FINAL PROJECT REPORT

MECH 6021 - Design of Industrial Control Systems

Paper - Robust Vehicle Suspension System by Converting Active & Passive Control of a Vehicle to Semi-Active Control System Analytically

Focus Area – Controlled Suspension System, Passive Controlled Suspension System, Active Controlled Suspension System

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3. Abstract

The research paper we chose for the project work focuses on semi-active control and discusses how effectively it controls the vibrations that are generated in the suspension system of a vehicle. For the purpose of study, the paper focuses on the once wheel of a car rather than entire vehicle, but the results can scale to entire vehicle. In this paper, the semi-active control of a vehicle is based on design of a Passive Vehicle Suspension System. A controller is introduced in passive suspension system which reads and controls the level of damping in real time in contrast to the previous mechanism which only had one degree of freedom and where sensitivity was always in the grey area. The major advantage achieved by this approach is that it adjusts the damping of the suspension system without the application of any actuator by using MATLAB® simulations. We also introduced three different methods to control the parameters for the controller and the same has been covered in our study of this paper. We have also integrated a PID controller in our semi-active suspension system which significantly enhanced system performance in comparison to passive suspension system. Here we used MATLAB Simulink for this section. We developed Simulink model of passive control system and semi-active suspension control system and compared them against different performance parameters.

4. Timeline and Work Description

Date:	Progress:
12-09-2021 - 06-10-2021	On 6th October we finalized the research paper after approval from professor.
07-10-2021 - 15-10-2021	Understanding the Research paper's concept. We also kept on covering the basics and kept on learning the concepts that were new to us. This is also the period when all the team members made them familiar with MATLAB and Simulink. We enrolled for courses on Udemy that helped us in understanding Simulink.
16-10-2021 - 22-10-2021	Since we got the basic idea of Simulink, we started decoding the mathematical model that was covered in the paper. We started decoding mathematical equations. Also, since we were understanding the paper, we also started reading the reference material to understand the procedure or research methodology adopted by the research group. We also explored the related literature and YouTube Videos which gave us conceptual clarity
22-10-2021 - 30-10-2021	We explored the MATLAB plots and results that were covered by the research team and tired replicating them. We also generated our own transfer function that shared same number of poles.
30-10-2021 – 04-11-2021	Mid-Term Week
5-11-2021 – 013-11-2021	Started exploring PID controllers and started exploring how can we use it in the Semi-Active Vehicle control system. Once we understood the how these are useful, we started exploring the literature that is focused on the same concept
13-11-2021 – 30-11-2021	We started generating the SIMULINK models. This was hard part as it took us sometime to understand how PID controller are used in different systems. We firstly started with developing the SIMULINK model for simple quarter car suspension or passive control suspension system. We then started working on the model of the semi-active control suspension system. Once that is done, we started testing the models and started comparing them against each other by providing different inputs and analyzing the obtained outputs. We also studied the state-space concept and started looking in developing the state-space equations for semi-active suspension system. We have covered state-space concept in the miscellaneous section as we still feel more work still needs to be done in that domain.

5. Introduction

a. Concept and Our Approach

The purpose of the vehicle suspension system is to ensure the comfort of the passengers, maximize the road grip of the tires and to provide maximum stability for the steering. An efficient suspension must maintain balance among all these factors. Vehicle suspension are mainly of three types, i.e., passive suspension, active suspension and semiactive suspension, and selection of between three suspension systems depends on how they address above stated factors.

A passive suspension control system is an older conventional system, where we use a combination of spring and damper to dissipate the energy. The spring and damper are uncontrolled entities, and no online feedback action is used. Since no energy can be added or dissipated from this system, there are various inherent limitations, and this system can only operate in specific range of operation conditions, which the vehicle is experiencing the most. Even though this system performs effectively at higher frequency, this system fails at providing optimum reduction in vibrations and maximum comfort since it consistently compromises between spring rate and damping characteristics. Hence this system cannot readjust itself with the variation in the road condition and causes fixed response for all type of road profiles.

Hence this drawback of the conventional passive suspension system led gain of popularity of the semi-active and active suspension systems as a choice for the suspension system. An adjustable system is thus required to cater various road disturbances that a vehicle can encounter while on road. A semi active suspension system does exactly this.

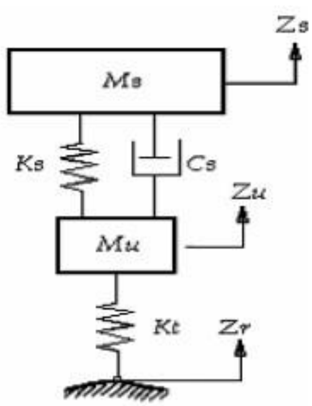
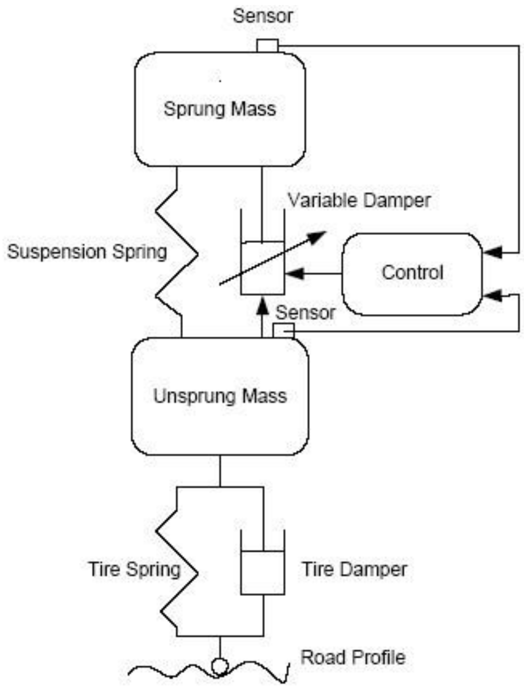


Figure 1- Figure 1- Free body schematics of Passive Suspension System

As we can see, Passive system has an element to absorb energy; a damper, and an element to store energy; a spring. Since there is no source for any additional energy in the system, therefore the system is called a "passive suspension system".



Semi Active Suspension System utilizes combination of sensors and controller to control the level of damping required from the shock absorber in a real time in response to input from the continuously varying road surfaces

Figure 2 - Fig. 2: Free body schematics of Semi Active Suspension System

Semi Active Suspension System is a remarkable as it uses fluid dampers, which can change their damping coefficient per the road disturbances. This auto-adjusted system is especially beneficial for the suspensions due to its low energy requirements. Normally, in semi active systems, an added system is introduced to modulate the damping force of the damper to achieve a range of damping force for multiple operating conditions. “Electro Rheological (ER)” and “Magnetic Rheological fluid dampers” are favored for the fact that they can efficiently change their damping stiffness coefficient.

The third kind of suspension system that exist is an active suspension system. In this system the conventional components are supplemented with the help of external actuators which can provide extra force to the sprung mass. The major setback of this type of system is the increase in the cost and unsprang weight caused by the added apparatus.

In this project work we focused on our main paper and supporting reference literature to understand the concept of semiactive suspension. A PID controller is designed for controlling the parameters of semiactive suspension system of a quarter car model and then we compare the response of the PID controlled model of semiactive suspension with the passively controlled model of suspension.

A proportional–integral–derivative controllers are widely used in control systems applications and a variety of other operations requiring continuously modulated control. A PID measures the error in real time as the difference between a desired system goals and process variable and comes up with corrective measures based on proportional, integral, and derivative terms hence the name.

In this research paper vehicle suspension model with semi active control system is established in MATLAB and SIMULINK and finally their performance is compared based on the result for the MATLAB.

In this paper the major advantage that is achieved by this system is that it adjusts the damping of the suspension system without the application of any actuator and just by using MATLAB Simulink and PID controller. The semiactive control is found to control the vibration of suspension system very well. We also tested the PID based semi-active control system and passive controlled suspension system against the different road profiles and compared their transient and steady state response. In order to come up with our project work we couldn’t rely on one single paper, so we explored other reference papers which could enhance our understanding of the concept and impart some modern knowledge in the field of the control systems. We also came up with the state e

The objectives of this paper align with the description of the project that is mentioned in the course outline and can be seen below:

1. The paper is about the mechanical system design in MATLAB to increase the comfort level of the vehicles with the help of semi active suspensions and overcome the limitations that were previously imposed on the passive suspensions system.
2. The solution given in the paper is in the form of the transfer function of the system that when applied to various controllers gives us the best among them.
3. The system can be modelled in MATLAB and Simulink and then there is scope of improvement in the equations that are expressed in the paper.

6. Modelling and Design of Suspension System

a. Literature Review

The mathematical model of quarter car passive and semi-active suspension has been derived by using the basic Newton's laws of motion.

The reference literature to understand the mathematical model of the system is taken from "K. A. Hair, Passive Vehicle Suspension System, Google Patents, 1994." And "A. M. Beard and A. H. V. Flotow, Active Vehicle Suspension System, Google Patents, 1997."

As done in previous research work, the researcher working on this paper also used a Quarter Suspension System (shown in *figure 3*) to analyze the performance of semi active suspension systems. They used linear spring with stiffness k_1 to model the stiffness of the tire. The quarter mass of the car that includes the axle, auxiliary components, and other moving parts is denoted by m_1. The mass of the wheel of the car is denoted by m_2 and k_2 is the spring constant of the suspension spring. Open loop transfer function that researchers considered for the paper is as follow.

$$T(s) = \frac{4s^2 + 3.333s + 3.333}{s^4 + 5.833s^3 + 9.833s^2 + 3.333s + 3.333}$$

We tried to explore the reference literature to find the approach that researchers might have followed to get this transfer function. Even though we were not able to find the method for generating the exact transfer function, but we successfully came up with the approach of generating the similar Transfer Functions as used by the research team. We analyzed a suspension model of a jeep with overall mass of around 2500kg. ***The transfer function we generated has the same number of poles as the transfer function that was considered by researchers for their research paper.***

b. Our Approach for Generating the Transfer Function

To obtain the transfer function $\frac{Y(s)}{U(s)}$ of the system shown in the figure 3, we considered vertical motion *W* at a point P as the input. We also assumed that the displacements *X1* & *X2* are measured from the respective equilibrium positions in the absence of the input *U*. We considered

M1 = Sprung mass (kg)

M2 = Un-sprung mass (kg)

x1 = Body Displacement (m)

x2 = Suspension Displacement (m)

b1 = Suspension damping coefficient (N.s/m)

b2 = Tyre damping coefficient (N.s/m)

k1 = Suspension spring coefficient (N/m)

k2 = Tyre spring coefficient (N/m)

W = Road profile

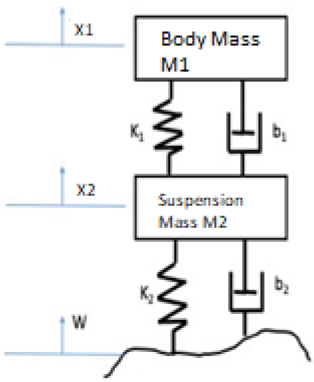


Figure 3 - Qarter Car Syspension Model

Taking Y(s) as output and U(s) as input for the system.

$$\frac{X1(s)}{W(s)} = \frac{Output}{Input}$$

Since change in X1 is related to change in W so from Newtons 2nd Law we can drive

$$M_1\ddot{x}_1 + b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = 0 \_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)$$
$$M_2\ddot{x}_2 + b_1(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) + b_2\dot{x}_2 + k_2x_2 = b_2\dot{w} + k_2w \_\_\_\_\_\_\_\_(2)$$

Taking Laplace Transform of equation 1 and 2

$$X(s)[m_1s^2 + bs + (k_1 + k_2)] = Y(s)[bs + k_2] + k_1U(s) \text{--------}(1)$$

$$Y(s)[m_2s^2 + bs + k_2] = X(s)[bs + k_2]$$

$$M_1S^2X_1(s) + b_1S(X_1(s) - X_2) + K_1\left(X_1(s) - X_2(s)\right) = 0\_\_\_\_\_\_\_\_\_\_ (3)$$

$$M_2S^2X_2(s) + b_1S(X_2(s) - X_1(s)) + k_2(X_2(s) - X_1(S)) + \ b_2SX_2(s) + k_2X_2(s) = b_2SW(s) +$$

$$k_2W(s)\_\_\_\_\_\_\_ (4)$$

Separating variables in equation (3) we obtained

$$X_1(s)\{M_1S^2 + b_1S + k_1\} = X_2(s)\{b_1S + k_1\} \_\_\_\_\_\_\_\_\_\_\_\_ (5)$$

Substituting (5) in equation (4) to obtain the transfer function

$$G(s) = \frac{X_1(s)}{W(s)} =$$

$$\frac{b_1b_2S^2 \ + (\ b_1k_2 + \ b_2b_1)S \ + \ k_1k_2}{M_1M_2s^4 + \{M_1(b_1 + b_2) + b_1M_2\}S^3 + \{M_1(k_1 + k_2) + b_1b_2 + k_1M_2\}S^2} + \frac{1}{(b_1k_2 + b_2k_1)S + k_2^2}$$

This is the Transfer Function for a Simplified Vehicle Suspension System.

In order to Transfer Function of the suspension system of a Vehicle that we have taken into consideration. Taking actual specifications of a particular vehicle with overall mass above 2000 kg.

M1 = 556 kg

M2 = 40 kg

b1 = 600 N.s/m

b2 = 800 N.s/m

k1 = 18000 N/m

k2 = 800 N/m

Now plugging in these values in the generated transfer function

$$\frac{Y(s)}{U(s)} = \frac{(4.80 * 10^5s^2 + 9.6 * 10^5s + 14.4 * 10^6)}{.22 * 10^5s^4 + 8 * 10^5s^3 + 1.11 * 10^7s^2 + 1.48 * 10^7s + 160000}$$

Simplifying the equation above

$$\frac{Y(s)}{U(s)} = \frac{(4.80s^2 + 9.6s + 144)}{.22s^4 + 8s^3 + 111s^2 + 148s + 1.6}$$

The above given equation is the Transfer Function of a jeep, and it is evident from the number of the poles and zeros that this transfer function is relatively like the transfer function that was considered by the research team for their study.

c. Analysis based on Transfer Function

The paper focuses on three approaches and for each approach a new transfer function is defined by the researchers. For each approach a

Controller Design Approach 1 - The researchers further observed that the transfer function leads to steady space error and in order to limit that they adapted the transfer function for closed loop system. This resulted in a modified transfer function

We performed the same conversion of transfer function from open loop system to closed loop system via MATLAB. Our code and command line window results are shown below.

MATLAB Code

```
% Defining Transfer function in Matlab

numer = [4 3.333 3.333];

denom = [1 5.833 9.833 3.333 3.333];

Tran_fun_open = tf(numer , denom)

% Converting closed loop function to open loop function.

Tran_fun_closed = feedback(Tran_fun_open,1)
```

Command Line Response

```
Tran_fun_open =

         4 s^2 + 3.333 s + 3.333

  ---------------------------------------------

  s^4 + 5.833 s^3 + 9.833 s^2 + 3.333 s + 3.333

Continuous-time transfer function.

Tran_fun_closed =

         4 s^2 + 3.333 s + 3.333

  ---------------------------------------------

  s^4 + 5.833 s^3 + 13.83 s^2 + 6.666 s + 6.666

Continuous-time transfer function.
```

Compute numerical values of the pole and zero locations of closed loop transfer function.

MATLAB Code

```
% Plotting and finding poles and zeros of closed loop tranfer function
```

pzmap(Tran_fun_closed)

grid

zeros_tf = zero(Tran_fun_closed)

polea_tf = pole(Tran_fun_closed)

% Column vectors containing the zero and pole will be returned

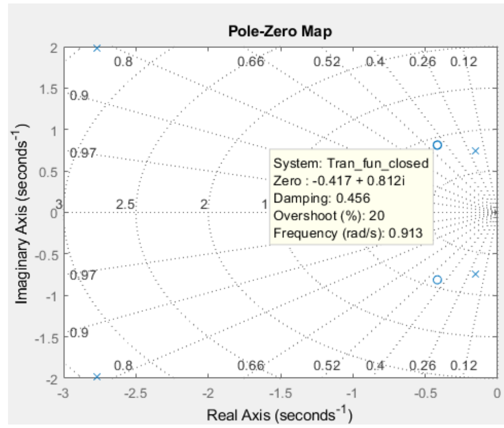


Figure 4 - MATLAB Zero-Pole Map

Command Line Response

zeros_tf =

-0.4166 + 0.8122i

-0.4166 - 0.8122i

poles_tf =

-2.7662 + 1.9858i

-2.7662 - 1.9858i

-0.1503 + 0.7432i

-0.1503 - 0.7432i

It is observed by the above generated matrix that poles lie in the left half of root locus diagram as shown in the Pole-Zero Map in fig. We hereafter generated the time step response of the closed loop transfer function as show in the figure.

MATLAB Code

%finding step response of the closed loop transfer function

t=0:0.1:10; step(numer,denom,t)

S = stepinfo(Tran_fun_closed)

Command Line Response

struct with fields:

RiseTime: 0.9163

SettlingTime: 21.1029

SettlingMin: 0.4093

SettlingMax: 0.6713

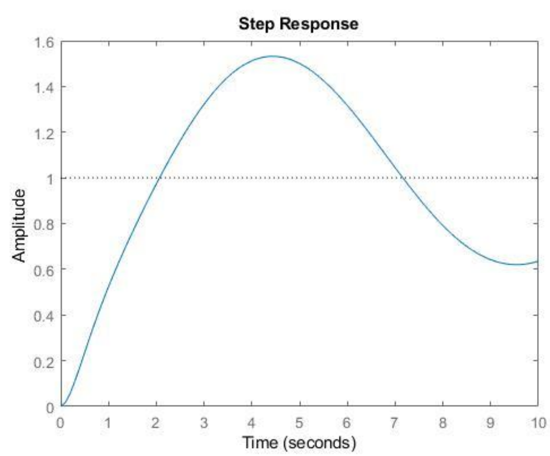


Figure 5 - Controller 1 Step Response Analysis

Overshoot: 34.2521

Undershoot: 0

Peak: 0.6713

PeakTime: 3.1631

Controller Design Approach 2 – From the step response passive suspension system is stable but can be further controlled by using either semi active or active system. Since we earlier discussed that Skyhook control is highly popular when it comes to automotive systems because its hardware is relatively lighter in comparison to active vehicle suspension system. For Skyhook control system, we introduced extra poles in the system so that location of all closed loop poles can be controlled.

Using the state space method, the research team came up with new transfer function with extra poles.

$$T(s) = \frac{1.421e\text{-}014\ s^3 + 4\ s^2 + 3.333\ s + 3.333}{s^4 + 11.64\ s^3 + 91.11\ s^2 + 76.72\ s + 67.71}$$

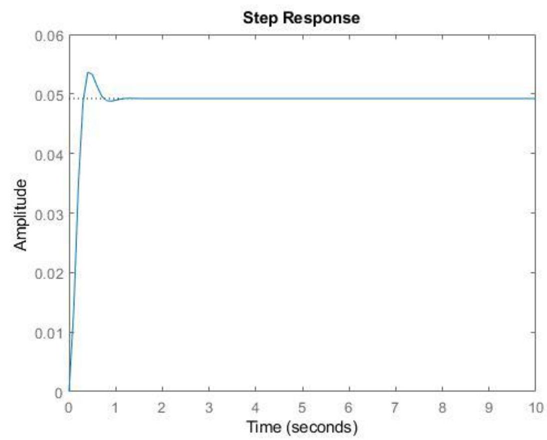


Figure 6 - Controller 2 Step Response

From the step response the skyhook controller revels a large steady state error but required time response. So, the system is controllable. Error can be reduced via integral control, but we switch to other system to avoid complexity.

Controller Design Approach 3 : The other approach that is preferred by the research group is of observer design. In the final paper, we will discuss the observer design thoroughly. It was observed that with observer design cost of equipment effectively limited. Moreover this approach is 10 times faster than previous design involving controller but at the same time the steady state error is too higher which implies that the system formed is highly unstable.

After understanding the controller design methodology i.e. is based on working with the transfer function, we also started looking into other approaches. We investigated the reference models and how modern control system are being designed. From the literature review we obtained the concept clarity and got a direction in which we should move our project work. We started exploring and understanding PID controller and started how this can be integrated into to a passive control suspension system to make it more responsive and efficient.

SIMULINK Model Design

The mathematical modelling has been done considering certain assumptions to keep the model effective yet simple. The passive suspension system and semi-active suspension system have been derived from basic Newton's second law of motion.

- The overall vehicle design is assumed to be a linear or uniform to support the quarter car model.
- Factors, like backlash and movement in gear systems, linkages and joints and the vehicle chassis flex are neglected to reduce the complexity.

a. Mathematical Passive Suspension System Model

The Simulink model of the Passive Suspension System is generated by the exact same mathematical approach that we discussed earlier while generating the transfer function for the passive suspension system.

M1 = Sprung mass (kg) = 556 kg

M2 = Un-sprung mass (kg) = 40 kg

x1 = Body Displacement (m)

x2 = Suspension Displacement (m)

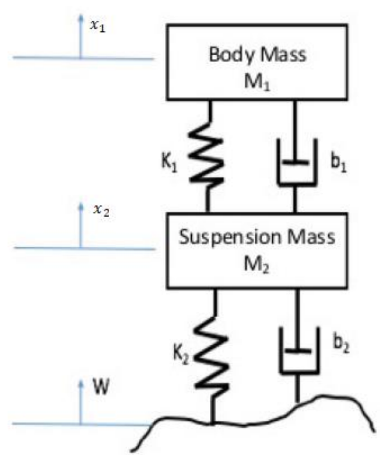
b1 = Suspension damping coefficient (N.s/m) = 600 N.s/m

b2 = Tyre damping coefficient (N.s/m) = 800 N.s/m

k1 = Suspension spring coefficient (N/m) = 18000 N/m

k2 = Tyre spring coefficient (N/m) = 800 N/m

W = Road profile



Using Newtons Second Law of motion, we obtained

$$M_1\ddot{x}_1 + b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = 0 \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$
$$M_2\ddot{x}_2 + b_1(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) + b_2\dot{x}_2 + k_2x_2 = b_2\dot{w} + k_2w \text{ \_\_\_\_\_\_\_\_(2)}$$

Taking Laplace Transformation of equation (1) and (2).

$$M_1S^2X_1(s) + b_1S(X_1(s) - X_2) + K_1\big(X_1(s) - X_2(s)\big) = 0 \text{\_\_\_\_\_\_\_\_\_\_ (3)}$$

$$M_2S^2X_2(s) + b_1S(X_2(s) - X_1(s)) + k_2(X_2(s) - X_1(S)) + b_2SX_2(s) + k_2X_2(s) = b_2SW(s) + k_2W(s) \text{\_\_\_\_\_\_\_ (4)}$$

Separating variables in equation (3) we obtained

$$X_1(s)\{M_1S^2 + b_1S + k_1\} = X_2(s)\{b_1S + k_1\} \_\_\_\_\_\_\_\_\_\_ (5)$$

Substituting (5) in equation (4) to obtain the transfer function

$$G(s) = \frac{X_1(s)}{W(s)} =$$

$$\frac{b_1b_2S^2 \ + (\,b_1k_2 + \ b_2b_1)S \ + \ k_1k_2}{M_1M_2s^4 + \{M_1(b_1 + b_2) + b_1M_2\}S^3 + \{M_1(k_1 + k_2) + b_1b_2 + k_1M_2\}S^2} + \frac{1}{(b_1k_2 + b_2k_1)S + k_2^2}$$

Block Diagram of Passive Suspension System

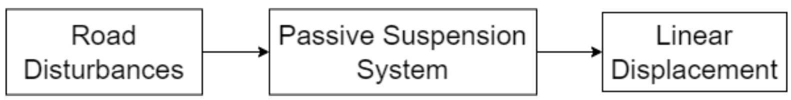


Figure 7- Block Diagram of Passive Suspension System

The block diagram of the passive suspension system shows a road profile, passive suspension system, and the linear displacement of vehicle body.

b. Simulink Model of Passive Suspension System Model

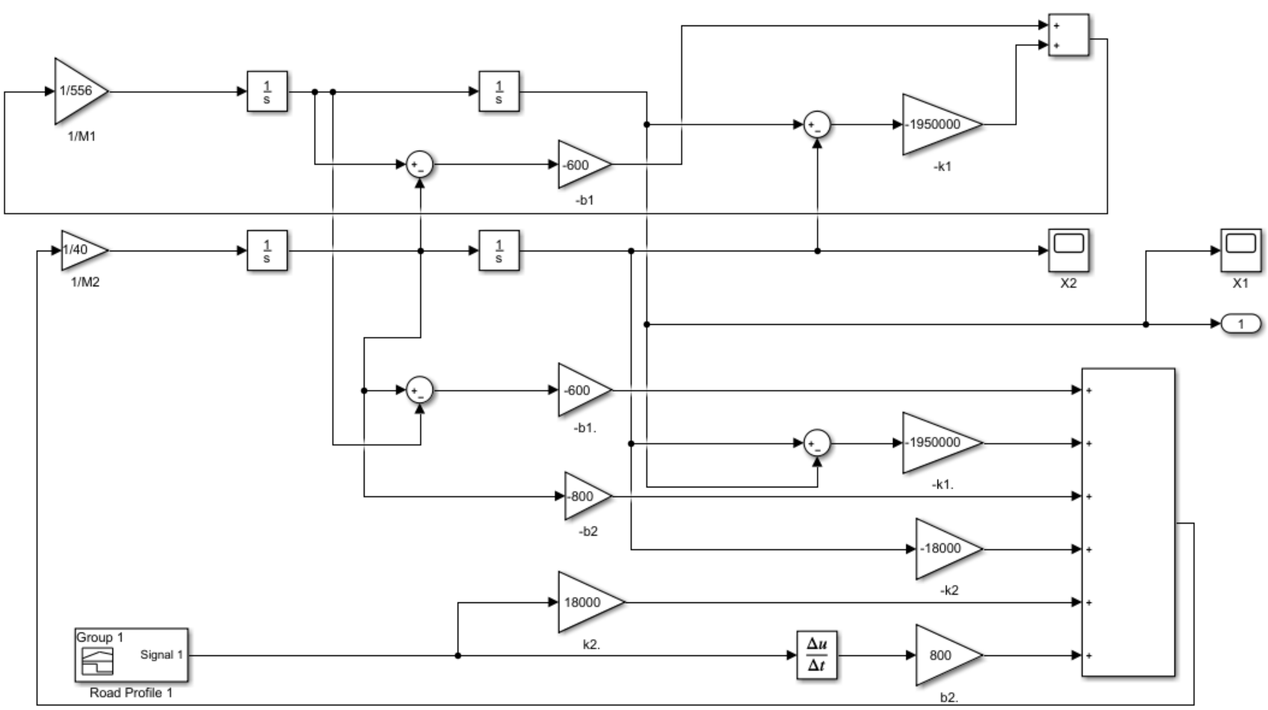


Figure 8- SIMULINK Model of Passive Suspension Control System

The mathematical equations shown in equation (1) and equation (2), were used to simulate the above Simulink model.

c. Mathematical Semi-active Suspension System Model

The equations for quarter car semi-active suspension system is also derived from Newton's laws of motion as follow

$$M_1\ddot{x}_1 + \bar{b}_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = 0 \_\_\_\_\_\_\_\_\_ (7)$$

$$M_2\ddot{x}_2 + \bar{b}_1(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) + b_2\dot{x}_2 + k_2x_2 = b_2\dot{w} + k_2w \_\_\_\_\_\_\_\_\_ (8)$$

Laplace transformation of Eqs. (7) and (8), we get,

$$M_1S^2X_1(s) + \overline{b_1}S(X_1(s) - X_2) + K_1\big(X_1(s) - X_2(s)\big) = 0 \_\_\_\_\_\_\_\_\_ (9)$$

$$M_2S^2X_2(s) + b_1S\big(X_2(s) - X_1(s)\big) + k_2\big(X_2(s) - X_1(S)\big) + b_2SX_2(s) + k_2X_2(s) = b_2SW(s) + k_2W(s)\_\_\_\_\_\_\_\_\_ (10)$$

Iterating equation (9)

$$X_1(s)\{M_1S^2 + \overline{b_1}S + k_1\} = X_2(s)\{\overline{b_1}S + k_1\} \_\_\_\_\_\_\_\_\_ (11)$$

In order to obtain the transfer function substituting equation (11) in equation (10)

$$G(s) = \frac{X_1(s)}{W(s)} = \frac{\overline{b_1}b_2S^2 + (\overline{b_1}k_2 + b_2\overline{b_1})S + k_1k_2}{m_1m_2s^4 + \{m_1(\overline{b_1} + b_2) + \overline{b_1}m_2\}S^3 + \{m_1(k_1 + k_2) + \overline{b_1}b_2 + k_1m_2\}S^2}$$

$$+ \frac{1}{(\overline{b_1}k_2 + \overline{b_2}k_1)S + k_2^2}$$

Semi-active suspension system makes use of a varying damping stiffness damper. The controller decides the level of the damping required by analyzing the road profile in order to adjusts the damper to achieve the optimum damping. Proportional Integral Derivative (PID) controller is designed and tested for three different road profiles and the suspension results were compared with a similar passive suspension system. For PID the proportionality constants k_p, k_i and k_d, are determined through the literature review. It must be noted that the literature that we took as reference for finding k_p, k_i and k_d, obtained their values by trial and error.

Block diagram of semi-active suspension system

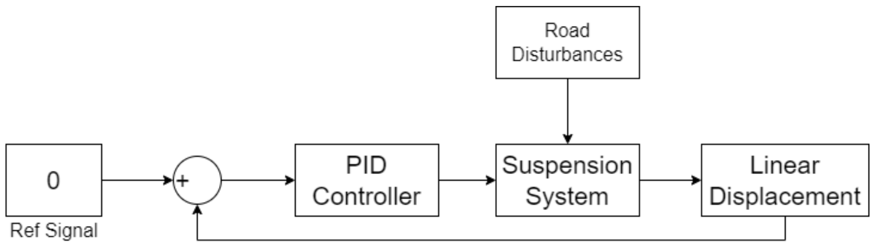


Figure 9 - Block Diagram of Semi-Active Suspension System

Contrary to the passive suspension system, the semi-active suspension system has a closed loop feedback to control the damping coefficient in the real time. The feedback response is fed into the PID controller which accordingly generates a response thus generating a modulated damping stiffness response.

d. Simulink Model of Semi-Active Suspension System Model

The mathematical equations shown in equation (1) and equation (2), were used to simulate the above Simulink model.

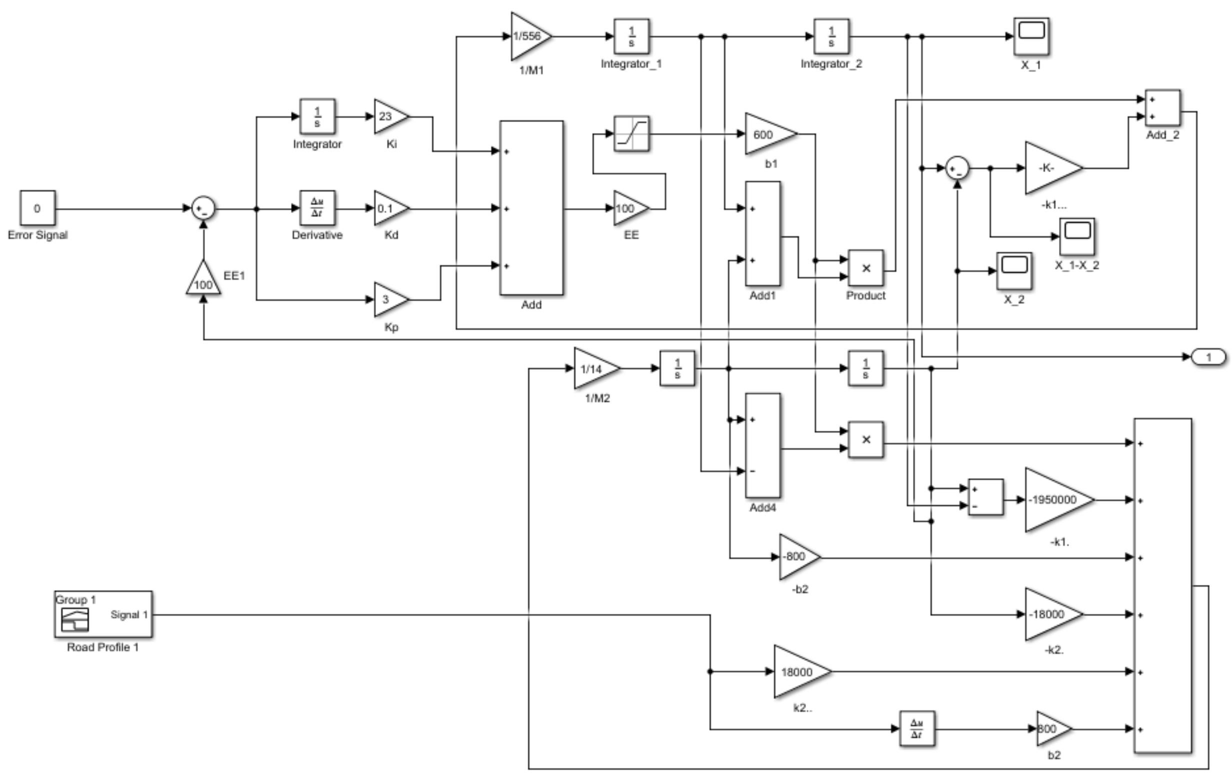


Figure 10 - SIMULINK Model of Semi-Active Suspension Control System

The mathematical equations shown in equation (7) and equation (8), were used to simulate the above Simulink model.

e. System Parameters and Road Conditions

As per the literature that we reviewed during our study we realized that most of the researchers were focusing on passengers' cars as their research subject. We following the same track considered a jeep as our research focus as we believe the vehicle like a jeep is subject to both smooth city rides and bumpy off-road activities.

We have already listed the system parameters that we considered however for convenience of reader we are again jotting it down.

i System Parameters

Parameters	Values
M1 Sprung mass (kg)	556
M2 Un-sprung mass (kg)	40
K1 (Suspension spring coefficient)	18000
K2 (Tyre spring coefficient)	1950000
b1 (Suspension damper coefficient)	600
b2 (Tyre damping coefficient)	800

Our main goal with the PID controller was to tune it manually such that the system overshoot is curtailed and settling time is minimized. The proportionality constants for PID controller were determined by hit-and-trial method. The one we considered were same as considered by the researchers.

Road profile 1 - $k_{p=10}$, $k_{i=70}$ and $k_{d=0.2}$

Road profile 2 - $k_{p=3}$, $k_{i=10}$ and $k_{d=0.2}$

Road profile 3 - $k_{p=}3$, $k_{i=23}$ and $k_{d=0.1}$

ii Road Conditions

In order to simulate the real life conditions, we considered three types of road conditions.

The road condition 1st is simulated such that it has a sinusoidal speed breaker of 10 cm spread through a 5 second interval as shown in the figure.

$$\omega = 1 \begin{cases} a[u(t-5)-(t-10)]\sin(0.2\pi t) & 5s \leq t \leq 10s \\ 0 & Otherwise \end{cases}$$

Where “a” (speed-breaker height) = 10 cm

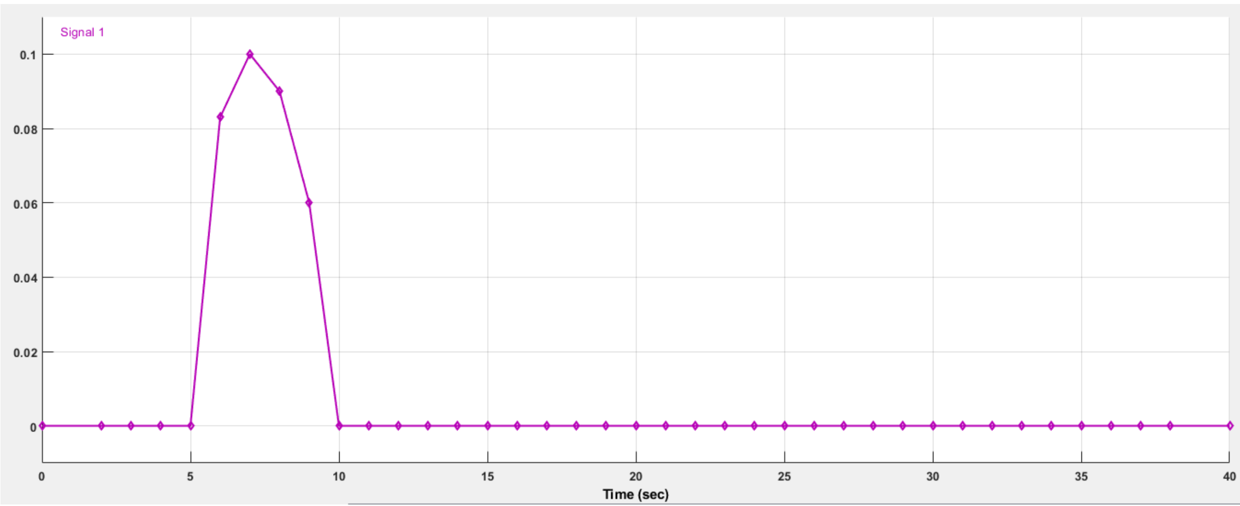


Figure 11 - Road Profile 1st

The road condition 2nd is simulated such that it mimics a step input signal taken as single step of 10 cm. It is like a sudden inclination in the road surface. The profile of this road is expressed by following expression.

$$\omega = \begin{cases} 0 & 5s \leq t \leq 10s \\ 0.1 & t \geq 5s \end{cases}$$

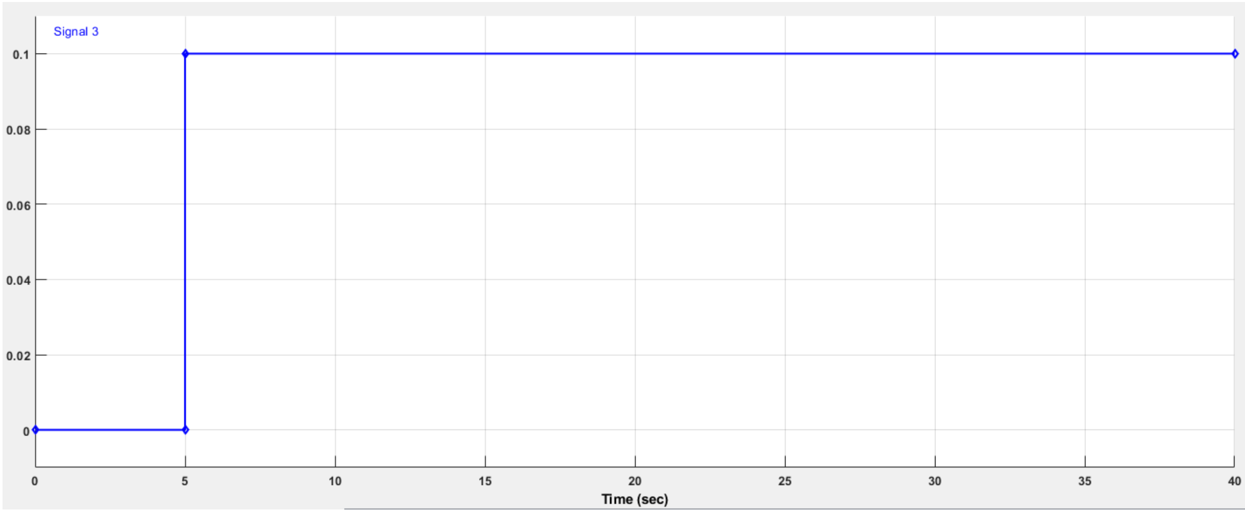


Figure 12 - Road Profile 2nd

The road condition 3rd is taken as two consecutive jerks within the time gap of 3 seconds. Such jerks are experienced by tires very frequently. This road profile can be expressed by following expression.

$$\omega = a \left\{ \frac{1 - \cos(8\pi t)}{2} \right\}$$

Where the value of "a" can be expressed as :

$$a = \begin{cases} 0.11m, & 0.55 \leq t \leq 0.75 \\ 0.55m, & 3.05 \leq t \leq 3.255 \end{cases}$$

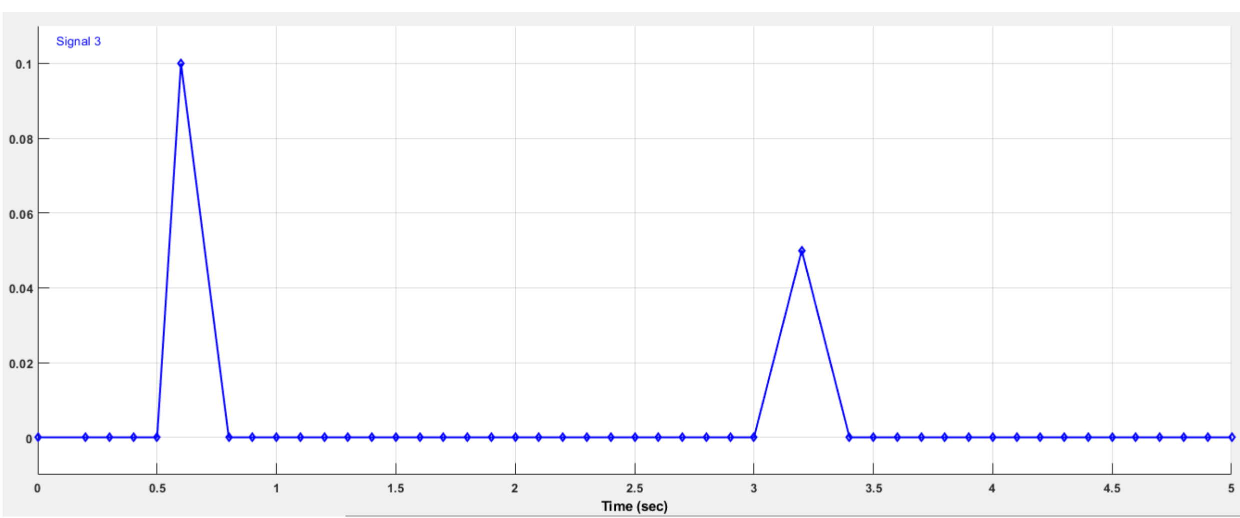


Figure 13 - Road Profile 3rd

RESULTS & DISCUSSIONS

From the simulation results that we obtained for both passive and semi-active vehicle suspension model we focused at suspension travel and car body travel in term of linear system as a major performance parameter. All the three above stated road disturbances were taken as input for both systems and results for all cases were observed separately. The performance criteria were taken in term

1. Rise Time (Tr) - The time taken for a signal to surpass from a specified value to a specified high value. We considered rise time as how long it takes for a signal to go from 10% to 90% of its final value.
2. Settling Time (Ts) - The settling time is the time needed to settle within a specific percentage of the input signal's amplitude. For second order system, we seek for which the response remains within 2% of the final value.
3. Percent Over-shoot (%OS) - Maximum value subtracted the step value divided by the step value

In general, the system which reports shorter settling time of amplitude and smaller magnitude of displacement is assumed to enhance driver comfort and vehicle stability.

The response of passive and semi-active suspension system for road profile-1 is as depicted as follow.

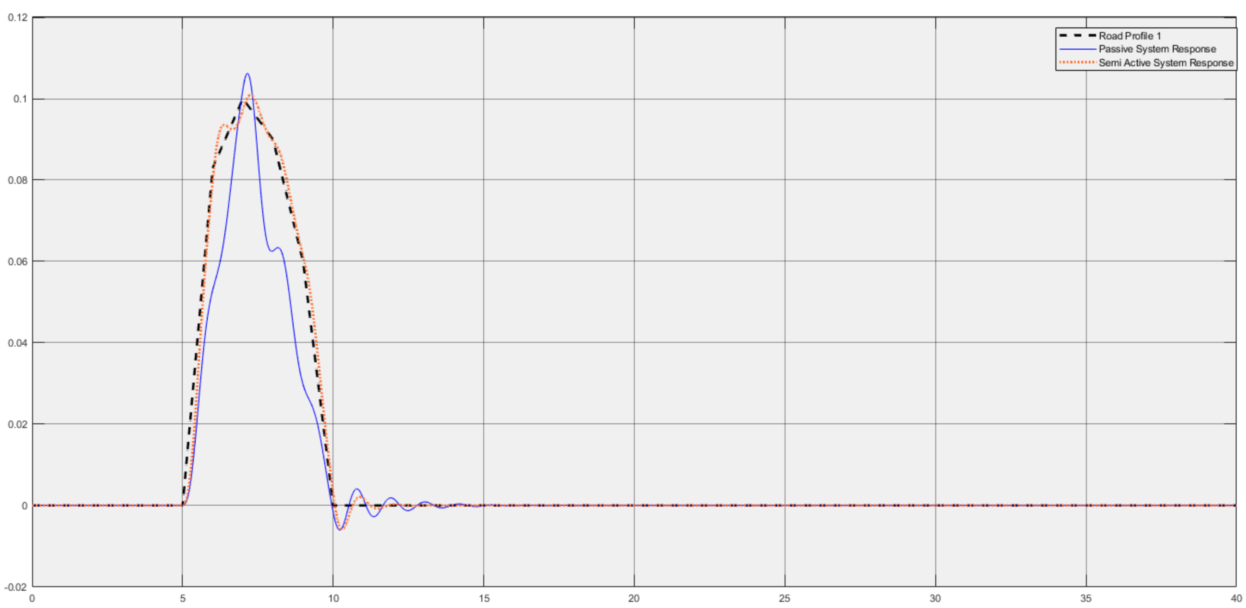


Figure 14 - Road Profile 1, Passive System Response, Semi-Active System Response

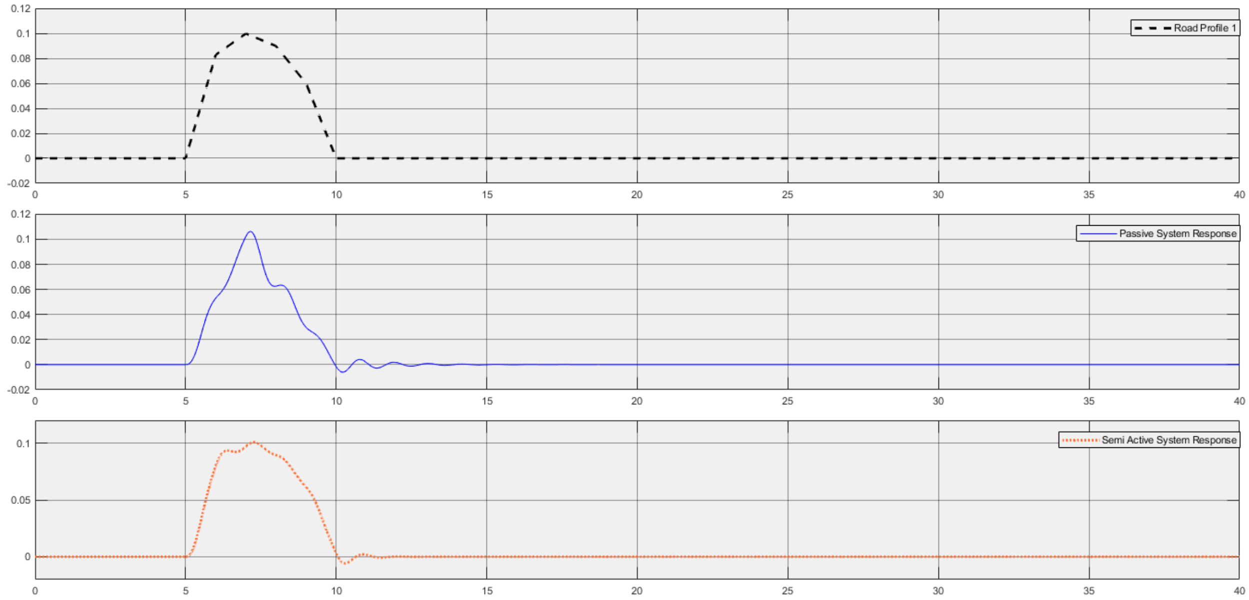


Figure 15 - Cascaded Road Profile 1, Passive System Response, Semi-Active System Response

Graph showcases a smooth and relatively long duration jump. From the figure 15 we can also observe that even passive system got a smooth response and gets a percentage over-shoot of approximately 2% and has a settling time of about 8 seconds. The suspension travel for semi active suspension system, shows settling time of about 5 seconds and 0.1% over-shoot.

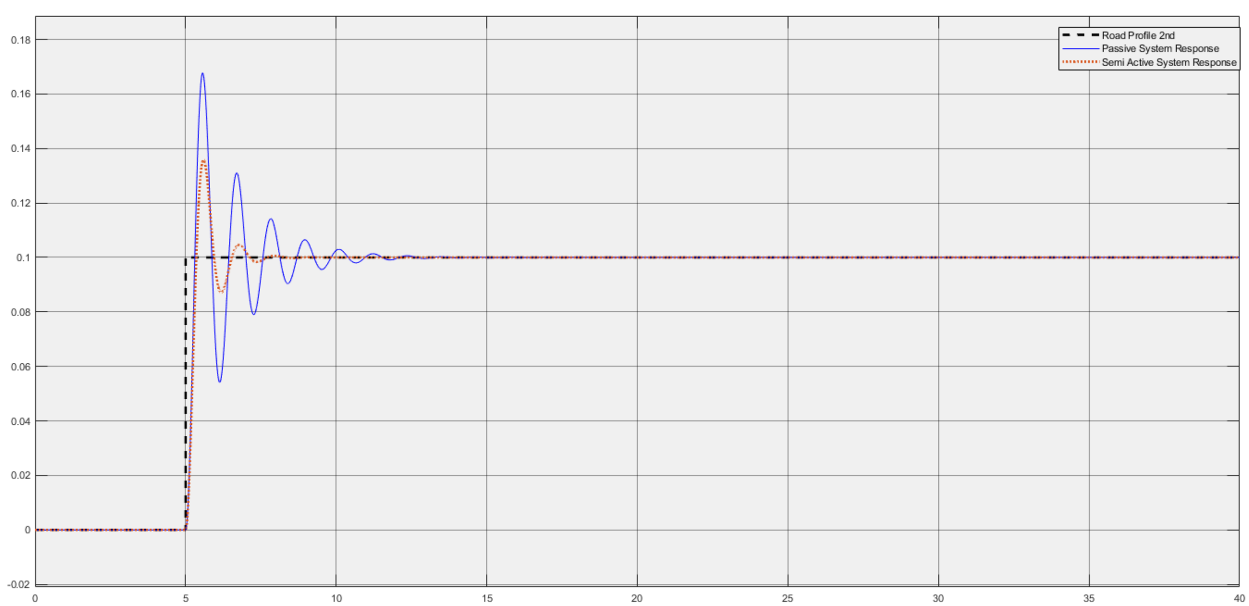


Figure 16 - Road Profile 2, Passive System Response, Semi-Active System Response

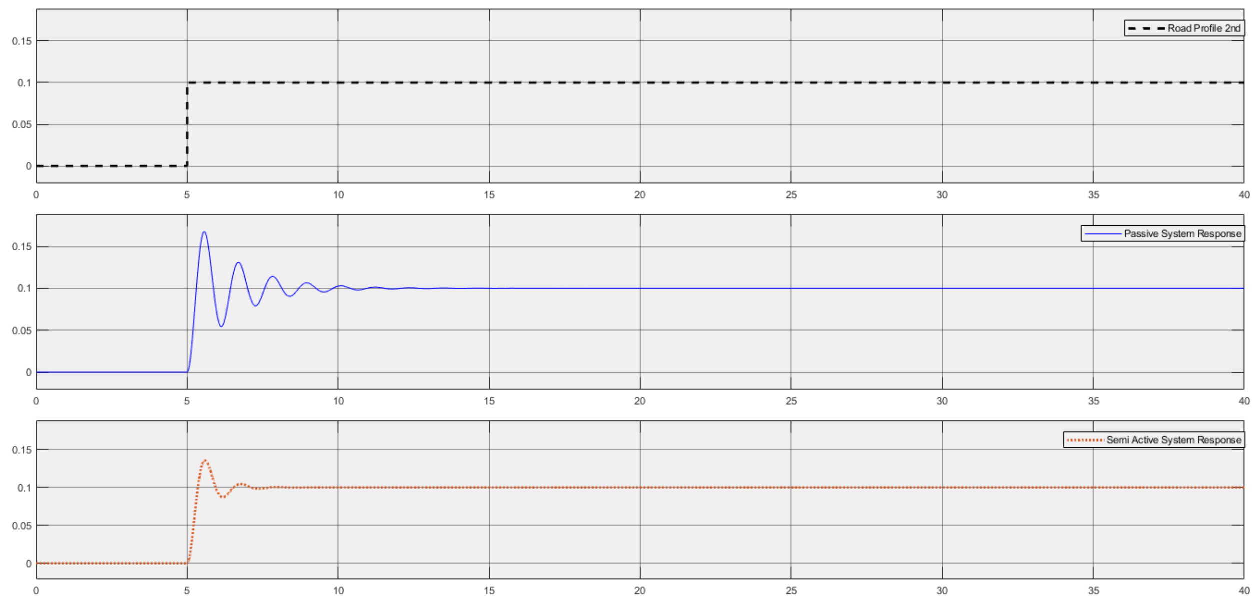


Figure 17 - Cascaded Road Profile 2, Passive System Response, Semi-Active System Response

Figure 16 and 17, shows the response of semi-active and passive system to the road profile 2nd. The percentage overshoot of passive system which is equal to 70% and 6 seconds and at the same time the semi-active system has percentage overshoot of 31%- and 3-seconds settling time.

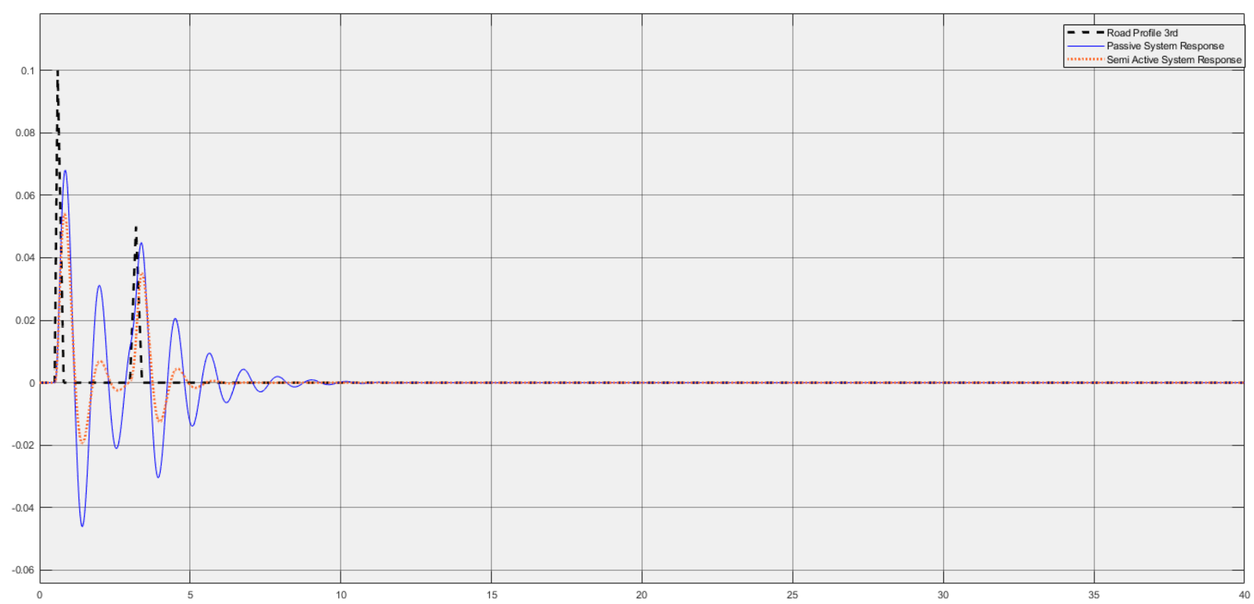


Figure 18 - Road Profile 3, Passive System Response, Semi-Active System Response

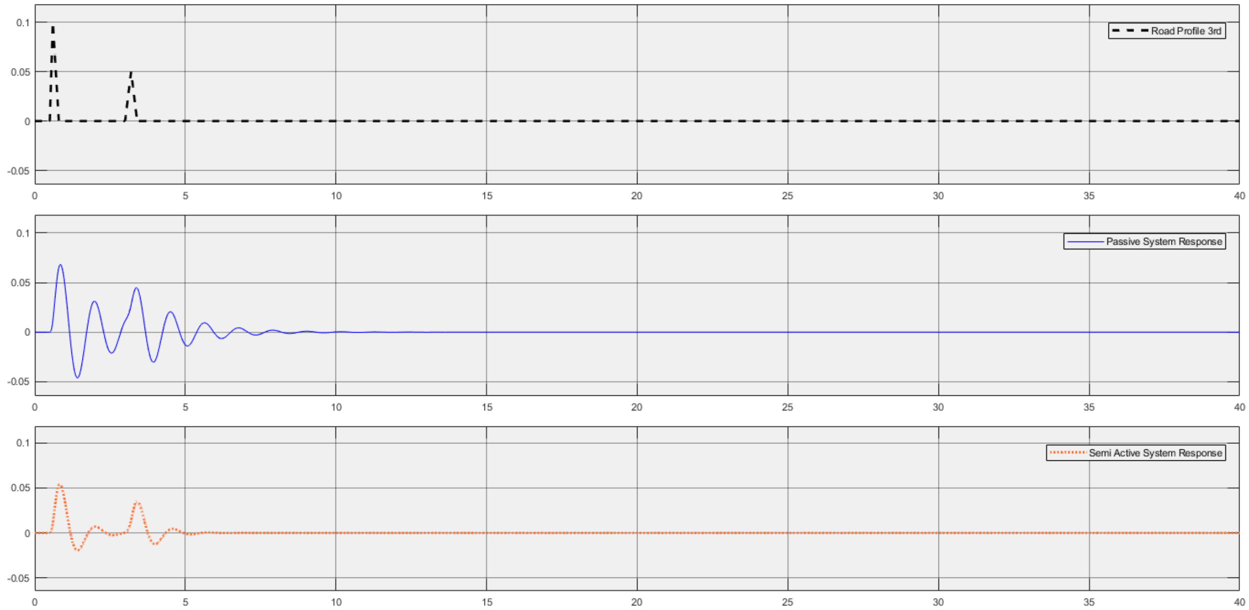


Figure 19 - Cascaded Road Profile 3, Passive System Response, Semi-Active System Response

For profile 3, the comparison of passive and semi-active suspension system is shown in the figure. For passive system the percent over-shoot of the body travel is 25% and settling time is 8 seconds, while on the other hand the percentage overshoot for semi-active vehicle suspension system is about 15% and settling time is 5 seconds

Road Profile	System Type	Tr (sec)	Ts (sec)	%OS	eSS
1st	Passive	2.50	8	2	0
	Semi Active	2.50	5	0.1	0
2nd	Passive	.45	6	70	0
	Semi Active	.44	3	31	0
3rd	Passive	.31	8	25	0
	Semi Active	.13	5	15	0

The above table clearly signifies that Semi-Active Suspension System are much efficient in comparison to the passive counterpart as the performance parameters obtained through Semi-Active Suspension System holds better prospect.

CONCLUSION

Semi active suspension systems and passive suspension systems were reviewed in our project and a very elaborative simulated comparison was done between both the systems. We introduced PID controller based semi active suspension system and examined it against the passive control suspension system. Our comparison was based on Rise Time (Tr), Settling Time (Ts), Percent Over-shoot (%OS) and Steady State Error (eSS). Our simulation results revealed us that PID controller used in the semi-active suspension system is highly efficient and offers huge advantage in terms of countering suspension travel and reducing the settling time. Therefore, we can state that that by addition of an active damping element, we can obtain positive response in terms of vehicle stability and a comfortable ride. We also feel that more work should be done in the direction of self-tuning PID controllers.

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Miscellaneous

Generating the State-Space Equation for Semi-Active Suspension System

Here the mathematical modelling of a two degree of freedom semi-active suspension system is based on newton's second law of motion, like we did earlier.

Taking the vehicle parameters and semi-active suspension system figure from page 15th into consideration.

$$M_2\ddot{Y} = -K_2(Y - X) - b_1(\dot{Y} - \dot{X}) - W(t) \text{............ (1)}$$

Unsprung mass equation:

$$-M_1\ddot{X} = K_2(Y - X) + b_1(\dot{Y} - \dot{X}) - K_1(X - U)(t)\text{-W(t)............ (2)}$$

Choosing State variables as

$$x_1(t) = [Y(t) - X(t)]$$
$$x_2(t) = [\dot{Y}(t) - \dot{X}(t)]$$
$$x_3(t) = \dot{Y}(t)$$
$$x_4(t) = \dot{X}(t)$$

From equation (1) we can see that

$$M_2\dot{x}_3(t) = -K_2x_1(t) - b_1[x_3(t) - x_4(t)]$$

From equation (2) we have:

$$M_1\dot{x}_4(t) = K_2x_1(t) + b_1[x_3(t) - x_4(t)] - K_1b_1[\dot{X}(t) - \dot{U}(t)]$$

Now with these equations we can also say that

$$\dot{x}_1(t) = x_3(t) - x_4(t)$$

$$\dot{x}_2(t) = x_4(t) - w(t)$$

$$\dot{x}_3(t) = \frac{1}{M_2}[-K_2x_1(t) - b_1x_3(t) + b_1x_4(t) - W(t)]$$

$$\dot{x}_4(t) = \frac{1}{M_1}[-K_2 x_1(t) + b_1 x_3(t) - b_1 x_4(t) - K_1 b_1(\dot{X}(t) - \dot{U}(t)) + W(t)]$$

State space equation can be written as form,

$$\dot{x}(t) = Ax(t) + Bw(t)$$

Hence;

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \\ \dot{x}3 \\ \dot{x}4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ \frac{-K_2}{Ms} & 0 & \frac{-b_1}{M_2} & \frac{b_1}{M_2} \\ \frac{K_2}{M_1} & \frac{-K_1}{M_2} & \frac{b_1}{M_2} & \frac{-b_1}{M_2} \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{-1}{M_2} \\ \frac{1}{M_1} \end{bmatrix} U + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} W$$

Where:

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ \frac{-K_2}{Ms} & 0 & \frac{-b_1}{M_2} & \frac{b_1}{M_2} \\ \frac{K_2}{M_1} & \frac{-K_1}{M_2} & \frac{b_1}{M_2} & \frac{-b_1}{M_2} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{-1}{M_2} \\ \frac{1}{M_1} \end{bmatrix}, B_w = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } D = [0; 0; 0; 0]$$

THANKS