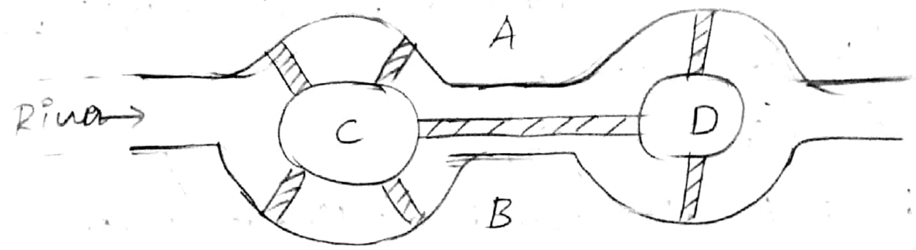
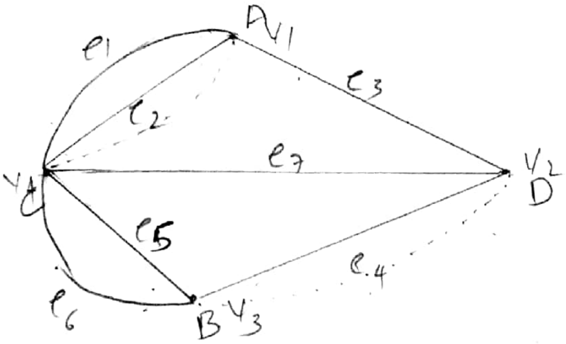
GRAPH THEORY

KONIGBERG BRIDGE PROBLEM (7 BRIDGE PROBLEM)





Graph theory was first introduced by Euler in 1736, through his research paper.

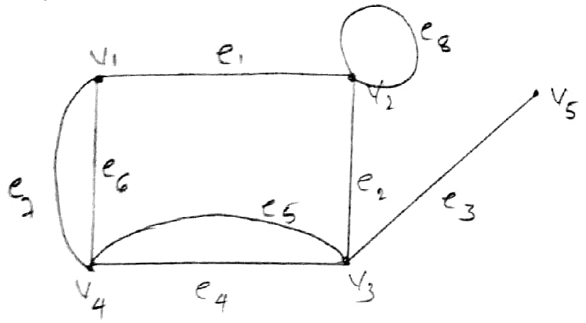
7 BRIDGE PROBLEM:

Two islands C & D formed by the Predge river is connected to each other. and the river bands A & B with 7 bridges as shown in the figure.

The problem was to start at any of the four land areas of the city A, B, C or D walk over each of the 7 bridges exactly one's and reach the starting point. Many people tried to solve this. But were unsuccessfull. Euler replaced each of the land areas by a point & each bridge by a line. This produced a diagram as shown in the previous page. Through this diagram Euler gave the concept of a graph.

GRAPH: A graph consists of objects called vertices & edges and is denoted by $G = G(V,E)$ where
$V(G) = \{v_1, v_2, v_3, \dots v_n\}$
$E(G) = \{e_1, e_2, e_3 \dots e_n\}$

A graph G is also called as linear graph



vertices are called points, nodes, or dots.

A vertex set of graph G is denoted by V(G)
$V(G) = \{V_1, V_2, V_3, V_4, V_5\}$.

EDGES: A line joining two vertices is called as an edge
It can be a st. line or an arc or a curved line. or a circle.
$E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$.

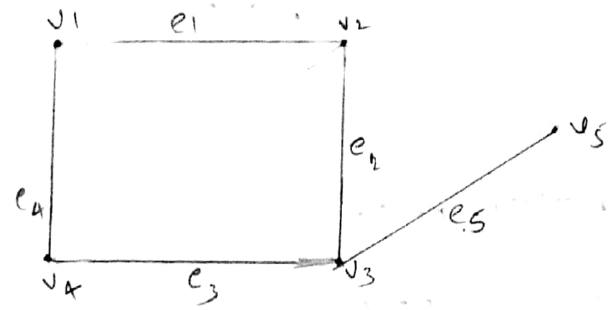
SELF LOOP: It is an edge having the same vertex as starting & ending point.
In the graph edge e_8 is a self loop.

PARALLEL EDGE: Given a pair of vertices may have a more than one edges such edges are called parallel edges
In the above graph e_5 & e_1, e_6 & e_7 are parallel edges.

SIMPLE GRAPH: A graph without self loop & parallel edges is called simple graph.

Ex:

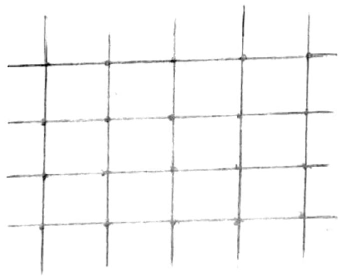
G_1:



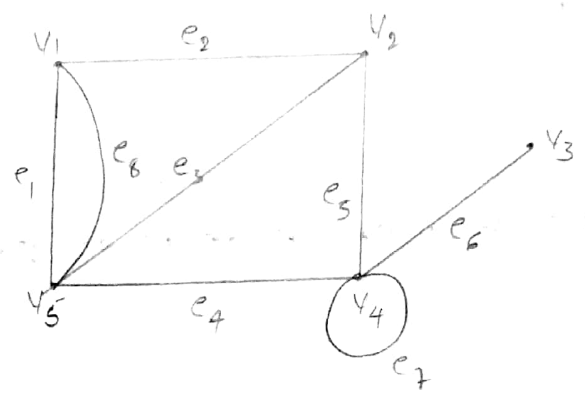
FINITE GRAPH: A graph G having finite no. of edges & vertices are called finite graph.

G_1 is a finite graph.

INFINITE GRAPH: A graph G having infinite no. of vertices is called infinite graph.

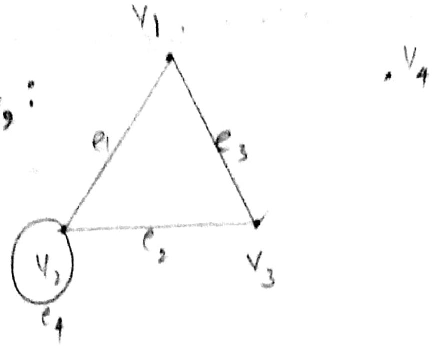


CONNECTED GRAPH: A graph G is said to be a connected graph if there is atleast one path b/w every pair of vertices otherwise G is called a disconnected graph.



This is called a connected graph.

G_2 :



$G_2(4,4)$

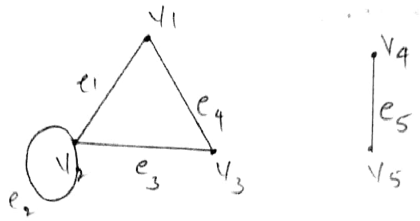
G_2 is a disconnected graph.

COMPONENTS: OF A GRAPH

A disconnected graph has two or more connected sub-graphs. Each of these connected subgraphs is called a component.

Ex : G_2 is a two component disconnected Graph.

G_3 :



ORDER AND SIZE OF A GRAPH:

The no. of vertices in a graph G is called it's order and the no. of edges is called it's size.

Ex. For the graph G_1,

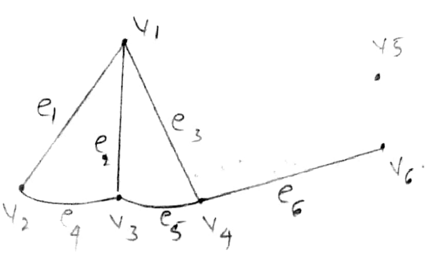
order(G_1) = $O(G_1)$ = 5

Size(G_1) = 8.

ISOLATED VERTEX:

A vertex having no incident edge is called an isolated vertex.

G_4 :



v_5 is an isolated vertex.

PENDENT VERTEX: (END VERTEX)

A vertex of degree 1 is called a pendent vertex.

In G_1 V_3 is the pendent vertex

loop contributes 2 edges to a vertex.

ADJACENT EDGES: (self loop 4 parallel can't be taken as adjacent)

Two non-parallel edges are said to be adjacent if they incident on a common vertex.

:ub.

In graph G_1 e_2 4 e_3 are adjacent edges because they

wed

incident on the same vertex. V_2.

ADJACENT VERTICES:

Two vertices are said to be adjacent if they are the end vertices of the same edge

i) Vertex V_1 4 V_2 are the end ~~per~~vertices of the edge e_2

∴ V_1 4 V_2 are the adjacent vertices

er ii) V_2 4 V_5 are the end vertices of the edge e_3

∴ V_2 4 V_5 are the adjacent vertices.

iii) V_3 4 V_5 are ~~the end~~ not the end vertices of the same edge.

∴ V_3 4 V_5 aren't the adjacent ~~edges~~. vertices.

lak. 1. Ex.

Consider the set $S=\{2, 3, 5, 8, 13, 21\}$. of 6 specific fibbinocci numbers (that) write the distinct pair of integer whose absoulute value of sum or difference belongs to S. Write the graph representing system.

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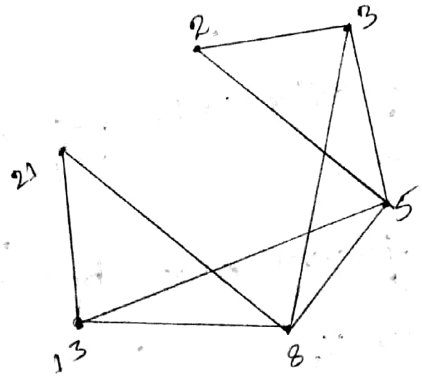
$V(G) = \{2, 3, 5, 8, 13, 21\}$

$E(G) = \{ \{2,3\}, \{2,5\}, \{3,5\}, \{5,8\}, \{8,13\}, \{3,8\}, \{5,13\}, \{13,21\}, \{8,21\} \}$.

GRAPH:



2. The 10 editors have decided on the 7 committee.

$C_1 = \{1, 2, 3\}$

$C_2 = \{1, 3, 4, 5\}$

$C_3 = \{2, 5, 6, 7\}$

$C_4 = \{4, 7, 8, 9\}$

$C_5 = \{2, 6, 7\}$.

$C_6 = \{8, 9, 10\}$

$C_7 = \{1, 3, 9, 10\}$.

They have set aside 3 time periods for the 7 committees to meet on those fridays. when all 10 editors are present. same pairs of committee cannot meet during the same period because one or two of the editors are on both committes. model the situation using the graph. Write edge set and vertex set.

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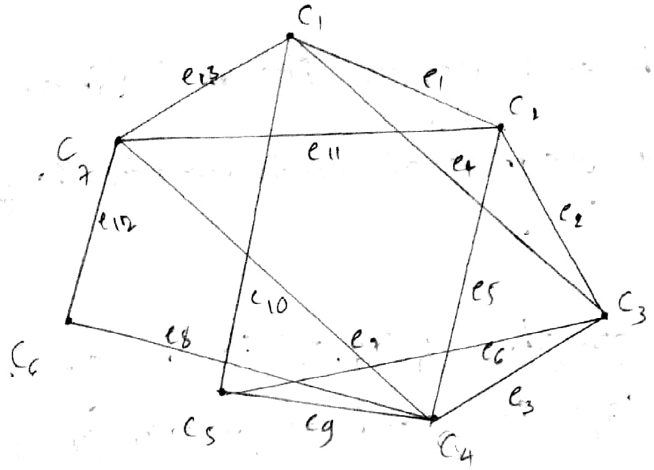
1 : C_1, C_2, C_7
2 : C_1, C_3, C_5
3 : C_1, C_2, C_7
4 : C_2, C_4.
5 : C_2, C_3
6 : C_3, C_4, C_7
7 : C_3, C_4, C_5
8 : C_4, C_6.
9 : C_4, C_6, C_7
10 : C_6, C_7



TRIVIAL GRAPH:

A graph with exactly one vertex is called a trivial graph.

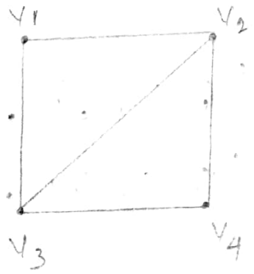
Ex: G_1: $\cdot V_1$

NON-TRIVIAL GRAPH:

A graph with more than one vertex is called non trivial graph.

Ex:

G_2:

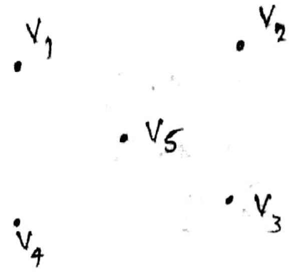


NULL GRAPH:

A graph which doesn't contain any edges is called a NULL graph.

Ex :

G_3



> In a NULL graph each vertex is an isolated vertex

> In a NULL graph degree of each vertex is zero.

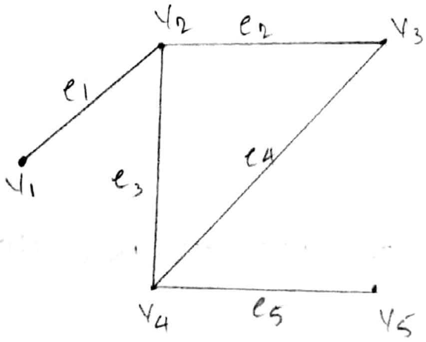
SUBGRAPHS:

Let $G(V,E)$ be a graph. $G_1(V_1,E_1)$ is called a subgr-aph of G if $V_1(G_1)$ is a subset of vertex set of G and

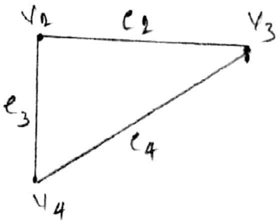
$E_1(G_1)$ is also a subset of $E(G)$. where each edge even is incident with vertices in V_1.

Ex:

G :



G_1 :

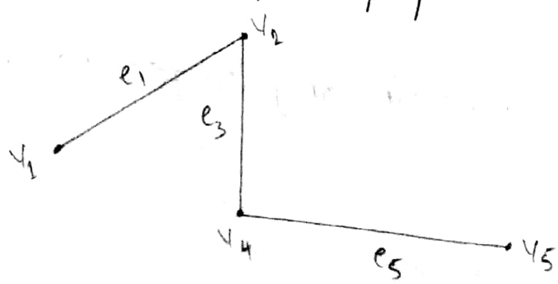


$V(G_1) = \{V_2, V_3, V_4\}$ $\therefore V(G_1) \subseteq V(G)$

$E(G_1) = \{e_2, e_3, e_4\}$ $\therefore E(G_1) \subseteq E(G)$

$\therefore G_1$ is a subgraph of G.

G_2 :



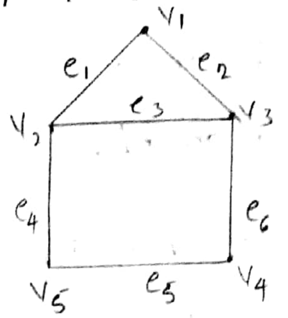
G_2 is a subgraph.

SPANNING SUBGRAPHS:

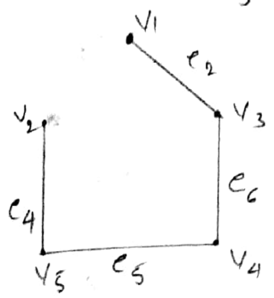
A subgraph G_1 of a graph G is called a spanning subgraph of G if G_1 contains all the vertices of G.

Ex:

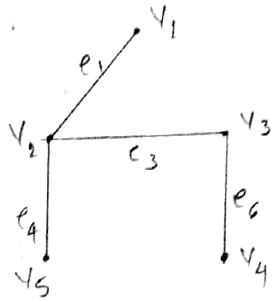
G :



G_1 :



G_2 :

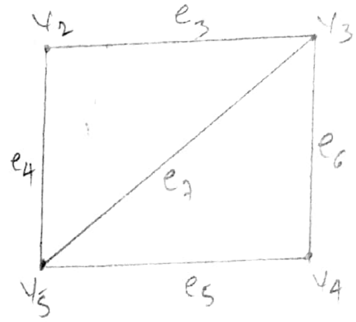


G_1 and G_2 are called spanning subgraph of graph G.

INDUCED GRAPH:

Let $G(V,E)$ be a graph. The subgraph F of the graph G is called induced subgraph by F, where the subgraph contains all the edges incidenting on the vertices of G.

F:



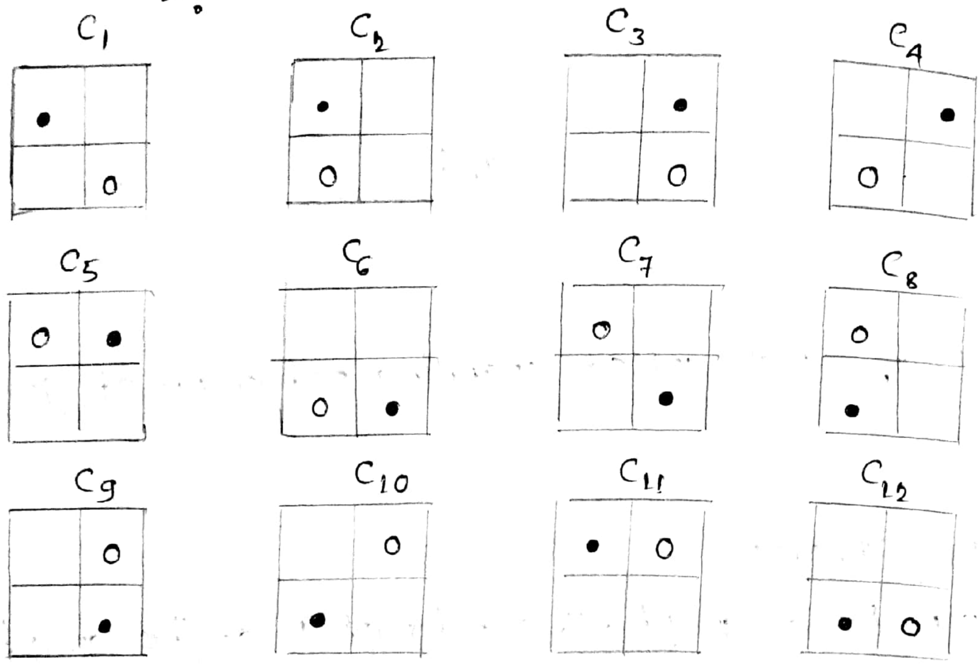
F is an induced subgraph of G.

Ex 1: Suppose we have two coins one silver and one gold placed on two of the four sequences of a 2×2 checkerboard. An arrangement c_i can be transformed into c_j such that if c_j can be obtained from c_i by performing exactly one of the following two steps:

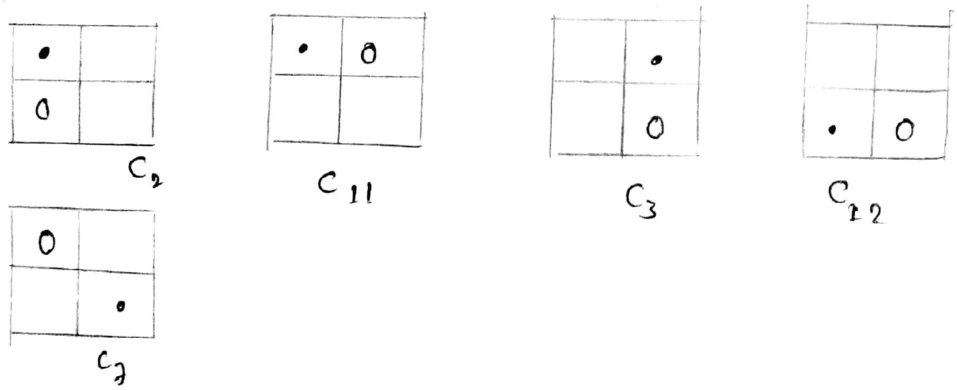
1. moving one of the coins C_i horizontally or vertically to an unoccupied square.
2. interchanging the two coins in C_i

Represent the transform in a graph.

$\left({}^4P_2 = \frac{4!}{2!} = 12 \text{ different combinations} \right)$



$C_1 \rightarrow C_2, C_3, C_7, C_{11}, C_{12}$



$C_2 \rightarrow C_1, C_4, C_8$

$C_3 \rightarrow C_{12}, C_4, C_9$

$C_4 \rightarrow C_2, C_3, C_5,$

$C_5 \rightarrow C_4, C_7, C_{12}$

$C_{12} \rightarrow C_1, C_6, C_{10}$

$C_6 \rightarrow C_4, C_7, C_{12}$

$C_7 \rightarrow C_1, C_5, C_6, C_8, C_9$

$C_8 \rightarrow C_2, C_7, C_{10}$

$C_9 \rightarrow C_3, C_7, C_{10}$

$C_{10} \rightarrow C_4, C_8, C_9, C_{11}, C_{12}$

$C_{11} \rightarrow C_1, C_5, C_{10}$

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WORD GRAPH:

Let w_1 be a word, the word w_1 can be transformed

ly into w_2 by following the two steps.

i) interchanging two letters of w_1

ii) Replacing a letter in w_1 by another letter.

The word w_1 can be adjacent to w_2. A graph obtained by following the above 2 steps is called a word graph.

Ex'1.

Given a collection of 3 letters English words say

ACT, AIM, ARC, ARM, ART

CAR, CAT, OAR, OAT, RAT, TAR

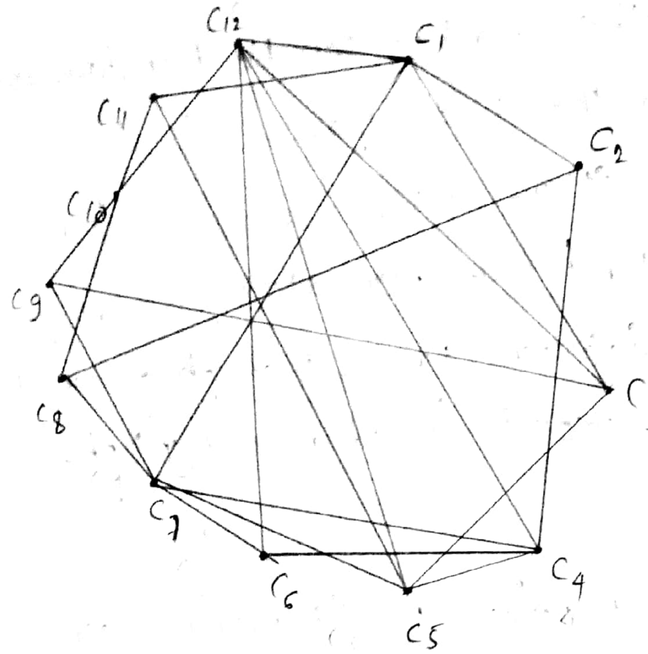
draw the word graph.

Soln:

ACT → CAT, ART

ART → ACT, RAT, ARM, TAR

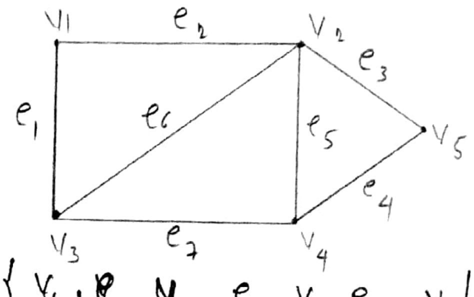
(INDUCED GRAPH)



WALK:

Let G be the a graph. A walk in G is defined as finite alternative sequence of vertices and edges, beginning & ending with vertices.

Ex:



A walk $\{V_1, e_2, V_2, e_6, V_3, e_7, V_4\}$

LENGTH OF A WALK: It is the no. of edges in the walk.

∴ A walk $\{V_3, e_1, V_1, e_2, V_2, e_6, V_3, e_7, V_4, e_4, V_3\}$ is of length 5.

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TRIVIAL WALK: A walk of length zero is called trivial walk.

CLOSED WALK: A walk is called closed walk if it begins and ends at the same vertex.

EX. A walk $\{v_4, e_4, v_5, e_3, v_2, e_6, v_3, e_7, v_4\}$ is a closed walk of length 4.

TRAIL: It is an open walk in which no edge is repeated.

A walk $\{v_1, e_2, v_2, e_6, v_3, e_7, v_4, e_4, v_5\}$ is a trail ~~is~~ of length 4.

A walk $\{v_3, e_6, v_2, e_5, v_4, e_7, v_3, e_6, v_2, e_2, v_1\}$ is not a trial.

NOTE: A trail is a walk but a walk need not to be a trail.

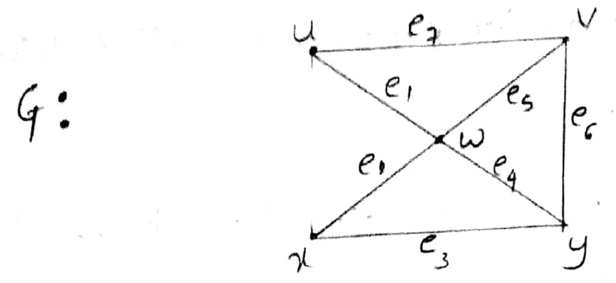
PATH: A path is an open walk in which no vertex is repeated.

A walk $\{v_1, e_1, v_3, e_7, v_4, e_5, v_2, e_3, v_5\}$ is a path of length 4.

CIRCUIT: A closed trail is called a circuit.

CYCLE: A closed path is called cycle.

THEOREM: "If a graph G, contains a u-v walk of length l then G contains a u-v path of length l".



Let P_1 be the smallest walk from u to v covering all the vertices. Then path P_1 is

$P_1 : \{u, e_1, e_2, x, e_3, y, e_6, v\}$ is the smallest walk covering all the vertices of G of length 4.

$\therefore$ Let $P_1 = \{u, u_1, u_2, \ldots\ldots u_l\}$ be the smallest walk covering all the vertices of G and having length say k.

Suppose that there is a path of length $l > k$. then the walk $P_2 : \{u, e_1, w, e_2, x, e_3, y, e_4, w, e_5, v\}$ is a walk of length 5.

i.e $P_2 : \{u, u_1, u_2, u_3 \ldots\ldots u_l\}$ be a walk of length k then definately some vertices are repeated.

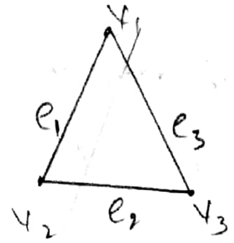
$\therefore$ The walk P_2 cannot be a path, thus P_1 is the path of length utmost l.

COMPLETE GRAPH:

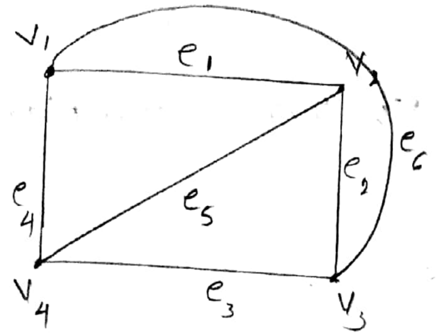
A simple graph in which there exists an edge b/w everywhere of vertices is called a complete graph.

A complete graph of 'n' vertices is denoted by K_n.

Ex: G_1 :



It is a complete graph of 3 vertices (K_3).



It is a complete graph of 4 vertices (K_4).

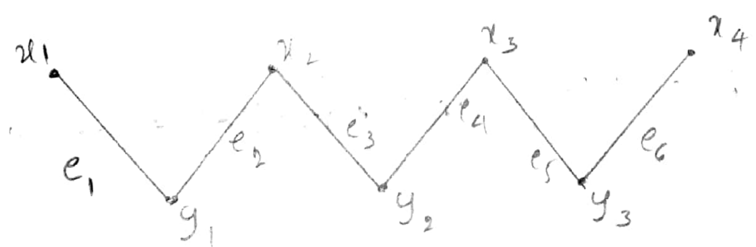
BIPARTITE GRAPH:

Let G be a graph if the vertex set $V(G)$ can be partitioned into 2 subsets V_1 & V_2 such that each edge of G has one end in V_1 and other end in V_2. Then the graph is called a bipartite graph.

In bipartite graph the vertex sets V_1 & V_2 satisfies the following property.

1. $V_1(G) \cup V_2(G) = V(G)$ & 2. $V_1(G) \cap V_2(G) = \phi$.

Ex.



$V_1(G) = \{x_1, x_2, x_3, x_4\}$

$V_2(G) = \{y_1, y_2, y_3\}$.

COMPLETE BIPARTITE GRAPH:

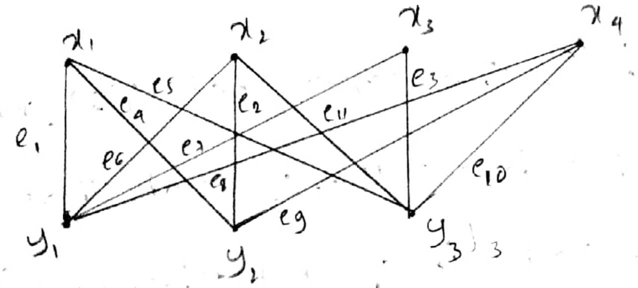
A bipartite graph G with each vertex of V_1 is joined with every vertex of V_2 then G is called complete bipartite graph.

If $|V_1(G)| = m$, $|V_2(G)| = n$ then a complete bipartite graph has

No. of vertices $= m+n$

No. of edges $= m \times n$

Ex.



$V_1(G) = \{x_1, x_2, x_3, x_4\}$ $\quad |V_1(G)| = 4$

$V_2(G) = \{y_1, y_2, y_3\}$ $\quad |V_1(G)| = 3$

No. of vertices $= m+n = 7$

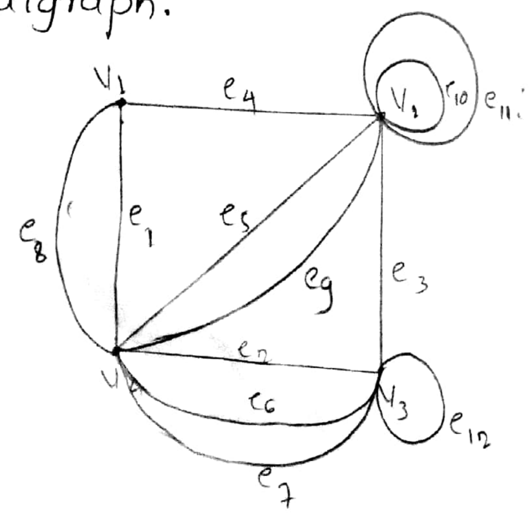
No. of edges $= m \times n = 12$

MULTIGRAPH:

A graph G with self loop and parallel edges called a multigraph.

Ex:

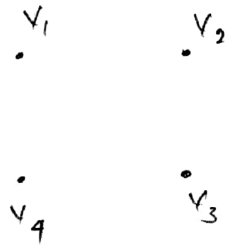
G:



REGULAR GRAPH:

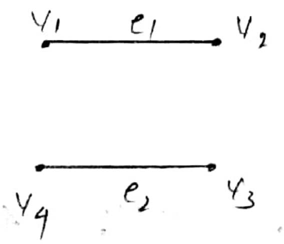
A graph in which all vertices are of equal degrees is called a regular graph.

G_1 :



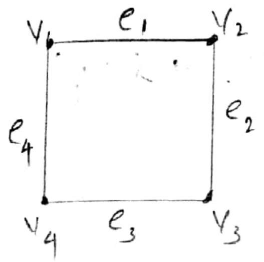
0-regular graph

G_2 :



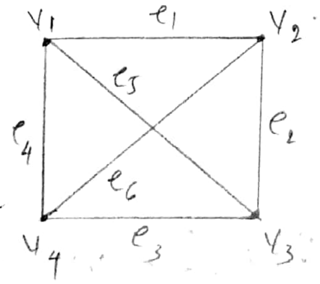
1-regular graph.

G_3 :



2-regular graph.

G_4 :



3-regular graph.

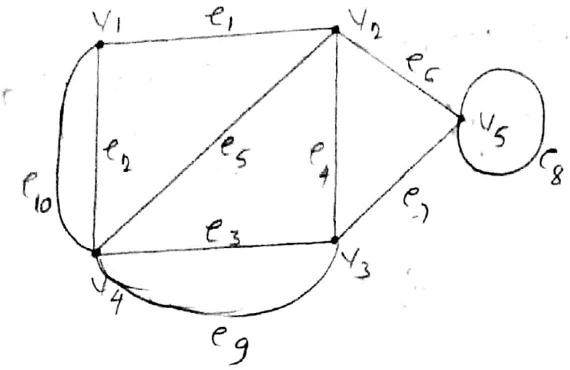
r-Regular Graph: If the degree of all vertices of the graph G is r, then the graph is called r-regular graph.

DEGREE OF A GRAPH:

The no. of edges incidenting on a vertex v_i with self loop counted twice is called degree of v_i.

denoted by $deg(v_i)$ $d(v_i)$

Ex:



$d(v_1) = 3$, $d(v_2) = 4$ $d(v_3) = 4$ $d(v_4) = 5$

$d(v_5) = 4$

$$\sum_{i=1}^{5} d(v_i) = d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5)$$

$$= 3 + 4 + 4 + 5 + 4$$

$$= 20.$$

$\Rightarrow$ 2 × No. of edges.

THEOREM:

1. "If G is a graph of size m, then $\sum d(v_i) = 2m$".

Each edge contributes two to the sum of degrees of vertices.

Since the graph has m edges, ∴ contributes exactly 2m to the sum of degrees of vertices.

∴ $\sum d(v_i) = 2m$ where 'm' is no. of edges.

(Hand shake property).

2. "Every graph has an even no. of odd degree vertices".

Let G has v_j no. of even degree vertices and v_k no. of odd degrees vertices.

$$\sum d(v_i) = \sum d(v_j) + \sum d(v_k)$$

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$\therefore \Sigma d(V_k)$ = even + even.

No. of odd degree vertices is even.

EXAMPLES:

1. A certain graph has order 14 & size 27. The degree of each vertex of G is 3, 4, or 5. There are 6 vertices of degree 4. How many vertices of G have degree 3? and how many have degree 5?

Soln: $|V| = n = 14$, $|E| = m = 27$

Let x be the no. of vertices of degree 3

$\Rightarrow 3x + 6\times4 + 5(8-x) = 2|E|$

$= \quad -2x = 54 - 40 - 24$

$x = 5.$

$8 - x = 8 - 5 = 3$

of vertices of degree 3 $= 5$

vertices of degree 5 $= 3$.

2. If G is a connected graph with $|E| = 17$ & $\deg(v) \geq 3$ for all $v \in V(G)$. what is the maximum value of $|V|$?

ces" Soln:

.. of In a connected graph G, sum of the degrees of vertices $= \Sigma d(v_i) = 2\times|E| = 2\times17$

$= 34$

Since the $d(v_i) \geq 3$.

$\therefore$ Maximum no. of vertices of the Graph G is given by $G \leq \lfloor 34 \rfloor = 11$

$\therefore$ Maximum no. of vertices, $|V| = 11$.

GRAPHICAL:

Let S be a finite sequence of non-negative integers. If S forms a degree sequence of some graph then S is called graphical.

Ex:

1 Which of the following sequence are graphical?

S_1 : 3, 3, 2, 2, 1, 1 is S_1 graphical?

S_2 : 6, 5, 5, 4, 3, 3, 3, 2, 2.

S_3 : 7, 6, 4, 4, 3, 3, 3

S_4 : 3, 3, 3, 1

PROCEDURE FOR A DEGREE SEQUENCE IS GRAPHICAL FOR A SIMPLE GRAPH

Step 1: Find the no. of vertices

Step 2: Find the no. of edges

Step 3: Find the no. of odd degree vertices.

Step 4: Highest degree of a vertex can be less than $(n-1)$.

Soln:

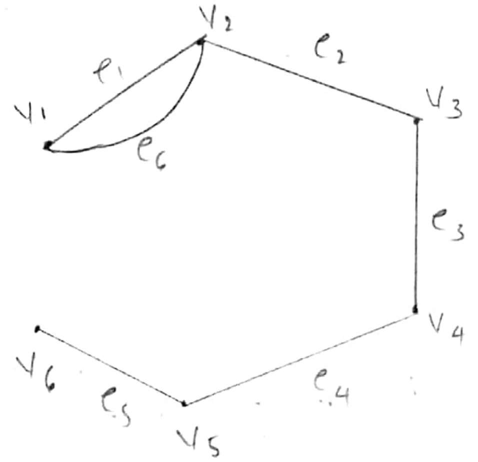
1. No of vertices $= 6$.

2. No. of edges $= \frac{6 \times 2}{2} = 6$.

3. No. of odd degree vertices $= 4$ (even)

4. Highest degree of a vertex < 5

$\therefore S_1 : 3, 3, 2, 2, 1, 1$ is graphical



Ex 2: Use Havel - Hakimi theorem whether the sequence $S_1 : 5, 1, 3, 3, 2, 2, 2, 1, 1, 4$ is graphical

Soln: $|V| = 10$

$\Sigma d(v_i) = 2 \times |E| \Rightarrow |E| = 12$

No. of odd degree vertices $= 6$ (even).

Highest degree is $< n-1 = 10-1 = 9$ (True).

$S : 5, 4, 3, 3, 2, 2, 2, 1, 1, 1$ (non-increasing order)

Deleting the first element 5 in the sequence

$S_1' : 4, 3, 3, 2, 2, 2, 1, 1, 1$

Subtracting 1 from each element till $d_i + 1 = 4 + 1$
$= 5$

$S_1' : 3, 2, 2, 1, 1, 2, 1, 1, 1$

Arrange S_1' in non-increasing order

$S_1' : 3, 2, 2, 2, 1, 1, 1, 1, 1.$

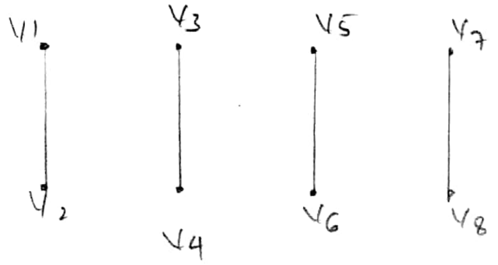
(repeat the process)

~~Subtracting~~ Delete the first element 3,

$S_1' : 2, 2, 2, 1, 1, 1, 1, 1$

Subtracting 1 from each element till $d_i + 1 = 2 + 1 = 3$

$S_2' : 1, 1, 1, 1, 1, 1, 1, 1$



$\therefore S_1 : 5, 4, 3, 3, 2, 2, 2, 1, 1, 1$ is graphical.

ADJACENCY MATRIX

Let G be a graph of order n and size m. The adjacency matrix of G is a $n \times n$ matrix whose elements are $a_{ij} = 1$ if $v_i, v_j \in E(G)$

$a_{ij} = 0$ if otherwise.

INCIDENCE MATRIX:

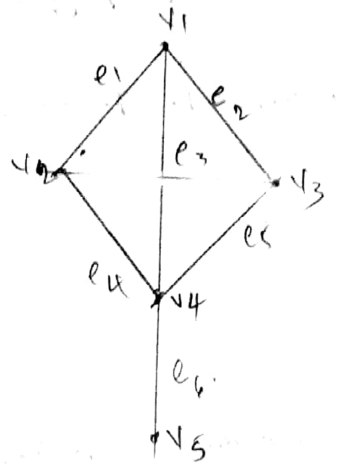
Let G be a graph of order n and size m. The incidence matrix of G is an $n \times m$ matrix where $b_{ij} = 1$ if v_i, is incident with e_j.

0 otherwise.

Ex. 1.

Write adjacency matrix & incidence matrix of the graph G given below.

G:



1. Adjacent matrix.

A =

	v_1	v_2	v_3	v_4	v_5
v_1	0	1	1	1	0
v_2	1	0	0	1	0
v_3	1	0	0	1	0
v_4	1	1	1	0	1
v_5	0	0	0	1	0

2. Incidence Matrix:

B =

	e_1	e_2	e_3	e_4	e_5	e_6
v_1	1	1	1	0	0	0
v_2	1	0	0	1	0	0
v_3	0	1	0	0	1	0
v_4	0	0	0	1	1	1
v_5	0	0	0	0	0	1

ISOMORPHISM:

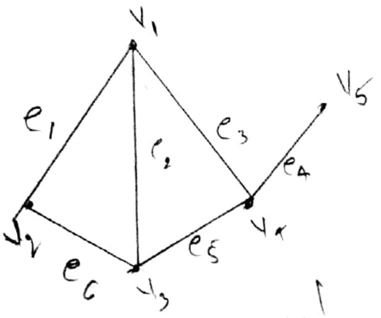
Two graphs G_1 & G_2 are said to be isomorphic to each other, if there is one-to-one correspondance b/w their vertices & their edges such that their incidence relationship is preserved. It is denoted by $G_1 \simeq G_2$.

Ex. 1.

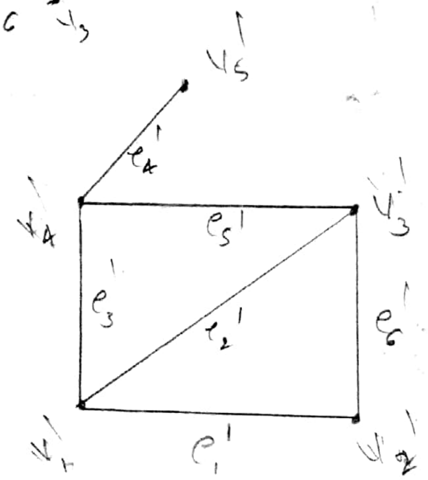
Show that the graphs given below are isomorphic to each other.

* label one graph.
* no. of vertices should (be equal)
* compare the vertices with same degree
* one-to-one correspondance b/w vertices & edges.
* incidence relationship

G_1:



G_2:



(i) The vertex v_1 in G_1, v_2, v_3, v_4, v_5 in G_1 corresponds to $v_1', v_2', v_3', v_4', v_5'$ in G_2 respectively.

$\therefore$ There is one-to-one correspondance b/w ~~G~~ vertex set of G_1 & G_2.

(ii) The edges $e_1, e_2, e_3, e_4, e_5, e_6$ in G_1 corresponds to $e_1', e_2', e_3', e_4', e_5', e_6'$ resp in G_2 respectively.

$\therefore$ There is one-to-one correspondance b/w edg $E(G_1)$ & $E_2(G_2)$.

(iii) The edge e_1 is incident on v_1 & v_2 in G_1
& edge e_1' is incident on v_1' & v_2' in G_1. ......

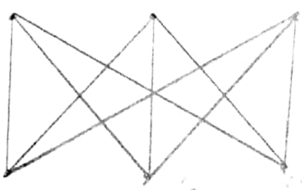
The edge e_2, e_3, e_4, e_5, e_6 is incident on v_1, v_3 in G_1
& edge e_2' is incident on v_1', v_3' in G_2 & so on....

Thus the edge incidente property is preserved.

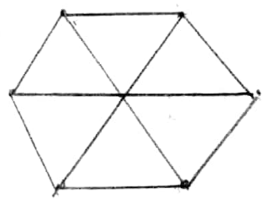
$\therefore G_1 \simeq G_2$.

2. Show that the following graphs are isomorphic to each other.

1. G_1:

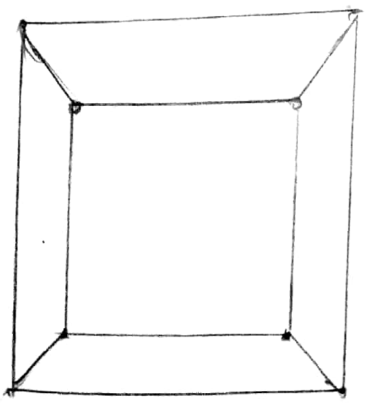


G_2:

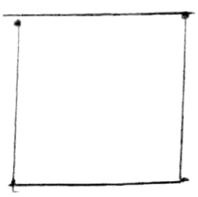


2.

G_1:



G_2:

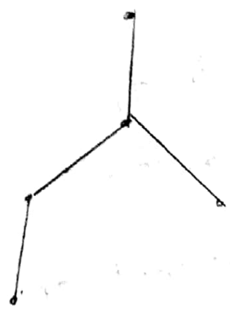


TREES:

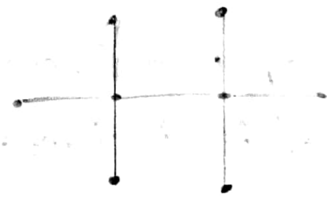
A graph G is called a tree if G is connected & contains no cycle. It is denoted by $T(V,E)$.

Ex:

T_1

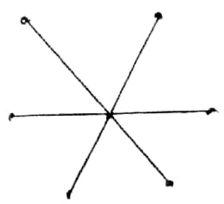


T_2:

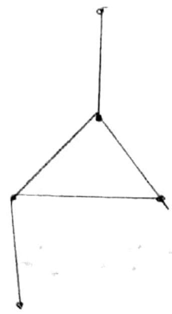


*impossible to have more than one path b/w 2 vertices

T_3:



T_4:



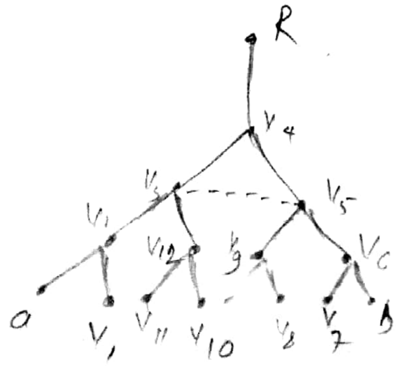
(n vertex n-1 edge)

T_1, T_2, T_3 are trees but T_4 is not a tree.

(n edges n+1 vertex)

THEOREM:

"A Graph G is a tree iff every two vertices of G are connected by a unique path."



Let G be a tree. We have to prove that there is a unique path in G. Suppose that there are two paths b/w the vertices a & b

conv

$P_1 = \{a, v_2, v_3, v_4, v_5, v_6, \ldots v_n, b\}$.

$P_2 = \{a, v_2, v_3, v_5, v_6 \ldots v_n, b\}$.

$P_3 = \{a, v_2, v_3, v_5, v_4, v_3, v_5, v_6 \ldots v_n, b\}$

in which some vertices are repeated. Then the graph has a cycle. ∴ G cannot be a tree.

Thus a connected graph G is a tree if every 2 vertices has a unique path.

Conversly,

suppose that every two distinct vertices has a unique path, then we have to prove that G is a tree.

Assume that G has two paths b/w a pair of vertices then in one of the paths some vertices are repeated which causes a cycle. Then G cannot be a tree.

∴ our assumption is wrong.

Thus a tree has unique path b/w every pair of vertices.

THEOREM: "A tree with n vertices has n-1 no. of edges"

EULER GRAPH:

EULER CIRCUIT OR EULER TOUR:

A closed walk through every edge of the graph G exactly once is called as an euler circuit or an Euler tour.

EULER GRAPH.

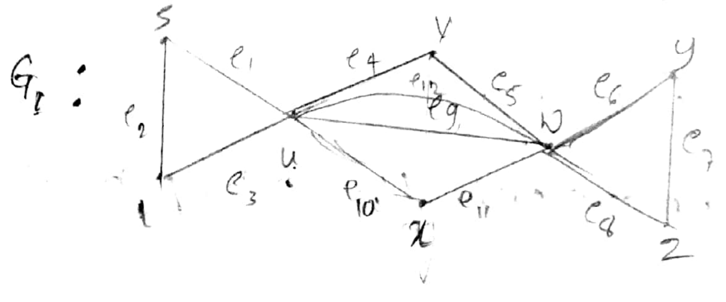
A graph that consists of an Euler circuit is called an Euler graph.

EULER TRIAL:

An open walk covering all the edges exactly once is called an Euler trial.

Ex:

G_1 :



(by a if all the vertices are even degree euler circuit is possible)

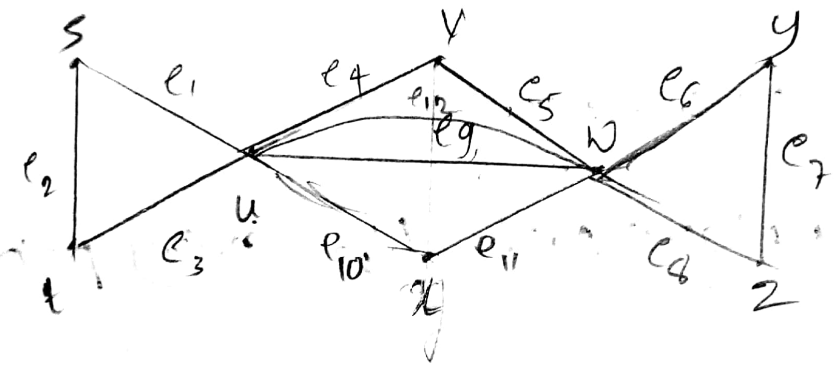
A open walk $\{u, e_1, s, e_2, t, e_3, u, e_4, v, e_5, w, e_6, y, e_7, z, e_8, w, e_9, u, e_{10}, x, e_{11}, w\}$ is an Euler trial.

EULER TRIAL:

An open walk covering all the edges exactly once is called an Euler trial.

Ex:

G_1 :



(by a if all the vertices are even degree, else circuit is possible)

A open walk $\{u, e_1, s, e_2, t, e_3, u, e_4, v, e_5, w, e_6, y, e_7, z, e_8, w, e_9, u, e_{10}, x, e_{11}, w\}$ is an Euler trial.

G_2:

A closed walk $\{u, e_1, s, e_2, t, e_3, u, e_4, v, e_5, w, e_6, y, e_7, z, e_8, w, e_9, u, e_{10}, x, e_{11}, w, e_{12}, u\}$.

This is called Euler's circuit. since the graph G_2 contains Euler's circuit. Therefore G_2 is an Euler graph.

SOLUTION OF KONIGSBERG BRIDGE PROBLEM:

Degree of (A) = deg (B) = deg (D) = 3

Deg (C) = 5.

G is not an Euler graph. Because it doesn't contain an Euler circuit.

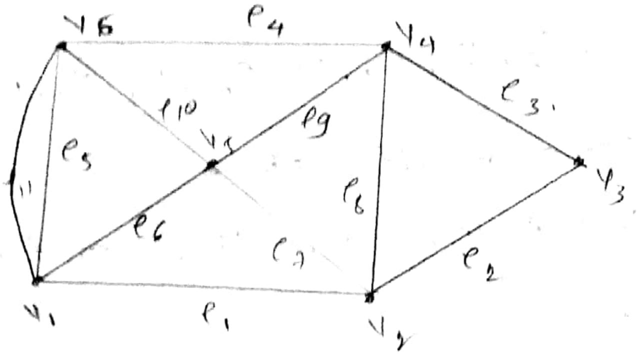
∴ It is not possible to have a closed walk, covering all the edges of G, exactly once. (Add two more bridges)

NOTE:

1. An Euler graph is always a connected graph.
2. In an Euler graph the degree of every vertex is even.

2,

THEOREM: "A non-trivial graph G is an Euler graph iff every vertex of G has even degree".

G: 

A closed walk $\{v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_6, e_5, v_1, e_6, v_5, e_7, v_2, e_8, v_4, e_9, v_5, e_{10}, v_6, e_{11}, v_1\}$

is an Euler circuit.

Let G has a Euler circuit then there exists a path for any v_i to v_j. Therefore G is a connected graph.

Let v_1 be the starting vertex of the Euler circuit. In tracing, an Euler circuit, every time the walk meet the vertex v_i and exit v_i. (annotation above v_i: "an entry")

This entry and exit contributes 2 degree to the vertex. Since the Euler circuit is a closed trial the trial terminates at the starting vertex v_1. Thus the degree of v_1 is also even.

Hence degree of every vertex in G is even.

conversely,

Let G be a connected graph, with every vertex of even degree. Construct an Euler circuit starting from v_1 and going through all the edges of G, such that no edge is traced more than once.

Since every vertex is of even degree, we can enter & exit every vertex and stop tracing at v_1 where v_1 is also of even degree.

Thus G is an Euler Graph.

HAMILTONIAL PATH & GRAPH:

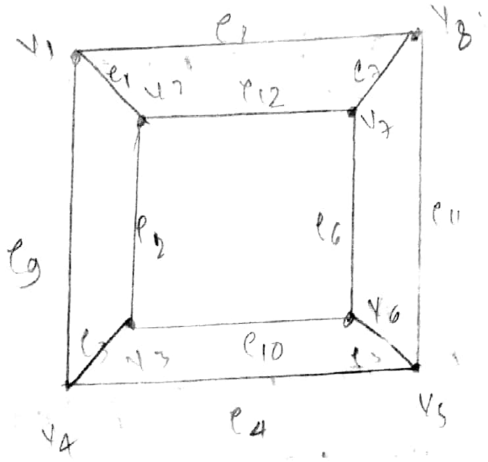
Let G be a connected graph. A hamiltonian path is an open walk in G that contains all the vertices of G

HAMILTONIAN CYCLE:

A hamiltonian cycle is a closed walk that contains all the vertices of graph G exactly once.

Ex:

G:

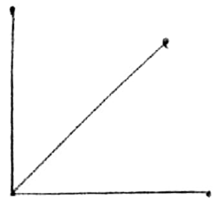


A closed walk $\{v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_5, e_5, v_6, e_6, v_7, e_7, v_8, e_8, v_1\}$. is Hamiltonian cycle.

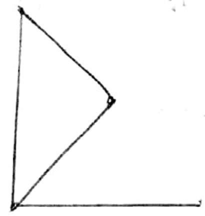
$\therefore$ G is a Hamiltonian graph.

Ex: Determine Hamiltonian path & cycle for the following graph.

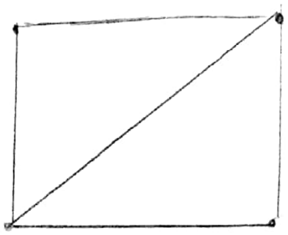
G_1:



G_2:



G_3:



PLANAR GRAPH:

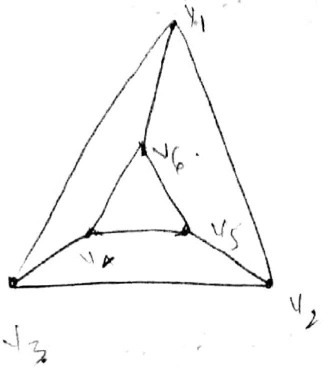
A graph G is said to be a planar graph, if G can be drawn on a plane such that no two of it's edges intersect.

NON-PLANAR GRAPH:

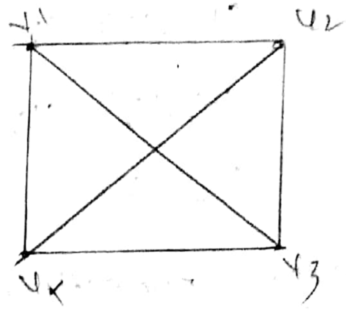
A graph that cannot be drawn on plane without edge intersection is called non-planar graph

Ex:

G_1:



G_2:



G_1 is a planar graph

G_2 - Non planar (Graph) representation.

KURATORKIS PROPERTIES ON PLANARITY:

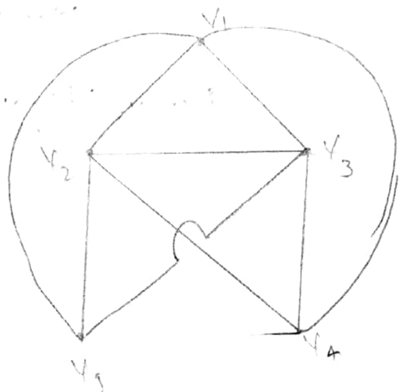
Kuratorkis is a polish Mathematician gave a unique property on a graph G to say that one of it's geometric representation is non-planar.

Property: I -

"A complete graph of 5 vertices (K_5) is non-planar."

Proof:

G:
(K_5)



Let the 5 vertices of of completegraph G be V_1, V_2, V_3, V_4, V_5. A complete graph is a simple graph. where every vertex is joined with every other vertex by an edge.

Construct a complete graph of 5 vertices.

Scanned by CamScanner

First connect (V_1, V_2), (V_1, V_3) (V_1, V_4) (V_1, V_5)

(V_2, V_3) (V_2, V_4) (V_2, V_5)

(V_3, V_4) $[(V_3, V_5)$

$(V_4, V_5)]$

the last edge connecting the vertices (V_3, V_5) cannot be drawn either from inside or from outside without edge cross over.

Thus, the graph cannot be embeded in a plane,

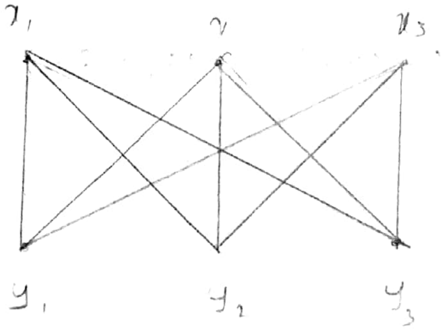
$\therefore$ K_5 is non-planar

PROPERTY 2 :

"A complete bipartite graph $K_{3,3}$ is non-planar"

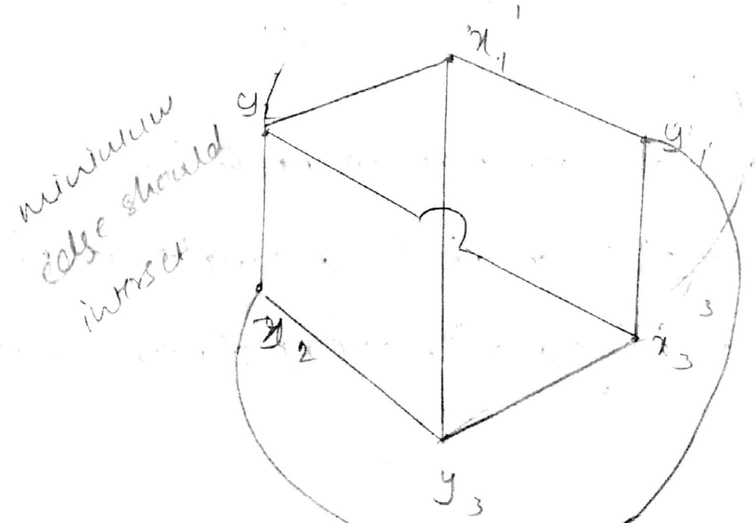
Proof:

$K_{3,3}$:



is a complete bipartite graph

Redrawing the graph,



An edge b/w the vertices (x_3, y_2) cannot be drawn either from outside or from inside without edge cross over.

$\therefore K_{3,3}$ is non-planar.

REGION:

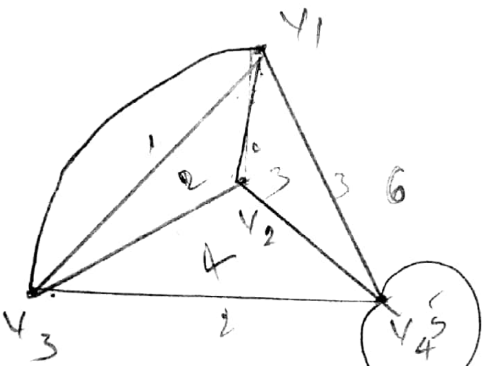
A planar representation of a graph divides the plane into regions.

A region is characterized by the set of edges forming it's boundary.

parallel edges are allowed, self loop also.

Ex:

G:



G is a planar graph dividing the plane into 6 regions. Euler's formula on