→ **Upper Bound :-**

An element $a \in A$ is called as upper bound of a subset B of A if $xRa, \forall x \in B$.

→ **Lower Bound:-**

An element $a \in A$ is called as lower bound of a subset B of A if $aRx, \forall x \in B$

→ **Supremum LUB :-**

An element $a \in A$ is called as Least Upper Bound of a subset B of A if

(i) a is upper bound of B

(ii) a is upper bound then aRa'

→ **Infemum LUB :-**

An element $a \in A$ is called as Greatest Lower Bound (GLB) of a subset B of A if

(i) a is a lower bound of B

(ii) if a' is lower bound of B then $a'Ra$

→ **PROBLEMS :-**

1) Consider a poset (A, R) given below if $B = \{c, d, e\}$ find

(i) all lower bounds of B

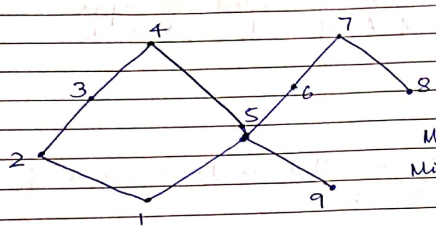
(ii) all upper bounds of B

(iii) the least upper bound of B

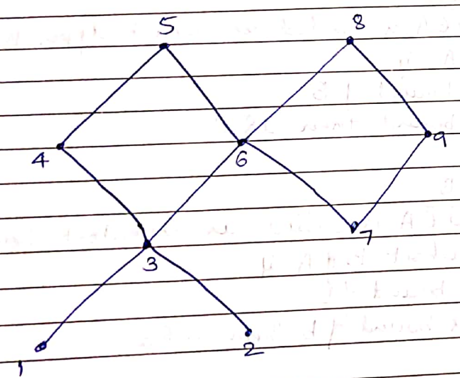
(iv) the greatest lower bound of B.

→ Contd.

→ Determine all maximal and minimal elements of the poset.

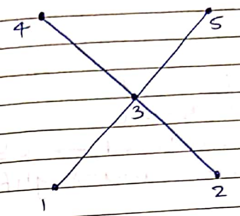


Maximal element: 4, 7
Minimal element: 1, 9, 8

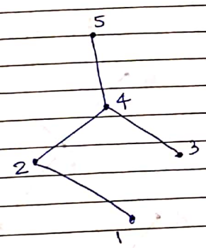


Maximal element : 5, 8
Minimal element : 1, 2, 7

→ Determine the greatest and least elements if they exist in the poset.



Greatest : None
Least : None



Greatest : 5
Least : None

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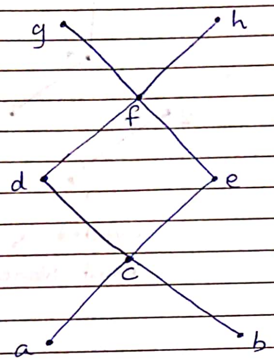
→ Continuation of problem 1

B = {c, d, e}



LUB → f
GLB → c
Lower bound → a b c
upper bound → g h f

Lower bound → a b c

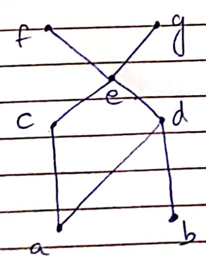
aRc	bRc	cRc
aRd	bRd	cRd
aRe	bRe	cRe

upper bound → g h f

cRg	cRf	cRh
dRg	dRf	dRh
eRg	eRf	eRh

2) Consider a poset whose hasse diagram given below. Find all lower bounds, upper bounds, LUB and GLB.

B = {c, d, e}



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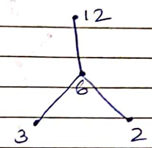
Lower bound → a
Upper bound → f, g, e
LUB → e
GLB → a

3) Consider the poset whose hasse diagram is shown below

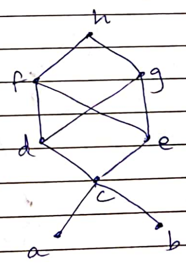
A = {2, 3, 6, 12}



Find

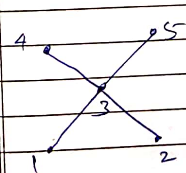
(i) LUB[2,3] → 6
(ii) GLB[2,3] → Null
(iii) LUB[2,12] → 12
(iv) GLB[6,12] → 6

4)



B_1 = {a, b}
B_2 = {c, d, e}

5)

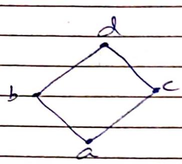


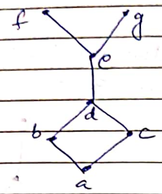
find
(i) A∩B
(ii) A∪B
(iii) LUB
(iv) GLB

B = {1, 2, 3, 4, 5}

→ Lattice :-

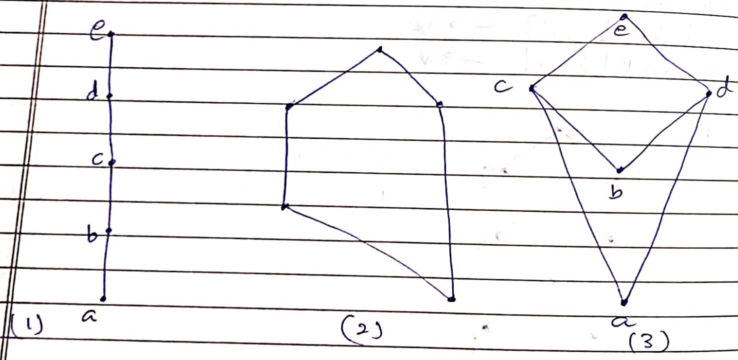
A poset A, R is said to be lattice if LUB and GLB exists.





B = {b, c}   B = {d, e}

Both are lattice as LUB and GLB exists for both

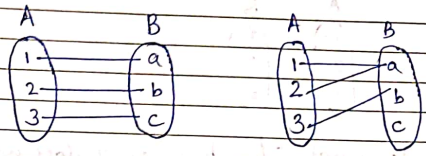


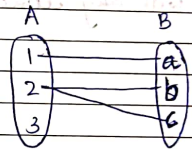
(1) B = {b, c, d}
(2) B = {c, d}
(3) B = {b, c, d}

UNIT-3
FUNCTIONS

function → f : A → B

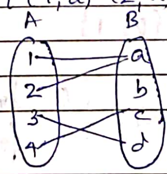
A = {1, 2, 3}   B = {a, b, c}





Let A and B be two non-empty sets.
A function f from A to B is a relation from A to B such that for each a in A there is a unique b in B such that (a, b) ∈ f.
The function is also called as Mapping, transformation.

1) Let A = {1, 2, 3, 4}
B = {a, b, c, d}
f = {(1, a) (2, a) (3, d) (4, c)}



$\boxed{f : A \longrightarrow B}$

A function f from A to B

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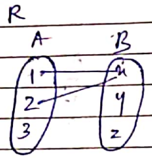
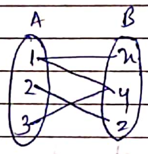
2) Let $A = \{1, 2, 3\}$
$B = \{x, y, z\}$
consider the relations $R = \{(1,x)(2,x)\}$
$S = \{(1,x)(1,y)(2,z)(3,y)\}$ and write the domain of ~~R/ax~~ and range for R and S.

for R



It is not a function as 3 does not have any unique relation with any element in B.

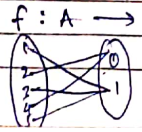
For S



It is not a function as elements of A cannot have two ~~relati~~ unique relations with the elements of B.

3) ~~Let $A = B = Z$~~ ~~$f : A \to B$~~
~~Defined by~~
~~$f(a) = a+1$ for $a \in A$~~

) Let $A = Z$ and $B = \{0, 1\}$
Let $f : A \longrightarrow B$



$f(a) = \begin{cases} 0 & \text{for even} \\ 1 & \text{for odd} \end{cases}$

$\boxed{f : A \longrightarrow A}$

Identity function

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4) Let $A = B = Z$.
and Let C be the set of even integers.
Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ be defined as
$f(a) = a+1$
$g(b) = 2b$
find gof.

$\downarrow$

f composition g

$g[f(x)] \Rightarrow g[a+1]$

$\downarrow$

$\underline{\underline{2(a+1)}}$

5) $f(x) = x^2$
$g(x) = 2x+1$
Find g composition f (fog).

$f[g(x)] \Rightarrow f[2x+1]$

$\downarrow$

$(2x+1)^2$

6) Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3\}$
Determine whether the ~~$~~ relation R from A to B is a function. $[R : A \to B]$
If it is a function give its range.

(i) $R = \{(a,1)(b,2)(c,1)(d,2)\}$
(ii) $R = \{(a,1)(b,2)(a,2)(c,1)(d,2)\}$
(iii) $R = \{(a,3)(b,2)(c,1)\}$
(iv) $R = \{(a,1)(b,1)(c,1)(d,1)\}$

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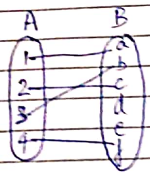
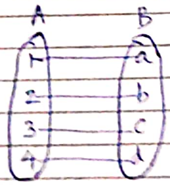
7) Let $A = B = C = R$ and let $f : A \to B$; $g : B \to C$ be defined by $f(a) = a + 1$ and $g(b) = b^2 + 2$. Find ~~g∘~~ $f \circ g$, $g \circ f$.

→ Special Types of Functions :-

1) One - to - One functions :-



2) Onto functions :-



3) Bijection or One-to-One Correspondence :-

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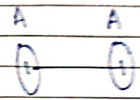
→ Identity function :-

A function $f : A \to A$ such that $f(a) = A$ for every $a \in A$ is called as identity function or identity mapping on A



→ One to One function :-

A function $f : A \to B$ is said to be one to one function if different elements of A have different images in B under f.

Note :-

One to one function is also called as injective function.

→ Onto function :-

A function $f : A \to B$ is said to be an onto function if every element of B there is an element of A such that $f(a) = B$

Note :-

Onto function is also called as ~~surjective~~ surjective function

→ One to One Correspondence or Bijection :-

A function which satisfies both one-to-one and onto function then it is called as one-to-one correspondence or Bijection

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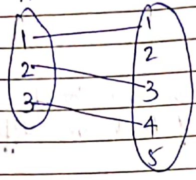
1) Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$ find the whether the following functions from $A \to B$ are one-to-one and onto.

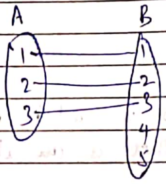
$f = \{(1, 1)\ (2, 3)\ (3, 4)\}$
$g = \{(1, 1)\ (2, 2)\ (3, 3)\}$

for f



one-to-one.
it does not satisfy onto

For g



one-to-one
it does not satisfy onto

2) Let $A = \{0, \pm 1, \pm 2, \pm 3\}$
Consider the $f: A \to R$, where R is the set of all real numbers defined by $f(x) = x^3 - 2x^2 + 3x + 1$ For $x \in A$. Find the range of f.

$f(0) = (0)^3 - 2(0)^2 + 3(0) + 1$
$f(0) = 1$

$f(1) = (1)^3 - 2(1)^2 + 3(1) + 1$
$1 - 2 + 3 + 1$
$f(1) = 3$

$f(-1) = (-1)^3 - 2(-1)^2 + 3(-1) + 1$
$= -1\ \cancel{4}\ 2 - 3 + 1$
$f(-1) = \cancel{-4}\ -5$

$f(2) = (2)^3 - 2(2)^2 + 3(2) + 1$
$= 8 - 8 + 6 + 1$
$f(2) = 7$

$f(-2) = (-2)^3 - 2(-2)^2 + 3(-2) + 1$
$= -8 - 8 - 6 + 1$
$f(-2) = -21$

$f(3) = (3)^3 - 2(3)^2 + 3(3) + 1$
$= 27 - 18 + 9 + 1$
$f(3) = 19$

27
+18
45
+9
54

$f(-3) = (-3)^3 - 2(-3)^2 + 3(-3) + 1$
$= -27 - 18 - 9 + 1$
$f(-3) = -53$

$R = \{(0, 1), (1, 3), (-1, -5), (2, 7), (-2, -21), (3, 19), (-3, -53)\}$

$Ran(f) = \{1, 3, -5, 7, -21, 19, -53\}$

3) Let a $f: R \to R$ be defined by $f(x) = x^2 + 1$ determine the images of the following subsets of R
(i) $A_1 = \{2, 3\}$
(ii) $A_2 = \{-2, 3, 0\}$

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(i) $f(2) = (2)^2+1 = 5$ $= \{5, 10\}$

$f(3) = (3)^2+1 = 10$

(ii) $f(-2) = (-2)^2+1 = 5$

$f(3) = (3)^2+1 = 10$ $= \{5, 10, 1\}$.

$f(0) = (0)^2+1 = 1$

4) Let $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{6, 7, 8, 9, 10\}$

If a $f: A \to B$ is defined by

$f = \{(1,7), (2,7), (3,8), (4,6), (5,9), (6,9)\}$

Determine $f^{-1}(6)$ and $f^{-1}(9)$.

If $B_1 = \{7, 8\}$ and $B_2 = \{8, 9, 10\}$.

Find the $f^{-1}(B_1)$ and $f^{-1}(B_2)$

$f^{-1}(9) = 5, 6$

$f^{-1}(6) = 4$

$f^{-1}(B_1) = 1, 2, 3$

$f^{-1}(B_2) = 3, 5, 6$

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5) Let $f: R \to R$ be defined as $f(x) = \begin{cases} 3x-5 & \text{for } x>0 \\ -3x+1 & \text{for } x \le 0 \end{cases}$

(i) Determine $f(0)$, $f(-1)$, $f\left(\frac{5}{3}\right)$, $f\left(\frac{-5}{3}\right)$

(ii) Determine $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(-1)$, $f^{-1}(3)$, $f^{-1}(-3)$, $f^{-1}(-6)$.

(iii) What are $f^{-1}(-5, 5)$, $f^{-1}(-6, 5)$

(i) $f(0) = -3(0)+1$

$= 1$

$f(-1) = -3(-1)+1$

$= 3+1$

$= 4$

$f(5/3) = 3\left(\frac{5}{3}\right) - 5$

$= 0$

$f(-5/3) = -\cancel{3}\left(\frac{-5}{\cancel{3}}\right) + 1$

$= 6$

(ii) $f^{-1}(0) = x$ $f^{-1}(0) = x$

$f^{-1}(x) = 0$ $f^{-1}(x) = 0$

$\cancel{3x}$

$3x - 5 = 0$ $-3x + 1 = 0$

$3x = 5$ $-3x = -1$

$x = 5/3$ ✓ $x = 1/3$ ✗

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$f^{-1}(1) = x$
$f^{-1}(x) = 1$

$3x-5 = 1$
$3x = 1+5$
$3x = 6$
$x = 2$

$f^{-1}(1) = x$
$f^{-1}(x) = 1$

$-3x+1 = 1$
$-3x = 0$
$x = 0$

$f^{-1}(-1) = x$
$f^{-1}(x) = -1$

$3x-5 = -1$
$3x = -4$
$x = \frac{-4}{3}$

$x = -1.3$

$f^{-1}(-1) = x$
$f^{-1}(x) = -1$

$-3x+1 = -1$
$-3x = -2$
$x = \frac{2}{3}$

$x = 0.6$

$f^{-1}(3) = x$
$f^{-1}(x) = 3$

$3x-5 = 3$
$3x = 3+5$
$3x = 8$
$x = \frac{8}{3}$

$f^{-1}(3) = x$
$f^{-1}(x) = 3$

$-3x+1 = 3$
$-3x = 2$
$x = \frac{-2}{3}$

$f^{-1}(-3) = x$
$f^{-1}(x) = -3$

$3x-5 = -3$
$3x = 2$
$x = 2/3$

$f^{-1}(-3) = x$
$f^{-1}(x) = -3$

~~$-3x+1 = -3$~~
~~$-3x = -4$~~
$x = 4/3$

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$f^{-1}(-6) = x$
$f^{-1}(x) = -6$

$3x-5 = -6$
$3x = -1$
$x = \frac{-1}{3}$

$f^{-1}(-6) = x$
$f^{-1}(x) = -6$

$-3x+1 = -6$
$-3x = -7$
$x = \frac{7}{3}$

(iii) $3x-5 = -5$
$x = 0$

$-3x+1 = -5$
$-3x = -6$
$x = 2$

$3x-5 = 5$
$x = \frac{10}{3}$

$-3x+1 = 5$
$-3x = 4$
$x = \frac{-4}{3}$

$f^{-1}[-5,5] = \left\{\frac{10}{3}, \frac{-4}{3}\right\}$

$3x-5 = -6$
$3x = -1$
$x = \frac{-1}{3}$

$-3x+1 = -6$
$-3x = -7$
$x = \frac{7}{3}$

$f^{-1}[-6,5] = \left\{\frac{10}{3}, \frac{-4}{3}\right\}$

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H.W

1) Consider the following relation on the set $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5\}$, a function $f : A \to B$ is defined by $f = \{(1,3), (2,3), (3,4), (4,5), (5,4)\}$

Find $f^{-1}(3)$, $f^{-1}(4)$, $f^{-1}(B_1)$, $f^{-1}(B_2)$ where $B_1 = \{3, 4\}$, $B_2 = \{4, 5\}$

2) If $f : Z \to R$ is defined by $f(x) = x^2 + 5$. Find $f^{-1}(6, 7)$, $f^{-1}(6, 10)$, $f^{-1}(-4, 5)$ & $f^{-1}(5, \infty)$

→ Composition of a function:-

Consider three non-empty sets A, B, C and the functions $f : A \to B$ and $g : B \to C$. The composition of these two functions is defined as the function gof from $gof : A \to C$ with $\forall a \in A$.

1) Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$ & $C = \{w, x, y, z\}$ with $f : A \to B$ and $g : B \to C$ given by $f = \{(1,a), (2,a), (3,b), (4,c)\}$ and $g = \{(a,x), (b,y), (c,z)\}$

Find f composition g.

$(gof)(1) = g[f(1)] = g(a) = x$

$(gof)(2) = g[f(2)] = g(a) = x$

$(gof)(3) = g[f(3)] = g(b) = y$

$(gof)(4) = g[f(4)] = g(c) = z$

$gof = \{(1,x)\ (2,x)\ (3,y)(4,z)\}$

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→ Consider the function f and g defined by

$f(x) = x^3$

$g(x) = x^2 + 1 \ \forall R$

find fog, gof, fof, gog

$\qquad\qquad\quad \downarrow \qquad \downarrow$

$\qquad\qquad\quad f^2 \qquad g^2$

$fog = f[g(x)]$

$= (x^2+1)^3$

$gof = g[f(x)]$

$= (x^3)^2 + 1$

$= x^5 + 1$

$fof = f[f(x)]$

$= (x^3)^3$

$= x^9$

$gog = g[g(x)]$

$\cancel{= (x^2)^2 + 1}$

$= (x^2+1)^2 + 1$

→ Let f and g be two functions defined on R by $f(x) = ax$ and $g(x) = 1 - x + x^2$. If $(gof)(x) = 9x^2 - 9x + 3$. Determine a and b.

$\cancel{gof = g[f(x)]}$

$\cancel{= 1 - (ax+b) + (ax+b)^2}$

$\cancel{= 1 - ax - b + a^2x^2 + b^2 + 2axb}$

$(gof)(x) = 9x^2 - 9x + 3$

$gof(x) = 9x^2 - 9x + 3$

$g(ax+b) = 9x^2 - 9x + 3$

$1 - (ax+b) + (ax+b)^2 = 9x^2 - 9x + 3$

$1 - ax - b + a^2x^2 + b^2 + 2axb = 9x^2 - 9x + 3$

$1 + a^2x^2 - b + b^2 + 2x(2ab - a) = 9x^2 - 9x + 3$

$a^2x^2 + (2ab - a)x + 1 - b + b^2 = 9x^2 - 9x + 3$

$a^2\cancel{x^2} = 9\cancel{x^2}$

$a^2 = 9$

$a = \pm 3$

$(2ab - a)\cancel{x} = -9\cancel{x}$

$2ab - a = -9$

$+3$	-3
$6a - a = -9$	$-6b + 3 = -9$
$5a = -9$	$-6b = -12$
$6b - 3 = -9$	$b = 2$
$6b = -6$	
$b = -1$	

$1 - b + b^2 = 3$

$1 - b + b^2 = 2$

$b(-1 + b) = 2$

$b = 2$, $-1 + b = 2$

$b = 3$

→ Let f and g be the functions defined on R given by $f(x) = ax + b$ and $g(x) = cx + d$. Find the relationship that satisfy $gof = fog$.

→ Let $f: a \to b$, $g: b \to c$, $h: c \to d$ be three functions, show that $(hog)of = ho(gof)$

$a \in A$, $b \in B$, $c \in C$, $d \in D$

$f(a) = b$, $g(b) = c$, $h(c) = d$

$f: a \to b$ $g: b \to c$ $h: c \to d$

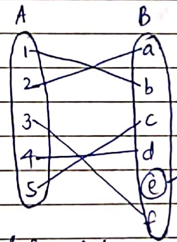
$((hog)of)(a) = (hog)of(a)$

$(hog)b$

$hog(b)$

$h(c) \to d$

→ Let f, g, h be the function on Z defined by $f(x) = x - 1$, $g(x) = 3x$, $h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$



co-domain $= B = \{a, b, c, d, e, f\}$

Range $= \{a, b, c, d, f\}$

$R = \{(1, b)(2, a)(3, f)(4, d)(5, c)\}$

Domain $= \{1, 2, 3, 4, 5\}$

The elements present in set A is called as a domain or domain set.

The elements present in set B is called as a codomain or codomain set.

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The elements of set B which has been mapped from set A are called as Range.

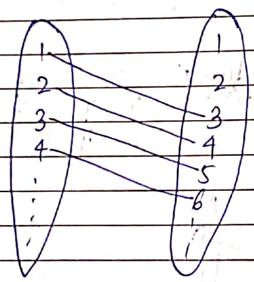
→ If $A = B = Z^+$, $f: a \to b$ is a function defined by $f(x) = x + 2$ for $x \in A$. If f is a function, find domain, co-domain and range.

$dom(f) = Z^+ = A$

$co\text{-}dom(f) = Z^+ = B$

$ran(f) = Z^+ - \{1, 2\}$

A B



→ Invertible Function (Inverse Function):-

Invertible Function or Inverse Function

Let $f: a \to b$, f is said to be an invertible function if $f^{-1}: b \to a$ exists.

Note:-

1) $f^{-1}: b \to a$ exist means $f^{-1}: b \to a$.
2) Chose $f^{-1} = g$ then $g: b \to a$ and g is called an inverse function of f.

→ Theorem:

Let f be a function from a to b $(f: a \to b)$ and g be a function from b to a $(g: b \to a)$ be functions such that $gof = I_A$ and $fog = I_B$ then f and g are one-to one and onto functions.

Proof:-

$f: A \to B$ be a function for $a \in A$,
$f(a) = b$, for $b \in B$, $g(b) = a$
$a = I_A(a) = gof(a) = g(f(a)) = g(b) = a$
$a = a$ (true)

$\therefore f: A \to B$ is one-to-one function.
every element of B is the image of one or more elements of set A.

$\therefore Ran(f) = B$
$f: A \to B$ is onto function.

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Similarly, $b = I_B(B) = fog(b) = f(g(b)) = f(a) = b$

$\therefore g: B \to A$ is one to one function.

Every element of A is the image of 1 or more element of B (co domain)

$Ran(g) = A$

$\therefore g: B \to A$ is onto function.

$\boxed{gof = I_A}$ $\boxed{fog = I_B}$

→ PROBLEMS:-

1) Let $A = B = \{1, 2, 3, 4\}$

f and g be functions from $A \to B$ and $B \to A$ respectively Given by $f = \{(1,4), (2,1), (3,2), (4,3)\}$ and $g = \{(1,2), (2,3), (3,4), (4,1)\}$.

Show that f and g are inverse to each other.

f and g are inverse to each other when $fog = I_B$ and $gof = I_A$

for $1 \in A \Rightarrow gof(1) = 4 \Rightarrow g(4) = 1$

$2 \in A \Rightarrow gof(2) = 1 \Rightarrow g(1) = 2$

$3 \in A \Rightarrow gof(3) = 2 \Rightarrow g(2) = 3$

$4 \in A \Rightarrow gof(4) = 3 \Rightarrow g(3) = 4$

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for $1 \in B \Rightarrow fog(1) = 2 \Rightarrow f(2) = 1$

$2 \in B \Rightarrow fog(2) = 3 \Rightarrow f(3) = 2$

$3 \in B \Rightarrow fog(3) = 4 \Rightarrow f(4) = 3$

$4 \in B \Rightarrow fog(4) = 1 \Rightarrow f(1) = 4$

~~gof = I_B and~~

$fog = I_B$ and $gof = I_A$ are satisfied.

$\therefore$ f and g are inverse to each other

2) Consider the function $f: R \to R$ defined by $f(x) = 2x + 5$ and $g: R \to R$ defined by $g(x) = \frac{x-5}{2}$ show that f is the inverse of g and g is the inverse of f when $gof = I_A$ and $fog = I_B$.

$f(x) = 2x + 5$

$g(x) = \frac{x-5}{2}$

$fog = I_B$ & $gof = I_A$

$fog(x) \Rightarrow f(g(x)) \Rightarrow f\left(\frac{x-5}{2}\right)$

$= \cancel{2}\left(\frac{x-5}{\cancel{2}}\right) + 5$

$= x - 5 + 5$

$= x$

$gof(x) = g(f(x)) \Rightarrow g(2x+5)$

$= \frac{2x + \cancel{5} - \cancel{5}}{2} = \frac{2x}{2} = x$

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Since $fog(x)$ and $gof(x)$ are equal, f and g are inverse to each other.

→ Theorem:-
A function $f: A \to B$ is invertible if and only if it is one-to-one and onto.

Proof:-

Let $f: A \to B$ is an invertible function then we have to prove that f is one-to-one and onto function.
Suppose that $f: A \to B$ is not invertible function
$f^{-1}: B \to A$ does not exist
$\therefore f^{-1}: B \to A$ is not a function.
An element $b \in B$ is not exactly assigned to exactly 1 element in A.

Let $b \in B$ is assigned to $a_1, a_2 \in A$
$f^{-1}(b) = a_1$, $f^{-1}(b) = a_2$
$f^{-1}: B \to A$ exist which contradicts our supposition for $b_1, b_2 \in B$.

$f^{-1}(b_1) = a_1 \Rightarrow f(a_1) = b_1$ $f^{-1}(b_2) = a_2 \Rightarrow f(a_2) = b_2$

$f(a_1) \neq f(a_2)$
$a_1 \neq a_2$

$f: A \to B$ is one-to-one function

$dom(f) = A$, $ran(f) = A$, $range(f^{-1}) = dom(f)$
$\forall$ element of B is the range of 1 or more element of set A
$f^{-1}: A \to B$ is an onto function.

Conversely:-

Let $f: A \to B$ be a function with one-to-one and onto functions for $a_1, a_2 \in A$

$f(a_1) \neq f(a_2) \Rightarrow a_1 \neq a_2$

$\therefore f: A \to B$ is one-to-one function

$Ran(f) = B$.
$f^{-1}: A \to B$ is onto-function.
$f: A \to B$ is an invertible function

→ PROBLEMS:-

1) If $f: A \to B$ and $g: B \to C$ are invertible function then gof is a function from $gof: A \to C$ is an invertible function.
Show that $(gof)^{-1} = f^{-1}og^{-1}$.

Since $f: A \to B$ is an invertible function then $f^{-1}: B \to A$ exists
$g: B \to C$ is an invertible function then $g^{-1}: C \to$ exists

then $f^{-1}og^{-1}: C \to A$ — 1

$f: A \to B$, $g: B \to C \Rightarrow$ ~~f/g~~ $gof = A \to C \Rightarrow$ [illegible]
$\Rightarrow C \to$
from 1 & 2
$(gof)^{-1} = f^{-1}og^{-1}$

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2) Let $A = \{x / x$ is a real number & $x \geq -1\}$

$B = \{x / x$ is a real number & $x \geq 0\}$

Consider $f : A \to B$ defined by $f(a) = \sqrt{a+1}$ $\forall a \in A$

(i) Is f an invertible function.

(ii) Find f^{-1}

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→ Big O Notation :-

$|f(n)| \le c . |g(n)|$ if $n \ge k$

where c & k are constants

$f(n) = \frac{1}{2}n^3 + \frac{1}{2}n^2$ and $g(n) = n^3$

$f(n) \le c\, g(n)$

$\frac{1}{2}n^3 + \frac{1}{2}n^2 \le \frac{1}{2}n^3 + \frac{1}{2}n^3$

→ PROBLEMS:-

1) If the function $f(n) = \frac{3}{2}n^3 + \frac{5}{2}n^2$ and and $g(n) = 2n^3$ is $f = O(g) = ?$

$\frac{3}{2}n^3 + \frac{5}{2}n^2 \le \cancel{2n^3}\ \frac{3}{2}n^3 + \frac{5}{2}n^3$

$\le \frac{8}{2}n^3$

$\le 4n^3$

$\le 2(2n^3)$

$C = 2$

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2) Show that $g(n) = n!$ is of order of n^n

$n^n = O(n^n)$

S.T $g(n) = n!$ is $O(n^n)$

$g(n) = n!$

$g(n) = n(n-1)(n-2) \ldots\ldots 3 \times 2 \times 1$

$g(n) = n . n\left(1-\frac{1}{n}\right) . n\left(1-\frac{2}{n}\right) \ldots\ldots n\left(\frac{3}{n}\right) . n\left(\frac{2}{n}\right) . n\left(\frac{1}{n}\right)$

$g(n) = n^n\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) \ldots\ldots \left(\frac{3}{n}\right)\left(\frac{2}{n}\right)\left(\frac{1}{n}\right)$

$= n^n$ (n is very large since $\frac{3}{n}, \frac{2}{n}, \frac{1}{n}, 1-\frac{1}{n}, 1-\frac{2}{n}$ ~~&~~ all these cases becomes very small)

$= n^n$ for $n \ge 1$

$|g(n)| \le c|n^n|$

$\therefore g(n) = O(n^n)$

3) Show that $h(n) = 1+2+3+4+\cdots n$ is an order of n^2

$f(n) = \frac{n(n+1)}{2}$

$= \frac{n^2+n}{2} = \frac{n^2}{2} + \frac{n}{2}$

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(or)

$\frac{n(n+1)}{2}$

$g(n) = n \cdot n \boxed{\left(\frac{1+1/n}{2}\right)}$

n is large or is 1

$g(n) = n^2$

4) If the function $f(n) = 5n^2 + 4n + 3$ and $g(n) = n^2 + \log n$ is $f = O(g)$.

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→ Big Θ Notation:

Let f and g be two functions defined on Z^+ if f and g have the same order then f is called $f = \Theta(g)$

Note:-

$f = \Theta(g)$ if f and g have the same order.

→ PROBLEMS:-

1) Show that $f(n) = \log_b n$ and $g(n) = \ln(n)$

$$f(n) = \frac{\log_e n}{\log_e b} \Rightarrow \frac{\ln n}{\ln b}$$

$$\frac{1}{\ln b} \times \ln n$$

$|f(n)| \le c\,|g(n)|$

$C = \frac{1}{\ln b}$ $n \geq 1$ $c = \frac{1}{\ln b}$ $k = 1$

$$g(n) = \ln n \Rightarrow \log_e n$$

$$\frac{\log_b n}{\log_b e}$$

$|g(n)| \le c\,|f(n)|$

$c = \frac{1}{\log_b e}$, $k = 1$

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→ Lower Bound Order :-

Let f and g be the two functions defined on Z^+ if f is an order of g but g is not order of f then f is lower order of g.

$f = O(g)$
$g \neq O(f)$

Note:- f is lower order than g means f grows more ~~slowly~~ slower than g.

→ PROBLEMS:-

1) Let $f(n) = 5n^6$ and $g(n) = 3n^8$ show that $f = O(g)$ [f is lower order than g]

$f(n) = 5n^6$

$f(n) \leq \frac{5}{3} \cdot 3n^8$

$|f(n)| \leq C \cdot |g(n)| \quad n \geq k$

$C = \frac{5}{3}$

$f = O(g)$

$g(n) = 3n^8$

$g(n) \leq \frac{3}{5} \times 5n^8$

$g(n) \leq \frac{3n^2}{5} \cdot 5n^6$

$|g(n)| \nleq C \cdot |f(n)|$

$C = \frac{3n^2}{5}$

$g \neq O(f)$

2) Let $f: a \to b$ $g: b \to c$ are invertible functions defined by $f(a) = 2a+1$ and $g(b) = \frac{b}{3}$

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

$f: a \to b$
$f(a) = b$
$f^{-1}: b \to a$
$f^{-1}(b) = a$

$f(a) = 2a+1$
$b = 2a+1$
$2a = b-1$
$a = \frac{b-1}{2}$

$f^{-1}(b) = \frac{b-1}{2}$

$g: b \to c$
$g(b) = c$
$g^{-1}: c \to b$
$g^{-1}(c) = b$
$g^{-1}(c) = 3c$

$g(b) = \frac{b}{3}$
$c = \frac{b}{3}$
$b = 3c$

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$f^{-1}og^{-1}(c)$

$f^{-1}g^{-1}(c)$

$f^{-1}(3c)$

RHS $(f^{-1}og^{-1})(c) = \frac{3c-1}{2}$ —— (1)

$gof : a \rightarrow c$

$gof(a) = c$

$(gof)^{-1} : c \rightarrow a$

$(gof)(c) = a$

~~f(a)~~ $(gof)(a) = g(f(a))$

$= g(2a+1)$

$c = \frac{2a+1}{3}$

$3c = 2a+1$

$3c-1 = 2a$

$a = \frac{3c-1}{2}$

$a = (gof)^{-1} c$

LHS $(gof)^{-1}(c) = \frac{3c-1}{2}$ —— (2)

From (1) & (2)

LHS = RHS

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→ **Functions on Boolean Algebra:-**

• **Boolean Polynomial (Boolean Expression):-**

Let $x_1, x_2, x_3, \dots x_n$ be a set of n symbols or variables. A boolean polynomial $P(x_1, x_2, x_3 \dots x_n)$ in the variable x grade is defined recursively as follows:-

(i) $x_1, x_2, x_3 \dots x_n$ are all boolean polynomial

(ii) The symbols 0 & 1 are boolean polynomials

(iii) If $P(x_1, x_2, x_3 \dots x_n)$ ~~and~~ $Q(x_1, x_2, x_3 \dots x_n)$ are two boolean polynomials then $P(x_1, x_2, x_3 \dots x_n) \vee Q(x_1, x_2, x_3 \dots x_n)$ and $P(x_1, x_2, x_3 \dots x_n) \wedge Q(x_1, x_2, x_3 \dots x_n)$ is true.

(iv) If $P(x_1, x_2, x_3 \dots x_n)$ is a boolean polynomial then $(P(x_1, x_2, x_3 \dots x_n))'$. By tradition $(0)'$ is denoted by $1'$, $(1)'$ is denoted by $1'$, $(x_n)'$ is denoted by x

(v) There are no boolean polynomial in the variables x_k other than those that can be obtained by repeated use of rules above.

→ **Rules for Boolean Algebra:-**

1) $x \le y$ iff $x \vee y = y$	1)' $A \subseteq B$ iff $A \cup B = B$
2) $x \le y$ iff $x \wedge y = x$	2)' $A \subseteq B$ iff $A \cap B = A$
3) a) $x \vee x = x$	3)' a) $A \cup A = A$
b) $x \wedge x = x$	b) $A \cap A = A$
4) a) $x \vee y = y \vee x$	4)' a) $A \cup B = B \cup A$
b) $x \wedge y = y \wedge x$	b) $A \cap B = B \cap A$

$f^{-1} o g^{-1}(c)$
$f^{-1} g^{-1}(c)$
$f^{-1}(3c)$

RHS $(f^{-1} o g^{-1})(c) = \frac{3c-1}{2}$ —— (1)

$gof : a \to c$
$gof(a) = c$
$(gof)^{-1} : c \to a$
$(gof)(c) = a$

$(gof)(a) = g(f(a))$

$= g(2a+1)$

$c = \frac{2a+1}{3}$

$3c = 2a+1$
$3c - 1 = 2a$
$a = \frac{3c-1}{2}$

$a = (gof)^{-1} c$

LHS $(gof)^{-1}(c) = \frac{3c-1}{2}$ —— (2)

From (1) & (2)
LHS = RHS

→ Functions on Boolean Algebra:-

• Boolean Polynomial (Boolean Expression):-

Let $x_1, x_2, x_3, \dots x_n$ be a set of n symbols or variables. A boolean polynomial $P(x_1, x_2, x_3 \dots x_n)$ in the variable x grade is defined recursively as follows:-

(i) $x_1, x_2, x_3 \dots x_n$ are all boolean polynomial
(ii) The symbols 0 & 1 are boolean polynomials
(iii) If $P(x_1, x_2, x_3 \dots x_n)$ ~~and~~ $Q(x_1, x_2, x_3 \dots x_n)$ are two boolean polynomials then $P(x_1, x_2, x_3 \dots x_n) \vee Q(x_1, x_2, x_3 \dots x_n)$ and $P(x_1, x_2, x_3 \dots x_n) \wedge Q(x_1, x_2, x_3 \dots x_n)$ is true.
(iv) If $P(x_1, x_2, x_3 \dots x_n)$ is a boolean polynomial then $(P(x_1, x_2, x_3 \dots x_n))'$. By tradition $(0)'$ is denoted by $0'$, $(1)'$ is denoted by $1'$, $(x_k)'$ is denoted by x_k'
(v) There are no boolean polynomial in the variables x_k other than those that can be obtained by repeated use of rules above.

→ Rules for Boolean Algebra:-

1) $x \le y$ iff $x \vee y = y$ 1)' $A \subseteq B$ iff $A \cup B = B$

2) $x \le y$ iff $x \wedge y = x$ 2)' $A \subseteq B$ iff $A \cap B = A$

3) a) $x \vee x = x$ 3)' a) $A \cup A = A$
b) $x \wedge x = x$ b) $A \cap A = A$

4) a) $x \vee y = y \vee x$ 4)' a) $A \cup B = B \cup A$
b) $x \wedge y = y \wedge x$ b) $A \cap B = B \cap A$

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5) a) $x \vee (y \vee z) = (x \vee y) \vee z$
b) $x \wedge (y \wedge z) = (x \wedge y) \wedge z$

5)' a) $A \cup (B \cup C) = (A \cup B) \cup C$
b) $A \cap (B \cap C) = (A \cap B) \cap C$

6) a) $x \vee (x \wedge y) = x$
b) $x \wedge (x \vee y) = x$

6)' a) $A \cup (A \cap B) = A$
b) $A \cap (A \cup B) = A$

7) $0 \leq x \leq 1$ for all x in L

7)' $\phi \subseteq A \subseteq S$ for all A in $P(S)$

8) a) $x \vee 0 = x$
b) $x \wedge 0 = 0$

9) a) $x \vee 1 = 1$
b) $x \wedge 1 = x$

10) a) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
b) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

11) Every element x has a unique complement x' satisfying
a) $x \vee x' = 1$
b) $x \wedge x' = 0$

12) a) $0' = 1$
b) $1' = 0$

13) $(x')' = x$

14) a) $(x \wedge y)' = x' \vee y'$
b) $(x \vee y)' = x' \wedge y'$

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1) Consider the Boolean polynomial $P(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \vee (x_2' \wedge x_3))$

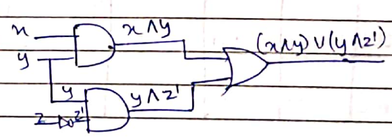
Construct a truth table for the boolean function $f: B_3 \longrightarrow B$. Determine this by boolean polynomial.

$$P(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \vee (x_2' \wedge x_3))$$

x_1	x_2	x_3	x_2'	c: $x_1 \wedge x_2$	a: $x_2' \wedge x_3$	b: $x_1 \vee a$	$c \vee b$
0	0	0	1	0	0	0	0
0	0	1	1	0	1	1	1
0	1	0	0	0	0	0	0
0	1	1	0	0	0	0	0
1	0	0	1	0	0	1	1
1	0	1	1	0	1	1	1
1	1	0	0	1	0	1	1
1	1	1	0	1	0	1	1

2) $P(x, y, z) = (x \wedge y) \vee (y \wedge z')$

x	y	z	z'	a: $x \wedge y$	b: $y \wedge z'$	$a \vee b$
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	1	0	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	1	0	1	1	1	1
1	1	1	0	1	0	1



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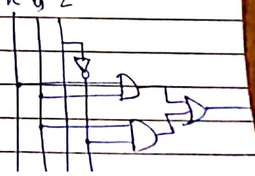
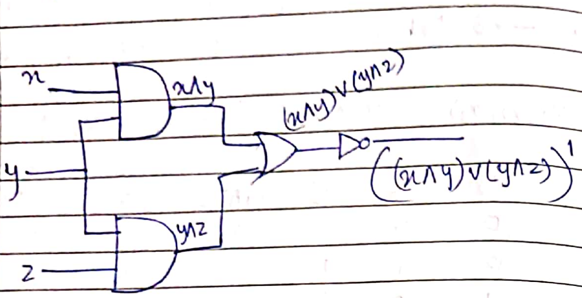
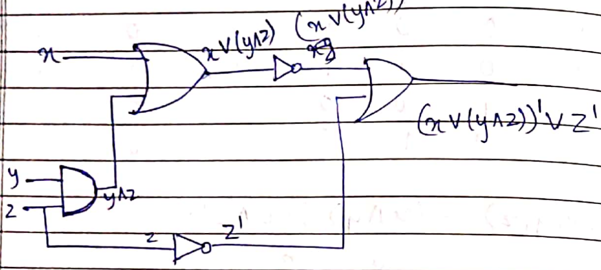


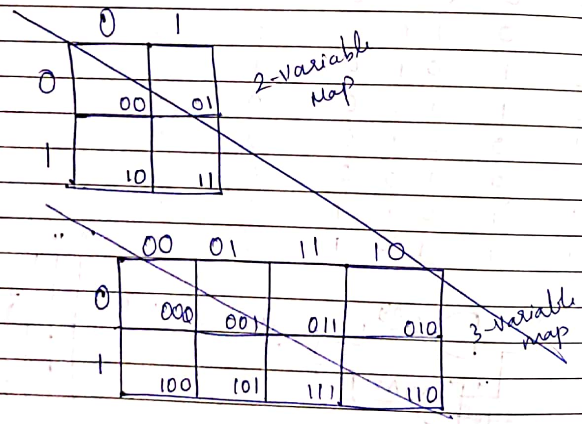
3) Give the Boolean Function described by the following logic diagram.

1)



2)



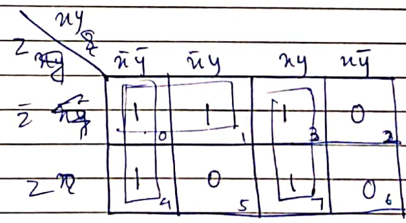


4) By using the K-map solve for the following equation

1) $(y' \wedge z') \vee (x' \wedge y') \vee (y \wedge z)$

	x	y	z	x'	y'	z'	a $y' \wedge z'$	b $x' \wedge y'$	c $y \wedge z$
0	0	0	0	1	1	1	1	1	0
1	0	0	1	1	1	0	0	1	0
2	0	1	0	1	0	1	0	0	0
3	0	1	1	1	0	0	0	0	1
4	1	0	0	0	1	1	1	0	0
5	1	0	1	0	1	0	0	0	0
6	1	1	0	0	0	1	0	0	0
7	1	1	1	0	0	0	0	0	1

	d $a \vee b$	$d \vee c$
0	1	1
1	1	1
2	0	0
3	0	1
4	1	1
5	0	0
6	0	0
7	0	1



$\bar{x}\bar{z} + \bar{x}\bar{y} + xy$

2) $(z' \wedge y') \vee (x' \wedge y' \wedge z) \vee (x \wedge y \wedge z \wedge w)$

	w	x	y	z	x'	y'	z'	d: $z' \wedge y'$	a: $x' \wedge y'$	e: $a \wedge z$	b: $x \wedge y$	c: $b \wedge z$	g: $c \wedge w$	f: $d \vee e$	$f \vee g$
0	0	0	0	0	1	1	1	1	1	0	0	0	0	1	1
1	0	0	0	1	1	1	0	0	1	1	0	0	0	1	1
2	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0
3	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
4	0	1	0	0	0	1	1	1	0	0	0	0	0	1	1
5	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0
6	0	1	1	0	0	0	1	0	0	0	1	0	0	0	0
7	0	1	1	1	0	0	0	0	0	0	1	1	0	0	0
8	1	0	0	0	1	1	1	1	1	0	0	0	0	1	1
9	1	0	0	1	1	1	0	0	1	1	0	0	0	1	1
10	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0
11	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0
12	1	1	0	0	0	1	1	1	0	0	0	0	0	1	1
13	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0
14	1	1	1	0	0	0	1	0	0	0	1	0	0	0	0
15	1	1	1	1	0	0	0	0	0	0	1	1	1	0	1

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	1 (0)	1 (1)	0 (3)	0 (2)
$\bar{w}x$	1 (4)	0 (5)	0 (7)	0 (6)
wx	1 (12)	0 (13)	1 (15)	0 (14)
$w\bar{x}$	1 (8)	1 (9)	0 (11)	0 (10)

5) Write the k-map for the following truth table

	x	y	z	w	$f(x,y,z,w)$
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	0

6) Construct a k-map for a function f for which

$S(f) = \{(0,0,1), (011), (101), (111)\}$

$S(\bar{f}) = \{(0001), (0011), (1010), (1101), (0100), (1000)\}$

7) a)

	$\bar{y}$ 0	y 1
$\bar{x}$ 0	1	0
x 1	0	1

b)

	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	1	1	1	1
x 1	0	0	1	0

c)

	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{w}\bar{x}$ 00	1	1	0	1
$\bar{w}x$ 01	1	1	0	1
wx 11	0	0	0	0
$w\bar{x}$ 10	1	0	0	1

Answers

5)

xy \ zw	$\bar{z}\bar{w}$	$\bar{z}w$	zw	$z\bar{w}$
$\bar{x}\bar{y}$	1 (0)	0 (1)	0 (3)	1 (2)
$\bar{x}y$	0 (4)	1 (5)	0 (7)	1 (6)
xy	1 (12)	0 (13)	0 (15)	1 (14)
$x\bar{y}$	0 (8)	0 (9)	0 (11)	0 (10)

$\bar{x}\bar{y}\bar{w} + xy\bar{w}$
$+ \bar{x}y\bar{z}w$
$+ \bar{x}yz\bar{w}$

~~[illegible] + $\bar{x}yzw$~~ $+ xyz\bar{w} + \bar{x}z\bar{w} + yz\bar{w}$

6) (i) $s(f) = \{(001), (011), (101), (111)\}$

	x	y	z	$s(f)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

$x \backslash yz$	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{x}$	0 (0)	1 (1)	1 (3)	0 (2)
x	0 (4)	1 (5)	1 (7)	0 (6)

z //

(ii) $s(f) = \{(0001), (0011), (1010), (1101), (0100), (1000)\}$

	w	x	y	z	$s(f)$
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

$wx \backslash yz$	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	0 (0)	1 (1)	1 (3)	0 (2)
$\bar{w}x$	1 (4)	0 (5)	0 (7)	0 (6)
wx	0 (12)	1 (13)	0 (15)	0 (14)
$w\bar{x}$	1 (8)	0 (9)	0 (11)	1 (10)

$\bar{w}\bar{x}z + w\bar{x}\bar{z} + \bar{w}x\bar{y}\bar{z} + wx\bar{y}z$

7)

a) $\bar{x}\bar{y} + xy$

b) $\bar{x}\bar{y} + yz + \bar{x}y$ or $\bar{x} + yz$

c) $\bar{w}\bar{y} + \bar{w}y\bar{z} + w\bar{x}\bar{z}$

UNIT - 4
GROUPS OR ABSTRACT ALGEBRA

→ Closure Property:-

$\boxed{\text{For } a, b \in G \quad , \quad a * b \in G}$

$2, 3 \in Z \quad , \quad 2 * 3 = 6 \in Z$

→ Associative Property :-

$\boxed{\begin{array}{l}\text{For } a, b, c \in G \\ (a * b) * c = a * (b * c)\end{array}}$

$2, 3, 4 \in Z$

$(2 * 3) * 4 = 2 * (3 * 4)$

$6 * 4 = 2 * 12$

$24 = 24 \in Z$

→ Identity Property:- → e

For $a \in G$ there exists an element $e \in G$ such that,

$\boxed{a * e = a} \Rightarrow 2 * 1 = 2 \in Z$

$\boxed{a * e = e * a}$

→ Inverse Element:-

For $a \in G$ there exists an element a' such that,

$\boxed{\begin{array}{l} a * a' = e \\ a * a' = a' * a = e \end{array}}$

$1 * 1 = 1 \in Z$

→ Abelian Group:-

Commutative Group:-

Let $(G, *)$ be a group, $(G, *)$ is called an Abelian Group if $(a, b) \in G$, then $\boxed{a * b = b * a}$

$2, 3 \in Z \quad , \quad 2 \times 3 = 3 * 2$

$6 = 6 \in Z$

→ Let G be a non-empty set and $*$ be a binary operation on G then $(G, *)$ is called as a group if it satisfies the following properties

→ PROBLEMS:-

1) Show that $(Z, +)$ where Z is a set of integers and show that whether it is an Abelian Group.

(i) Closure property:-

$2, 3 \in Z$

$a, b \in G \quad , \quad a + b \in G$

$2 + 3 = 5 \in Z$

(ii) Associative property:-

$2, 3, 4 \in Z$

$a, b, c \in G \quad , \quad (a + b) + c = a + (b + c)$

$\cancel{a *}\ (2 + 3) + 4 = 2 + (3 + 4)$

$5 + 4 = 2 + 7$

$9 = 9 \in Z$

(iii) Identity property:-

$2 \in Z$

$a + e = e + a \quad , \quad a + e = a$

$2 + e = 2$

$2 + 0 = 2 \in Z$

(iv) Inverse element:-

$2 \in Z$

$a + a' = e$

$2 + (-2) = 0$

$2 - 2 = 0$

(v) Abelian Group :-

$2, 3 \in Z$

$a, b \in G \quad , \quad a + b = b + a$

$2 + 3 = 3 + 2$

$5 = 5 \in Z$

2) For $(Z, -)$

satisfies

(i) $2, 3 \in Z$
$a, b \in G \quad a - b \in G$
$2 - 3 = -1 \in Z$

(ii) $2, 3, 4 \in Z$
$a, b, c \in G \quad (a-b)-c = a-(b-c) \in G$
$(2-3)-4 = 2-(3-4)$
$-1-4 = 2-(-1)$ does not satisfy
$-5 = 2+1$
$-5 \neq 3$

(iii) $2 \in Z$
$a - e = e - a \quad , \quad a - e = a$
$2 - e = 2$
$2 - 0 = 2 \in Z$ satisfies

(iv) $2 \in Z$
$a - a' = e$
$2 - (+2) = 0$
$2 \mp 2 = 0 \in Z$ ~~does not satisfy~~ Satisfies

(v) $2, 3 \in Z$
$a, b \in G \quad a - b = b - a \in G$
$2 - 3 = 3 - 2$
$-1 \neq 1$ does not satisfy

Date: / /

3) Let $A = \{0, 1\}$ and the operation $*$ on A be defined by the following table.

*	0	1
0	0	1
1	1	0

If $(A, *)$ belong to Abelian Group.

(i) $0, 1 \in Z$
$a, b \in G \quad a * b \in G$ S
$0 * 1 = 1 \in Z$

(ii) $0, 1, 0 \in Z$
$a, b, c \in G \quad (a * b) * c = a * (b * c) \in G$
$(0 * 1) * 0 = 0 * (1 * 0)$
~~$0 = 0 \in Z$~~ $1 = 1 \in Z$ S

(iii) $1 \in Z$
$a * e = e * a \quad , \quad a * e = 1$
$1 * e = 1$ S
$1 * 0 = 1 \in Z$

(iv) $1 \in Z$
$a * a' = e$ S
$1 * (1) = 0$
$1 * 1 = 0 \in Z$

(v) $0, 1 \in Z$
$a, b \in G \quad a * b = b * a \in G$
$0 * 1 = 1 * 0$ S
$1 = 1 \in Z$

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4) Let G be the set of all non-zero real numbers and let $a * b = \frac{1}{2} ab$. Is G an Abilian Group?

(i) Closure:-

Let $1, 2 \in G$

$1 * 2 = \frac{1}{2} * 1 * 2 = 1 \in G$

$\therefore$ satisfied.

(ii) Associative:-

Let $1, 2, 4 \in G$

$(1*2)*4 = 1*(2*4)$

$\left(\frac{1}{2} * 1 * 2\right) * 4 = 1 *$