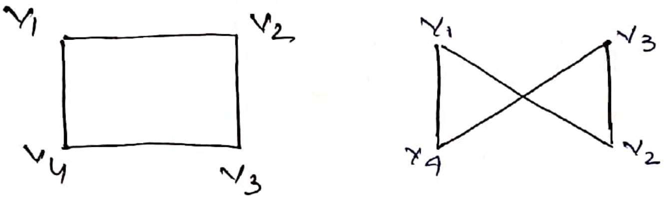
Isomorphism :-

Two graphs G and G' are said to be are said to be isomorphic if there is a same no. of vertices, same no. edges, equal no. vertices with a given degree and also adjacency of vertices → are preserved.

Eg:

Q. Prove that the graph shown below are iso-morphic.

$n=4$
$m=4$
$d=2$

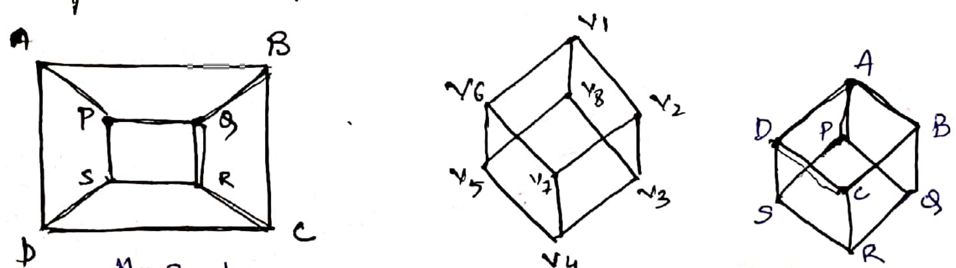


$n=4$
$m=4$
$d=2$

→ • no of vertices = same.
• no. of edges = same.
• equal degree.
• adjacency preserved.

∴ Two graphs are isomorphic.

Q. Prove that two graphs are shown below isomorphic.

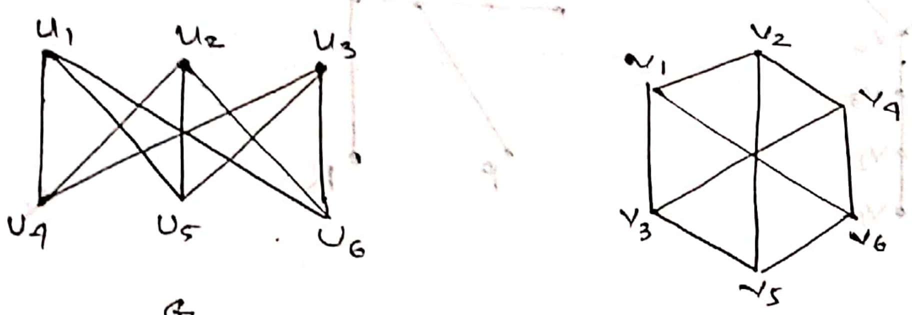


$n=8$
$m=12$ | $d=3$

$n=8$, $m=12$
$d=3$

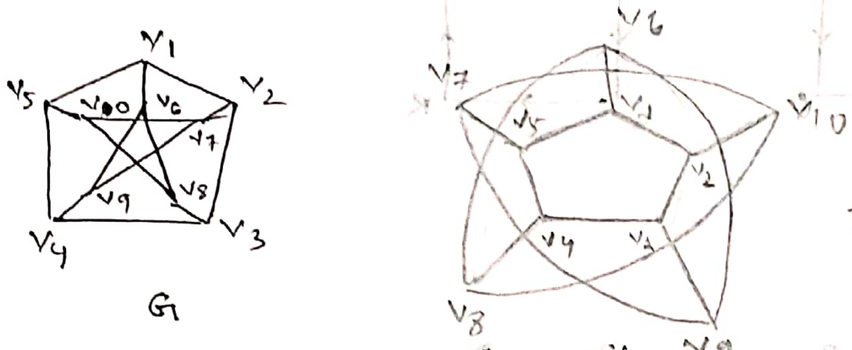
3. Show that following graphs are Isomorphic

(i)



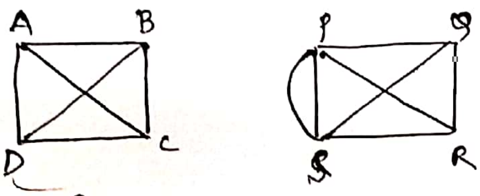
G G' I

(ii)



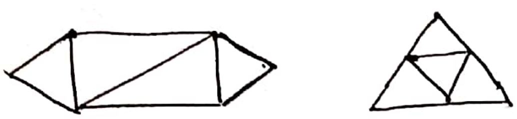
G G' I

(iii)



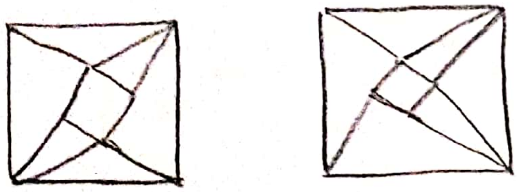
G G' I

(iv)



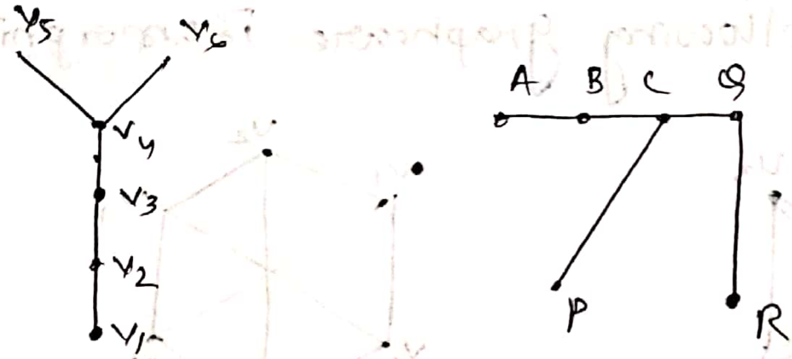
G G' NI

(v)

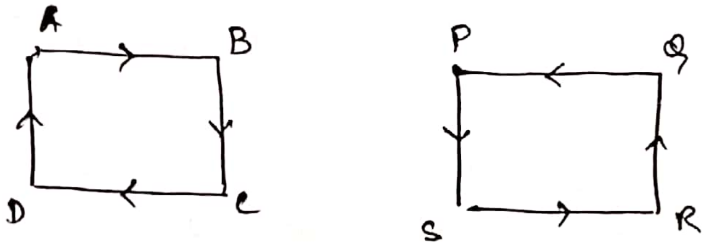


NI Adjacency not preserved

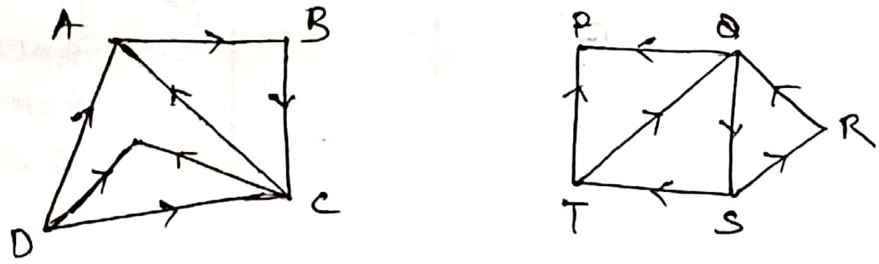
(6)



(7)



(8)



Group Orders :- (7c/8c → Que in SEE)

Q. The word $c = 1010110$ is transmitted through a channel if $e = 0101101$ is the error pattern, find the word ~~the~~ is 'r' received. if $p = 0.05$ is the probability that a signal is incorrectly received find the probability with r is received.

→ $r = c + e$

= error in 2, 4, 5, 7 position

i.e 4 bit error $= k$

n = total no. of bits in r

$$P = p^k (1-p)^{n-k}$$

$$= 0.05^4 (1-0.05)^{7-4}$$

$$= (0.05)^4 (1-0.05)^3$$

$$= 5.35 \times 10^{-6}.$$

$c = 1010110$

$e = 0101101$

$\boxed{} \; 1111011$

arithmetic mod 2

$0+1 = 1$, $1+1 = 2$

$1 \bmod 2 = 1$

$2 \bmod 2 = 0$

$x \equiv y \pmod{n}$

remainder

~~....~~ →

$1 \bmod 2 = 1$

Q. the word $c = 1010110$ is send ~~to~~ through a binary channel if $p = 0.02$ is a probability of incorrect receive of a signal find the probability that c is received at $r = 1011111$. also Determine error ~~factor~~. pattern.

→ $r = c + e$.

→ $e = r - c$

$= 0001001$

$r = 1011111$

$c = 1010110$

0001001

1 3 5 6

$\left\{ \begin{array}{l} r = 1011111 \\ c = 1010110 \\ \quad\quad 4 \quad 7 \end{array} \right.$

$P = 0.02$.

$n = 7$

$k = 2$

$$\therefore \; P = p^k (1-p)^{n-k}$$

$$= (0.02)^2 (1-0.02)^{7-2}$$

$$= 3.615 \times 10^{-4}$$

Encoding Function :-

Let $E: Z_2^m \to Z_2^{m+1}$ be an encoding function defined as follows →

$$W_{m+1} = \begin{cases} 0, & \text{if } \omega \text{ contains even no of 1's} \\ 1 & \text{if } \omega \text{ contains odd no of 1's} \end{cases}$$

Q. Find the code word for the following :-

a) $E: Z^3 \to Z^4$ for -
000, 001, 011, 100, 110, 101, 111, 010

b) $E: Z^4 \to Z^5$ for - 0000, 0001, 0101, 1111, 1010, 1100, 1101, 1001

→ (a) E(000) = 0000
E(001) = ~~000 0001~~ 0011
E(011) = 0110
E(100) = 1001
E(110) = 1100
E(101) = 1010
E(111) = 1111
E(010) = 0101

(b) E(0000) = 00000
E(0001) = 00011
E(0101) = 01010
E(1010) = 10100
E(1100) = 11000
E(1101) = 11011
E(1001) = 10010

Q. $(m, 3m)$ → encoding function. If $E: Z^m \to Z^{3m}$ find the code word assigned by encoding function E for 000, 001, 010, 100, 011, 101, 110, 111

→ E(000) = 000 000 000 000
E(001) = 001 001 001 001
E(010) = 010 010 010 010
E(100) = 100 100 100 100
E(011) = 011 011 011 011
E(101) = 101 101 101 101
E(110) = 110 110 110 110
E(111) = 111 111 111 111

Decoding Functions :-

- $(3m, m)$ - decoding function
i.e $E: Z^{3m} \to Z^m$ is a decoding function defined by $D(r) = s_1, s_2, s_3 \ldots s_n$ where

$$s_i = \begin{cases} 1; & \text{if } r_i, r_{i+2m} \ldots \text{ has majority of 1's} \\ 0; & \text{if } r_{i+m}, r_{i+3m} \ldots \text{ has majority of 0's.} \end{cases}$$

Q. Find the decoded word assigned by $D: Z^6 \to Z^2$ for 111111, 101010, 010101, 100100, 010011, 110110, 010110, 000111

~~D(111111)~~ $D(r) = s_1 s_2$
where $s_1 = r_1 r_3 r_5$
$s_2 = r_2 r_4 r_6$

D(111111) = 11
D(101010) = 10
D(010101) = 01
D(100100) = 00
D(010011) = 01

$D(110110) = 11$

$D(010111) = 01$

$D(000111) = 101$

Q. $D: Z^9 \to Z^3$ for 000000000, 011011011, 010010010, 100100100, 001001001, 101101101, 110110110, 111111111

→ $D(r) = s_1 s_2 s_3$

where $s_1 = r_1, r_4, r_7$ < 1 – majority of 1 / 0 – " " 0

$s_2 = r_2, r_5, r_8$ < 1 / 0

$s_3 = r_3, r_6, r_9$ < 1 / 0 same

D (000000000) = 000

D (011011011) = 011

D (010010010) = 010

D (100100100) = 100.

D (001001001) = 001

D (101101101) = 101

D (110110110) = 110

D (111111111) = 111

Q. Hamming distance :-

Let x & y be words in B^m where $x = x_1, x_2, \dots, x_m$ and $y = y_1, y_2, \dots, y_m$ and $x_i \neq y_i$

The hamming distance b/w x & y is the weight of $x+y$ it is denoted by $d(x,y)$

$$d(x,y) = H(x,y) = \text{weight}(x+y)$$

Q. Find the distance b/w x & y where $x = 110110$, $y = 000101$

$$\begin{array}{l} 110110 \\ 000101 \\ \hline H(x,y) = 110011 \end{array}$$

same 0, different 1

$W(x,y) = 4$ (total no. of 1 present in $H(x,y)$)

Q. $x = 001100$

$y = 010110$

$H(x,y) \to 011010$

$\therefore W(x+y) = 3$.

□ Minimum distance :-

min. distance of an encoding function $E: Z^m \to Z^n$ is the minimum of → all distinct pairs of code words.

$$\text{min dist} = \min\left(d(E(x), E(y)) \mid x, y \in B^m\right)$$

Q. Find the minimum distance of $E: Z^2 \to Z^5$ encoding function $E(00) = 00000$; $E(10) = 00111$, $E(01) = 01110$, $E(11) = 11111$

→ $E(00) = 00000$

	$H(x,y)$	$W(x,y)$
$d(E(00), E(10))$ =	00111 →	3
$d(E(00), E(01))$ =	01110 →	3
$d(E(00), E(11))$ =	11111 →	5
$d(E(10), E(01))$ =	01001 →	2
$d(E(10), E(11))$ =	11000 →	2
$d(E(01), E(11))$ =	10001 →	2

min distance = 2

Generating matrix ~~zero~~ $G = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mr} \end{bmatrix}$

An ~~end~~ encoding function of G is donoted as $E: Z^m \to Z^n$ suchthat $E(b) = x$ where $E = (b_1, b_2, \ldots, b_m)$ and $x = b_1, b_2, b_3, \ldots, b_m$ ~~follow~~ followed by $x_1, x_2, \ldots, x_r$ to determine $x_1, x_2, x_3, \ldots,$ $(r+m) = n$.

following relⁿ are given by →

$$x_1 = b_1 a_{11} + b_2 a_{21} + \cdots\cdots + b_m a_{m1}$$
$$x_2 = b_1 a_{12} + b_2 a_{22} + \cdots + b_m a_{m2}$$
$$x_3 = b_1 a_{13} + b_2 a_{23} + \cdots + b_m a_{m3}$$
$$\vdots$$
$$x_r = b_1 a_{1r} + b_2 a_{2r} + \cdots + b_m a_{mr}.$$

Q. An encoding function $E: Z^2 \to Z5$ is given by the the generator matrix $G = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

determine all code words of Z^2

⇒ $Z^2 = \{\overset{b_1b_2}{00}, \overset{b_3b_4}{01}, \overset{b_5b_6}{10}, \overset{b_7b_8}{11}\}$

$E(00) = 00x_1x_2x_3 = 00000$
$E(01) = 01\,x_1x_2x_3 = 01011$
$E(10) = 10\,x_1x_2x_3 = 10110$
$E(11) = 11\,x_1x_2x_3 = 11101$

For $E(00)$
$x_1 = 0\times1 + 0\times0 = 0$
$x_2 = 0\times1 + 0\times1 = 0$
$x_3 = 0\times0 + 0\times1 = 0$

$E(01)$
$x_1 = 0\times1 + 1\times0 = 0$
$x_2 = 0\times1 + 1\times1 = 1$
$x_3 = 0\times0 + 1\times1 = 1$

$E(10)$
$x_1 = 1\times1 + 0\times0 = 1$
$x_2 = 1\times1 + 1\times0 = 1$
$x_3 = 1\times0 + 0\times1 = 0$

$E(11)$
$x_1 = 1\times1 + 1\times0 = 1$
$x_2 = 1\times1 + 1\times1 = 0$ → (mod 2) (remainder)
$x_3 = 1\times0 + 1\times1 = 1$

Word Graph :-

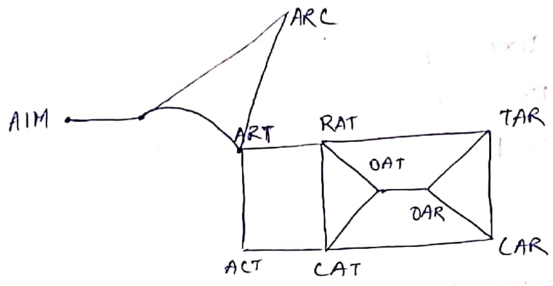
Given words are the vertices of G and two vertices are representing G if the corresponding words can be transformed into eachother

- Suppose that we have a collection of 3 letters english words say ACT, AIM, ARC, ARM, ART, CAR, CAT, OAR, OAT, RAT.

→ A word w_1 can be transformed into a word w_2 if w_2 can be obtained from w_1 by performing exactly one of the following two steps.

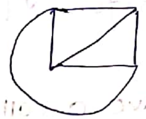
(i) Interchanging 2 letters of w_1
(ii) Replacing a letter in w_1 by another letter.

⇒
ACT — CAT, ART
AIM — ARM, AIR
ARC — ARM, ART
RAT — ART, CAT, TAR, OAT
ART — RAT, ARC, ACT, ARM
ARM — ART, ARC



☐ Planner Graph: A graph without edge intersection Path b/w every pair of vertices (connected graph)

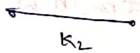
Eg: Plannar graph.



A graph is said to be planner graph iff drawing a graph without edge intersection otherwise graph is said to be non-planner.

☐ Theorem:- Prove that K_2 is planner

Two vertices & one edge.



There is no intersection b/w the edge hence K_2 is planner.

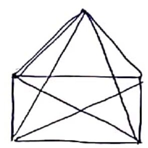
2. Prove that K_3 & K_4 are planner

K_3 =  K_4 = 

There is no intersection b/w edges hence K_3 & K_4 are planner.

3. Prove that K_5 / Kuratowski's 1st graph is non-planar.

→



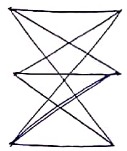
Kuratowski's 1st graph is non-planar because K_5 is a complete graph of ordered 5.

A complete graph is a graph in which there exists an edge b/w every pair of vertices. In the above graph, we cannot redraw in such a way that there is no edge intersection

∴ K_5 is a non-planar.

4. Prove that Kuratowski's 2nd graph is non planar.

→ $K_{3,3}$ → Biparted graph.



This graph is complete biparted graph

A biparted graph is said to be complete biparted graph ∃ an edge b/w every vertex in V_1 and every vertex in V_2 where V_1 and V_2 are the two partitions of H vertex set E'

In alone graph we cannot redraw without edge intersection. Hence $K_{3,3}$ is non-planar.

$\therefore K_{a,b}$ $\therefore$ No. of vertices $(a+b)$

In $K_{a,b}$ where $(a+b)$ is the total no. of vertices and $(a \times b)$ is the total no. of edges in biparted graph.