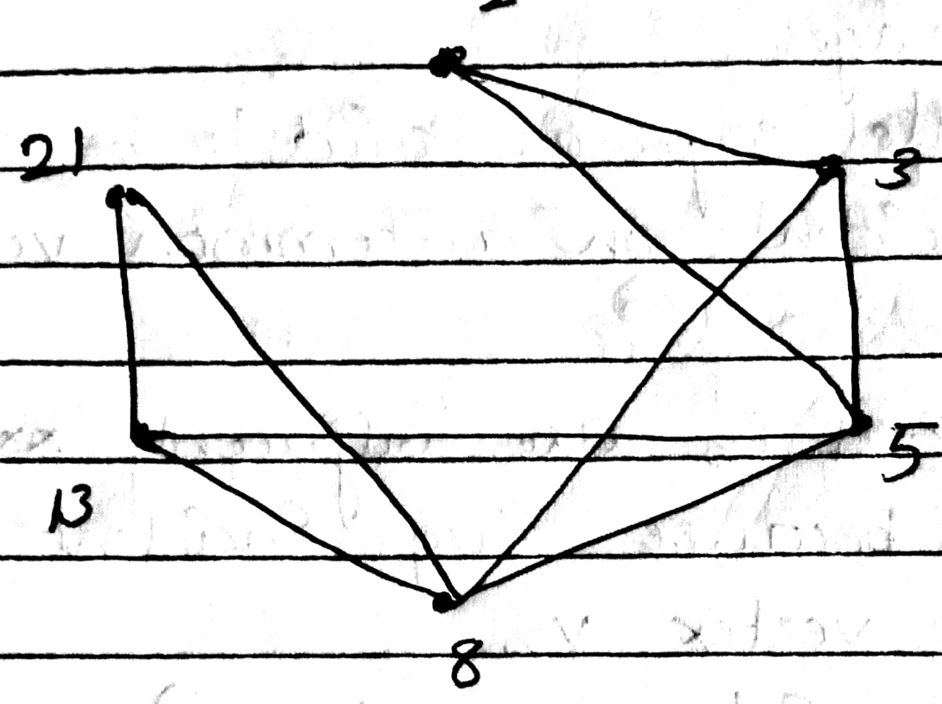
* Adjacent vertices.
2 vertices are said to be adjacent if they are end vertices of same edge

Ex: In G_1 v_1 & v_2 are end vertices of edge e_2
∴ v_1 & v_2 are adjacent vertices

v_3 & v_5 are not adjacent vertices

1) Consider set $S = \{2, 3, 5, 8, 13, 21\}$ of 6 specific fibonacci number. Write the distinct pair of integers whose absolute value of sum or difference belongs to S. Write the graph representing the system.

$V(G) = \{2, 3, 5, 8, 13, 21\}$
$= \{\{2,3\} \{2,5\} \{3,5\} \{3,8\} \{5,8\} \{5,13\} \{8,13\} \{8,21\} \{13,21\}\}$


2
21
3
5
13
8