Unit : 01 10/08/15

Logics and Proofs.

logic is a discipline that deals with the methods of reasoning. Logic provides rules and techniques for determining whether a given argument is valid. Logical reasoning is used in mathematics to prove theorems, In computer science to verify the correctness of programms and to programms and to programms and to prove theorems in the natural and physical sciences to draw conclusions in experiments.

Propositional logic:

A proposition is a declarative sentence i.e., either true or false not both.

Eg: $1+1=3$.

Ravi is studying in 3rd sem CSE.

Bangalore is the capital city of India.

Eg: Sit down

me here

what is the time now?

$x+1=2$

} not proposition.

logical expressi operations:

01. Negation:

'p' be a proposition, then the negation of 'p' denoted by '$\neg p$' or '$\sim p$', or '$\bar{p}$' which means "It is not the case of p". Truth table.

P	$\neg$P
T	F
F	T

Eg: p: Today is monday.
¬p: Today is not monday.
q: Atleast ten inches of rainfall today in Mangalore.
¬q: Less than ten inches of rainfall today in Mangalore.

02. Conjunction: '∧' Equivalent to AND.

Truth table

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Let p and q be propositions then the conjunction of p and q denoted by '∧' is the proposition "p & q". It is true when both p & q are true and is false other wise.

03. Disjunction: 'v' Equivalent to OR.

The disjunction of p and q denoted by 'v' is a proposition "p or q". It is false when both p & q are false when both p and and is true otherwise.

Truth table:

p	q	pvq
T	T	T
T	F	T
F	T	T
F	F	F

Eg: p: Today is Friday.
q: Today is rainy.
p∧q: Today is friday and it is rainy
pvq: Today is friday or it is rainy

21/01/15.

04. Exclusive OR ⊕

Exclusive OR is denoted by p⊕q is a proposition that is true when exactly one of p and q is true and is ~~flase~~ false otherwise.

Truth table:

p	q	p⊕q
T	T	F
T	F	T
F	T	T
F	F	F

Abhishek C

05. Conditional statement:-

The conditional statement p→q is the proposition "if p then q" and is false when p is true, q is false and true otherwise.

Truth table:

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

06. Biconditional statement: ↔

The biconditional statement p↔q is the proposition "p if and only if q" is true when bothe p & q receives same truth value, and false otherwise.

Truth table:

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

Ex: Consider, p: Vinay learns DM.
q: Vinay will find a good job.

$p \rightarrow q$: If Vinay learns DM then he will find a good job.

$p \leftrightarrow q$: ~~[Vinay learns DM will find a good job if and only if he learns DM.]~~

$p \leftrightarrow q \equiv q \leftrightarrow p$. Vinay learns DM if & only if he find a good job.

Converse, Contrapositive & inverse of a conditional statements.

The proposition $q \rightarrow p$ is called a converse of $p \rightarrow q$.

$\neg q \rightarrow \neg p$ is contrapositive of $p \rightarrow q$.

$\neg p \rightarrow \neg q$ is the inverse of $p \rightarrow q$.

Note: $p \rightarrow q \not\equiv q \rightarrow p$.

but, $p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \rightarrow q \equiv \neg p \rightarrow \neg q$.

Eg: "The home team wins whenever it is raining."

p: It is raining.

q: The home team wins.

$p \rightarrow q$ If it is raining then the home team wins.

Converse : $q \rightarrow p$

If the home team wins then it is raining.

Contrapositive:

$\neg q \rightarrow \neg p$: If the home team does not wins then it is not raining.

Inverse:-

$\neg p \rightarrow \neg q$: If it is not raining then the home team does not wins.

Operator precedence:-

$\neg$	1
$\wedge$	2
$\vee$	
$\rightarrow$	3

$\neg \longrightarrow$ negation.

$\wedge \longrightarrow$ Conjunction (AND)

$\vee \longrightarrow$ Disjunction (OR)

$\rightarrow \longrightarrow$ Conditional

$\leftrightarrow \longrightarrow$ Biconditional.

Construct a truth table each of the following compound proposition.

① $p \rightarrow (\neg q \vee r)$.

p	q	r	$\neg q$	$\neg q \vee r$	$p \rightarrow (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

- $\neg p \rightarrow (q \rightarrow r)$.
- $(p \rightarrow q) \vee (\neg p \rightarrow r)$
- $(p \rightarrow q) \wedge (\neg p \rightarrow q)$
- $(p \oplus q) \rightarrow (p \wedge q)$
- $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$.
- $(p \oplus q) \wedge (p \oplus \neg q)$

Translating English sentences into logical equations.

let p and q be the propositions,

q: Sharks have been spotted near the shore.

p: Swimming at the shore is allowed.

Express each of the following compound propositions as an english sentences.

a) $\neg q$: Sharks have not been spotted near the shore.

b) $p \wedge q$: Swimming at the shore is allowed and sharks have been spotted near the shore.

c. $\neg p \vee q$: Swimming at the shore is not allowed or sharks have been spotted near the shore.

d. $p \to \neg q$: If swimming at the shore is allowed then sharks have not been spotted near the shore.

e. $\neg p \to q$: If swimming at the shore is not allowed then sharks have been spotted near the shore.

f. $p \leftrightarrow \neg q$: Swimming at the shore is allowed if and only if sharks have not been spotted near the shore.

g. $\neg p \wedge (p \vee \neg q)$:

$(p \vee \neg q)$: Swimming at the shore is allowed or sharks have not been spotted near the shore.

$\neg p \wedge (p \vee \neg q)$: Swimming at the shore is not allowed and soi either swimming at the shore is allowed or sharks have not been spotted near the shore.

14 p. from

let p: It is below freezing &

q: It is snowing.

Write the following propositions using p & q & logical connectivities.

1. It is below freezing and snowing.
   $p \wedge q$

2. It is below freezing but not snowing.
   $p \wedge \neg q$

3. It is not below freezing and it is not snowing.
   $\neg p \wedge \neg q$

4. It is either snowing or below freezing.
   $p \vee q$

5. If it is below freezing, it is also snowing.
   $p \to q$

6. If it is either below freezing or it is snowing but it is not snowing if it is below freezing.
   $p \to (p \vee q) \wedge \neg q$

7. That it is below freezing is necessary and sufficient for it to be snowing.
   $q \leftrightarrow p$

- Construct a truth table, each of the following compound propositions.

② $\neg p \to (q \to r)$

p	q	r	$\neg p$	$q \to r$	$\neg p \to (q \to r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

③ $(p \to q) \vee (\neg p \to r)$

p	q	r	$\neg p$	$p \to q$	$\neg p \to r$	$(p \to q) \vee (\neg p \to r)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T

p	q	r	¬p	p→q	¬p→r	(p→q)∨(¬p→r)
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

④ $(p\to q)\wedge(\neg p\to q)$

p	q	¬p	p→q	¬p→q	(p→q)∧(¬p→q)
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

⑤ $(p\oplus q)\to(p\wedge q)$

p	q	p⊕q	p∧q	(p⊕q)→(p∧q)
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

⑥ $(p\to q)\leftrightarrow(\neg q\to\neg p)$

p	q	¬p	¬q	p→q	¬q→¬p	(p→q)↔(¬q→¬p)
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

⑦ $(p\oplus q)\wedge(p\oplus\neg q)$

p	q	¬q	p⊕q	p⊕¬q	(p⊕q)∧(p⊕¬q)
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

3) Let p, q and r be the propositions
p: You have the flu
q: You miss the final examination
r: You pass the course.

01. $p\to q$: If you have the flu then you miss the final examination.

02. $\neg q\leftrightarrow r$: You will not miss the final exam if and only if you pass the course.

03 $q\to\neg r$: If you miss the final examination then you will not pass the course.

04. $(p\to\neg r)\vee(q\to\neg r)$: If you have the flu then you will not pass the course but if you miss final examination then you will not pass course.

05. $(p\wedge q)\vee(\neg q\wedge r)$: You have the flu and you miss the final examination or you will ~~miss~~ the final examination and pass the course.

4) Let p and q be the propositions.
p: You drive over 65 miles per hour.
q: You get a speeding ticket.

01. You donot drive over 65 miles per hour : $\neg p$.

02. You drive over 65 miles per hour, but you donot get a speeding ticket : $(p\vee\neg q)$

03. You will get a speeding ticket if you drive over 65 miles per hour : $p\to q$.

04. If you donot drive over 65 miles per hour, then you will not get a speeding ticket : $\neg p\to\neg q$.

05: Driving over 65 miles per hour is sufficient for getting a speeding ticket : $p\to q$

06. You get a speeding ticket, but you donot drive over 65 miles per hour : $q\wedge\neg p$.

07. Whenever you get a speeding ticket, you are driving over 65 miles per hour : $q\leftrightarrow p$

5) Let p, q and r be the propositions
p: You get an A on the final exam.
q: You do every exercise in this book.
r: You get an A in this class.

01. You get an A in this class, but you donot do every exercise in this book : $r\vee\neg q$.

12/08/15

System specifications & consistency

System & software engineers take requirements in natural language and produce precise and unambigious specifications that can be used as the basis for the system development.

System specification is said to be consistent if they should not contain conflicting statements.

Eg:- Determine where the following system specifications are consistent.

① → "The diagonostic message is stored in buffered or it is transmitted."

→ "The diagonostic message is not stored in the buffer."

→ "If the diagonostic message is stored in buffer then it is transmitted."

Let:

P: The diagonostic message is stored in the buffer.

q: Messages is transmitted.

Given, 1. $p \vee q$

2. $\neg p$

3. $p \rightarrow q$

Truth table.

p	q	$\neg p$	$p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	T

For p = F, q = T, all the given system specification are true ∴ it is consistent.

② Express, are these system specification is consistant

"The system is in multiuser state iff it is operating normally."

"If the system is operating normally, the kernel is functioning."

"The kernel is not functioning or the system is in interrupted mode."

"If the system is not in multiuser state, then it is in interrupt mode."

"If the system is not in interrupt mode."

Let:-

p: The system is in multiuser state.

q: The system is operating normally.

r: The kernel is functioning.

The system is inconsistent

p	q	r	$\neg p$	$\neg q$	$\neg r$	$p \leftrightarrow q$	$p \rightarrow r$	$\neg r \vee \neg q$	$p \rightarrow q$
1	1	1	0	0	0	T	1	0	T
1	1	0	0	0	1	1	0	1	1
1	0	1	0	1	0	0	1	1	1
1	0	0	0	1	1	0	0	1	1
0	1	1	1	0	0	0	1	0	0
0	1	0	1	0	1	0	1	1	0
0	0	0	1	1	0	1	1	1	1
0	0	0	1	1	1	1	1	1	1

Joday there is possible rain

nsistant
is operating

he kernel is

tem is in

then it is in

c."

inconsistant

q ¬p→¬q
1
1
1
0
0
1
1

All given system specifications are not satisfied,
∴ The system is inconsistent.

Q. pg no 30; Q no 47, 48, 50, 51, 52.

Logic Puzzels.

Puzzels that can be solved using logical resoning are known as logic puzzels.
Computer programs design to carry out logical resoning often use well known puzzels to illustrate their capabilities.

Consider a puzzel about an island that has two kinds of inhabitants,
knights : who always tells truth.
knaves : who always tells lies.

⇒ You encounter two people A & B, what are A & B. if "A says B is knight" & B says the 2 of us are opposite types" ?

Let p : A is knight
q : B is knave.

Assume.
If p is True

Home workrms.

Q. Express these system specifications using the propositions p "The message is scanned for viruses" & q "The message was sent from an unknown system" together with logical connectives.

a) "The message is scanned for viruses whenever the message was sent from an unknown system."

b) "The message was sent from an unknown system but it was not scanned for viruses."

c) "It is necessary to scan the message for viruses whenever it was not scanned for vi it was sent from an unknown system.

d) "When a message is not sent from an unknown system it is not scanned for viruses."

a) $p \rightarrow q$. b) $q \wedge \neg p$ c) $p \leftrightarrow q$ d) $\neg p \rightarrow \neg q$.

p	q	r
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

(Q) Express these system specifications using the propositions p "The user enters a valid password," q "Access is granted," and r "The user has paid the subscription fee" and logical connectives.

a) "The user has paid the subscription fee, but does not enter a valid password."

b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."

c) "Access is denied if the user has not paid the subscription fee."

d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

a) $r \wedge \neg p$ b) $(r \wedge p) \rightarrow q$ c) $\neg q \rightarrow \neg r$ d) $(\neg p \wedge r) \rightarrow q$

p	q	r
1	1	1
1	1	0
1	0	1
1	01	0
0	1	1
0	01	0
0	0	1
0	0	0

X5p. Are these system specification consistent? The system multiuser state if and only if it is operating normally the system is operating normally X

3. Are these system specification consistent? "Whenever

note it in terms of letters p.

nt ? The system is
erating normally

t ? "Whenever

...the puzzle.

Let p: A is Knight
q: B is Knave.

A says "B is Knight" → ①

B says "Two of us are opposite type." → ②

Let p is true ⇒ A is knight (which means whatever he says is true) → ③

Now from ① A says B is knight then
¬q is True →
i.e., B is knight → ④

From ③ & ④, both A and B are knights

The above statement contradicts statement ②.

Our assumption is wrong.
p is false.
i.e, A is a knave.

⇒ Now from ①
B is a knave.

∴ In ②, both A, B are knave.

② Let p: A is knight
q: B is knave.

A says "I am a knave or B is knight"

B says "Nothing."

Consider,

A says "I am a knave or B is knight." → ①

B says "Nothing." → ②

Assume, p / p is false.

There are 2 situations:-

Assume,
1) A is a knave.
From statement ① A says I am a knave, then from our assumption A says wrong.
i.e., thus we have a contradiction.

2) A is a knight.
If A is a knight, then statement ① is true.
Thus A is either a knave (false) or B is a knight.
From statement ②. A is not a knave then both A & B are knight.

② p: A is knight (OR)
⇒ given statement is $\neg p \vee \neg q$.
If p is T ⇒ A says True
then ¬p is False.
Now, $\neg p \vee \neg q = T$
$F \vee \neg q = T$
from above statement
¬q is true then q is false.
∴ B is knight.

③. A says "we are both knight knaves"
B says "Nothing."

given statement : $\neg p \wedge q$.

p is true, $\rightarrow$ A says true.

$\neg p \wedge q \equiv T$

then $\neg p$ is False.

$F \wedge q \equiv T$ (and) is a contradiction statement.

$\therefore$ q is true.

then ~~r~~

$\therefore$ Take p is $F \Rightarrow \neg p \wedge q \equiv F$

$T \wedge q \equiv F$

q is F $\therefore$ B is a knight.

$\therefore$ A is a knave, B is a knight.

13/08/15.

Logic and Bit operators.

Computer bit-operators correspond to the logical connectives, here True is replaced by 1 & False is replaced by 0, $\vee$ (disjuction) is replaced by OR, $\wedge$ (conjuction) is replaced by AND, $\oplus$ replaced by XOR.

Q. Find the bitwise OR and bitwise XOR 0110110110 and 1100011101

A:

```
         0 1 1 0 1 1 0 1 1 0
         1 1 0 0 0 1 1 1 0 1
         -------------------
bit AND  0 1 0 0 0 1 0 1 0 0
Bit OR   1 1 1 0 1 1 1 1 1 1
bit XOR  1 0 1 0 1 0 1 0 1 1
```

Propositional equivalence.

A compound proposition i.e., always true, no matter what the truth values of the proposition that occurs is called tautology.

A compound proposition i.e., always false, is called contradiction.

A compound proposition i.e., neither a tautology nor a contradiction is called a contigency.

The compound proposition p and q are called logically equivalent if $p \longleftrightarrow q$ is a tautology, denoted $p \equiv q$ or $p \Longleftrightarrow q$.

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ is logically equivalent.

p	q	$\neg p$	$\neg q$	$p \vee q$	A $\neg(p \vee q)$	B $\neg p \wedge \neg q$	$A \leftrightarrow B$
0	0	1	1	0	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1
1	1	0	0	1	0	0	1

$\therefore \neg(p \vee q) \equiv \neg p \wedge \neg q$

Assignment:

Show that ① $p \longrightarrow q \equiv \neg p \vee q$

② $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

③ S.T $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology

④ Verify for logical express equivalence.

(i) $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$

(ii) $(p \rightarrow r) \wedge (q \rightarrow r)$ & $(p \vee q) \longrightarrow r$

① $p \rightarrow q \equiv \neg p \vee q$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

$\therefore p \rightarrow q \equiv \neg p \vee q$.

② $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$p \vee q$	$p \vee r$	$q \wedge r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

$\therefore p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

③ $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$

p	q	r	$\neg p$	$p \vee q$	$\neg p \vee r$	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$	$(p \vee q) \wedge (\neg p \vee r) \rightarrow q \vee r$
0	0	0	1	0	1	0	0	1
0	0	1	1	0	1	0	1	1
0	1	0	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	0	0	1
1	0	1	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1
1	1	1	0	1	1	1	1	1

It is a tautology.

④ $(p \leftrightarrow q)$ & $(p \wedge q) \vee (\neg p \wedge \neg q)$

p	q	$p \leftrightarrow q$	$p \wedge q$	$\neg p$	$\neg p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
0	0	1	0	1	1	1	1
0	1	0	0	1	0	0	0
1	0	0	0	0	1	0	0
1	1	1	1	0	0	0	1

$\therefore p \leftrightarrow \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

⑤ $(p \rightarrow r) \wedge (q \rightarrow r)$ & $(p \vee q) \rightarrow r$.

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$	$(p \vee q) \rightarrow r$
0	0	0	0	01	1	1	1
0	0	1	0	01	1	1	1
0	1	0	1	1	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	1	0	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

$\therefore (p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

17/08/18

Laws of logical equivalence

1. Identity laws:
   ① $p \wedge T \equiv p$
   ② $p \vee F \equiv p$

2. Domination laws:
   $p \vee T \equiv T$
   $p \wedge F \equiv F$

3. Idempotent laws.

$p \vee p \equiv p$

$p \wedge p \equiv p$

4. Double negation laws:

$\neg(\neg p) \equiv p$

5. Commutative law.

$p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$.

6. Associative laws:

$p \vee (q \vee r) \equiv (p \vee q) \vee r$.

$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$.

7. Distributive law:

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

8. Demorgan's law:

$\neg(p \vee q) \equiv \neg p \wedge \neg q$.

$\neg(p \wedge q) \equiv \neg p \vee \neg q$.

9. Absorption laws:

$p \vee (p \wedge q) \equiv p$ $\quad (p \vee p) \wedge (p \vee q)$

$p \wedge (p \vee q) \equiv p$ $\quad p \wedge (p \vee q) = (p \wedge p) \vee (p \wedge q)$

10. Negation laws:

$p \vee \neg p \equiv T$

$p \wedge \neg p \equiv F$

11. Demorgan's law extended to 'n' propositions.

$\neg(p_1 \vee p_2 \vee p_3 \cdots \vee p_n) \equiv \neg p_1 \wedge \neg p_2 \wedge \neg p_3 \cdots \wedge \neg p_n$

$\neg(p_1 \wedge p_2 \wedge p_3 \cdots \wedge p_n) \equiv \neg p_1 \vee \neg p_2 \vee \neg p_3 \cdots \vee \neg p_n$

Some standard logical equivalence involved in cond statements:-

① $p \to q \equiv \neg p \vee q$.

② $p \to q \equiv \neg q \to \neg p$.

③ $p \vee q \equiv \neg p \to q$.

④ $p \wedge q \equiv \neg(p \to \neg q)$ $\quad p \wedge (p \to q) \vee r$.

⑤ $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$ $\quad (p \vee q) \to r$

⑥ $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$.

⑦ $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

⑧ $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$.

Logical equivalence involving biconditionals.

① $(p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p)$

② $(p \leftrightarrow q) \equiv \neg p \leftrightarrow \neg q$

③ $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$.

④

~~Prove that the negation~~

Using laws of logic.

1. Show that $\neg(p \to q) \equiv p \wedge \neg q$.

LHS, $\neg(p \to q) \equiv \neg(\neg p \vee q)$ $\quad \because (p \to q \equiv \neg p \vee q)$

$\equiv \neg(\neg p) \wedge \neg q$ $\quad (\because$ Demorgan's law)

$\equiv p \wedge \neg q = \text{RHS}$ $\quad (\because$ Double negation law)

2. $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$.

LHS, $(p \to q) \wedge (p \to r) \equiv (\neg p \vee q) \wedge (\neg p \vee r)$ $\quad \because (p \to q \equiv \neg p \vee q)$

$\equiv \neg p \wedge (\neg p \vee r) \vee q \wedge r$ (Distributive law)

$\equiv \neg p \vee (q \wedge r)$ $\quad \because (p \to q \equiv \neg p \vee q)$

$\equiv \neg p \to (q \wedge r)$.

3. $\neg p \to (q \to r) \equiv q \to (p \vee r)$

LHS, $\neg p \to (q \to r) \equiv \neg p \to (\neg q \vee r)$ $\because p \to q \equiv \neg p \vee q$.

$\equiv \neg p$ q

we have $\neg p \to q \equiv p \vee q$.

$\equiv p \vee (\neg q \vee r)$ $\because$ Associative law

$\equiv (p \vee \neg q) \vee r$.

$\equiv p \vee r \vee \neg q$

$\equiv \neg q \vee (p \vee r)$ $\because \neg p \vee q \equiv p \to q$.

$\equiv q \to (p \vee r) = \text{RHS}$

4. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

LHS, $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ $\because p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$.

$\equiv \underset{A}{(\neg p \vee q)} \wedge \underset{B}{(\neg q \vee p)}$ $\because p \to q \equiv \neg p \vee q$

$\equiv (\neg p \vee \neg q) \vee (\neg p \vee p)$ $\because$ Distributive law.

$\equiv A \wedge (\neg q \vee p)$ $\because$ Distributive law $A = \neg p \vee q$

$\equiv (A \wedge \neg q) \vee (A \wedge p)$

$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$

$\equiv [(\neg p \wedge \neg q) \vee (q \wedge \neg q)] \vee [(\neg p \wedge p) \vee (q \wedge p)]$

$\equiv [(\neg p \wedge \neg q) \vee F] \vee [F \vee (q \wedge p)]$ Negation law [Domination Identity law]

$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$

commutative law

$\equiv \underbrace{(\neg p \wedge \neg q) \vee (p \wedge q)}_{\text{commutative law}}$

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q) = \text{RHS}$

NOTE: $p \to q$ can be expressed in the following forms.

1. If p then q
2. q whenever p
3. If p, q.
4. p sufficient for q.
5. q if p.
6. q when p
7. a necessary condition for p is q.
8. q unless ¬p
9. p implies q.
10. p only if q
11. A sufficient condition for q is p.
12. (q whenever p)
13. q is necessary for p.
13. q follows p

whenever

wh

unless p

13/8/15.

Predicates and Quantifiers.

A Predicate is a property that variable or a finite collection of variables can have. a predicate becomes a proposition when a specific values are assigned to the variables.

$P(x_1, x_2, \ldots x_n)$ is called a predicate of n variables.

Eg: 1) $P(x): x > 3$

2) $P(x,y): x+y=3$.

$x+y=z$

3) $P(x)$: x is a student of NMIT → predicates.

P(vinay): x is a student of NMIT → proposition.

4) $Q(x,y)$: x is a student of y.

Quantifiers

Quantification expresses the extend to which predicate is true over a ronge of elements.

In english, the words 'all', 'some', 'mony', 'none', 'few' are used in quontification.

Types of quantification:-

01. Universal quantification:

The universal quantification of $P(x)$ is a statement "$P(x)$ for all values of x in the domain"

$\forall x\ P(x)$ is the notation. "$\forall$ read it as for all $x\ P(x)$"

OR

"$\forall$ read as for every x $P(x)$"

02. The existential quantification:

The existential quantification of $P(x)$ is a statement "there exist an element x in the domain such that $P(x)$".

The notation is $\exists x\ P(x)$, here $\exists$ is called existential quantifier.

$P(x): x>3$

Eg: ① $\forall x\ P(x)$ is False

$\exists x\ P(x)$ is True.

② $P(x,y)$: x is a student of y.

$\forall x\ \forall y\ P(x,y)$ is False.

$x \in$ student set.

$y \in$ Set of Collegu.

$\exists x \exists y\ P(x,y)$ is True.

03. Other quantifiers.

1) Uniquenoos quantifiers:

Denoted by $\exists!$ or $\exists_1$

The notation $\exists! x\ (P(x))$ states that there exist unique x such that $P(x)$ is true.

Translate the following statements into english where

$C(x)$: "x is a comedian"

$F(x)$: "x is funny".

Domain is consists of all people.

1. $\forall x\ (C(x) \rightarrow F(x))$.

Every comedian is funny.

2. $\forall x\ (C(x) \land F(x))$

All people are Every parson is a comedian & funny

3. $\exists x\ (C(x) \rightarrow F(x))$.

There exist a parson such that if he is comedian then he is funny.

4. $\exists x\ (C(x) \land F(x))$

There exist a parson who is comedian and funny.

Ex. $P(x)$ = "x can speak russian."

$Q(x)$ = "x knowns computer language C++."

Express each of the foll^g sentence $P(x)$ & $Q(x)$ quantifiers and logical connectives.

The Domain for quantifier consists of all students of NMIT.

who knows C++: $\exists x\ (P(x) \wedge Q(x))$

2. There is a student at NMIT who can speak russian but who donot know C++: $\exists x (P(x) \wedge \neg Q(x))$

3. Every student at NMIT either can speak russian ~~but~~ or knows C++: $\forall x\ (P(x) \vee Q(x))$

4. No students at NMIT can speak russian or knows C++:

$$\cancel{\forall x(\neg P(x) \wedge \neg Q(x))} \quad \forall x\ \neg(P(x) \vee Q(x))$$

Note:- Logical equivalence involving quantifiers:

01. $\forall x(P(x) \wedge Q(x)) \equiv \forall x\ (P(x)) \wedge \forall x\ Q(x)$
02. $\forall x\ (P(x) \vee Q(x)) \not\equiv \forall x\ (P(x)) \vee \forall x\ Q(x)$
03. $\exists x\ (P(x) \wedge Q(x)) \not\equiv \cancel{\exists x\ P(x) \wedge \exists x\ Q(x)}$

Negation of quantified expressions:

$\neg \exists x\ P(x)$

Negation - Equivalent statement:

$\neg \exists x\ P(x) \equiv \forall x\ \neg P(x)$

$\neg \forall x\ P(x) \equiv \exists x\ \neg P(x)$

Find write the negation of quantified expressions.

1. Every student in your class has taken a course in DMS.

Given, $\forall x\ P(x)$ x is in the domain of all students of your class.

$P(x)$: x has taken a course in DMS.

Given state, $\forall x\ P(x)$.

$$\neg(\forall x\ P(x)) \equiv \exists x\ \neg P(x).$$

There exists is a student in your class who doesn't was taken a course in DMS.

2. There is a honest Politician.

Domain :- All politician.

$P(x)$: x is a honest politician.

given, $\exists x\ P(x)$.

$$\neg(\exists x\ P(x)) \equiv \forall x\ \neg P(x)$$

All politicians are not honest.

OR

Every politicians are corrupt.

3. All indians eat sweets.

Domain: All indians.

$P(x)$: x eat sweets.

given, $\forall x\ P(x)$

$$\neg(\forall x\ P(x)) \equiv \exists x\ \neg P(x).$$

All ~~poli~~ indians donot eat sweets.

There is an indian who doesnt eat sweets.

QNo: 9; Pg No: 172; Q. No. 1.3

let $P(x)$ be a statement,

$P(x)$ = "$x = x^2$" if the domain consists of integers what are the ~~t~~ truth values.

① $P(0) \longrightarrow T$

3. $P(2) \longrightarrow F$
2) $P(-1) \longrightarrow F$
5. $\exists x P(x) \longrightarrow T$
6. $\forall x P(x) \longrightarrow F$

Determine the truth to value of each of the following statements if the domain consist of all integers.

1. $\forall n(n+1>n) \rightarrow T$
2. $\exists n(2n=3n) \rightarrow T$ $(\because n=0)$
3. $\exists n(n=-n) \rightarrow T$ $\because$ for $(n=0)$
4. $\forall n(n^2 \geq n) \rightarrow T$

hw Q50, Pg 47; Q No 15, 17, 19. h.w.

Translate into the statement to logical expressions using predicates, quantifiers & logical connectives, condering,

(a) domain consists of students in your class.
(b) domain consist of all people.

State

(i) Some one in your class can speak hindi.
(ii) Every one in your class is friendly.
(iii) There is a person in your class who was not born in California.
(iv) A student in your class has been in a movie.
v). No student in your class taken a course in logic program.

1) Let $P(x)$: x can speak hindi.
$\exists x\, P(x)$

2) let $P(x)$: x is friendly in your class.
$\Rightarrow \forall x\, P(x)$

3) let $P(x)$: x was not born in California.
$\Rightarrow \exists x\, \neg P(x)$

4) let $P(x)$: x has been in a movie.
$\Rightarrow \exists x\, P(x)$

5) let $P(x)$: x has taken a course in logic program
$\Rightarrow \cancel{\nexists} \forall x \neg P(x)$ $\quad \neg \exists x\, P(x)$

b. Domain consists of all people.

1) let $P(x)$: x can speaks hindi.
$Q(x)$: x is in your class.
$\Rightarrow \exists x\,(Q(x) \wedge P(x))$

2) let $P(x)$: x is friendly
$Q(x)$: x is in your class.
$\Rightarrow \forall x\,\cancel{(Q(x) \wedge P(x))}$
$Q(x) \rightarrow P(x)$

3) let $P(x)$: x was not born in California.
$Q(x)$: x is in your class.
$\Rightarrow \exists x\,(Q(x) \wedge \neg P(x))$

4) Let $P(x)$: x has been in a movie.
$Q(x)$: x is in your class.
$\Rightarrow \exists x\,(Q(x) \wedge P(x))$

5) Let $P(x)$: x has taken a course in logic program
$Q(x)$: x is in your class.
$\Rightarrow \forall x\,(Q(x) \longrightarrow \neg P(x))$

Pg No 48 Q No 15, 21

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System specification using predicates and quantifiers.

~~Use~~ predicates and quantizers to express the system specification.

1. At least the one main message, among the non empty set of messages can be saved if there is a disk, with more than 10k bytes of free space.

Given, $P(m)$: Message m can be saved.

$Q(x, y)$: x is a disk with more than y KB of free space.

$m \to$ set of messages.
$x \to$ disk $y \to$ set of bytes

$\exists m \quad \exists x\, Q(x, 10) \longrightarrow \exists m\, P(m)$.

2. Whenever there is an active alert all ~~cate~~ queued messages are transmitted. $x \to$ set of message

Given; $P(x)$ = x is an active alert.

$Q(x)$ = x is queued.

$S(x)$ = x is transmitted.

$\exists x\, (P(x)) \Longrightarrow \forall x\, (Q(x) \to S(x))$

Domain All Alert message   Domain All message

3. Diagnostic monitor ~~diags~~ tracks the status of all systems except the main console

$P(x)$: The diagnostic monitor tracks the status of system x.

$\forall x\, (x \neq \text{main console}) \longrightarrow P(x)$.

OR

$\forall x\, (\neg Q(x) \to Px)$ where, x is main console = $Q(x)$.

4. Each participant on the conference call whom the host of the call did not put on a special list was ~~bilded~~ billed.

Let, $P(x)$ = The host of conference call put participant x on a special list

$Q(x)$ = x was billed.

$\forall x\, (\neg(P(x)) \longrightarrow Q(x))$ Ex: P43, QNo: 85, 25, 27.

~~Nested~~ quantifiers :-

Two or more quantifiers are nested if one is still within the scope of other.

Eg: $\forall x \exists y\, (x + y = 0)$

$\forall x \forall y\, \forall z\, (x + (y+z) = (x+y) + z)$

Translate into English statement

$\forall x \forall y\, ((x > 0) \wedge (y < 0) \longrightarrow (xy < 0))$

For all real nos x & y, if x is positive & y is negation then xy is negative.

(OR)

The product of a +ve real no and a -ve real no is always a negative real no.

1. $\forall x \exists y\ (x<y)$

2. $\forall x \forall y\ ((x \geq 0) \wedge (y \geq 0)) \rightarrow (xy \geq 0))$

3. $\forall x \forall y\ \exists z\ (xy = z)$

1. For all real no x there exist real no y, such that x is less than y.

2. For all real no x and real no y if x and y are +ve or of zero then

For all real no x and real no y if x is non-+ve and y is non-+ve then their product of is non-ve.

3. For all real no x and real no y, There exist a real no z such that the product of x & y is equal to z.

4. $\forall x (C(x) \vee \exists y (C(y) \wedge F(x,y)))$

where $C(x)$: x has a computer.

$F(x,y)$: x and y are friends.

Domain, x and y is set of all students in your class.

~~every student~~

For all x student in your class, x has a computer or there is a student y such that y has a computer and x and y are friends.

(OR)

Every student in your class has a computer or has a friend who has a computer.

6. $\exists x \forall y \forall z ((F(x,y) \wedge F(x,z) \wedge (y \neq z)) \rightarrow \neg F(y,z))$

where $F(a,b)$: a & b are friends.

Domain of x, y, z is set of all students in your class.

Soln:- There exist a student x, for all y and z students other than x in your class, if x and y are friends and x & z are friends and then y and z are not friend.

Q. Translate english statements to quantifiers.

1. Express.

"If a person is a female and a parent then, there is a person y such that person x is mother of person y."

This person is someones mother.

Let $P(x)$: x is a female.

$Q(x)$: x is a parent.

$R(x,y)$: x is a mother of y.

Domain of x & y set of all persons.

$\forall x (P(x) \wedge Q(x)) \longrightarrow \exists y (R(x,y)))$. (OR)

$\forall x \exists y\ ((P(x) \wedge Q(x)) \rightarrow R(x,y))$

2. Every one has exactly one best friend.

Let $P(x,y)$: y is a best friend of x.

$\forall x \exists y (P(x,y) \wedge \forall z ((z \neq y)) \longrightarrow \neg P(x,z)$

Negating nested quantifiers.

Use quantifiers

Express the negation of statements.

$\forall x \exists y (xy = 1)$

Soln: $\neg(\forall x \exists y (xy=1)) \equiv \exists x \forall y (xy \neq 1)$

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2. Use quantifiers to express the following statement.

There does not exists a women who has taken a flight on every airline in the world.

Leaving the negation, there is a women, who has taken a flight on every airline in the world.

P(w,f): "Wom has taken the flight f".

Q(f,a): "f is a flight on a".

w – women
f – flight
a – airline
} Domains.

$\neg(\exists w \exists f \forall a (P(w,f) \wedge Q(f,a)))$

OR

$\forall w \forall f \exists a (\neg P(w,f) \vee \neg Q(f,a))$

$\exists w \exists f \forall a (P(w,f) \wedge Q(f,a)) \longrightarrow ①$

Taken the negation of ①.

$\neg(\exists w \exists f \forall a (P(w,f) \wedge Q(f,a)))$

$\equiv \forall w \forall f \exists a (\neg P(w,f) \vee \neg Q(f,a))$

24/08/2015

3. Let ~~P(x,y)~~ be that

$T(x,y)$: "x likes cuisine y."

where Domain for x consists of all student in your class. & the domain for y consists of all cuisine.

Express each of the following the english sentence.

① $\neg T(Mohan, Japanese)$

Mohan does not like cuisine Japanese.

② $\exists x\, T(x, Korean) \wedge \forall x\, T(x, Mexican)$.

There exist a student in your class who likes Korean cuisine and for every one student in your class likes Mexican cuisine.

③ $\exists y (T(Anil, y) \vee T(Jay, y))$.

There is some cuisine which is liked by Anil or Jay. (OR)

There is a some cuisine which which is either Anil or Jay likes it.

④ $\forall x \forall z \exists y ((x \neq z) \rightarrow \neg(T(x,y) \wedge T(z,y)))$

For every pair of distinct students in your class, there is a cuisine such that atleast one of them donot like it.

⑤ $\exists x \exists z \forall y (T(x,y) \leftrightarrow T(z,y))$

There are two students in your class (for all cuisine) who both likes exactly same set of cuisine.

OR

For a student in your class there is another student such that one likes any cuisine iff other does like it.

⑥ $\forall x \forall z \exists y (T(x,y) \leftrightarrow T(z,y))$.

For all your pair of student in your class, there is some cuisine either they like it or they donot like it. (OR)

About which they have the same opinion.

Pg No 59, Q No 9, 10.

English → logical.

01. let $S(x)$ = "x is a student"
$F(y)$ = "y is a faculty member"
$A(x,y)$ = "x has asked y a question.

Domain consists of all people associated in your college use quantifiers to express the following statements.

i. Karan asked professor vijaya shetty a question.

$$\exists x (A(x, y)) \quad \text{or} \quad (A(\text{karan}, \text{vijaya shetty}))$$

ii. Every student asked professor shobha a question.

$$\forall x\, (A(x, \forall y\ \text{shobha}))$$

∵ domain is all people

$$\forall x (S(x) \longrightarrow A(x, \text{shobha}))$$

iii. Every faculty member ~~ask~~ has either asked Prof. thippeswamy or a question or ~~being~~ been asked a either a question by prof. thippe a.

$$\forall y (F(y) \rightarrow A(x, \text{Prof. Thippe}) \vee (F(\text{Prof. Thippe}, y)))$$

iv. Some student has not asked any faculty member a question.

$$\exists x (S(x) \leftrightarrow \neg A(x, y))$$

$$\exists x (S(x) \wedge \forall y (F(y) \longrightarrow \neg A(x, y)))$$

OR.

$$\exists x \forall y\, (S(x) \wedge (F(y) \longrightarrow \neg A(x, y)))$$

v. There is a faculty member who is n(ever) a question by a student.

$$\exists y (F(y) \wedge \forall x (S(x) \longrightarrow \neg A(x, y)))$$

vi. Some student has asked every faculty mem(ber) question.

$$\exists x (S(x) \wedge \forall y (F(y) \rightarrow A(x, y)))$$

OR.

$$\forall y\, (F(y))$$

vii. There is a faculty member who has as(ked) ~~who~~ every other faculty member a quest(ion)

$$\exists y (F(y) \wedge \forall z ((y \neq z) \wedge F(x)) \rightarrow A(y$$

viii. Some student has never been asked a (question) by a faculty member.

$$\exists x (S(x) \wedge \forall y (F(y) \longrightarrow \neg A(x, y))).$$

59

Q.No. 13, 60. 17, 18.

Rule of inference:
consider the following statements.

1. If you have a current password, then you can logon to the network.
2. You have current password.

Therefore, you can logon to the network.

Argument means a sequence of statement (each statement called as premises, hypothesis) that [ends] with a conclusion.

Consider a general format of an argument.

p : hyp 1
q : hyp 2
r : hyp 3
s : conclusion. where, p, q, r, s are compound statement.

$p \wedge q \wedge r \to s$

The above argument is said to be valid, when $(p \wedge q \wedge r) \longrightarrow s$ is Tautology.

Rules of Inference : Name of Inference

① p, $p \to q$, $\therefore q$ — Modus Ponens.

② $\neg q$, $p \to q$, $\therefore \neg p$ — Modus Tollens.

③ $p \to q$, $q \to r$, $\therefore p \to r$ — Hypothetical Syllogism.

④ p, $\therefore p \vee q$ — Addition rule.

⑤ $p \wedge q$, $\therefore p$ — Rule of simplification.

⑥ p, q, $\therefore p \wedge q$ — Rule of conjuntion.

⑦ $p \vee q$, $\neg p \vee r$, $\therefore q \vee r$ — Rule of resolution.

Determine the rule of inferences applicable in the following cases:-

① a. If you have access to the network, then you can change your grade.
b. You have access to the network.
Therefore you can change your grade.

Let p: You have access to the network.
q: You can change your grade.

a: $p \to q$
b. p
conclusion: q.

$\therefore$ $p \to q$, p, $\therefore q$ is the modus Ponens.

②a. It is below freezing now. ~~Therefo~~

b. Therefore it is either ~~below~~ freezing or raining now.

Let : p: It is below freezing now.

q: It is raining now.

⇒ a. p.

conclusion: $p \vee q$

$$\frac{p}{\therefore p \vee q} \rightarrow \text{Rule of Addition.}$$

③a. It is below freezing & raining now.

b. Therefore, it is below freezing now.

Let p: It is below freezing now.

q: It is raining now.

a. $p \wedge q$

conclusion p

$$\frac{p \wedge q}{\therefore p} \rightarrow \text{Rule of simplification}$$

④ a. If it rains today, then we will not have a barbeque today.

b. If we donot have a barbeque today, then we will have a barbeque tomorrow.

Therefore, if it rains today, then we have a barbeque tomorrow.

It is both a cooking method and cooking apparators.

Let ; p: It rains today.

q: We will not have a barbeque today.

r: We will have a barbeque tomorrow.

Step No		Reasons
01.	$\neg p \wedge q$	hyp 1.
02.	$\neg p$	Rule of simplification.
03.	$r \to p$	hyp 2.
04	$\neg r$	~~Mode~~ Modus Tollens.
05.	$\neg r \to s$	hyp 3.
06.	s	Modus Tollens.
07.	$s \to t$	hyp 4
08.	t	Modus Tollens.

26/03/15

Show that the hypothesis is valid.

If you sent me and email message, then I will finish writing the program. If you donot sent me an email message then I will go to sleep early. If I go to sleep early, then I will wake up ~~ea~~ ~~dearly~~ feeling refreshed.

∴ If I donot finish writing the program, then I will wake up ~~early~~ feeling refreshed.

Let: p: You send me an email message.

q: I will finish writing the program.

r: I will go to sleep early.

s: I will wake up feeling refreshed.

$p \to q$

$\neg p \to r$

$r \to s$

$\therefore \neg q \to s$

Step No.		Reasons
01.	$p \to q$	hyp 1.
02.	$\neg q \to \neg p$	Contrapositive of ①.
03.	$\neg p \to r$	hyp 2.
04.	$\neg q \to r$	Hypothetical syllogism ②&③
05.	$r \to s$	hyp 3.
06.	$\neg q \to s$	~~Hypothetical~~ Hypo the syti cal syllogism ④&⑤

07. ∴ The above hypothesis are valid.

Use rule of inference to S.T the hypothesis is valid -

Vinay works hard. If vinay works hard, then is a dull boy. If vinay is dull boy then he will not get job. Therefore Vinay will not get the job.

Given, let p: Vinay works hard.

q: Vinay is a dull boy.

r: Vinay will not get the job.

p

$p \to q$

$q \to r$

$\therefore r$

		Reasons
01.	p	hyp1.
02.	$p \to q$	hyp 2.
03.	q	Modus Ponens ①&②
04.	$q \to r$	hyp3.
05.	r	Modus Ponens ③&④

∴ It is valid.

Combining rule of inferences of positive propositions and quantifiers.

Universal instantiation.

for a given premise $\forall x\, P(x)$ universal instantiation is used for a particular instant.

Eg: All woman are wise.

Universal instantiation for above statement is "Kiran Bedi is wise" is ca

Universal generalization.

Existential instantiational:

Is a rule that allows us to control conclude that there is an element 'a' in the domain for which $P(a)$ is true if we know that ∃x true.

Existential generalization.

Is a rule that is used to conclude there $\exists x\, P(x)$ when a particular element a with $P(a)$ is true.

Rule of Inference.

① $\dfrac{\forall x\, P(x)}{\therefore P(a)}$ Universal instantiation

② $\dfrac{P(a) \text{ for any arbitrary } a}{\therefore \forall x\, P(x)}$ Universal generalization

③ $\dfrac{\exists x\, P(x)}{\therefore P(a)}$ Existential instantiation

④ $\dfrac{P(a) \text{ for an some } a}{\therefore \exists x\, P(x)}$ Existential generalization

$\forall x$ $P(a)$ $\exists x$ $P(a)$ $\forall x$ $\exists x$

Q. Prove Show that the following argument are valid.
Everyone in this DMS class has taken a course in Computer Science. Niranjan is a student in this class implies the conclusion. Niranjan has taken a course a course in CS.

Let $P(x)$: x is in DMS class. $x \in$ to the domain all student in NMN.

$Q(x)$: x has taken a course in CS.

$\forall x\ (P(x) \rightarrow Q(x))$
P(Niranjan)

$\therefore$ Q(Niranjan).

1.	$\forall x\ (P(x) \rightarrow Q(x))$	premise 1
2.	$P(N) \rightarrow Q(N)$	Universal instantiation
3.	$P(N)$	premise 1
4.	$Q(N)$	Modus Ponens ② & ③

It is true.

2. S.T the premises is true.
A student in this class has not read the book. Everyone in this class passed the 1st exam imply the conclusion someone who passed the 1st exam doesnot read the book.

Given, Let $P(x)$: x is a student in this class.
$q(x)$: x has not read the book.
$r(x)$: x has passed the 1st exam.

$\exists x\ (P(x) \wedge \neg q(x))$
$\forall x\ (P(x) \rightarrow r(x))$

$\therefore \exists x\ (r(x) \wedge \neg q(x))$

Step No.		
1.	$\exists x\ (P(x) \wedge \neg q(x))$	hyp 1
2.	$P(a) \wedge \neg q(a)$	Existential instantiation. (where a is a student)
3.	$P(a)$	Rule of simplification ②
4.	$\neg q(a)$	Rule of simplification
5.	$\forall x\ (P(x) \rightarrow r(x))$	hyp 2
6.	$P(a) \rightarrow r(a)$	Universal instantiation.
7.	$r(a)$	Modus Ponens ③ & ⑥
8.	$r(a) \wedge \neg q(a)$	Rule of conjuntion ④ & ⑦
9.	$\exists x\ (r(x) \wedge \neg q(x))$	Existential Generalization

What rule of inference are used in the following famous argument.
All men are moral. Socrates is a man.
Therefore Socrates is mortal.

$P(x)$: x is man.
$Q(x)$: x is moral.
$r(x)$: a = Socrates.

$\forall x\ (P(x) \rightarrow q(x))$
P(Socrates).

$\therefore$ q(Socrates).

Step No		Reasons
01.	$\forall x\,(p(x) \to q(x))$	hyp 1.
02.	$p(s) \to q(s)$	Universal instantiation.
03.	$p(s)$	hyp 2.
04.	$q(s)$	Modus Ponens ② & ③.

03. The rule of inference are used is Universal instantiation & Modus Ponens & is valid.

For each of the following argument explain which rule of inference are used for each step. [Verify the validity for foll hyp with reason]

→ Niranjane is a student in this class, knows how to write program in java.
Everyone who knows how to write program in java can get a high ~~wage~~ paying job. Therefore someone in this class can get a high paying job.

Let $p(x)$: x is a student in this class

$q(x)$: x knows how to write program in java.

$r(x)$: x can get a high paying job.

$\therefore \exists x\,(p(N) \wedge q(N))$. N→ Niranjan

$$\frac{\begin{array}{c} p(N) \wedge q(N) \\ \forall x\,(q(x) \to r(x)) \end{array}}{\therefore \exists x\,(p(x) \wedge r(x))}$$

Step No:		Reason
01:	$p(N) \wedge q(N)$	
02.	$\exists x\,(p(x) \wedge q(x))$	hyp 1

01.	$p(N) \wedge q(N)$	hyp 1
02.	$p(N)$.	Rule of s
03.	$q(N)$	
04.	$\forall x\,(q(x) \to r(x))$	hyp 2.
05.	$q(N) \to r(N)$	Universal instantiation
06.	$r(N)$	Modus Ponens ③ & ⑤
07.	$p(N) \wedge r(N)$	Rule of conjunction ② & ⑥
08	$\exists x\,(p(x) \wedge r(x))$	Existential Generalization

Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution.
Given.

$p(x)$: x is a student in this class.

$q(x)$: x enjoys whale watching.

$r(x)$: x cares about ocean pollution.

$$\begin{array}{l} \exists x(p(x) \land q(x)) \\ \forall x(q(x) \to r(x)) \\ \hline \therefore \exists x(p(x) \land r(x)) \end{array}$$

Step No.		Reasons
01.	$\exists x(p(x) \land q(x))$	hyp 1.
02.	$p(a) \land q(a)$	Existential instantiational
03	$p(a)$	Rule of simplification
04	$q(a)$	Rule of simplification
06	$\forall x(q(x) \to r(x))$	hyp 2.
06	$q(a) \to r(a)$	Universal Instantiation
07	$r(a)$	Modus Ponens 4 & 6
08	$p(a) \land r(a)$	Rule of conjuction 3 & 7
09.	$\exists x(p(x) \land r(x))$	Existential Generalization.

It is valid. pg 72, 1,2,3.

7

2. In proof by contraposition, we take contrapositive of statement $p \to q$ i.e., $\neg q \to \neg p$ then we give direct proof to $\neg q \to \neg p$ as $\neg q \to \neg p \equiv p \to q$. Hence the result.

3. In proof by contraposition method, let $p = T$. Actually we have to prove q is true, but on the contrary we assume q = False. Then we analyse and argue with definition and properties of q, we get p = False.
This is contradiction. Therefore our assumption q = False is wrong. Hence the theorem.

Eg: Give a direct proof of the theorem. "If n is odd integer then n^2 is odd.

$\Rightarrow$ p: n is odd
q: n^2 is odd.

We have to prove $p \to q$
Consider n is odd.
$n = 2k+1$ where $k \in Z$

Squaring on both side for the above eqn.

$$n^2 = (2k+1)^2$$
$$n^2 = 4k^2 + 1 + 4k$$
$$= 4(k+1)k + 1$$
$$= 2(2k^2+2k) + 1$$
$$n^2 = 2\underbrace{\quad}_{k_1} + 1 \qquad k_1 \in Z,\ k_1 = 2k^2 + 2k$$

$\therefore$ n^2 is also odd. Hence theorem is proved.

Prove that $\sqrt{2}$ is a irrational number by giving a proof by contradictive.

$P: \sqrt{2}$ is irrational, Assume $\neg P = T$.

i.e, On the contrary, assume $\sqrt{2}$ is irrational.

then, $\sqrt{2} = \frac{a}{b}$ $b \neq 0$ $a, b \in Z$ and have no common factor. ①

square on b.s.

$2 = \frac{a^2}{b^2}$

$2b^2 = a^2$ ② (Here 2 is multiple with b^2).

∴ the a^2 is also even.

$\Rightarrow a^2$ is even.

By defination and properties of even integer.

a is even.

$\Rightarrow a = 2k$. for some $k \in Z$

Eqn ② becomes.

$2b^2 = 4k^2$.

$b^2 = 2k^2$.

By defination and properties of even intega.

b^2 is even.

$\Rightarrow b$ is even.

∴ $b = 2k_1$ then eqn ① becomes.

$\sqrt{2} = \frac{a}{b} = \frac{2k}{2k_1}$ [∵ common factor)

The above eqn is contradiction to eqn ①.

that is a & b have no common factor.

$\Rightarrow$ Our assumption is $\sqrt{2}$ is rational is false.

∴ $\sqrt{2}$ is irrational.

Hence the proof.

Give a proof by contradiction of the theorem if $3n+2$ is odd then n is odd.

Proof: p: $3n+2$ is odd.

q: n is odd.

we need to prove, $P \rightarrow q = T$.

for contradiction proof Assume $\neg q = T$ i.e, n is even } eqn ①.

$n = 2k$.

Consider, $3n+2 = 3(2k)+2$.

$= 2(3k+1)$

$3n+2$ = even. for $(3k+1) \in Z$.

i.e., p is False.

By contradiction to p. i.e.

$\Rightarrow$ Our assomption, n is even is false.

∴ n is even.

i.e., $(3n+2$ is odd$) \rightarrow (n$ is odd$)$ is True.

Hence the proof.

Given, P.T th if n is an integer and n^3+5 is odd then, n is even using i) contradiction proof

ii) contrapositive proof.

q: n is even. $p \to q$

For contradiction proof take $\neg q$ is ~~false~~ True.

then, n is odd. $\to$ ①

By defination of odd integer.

$$n = 2k+1$$

Consider $n^3+5 = (2k+1)^3 + 5$

$$= 8k^3 + 1 + 3(2k)(1)(2k+1) + 5$$

$$= 8k^3 + 6k(2k+1) + 5$$

$$= 8k^3 + 12k^2 + 6k + 1 + 5$$

$$= 8k^3 + 12k^2 + 6k + 6$$

$$n^3+5 = 2(\underbrace{4k^3 + 6k^2 + 3k + 3}_{\in Z})$$

$\Rightarrow n^3+5 =$ even.

$\therefore$ The above eqn is contradiction to p: n^3+5 is odd.

$\therefore$ Our assumption that n is odd ($q = F$) is false the $q = T$.

i.e., n is even. Hence the proof.

ii) Contrapositive proof.

p: n^3+5 is odd.

q: n is even.

$p \to q$. The contrapositive of $p \to q$ is $\neg q \to \neg p$.

let $\neg q = T \to$ ①

n is odd.

$\Rightarrow n = 2k+1$ for some $k \in Z$.

Consider, $n^3+5 = (2k+1)^3 + 5$

$$n^3+5 = 2(4k^3 + 6k^2 + 3k + 3)$$

$\Rightarrow n^3+5$ is even

i.e., $\neg p = T \longrightarrow$ ②

$\Rightarrow \neg q \to \neg p$ is T

but $\neg q \to \neg p \equiv p \longrightarrow q$. $p \to q = ($

$\therefore p \to q$ is True

Hence the proof.

pg no.

B, 15,

i) P.T. theorem is n is +ve integer then n is odd iff n^2 is odd. III^{ly}.

iii) n is even iff n^2 is even.

Given, i) p: n is odd.

q: n^2 is odd.

$p \longleftrightarrow q = T$.

Since $p \longleftrightarrow q \equiv (p \to q) \wedge (q \to p) \to$ ①

Consider $(p \to q) = T$.

Direct proof.

Let $p = T$.

n is odd.

By defination of property of odd.

$n = (2k+1)$ $\quad n^2 = (2k+1)^2$

squaring on b.s.

$n^2 = 4K^2 + 1 + 4K$

$= 2(2K^2 + 2K) + 1$

$n^2 = 2K_1 + 1$ for $K_1 = 2K^2 + K \in Z$

$\Rightarrow n^2$ is odd.

$\therefore \underline{q = T}$

$\therefore p \to q = T \to (2)$

Consider $q \to p = T$.

then ~~$q = F$~~, $\neg q = T$. In contrapositive

~~i.e., n^2 is odd~~.

In this case direct proof will fails, then we make use of contrapositive method.

Assume $q \to p$ contrapositive is $\neg p \to \neg q$.

$\neg p$ is True $\to (3)$

n is even

$\Rightarrow n = 2K$

squaring on b.s

$n^2 = 4K^2$

$n^2 = 2(2K^2)$

By defination of even property

n^2 = even

$\neg q$ is True $\to (4)$.

from (3) & (4).

$\neg p \to \neg q$ is true.

Then its contrapositive is also true.

i.e., $q \to p = T \to (3)$.

$\therefore$ from (2) & (3)

$(p \to q) \wedge (q \to p)$ is True.

$\Rightarrow (p \leftrightarrow q)$ is True. $[\because (p \leftrightarrow q) = (p \to q) \wedge (q \to p)]$

Hence the proof.

ii). p: n is even,

q: n^2 is even.

$p \leftrightarrow q = T$ from given,

wkt

$p \leftrightarrow q = (p \to q) \wedge (q \to p) = T \to (1)$.

Consider, $p \to q = T$

Direct method,

$p = T$

i.e., n is even,

$n = 2k$ P-85,

Squaring on b.s. Q-26, 27

$n^2 = 4k^2$ for some $K \in Z$ 2 and ex

$n^2 = 2(2k^2)$ P-80, Q-23,

from this, Q-3.

n^2 is even,

q is T

$\therefore p \to q$ is True $\to (2)$

Consider, $q \to p = T$.

Contrapositive method,

$\neg p \to \neg q$

$\neg p$ is true.

$\Rightarrow$ n is odd.

n is $= 2k+1$

$n^2 = 4k^2+1+4k$

$\therefore n^2 = 2\underbrace{(2k^2+2k)}_{k_1 \in \mathbb{Z}}+1$

$n^2 = 2k_1+1 \Rightarrow n^2$ is odd.

$\therefore \neg q$ is true.

$\therefore \neg p \to \neg q = T \Rightarrow q \to p = T \to$ (3)

$\therefore p \leftrightarrow q$ becomes from (2) & (3).

Hence the proof.

ST the following statements above integer n are equivalent.

i) p: n is even

ii) q: $n-1$ is odd

iii) r: n^2 is even.

W.k.t. p & q & r are 3 proposition then,

$p \to q$

$q \to r$

$r \to p$

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Unit-03.

If A and B are non-empty set then the product set (or) cartesian product $A \times B$ is the set of all ordered pair (a,b) with $a \in A$ and $b \in B$.

i.e., $A \times B = \{(a,b) / a \in A; b \in B\}$.

Eg: $A = \{1,2,3\}$, $B = \{a,b\}$.

$A \times B = \{(1,a)(1,b)(2,a)(2,b),(3,a),(3,b)\}$.

$B \times A = \{(a,1),(a,2)(a,3),(b,1)(b,2),(b,3)\}$.

Notes: $A \times B \neq B \times A$.

$|A \times B| = |A| \cdot |B|$.

1. A car manufacturer make 3 different types of car frames and 2 type engines.

Frame types: sedan(s), cape(c), van(v).

Engine types: gas(g), deesel(d), How many models are there & list them.

soln:

$A = \{s, c, v\}$ $B = \{g, d\}$

$A \times B = \{(s,g),(s,d),(c,g),(c,d),(v,g),(v,d)\}$

$|A \times B| = |A| \cdot |B| = 3 \times 2 = 6$. Totally they are manufacturing 6 cars models.

2. A software firm provides the following 5 characteristics for each program that it sells.

Soln.

language: Fortran(f), PASCAL(p), LISP(l).

Memory: 2meg(2), 4meg(4), 8meg(8).

O.S: UNIX(u), DOS(d).

A: (f, p, l)

B: (2, 4, 8)

C: (u, d).

$A \times B \times C = \{(f,2)(f,4)(f,8),(p,2)(p,4)(p,8)(l,2)(l,4)(l,8)\} \times C$

$= \{(f,2,u)(f,2,d)(f,4,u)(f,4,d)(f,8,u)(f,8,d)(p,2,u)(p,2,d)(p,4,u)(p,4,d)(p,8,u)(p,8,d)(l,2,u)(l,2,d)(l,4,u)(l,4,d)(l,8,u)(l,8,d)\}$.

$\therefore A \times B \times C = 18$

Relation:

A relation R from A to B is a subset of $A \times B$. If R is subset of $A \times B$, $(a,b) \in R$ then we say that a is related to b by R, it is denoted by aRb.

If A not related to B by R, then it can be denoted as $a \not R b$ or $(a,b) \notin R$.

Representation of relations in matrix and digraphs:

$$M_R = \begin{array}{c|cccc} & 0 & 1 & 5 & 8 \\ \hline 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 & 1 \\ 5 & 0 & 0 & 0 & 1 \end{array}$$

By eg: $A = \{2,3,5\}$, $B = \{0,1,5,8\}$.

$R: A \to B$ is $\{(2,5)(2,8)(3,5)(3,8)(5,8)\}$.

Digraph:

If $A = \{a_1, a_2 \dots a_m\}$, $B = \{b_1, b_2, \dots b_n\}$.

... matrix denoted by:

$M_R = [m_{ij}]$ where m_{ij} is $= \begin{cases} 1 & \text{if } a_i R b_j \\ 0 & \text{if } a_i \not R b_j \end{cases}$

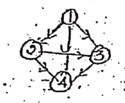
Eg:- Digraph:-

Eg: $A = \{1, 2, 3, 4\}$ draw R where is a is lessthan.

i.e., aRa if $a<a$.

$R : A \longrightarrow A$.

$R = \{(1,2)(1,3)(1,4)(2,3)(2,4)(3,4)\}$.



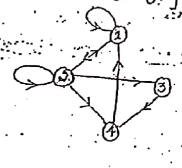
1. Let $A = \{1, 2, 3, 4\}$, $R = \{(1,1)(1,2)(2,1)(2,2)(2,3)(2,4)(3,4)(4,1)\}$ where R is a relation defined on A. Find the domain, range, matrix, draw its digraph.

Soln: Domain $= \{1, 2, 3, 4\}$ $R: A \to A$.

Range $= \{1, 2, 3, 4\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Digraph.

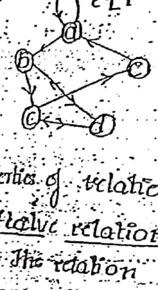


2. For the following matrix give the relation R, digraph & draw its digraph. $A = \{a, b, c, d, e\}$

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

w.r.t. $M_R =$
$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 1 & 1 & 0 & 0 & 0 \\ b & 0 & 0 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 1 & 1 \\ d & 0 & 1 & 10 & 0 & 0 \\ e & 1 & 0 & 0 & 0 & 0 \end{array}$$

$R = \{(a,a), (a, ...$
$(c,d)(c,e)$,
$(e,a)\}$.



HW-
pg No:
Q. All qu...

Properties of relations:

1. Reflexive relations:

* A The relation R on a set A is reflexive if ... $\forall a \in A$.

i.e., $aRa \ \forall a \in A$

→ Irreflexive relations:

* A relation R on a set A is irreflexive, if ... $\forall a \in A$.

2. Symmetric relations

* A relation R on a set A is symmetric if, whe... aRb then bRa $\forall a, b \in A$

→ OR is not symmetric if we have some $a, b \in A$ with aRb but $b \not R a$.

aRb bRa

Asymmetric relation:

A relation on a set A is asymmetric relation if, whenever aRb then $b \not R a$ $\forall a,b \in A$.

Antisymmetric relation:

A relation on a set A is said to be antisymmetric if aRb and bRa then $a=b$.

3. **Transitive relation:**

A relation on a set A is transitive if whenever aRb and bRc then aRc $\forall a,b,c \in A$.

Not transitive

A relation R on a A is not transitive if there exist $a,b,c \in A$ with aRb and bRc but $a \not R c$

Determine whether the relation R on the set A is reflexive, ~~sy~~ irreflexive, symmetric, asymmetric relation, transitive relation etc.;

(i) $A = Z$; aRb if and only if $a \le b+1$

i) Reflexive For some arbitrary $a \in Z$.

$a < a+1$ (True) both +ve & -ve integers

$\Rightarrow aRa.$ $\forall a \in Z$.

R is reflexive.

R is not irreflexive.

ii) Symmetric: For some arbitrary $a,b \in Z$.

let aRb.

$\Rightarrow a \le b+1$

but $b \nleq a+1$ $\forall a,b \in Z$.

$\Rightarrow b \not R a.$

Eg: $(2,1) = (a,b)$ $(2,4) \in R$

$\Rightarrow 2 \le 1+1$ (True)

but $4 \nleq 2+1$ $\forall 2,4 \in Z$.

$\Rightarrow 4 \not R 2.$ $\therefore 2R4$ but $4 \not R 2$.

But some it exists, for

$\therefore$ R is not symmetric.

Asymmetric relation:-

for $a=0$ $b=1$; $a \le b+1$.

aRb.

$\Rightarrow b \le a+1$

$1 \le 0+1$

$\Rightarrow bRa$ $\therefore$ It is not asymmetric.

Antisymmetric:

For $a,b \in Z$

Let aRb and bRa.

$\Rightarrow a \le b+1$, $b \le a+1$

$a \neq b$ from ex $a=0, b=1$.

aRb and bRa, but $a \neq b$.

$\therefore$ It is not anti-symmetric.

Transitive: For some arbitary element $a, b, c \in Z$.

let aRb & bRc.

Given, $a \le b+1$ $\quad b \le c+1$

$\Rightarrow a \le c+1+1$

$a \le c+2$.

$\Rightarrow a \not R c$ $\quad \therefore R$ is transitive.

given, $A = Z$; aRb iff $|a-b| \le 2$.

Soln: For $a \in Z^+$

$|a-a| = 0 \le 2$

$\Rightarrow aRa \quad \forall a \in Z^+$

$\therefore R$ is reflexive.

and R is irreflexive.

Symmetric: For $a, b \in Z^+$

let aRb.

$\Rightarrow |a-b| \le 2$. then

$|b-a| \le 2 \Rightarrow bRa$.

$\therefore \Rightarrow aRb \Rightarrow bRa \quad \forall a, b \in Z^+$

R is symmetric.

R is not asymmetric.

Anti-symmetric: For $a, b \in Z^+$

let aRb $\quad bRa$.

$|a-b| \le 2$ $\quad |b-a| \le 2$.

eg: $a=3$, $b=1$

$|a-b| \le 2$ $\quad |b-a| \le 2$.

but $a \ne b$.

$\therefore R$ is not anti-symmetric.

Transitive: For $a, b, c \in Z^+$

let aRb and bRc

$|a-b| \le 2$ $\quad |b-c| \le 2$.

Consider, $|a-c| = |a-b+b-c|$.

$\le |a-b| + |b-c|$

$\le 2+2 = 4$.

if $a=4$ $\quad b=2$ $\quad c=1$.

$|a-b| = |4-2| = 2 \le 2$.

$|b-c| = |2-1| = 1 \le 2$.

but $|a-c| = |4-1| = 3 \not\le 2$.

* Partial order: R is reflexive, antisymmetric, transitive is called partial ord

3. $A = Z$; aRb iff $|a-b| < 2$.

Soln: Reflexive: For $a \in Z$

ie $|a-a| = 0 < 2$

$\Rightarrow aRa \quad \forall a \in Z$.

$\therefore R$ is reflexive.

Symmetric: For $a, b \in Z$

eg/ $a=-3$, $b=1$ $\quad$ let aRb

$|-3-1| \not< 2$, $|1-(-3)| \not< 2$ $\quad$ i.e. $|a-b| < 2$

$\Rightarrow |b-a| < 2 \Rightarrow bRa$.

Operations on Relations.

Complementary relations :

complement of relation R is denoted by as complementary relations: if $a\bar{R}b$ iff $a \not R b$.

i.e., $(a,b) \in \bar{R} \Rightarrow (a,b) \notin R$.

Inverse relations: If aRb then $aR^{-1}b$. If R is a relation A to B then inverse of R (inverse relation) is denoted is defined by : $bR^{-1}a$ iff aRb.

i.e., $(b,a) \in R^{-1}$ iff $(a,b) \in R$.

Composition of relations:

If let $R: A \to B$ & $S: B \to C$ then composition is $RoS: A \to C$.

if $(a,b) \in R$, $(b,c) \in S$ then $(a,c) \in RoS$.

1. let R & S be the relation from A to B where i) A = B = {1

$R = \{(1,1),(1,2)(2,3)(3,1)\}$ ii) A = {a,b,c

$S = \{(2,1),(3,1)(3,2),(3,3)\}$ B = {1,2,3

$R = \{(a,1)(b,1)(c,2)(c,3$

$S = \{(a,1)(a,2)(b,1)(b,2$

Compute a) $\bar{R}$ b) $R \cap S$ c) $R \cup S$ d) S^{-1}

Soln: i) a) $\bar{R} = A \times B - (R)$

$\bar{R} = \{(1,3),(2,1)(2,2),(3,2)(3,3)\}$

b) $R \cap S = \{(1,1)(1,2)(2,3)(3,1)\} \cap \{(2,1)(3,1)(3,2)(3,3$

$= \{(3,1)\}$

c) $R \cup S = \{(1,1),(1,2),(2,3)(3,1),(2,1),(3,2)(3,3)\}$.

d) $S^{-1} = \{(1,2),(1,3),(2,3)(3,3)\}$.

ωgK- HXB - R.

$= \{(a,1)(a,2)(a,3)\ (b,1)(b,2)\ (b,3)\ \ (c,1)(c,2)(c,3)\} - \{(a,1)(b,1)(c,2),(c,3)\}$

$= \{(a,2)(a,3)(b,2)(b,3)\ (c,1)\}$

b) $R\cap S = \{(a,1)(b,1)(c,2)\ (c,3)\} \cap \{(a,1)(a,2)(b,1)(b,2)\}$

$= \{(a,1)\},(b,1)\}$

c) $R\cup S = \{(a,1),(b,1)(b,2)(c,3)\} \cup \{(a,1)(a,2)(b,1)(b,2)\}$

$= \{(a,1)(a,2)(b,1)(b,2)(c,2)(c,3)\}$

d) $S^{-1} = \{(1,a)(1,b),(2,a)(2,b)\}$

2. $A = \{2,3,6,12\}$ let R and S with the relation on A such that xRy iff $2/x-y$, xSy iff $3/x-y$

compute, $\bar{R}$, $R\cap S$, $R\cup S$, S^{-1}

Given, $R = \{(2,6),(2,12)\},(2,2)\ (3,3)\ (3,12)\ (6,12)\ (6,6)(12,12)$
$(6,2)\ (12,2)\ (12,6)\}$

$S = \{(2,2)\ (3,3)(3,6)(3,12)\ (6,12)(6,6)(12,12)(6,3)$
$(12,3)\ (12,6)\}$

$\bar{R} = A\times A - R$ $\quad A = \{2,3,6,12\}$ $\quad A = \{2,3,6,12\}$

$= \{(2,3)\ (3,6)(3,12),(3,2)\ (6,3)\ (12,3)\}$

$R\cap S = \{(2,2)(3,3)(6,6)(12,12)(6,12)(12,6)\}$

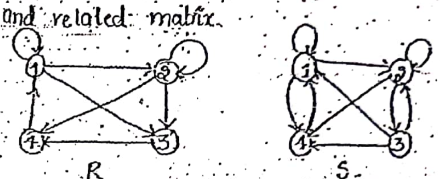
$R\cup S = \{(2,2)\ (2,6)(2,12)(3,3)(3,6)(3,12)\ (6,12)(6,6)(6,3)$
$(6,2)(12,12)(12,2)(12,6)\}\ (12,3)\}$

$S^{-1} = \{(2,2)\ (3,3)(6,3)(12,3)\ (12,6)\ (6,6)\ (12,12)\ (3,6),$
$12,12\ (3,12)\ (6,12)\}$

3. let A = set of people. let aRb iff a is the father b, aSb iff a is the mother of b. Describe $R\cup S$.

Given, $a(R\cup S)b$ iff a is a parent of b.

4. let R & S be two relations whose corresponding digraph are shown. Compute $\bar{R}$, $R\cap S$, $R\cup S$, S^{-1} & write their digraphs and related matrix.



R $\quad$ S

$R = \{(1,1)(1,3)(1,2),(2,2)(2,3)(2,4),(3,4)(4,1)\}$

$S = \{(1,1)\ (1,2)\ (1,4)\ (2,2)(2,3)(2,4)(3,2)(3,4)\ (4,1)\}$
$(3,1)$

$\bar{R} = A\times A - R$ $\quad A = \{1,2,3,4\}$

$= \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)(2,4)(3,1)(3,2)(3,3)(3,4),$
$(4,1)(4,2)(4,3)(4,4)\} - \{(1,1)(1,3)(1,2)(2,2)(2,3)(2,4)$
$(3,4)(4,1)\}$

$\bar{R} = \{(1,4)(2,1)(3,1)(3,2)(3,3)(4,2)(4,3)(4,4)\}$

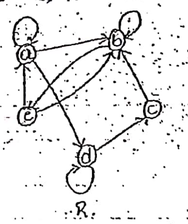
$R\cup S = \{(1,1)(1,2)(1,3)(1,4)(2,2)(2,3)(2,4)(3,1)(3,2)(3,4)$
$(4,1)\}$

$R\cap S = \{(1,1)(1,2)\ (2,2)(2,3)\ (2,4)\ (4,1)\ (3,4)\}$

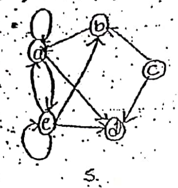
$S^{-1} = \{(1,1)$

diagraph is given by below.

compute a) $\overline{R}$ b) $R \cap S$ c) $R \cup S$ d) S^{-1}



R.



S.

Given, $R = \{(a,a)\ (a,b), (b,b)\ (b,c)\ (c,b)\ (d,d)\ (a,d)\ (e,b)$
$(d,c)\ (e,a)\}$.

$S = \{(a,a)\ (a,e), (b,a)\ (c,b), (c,d)\ (a,d)\ (e,d)\ (e,e)$
$(e,a)\ (e,b)\}$

$A = \{a, b, c, d, e\}$

$\overline{R} = \{A \times A\} - R$ / $A = \{a, b, c, d, e\}$

$= \{(a,a)\ (a,b)\ (a,c)\ (a,d)\ (a,e)\ (b,a)\ (b,b)\ (b,c)\ (b,d)$
$(b,e)\ (c,a)\ (c,b)\ (c,d)\ (c,c)\ (c,e)\ (d,a)$
$(d,b)\ (d,c)\ (d,d)\ (d,e)\ (e,e)\ (e,a)\ (e,b)\ (e,c)$
$(e,d)\} - R$

$\overline{R} = \{(a,c)\ (a,e), (b,a)\ (b,e)\ (b,d)\ (c,a)\ (c,d)\ (c,c)\ (a,c)$
$(d,a)\ (d,b)\ (d,e)\ (c,e)\ (e,c)\ (e,d)\}$

$R \cap S = \{(a,a)\ (c,b)\ (e,b)\ (e,a)\}$.
(a,d).

$R \cup S = \{(a,a)\ (a,b), (a,e)\ (a,d)\ (b,b)\ (b,e)\ (b,a)\ (c,b)\ (d,d)$
$(c,d)\ (d,c)\ (e,b)\ (e,e)\ (e,d)$

$(e,d)\}$.

$S^{-1} = \{(a,a)\ (e,a), (a,b)\ (b,c)\ (d,c)\ (d,a)\ (d,e)$
$(a,e)\ (b,e)\}$.

6. Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$, let R & S be relations from $A \to B$ whose matrices are given compute
$\overline{R}$, $R \cap S$, $R \cup S$, S^{-1}, R^{-1}

Given. i) $M_R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$ $M_S = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$

$R = \{(1,1)\ (1,2)\ (1,4)\ (2,4)\ (3,1)\ (3,2)\ (3,3)\}$.

$S = \{(1,2)\ (1,3)\ (2,1)\ (2,4)\ (3,1)\ (3,2)\}$.

$\overline{R} = A \times B - R$.
$= \{(1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,4)\}$.

$R \cap S = \{(1,2)\ (3,1)\ (3,2)\ (2,4)\}$

$R \cup S = \{(1,1)\ (1,2)\ (1,4)\ (1,3)\ (2,1)\ (2,4)\ (3,1)\ (3,2)\ (3$

$S^{-1} = \{(2,1)\ (3,1)\ (1,2)\ (4,2)\ (1,3)\ (2,3)\}$.

7. Given, M_R & M_S matrices for relations R and S from A to B, $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$

compute, $M_{R \cap S}$ $M_{R \cup S}$ $M_{R^{-1}}$ $M_{\overline{S}}$.

Given, $M_R = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$ $M_S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$M_{R\cap S} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad M_{R\cup S} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad M_{S^{-1}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Pg No 148 149
Q1 to 13

Theorem 1:

Suppose R and S are relations from A to B then prove the following

1. If $R \subseteq S$ then, $R^{-1} \subseteq S^{-1}$
2. If $R \subseteq S$ then, $\overline{S} \subseteq \overline{R}$.
3. i) $(R\cap S)^{-1} = R^{-1} \cap S^{-1}$ and
   ii) $(R\cup S)^{-1} = R^{-1} \cup S^{-1}$
4. $\overline{R\cap S} = \overline{R} \cup \overline{S}$ and
   $\overline{R\cup S} = \overline{R} \cap \overline{S}$.

Proof 1:- let $(a,b) \in R^{-1} \rightarrow ①$

$\Rightarrow (b,a) \in R$ ($\because$ by defination of R^{-1})

$\Rightarrow (b,a) \in S$ (Given $R \subseteq S$)

$\Rightarrow (a,b) \in S^{-1}$ ($\because$ by defination of S^{-1}) $\rightarrow ②$

From ① & ②

$(a,b) \in R^{-1} \Rightarrow (a,b) \in S^{-1}$

$R^{-1} \subseteq S^{-1}$

Proof 2: let $(a,b) \in \overline{S} \rightarrow ①$

$\Rightarrow (a,b) \notin S$ ($\because$ by defination of $\overline{S}$)

given, $R \subseteq S$.

$\Rightarrow (a,b) \notin R$.

$\Rightarrow (a,b) \in \overline{R} \rightarrow ②$ ($\because$ defination of $\overline{R}$).

$\Rightarrow \overline{S} \subseteq \overline{R}$.

Proof 3. i) $(R\cap S)^{-1} = R^{-1} \cap S^{-1}$

let $(a,b) \in (R\cap S)^{-1} \rightarrow ①$

$\Leftrightarrow (b,a) \in R\cap S$ ($\because$ by defination of Inverse)

$\Leftrightarrow (b,a) \in R$ & $(b,a) \in S$

by defn R^{-1} & S^{-1}

$\Leftrightarrow (a,b) \in R^{-1}$ & $(a,b) \in S^{-1}$

$\Leftrightarrow (a,b) \in R^{-1} \cap S^{-1} \rightarrow ②$

from ① & ② $(R\cap S)^{-1} \subseteq R^{-1} \cap S^{-1}$ and

$R^{-1} \cap S^{-1} \subseteq (R\cap S)^{-1}$

$\therefore (R\cap S)^{-1} = R^{-1} \cap S^{-1}$

Proof 3 ii) $(R\cup S)^{-1} = R^{-1} \cup S^{-1}$

let, $(a,b) \in (R\cup S)^{-1} \rightarrow ①$

$\Leftrightarrow (b,a) \in (R\cup S)$ ($\because$ by defination of inverse)

$\Leftrightarrow (b,a) \in R$ or $(b,a) \in S$.

$\Leftrightarrow (a,b) \in R^{-1}$ or $(a,b) \in S^{-1}$ ($\because$ by defn of R^{-1} & S^{-1})

$\Leftrightarrow (a,b) \in (R^{-1} \cup S^{-1}) \rightarrow ②$

from ① & ② $(R\cup S)^{-1} \subseteq R^{-1} \cup S^{-1}$ & $(R^{-1} \cup S^{-1}) \subseteq (R\cup S)^{-1}$

$(R\cup S)^{-1} = R^{-1} \cup S^{-1}$

Proof 4 i) $\overline{R \cap S} = \overline{R} \cup \overline{S}$

let $(a,b) \in \overline{R \cap S}$

then $(a,b) \notin R \cap S$ ($\because$ by defn of complement)

$\Leftrightarrow (a,b) \notin R$ or $(a,b) \notin S$

$\Leftrightarrow (a,b) \in \overline{R}$ or $(a,b) \in \overline{S}$

$\Leftrightarrow (a,b) \in \overline{R} \cup \overline{S}$

$\therefore \overline{R \cap S} \subseteq \overline{R} \cup \overline{S}$ & $\overline{R} \cup \overline{S} \subseteq \overline{R \cap S}$

$\therefore \overline{R \cap S} = \overline{R} \cup \overline{S}$

ii) $\overline{R \cup S} = \overline{R} \cap \overline{S}$

let $(a,b) \in \overline{R \cup S} \rightarrow ①$

$\Leftrightarrow (a,b) \notin R \cup S$

$\Leftrightarrow (a,b) \notin R$ and $(a,b) \notin S$

$\Leftrightarrow (a,b) \in \overline{R}$ and $(a,b) \in \overline{S}$

$\Leftrightarrow (a,b) \in \overline{R} \cap \overline{S} \rightarrow ②$

$\therefore$ from ① & ②, $\overline{R \cup S} \subseteq \overline{R} \cap \overline{S}$ & $\overline{R} \cap \overline{S} \subseteq \overline{R \cup S}$

then $\overline{R \cup S} = \overline{R} \cap \overline{S}$

Theorem 02:

Let R and S be the relations on set A. Then prove that

(a) if R is reflexive, so is R^{-1}

(b) If R & S are reflexive, so are $R \cap S$ and $R \cup S$.

(c) ~~If~~ R is reflexive iff $\overline{R}$ is irreflexive.

Q. Let R

Theorem 03:

Let R be a relation on set A then prove the following

(a) R is symmetric iff $R = R^{-1}$

(b) R is assymmetric iff $R \cap R^{-1} = \phi$.

Proof: a) Let R is symmetric, then,

let $(a,b) \in R$

$\Rightarrow (b,a) \in R$; ($\because$ R is symmetric)

By defn R^{-1}

$(a,b) \in R^{-1}$

$\Rightarrow R \subseteq R^{-1} \rightarrow ①$ & $R^{-1} \subseteq R$ / then, / $R = R^{-1}$

then consider, $(a,b) \in R^{-1}$

Conversely, Let $(a,b) \in R^{-1}$

By defination of R^{-1}

$\Rightarrow (a,b) \in R$

Since R is symmetric

$\Rightarrow (a,b) \in R$

$\therefore R^{-1} \subseteq R \longrightarrow ②$

then, $R = R^{-1}$ $\because$ from ① & ②.

Conversely, given, $R = R^{-1}$.

we have to prove R is symmetric.

let $(a,b) \in R \rightarrow ③$

by defn of R^{-1}:

$(b,a) \in R^{-1}$.

$\therefore R^{-1} = R$.

$(b,a) \in R \rightarrow ④$.

from ③ & ④. R is symmetric.

Proof b): let R is assymmetric.

let, $(a,b) \in R$.

$\Rightarrow (b,a) \notin R$ then, $\because$ R is assymmetric.

$\Rightarrow (b,a) \in R^{-1}$ $\because$ by defn of R^{-1}

i.e., $(a,b) \in R \Rightarrow (b,a) \in R^{-1} \rightarrow ②$

from ① & ② $\therefore R \cap R^{-1} = \phi$.

Conversely. $R \cap R^{-1} = \phi$.

let $(a,b) \in R \rightarrow ③$

By defn of R^{-1}.

$(b,a) \in R^{-1}$. By defn of given, $R \cap R^{-1} = \phi$

$\Rightarrow (b,a) \notin R \rightarrow ④$.

from ③ & ④ $(a,b) \in R$ $(b,a) \notin R$ $\forall (a,b) \in R$.

R is assymmetric.

Composition Relation

Let A, B and C are the sets, R is a relation from A to B and $S: B \rightarrow C$, then the composition of R & S is written as 'SoR', is a relation from $A \rightarrow C$ and is defined / given by.

$$SoR = \{(a,c) / \exists b \in B \ \exists (a,b) \in R \ \& \ (b,c) \in S\}.$$

i.e., $R: A \rightarrow B$ $(a,b) \in R$

$S: B \rightarrow C$ $(b,c) \in S$ then, $\Rightarrow (a,c) \in SoR$.