20171157 Harsh Sharma

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Ass_06

Q1 Ans $F_c = 1KHz$, $F_s = 4KHz$,

As we know, the characteristics of low-pass filter as

$$H_d(e^{j\omega}) = \begin{cases} 1 & -\omega_c \le \omega \le \omega_c \\ 0 & \text{else} \end{cases}$$

So, firstly we have to find ω_c,

$$\Rightarrow \omega_c = 2\pi \frac{F_c}{F_s} = \pi/2.$$

$$\Rightarrow H_d(e^{j\omega}) = \begin{cases} 1 & -\pi/2 \le \omega \le \pi/2 \\ 0 & \text{else.} \end{cases}$$

Now, we have to find $h_d(n)$,

$$\Rightarrow h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1 \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{1}{jn} e^{j\omega n}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{\pi n}\left(\frac{e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}}{2j}\right)$$

$$\boxed{h_d(n) = \frac{\sin\left(\frac{\pi}{2}n\right)}{n\pi}}$$

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Now, as given, $N = 11$,

So, we will define a new func^n $h(n)$ as.

$$h(n) = \begin{cases} h_d(n) & |n| \le 5 \\ 0 & \text{else} \end{cases}$$

$$= \begin{cases} \frac{1}{n\pi} \sin\left(\frac{\pi}{2} n\right), & |n| \le 5 \\ 0 & \text{else} \end{cases}$$

Now, we have to find value of $h(n)$, at $n = \{\pm 5, \pm 4, \pm 3, \pm 2, \pm 1, 0\}$.

$\Rightarrow$ ~~find as we~~

And as we can see $h(n)$, is a symmetric func^n.

$\Rightarrow h(n) = h(-n)$

$\Rightarrow$
$h(1) = h(-1) = 0.3183$
$h(2) = h(-2) = 0$
$h(3) = h(-3) = -0.106$
$h(4) = h(-4) = 0$
$h(5) = h(-5) = 0.0636.$

And to find freq. response of $h(n)$, firstly we have to find $H(z)$, then we will put $z = e^{j\omega}$, to get $H(e^{j\omega})$ (new),

$$H(z) = \sum_{n=-5}^{5} h(n) z^{-n}$$

$$= h(0) + \sum_{n=1}^{5} h(n) [z^{-n} + z^{n}]$$

$$H(z) = 0.5 + 0.3183 [z^{-1} + z^{1}] + 0.0636 (z^{-5} + z^{5}) - 0.106 (z^{-3} + z^{3}).$$

Now, as for $H(z)$, to be causal, we have to shift $h(n)$ by 5, so, the new $H(z)$, $H'(z)$, equals to,

$$H'(z) = z^{-5} [0.5 + 0.3183 [z^{-1} + z^{1}] - 0.106 (z^{-3} + z^{3}) + 0.0636 (z^{-5} + z^{5})]$$

$$= \cancel{(e^{-j\omega 5}) [\;1.2\; 0.95}$$

Now replace $z = e^{j\omega}$

$$H'(e^{j\omega}) = (e^{-j\omega 5}) [0.5 + 0.3183 (e^{j\omega} + e^{-j\omega}) - 0.106 (e^{3j\omega} + e^{-3j\omega}) + 0.0636 (e^{5j\omega} + e^{-5j\omega})]$$

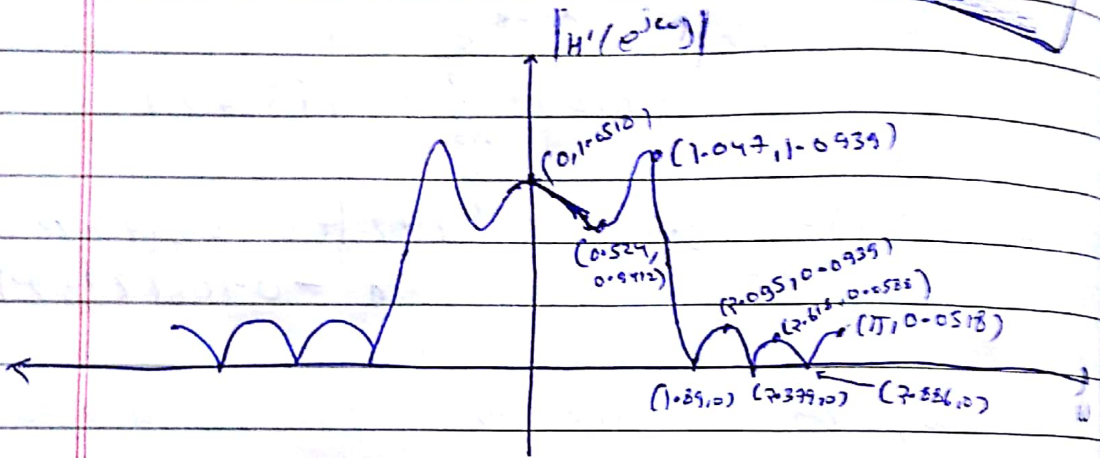
$$\cancel{= e^{-5j\omega} [\; 1.2\; 0.6366}$$

$$H'(e^{j\omega}) = e^{-5j\omega} [0.5 + 0.6366 (\cos\omega) - 0.212 \cos(3\omega) + 0.1272 \cos(5\omega)]$$

$$\boxed{|H'(e^{j\omega})| = |0.5 + 0.6366\cos\omega - 0.212\cos 3\omega + 0.1272\cos 5\omega|.}$$

$$\Rightarrow \cancel{H(e^{j\omega}) = j\cdot e^{-j\omega 5}[0.5 + 0.6366\cos\omega - 0.212\cos 3\omega + 0.1272 \ldots}$$

$$\Rightarrow \boxed{|H'(e^{j\omega})| = [0.5 + 0.6366\cos\omega - 0.212\cos(3\omega) + 0.1272\cos(5\omega)]}$$



Ans 3

$\gamma = 4$ (Phase delay).

$\alpha_s \geq 40\,dB.$

The linear Phase FIR filter is normalized means its cut-off freq. is of ~~wc=1~~ $\boxed{\omega_c = 1}$ rad/sample.

length of filter $= \frac{N-1}{2} = \gamma$.

$\gamma = 4. \Rightarrow \underline{N = 9}.$

$$\Rightarrow h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

and . $$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\left(\frac{M-1}{2}\right)} & |\omega| \le \omega_c \\ 0 & else \end{cases}$$

$$\Rightarrow \quad H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\gamma} & |\omega| \le \omega_c \\ 0 & else \end{cases}$$

$$\Rightarrow \quad H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\gamma} & |\omega| \le 1 \\ 0 & else \end{cases}$$

$$\Rightarrow h_d(n) = \frac{1}{2\pi}\int_{-1}^{1} e^{-j\omega\gamma} \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} e^{j\omega(n-\gamma)}\, d\omega$$

$$= \frac{1}{2\pi} \cdot \frac{1}{(n-\gamma)j}\left[e^{j\omega(n-\gamma)} \right]_{-1}^{1}$$

$$= \frac{1}{2\pi(n-\gamma)j}\left[e^{j(n-\gamma)} - e^{-(n-\gamma)} \right]$$

$$= \frac{1}{\pi(n-\gamma)} \cdot \sin(n-\gamma).$$

$$\boxed{h_d(n) = \frac{1}{\pi(n-\gamma)} \sin(n-\gamma)}$$

As we can see the func' $hd(n)$ is symmetric about $n = 4$.

$\Rightarrow$ $hd(0) = hd(8) = -0.06023$

$hd(1) = hd(7) = 0.0149$

$hd(2) = hd(6) = 0.1447$

$hd(3) = hd(5) = 0.267$

$hd(4) = 0.3183$

Now, $w(n) = 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)$

~~but 8bits~~

As stopband requires 40dB, $\Rightarrow$ it matches the req. for hanning window, so we choose hanning window

Now, $w(n) = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{N-1}\right)\right)$, $n \in [0, N-1]$ (Given)

$\Rightarrow$ $w(0) = 0$, $w(3) = 0.8535$

$w(1) = 0.1464$, $w(4) = 1$

$w(2) = 0.5$, $w(5) = 0.8535$

$w(6) = 0.5$, $w(7) = 0.1464$

$w(8) = 0$

$\Rightarrow$ $h(n) = hd(n) \cdot w(n)$

$\Rightarrow$ $h(0) = 0 = h(8)$

$h(1) = h(7) = 0.00218$

$h(2) = h(6) = 0.07235$

$h(3) = h(5) = 0.22788$

$h(4) = 0.3183$

Now, to find freq response, we firstly find z-transform of the filter.

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}.$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} + h(6) z^{-6} + h(7) z^{-7} + h(8) z^{-8}.$$

$\Rightarrow$ ~~$h(0) + h(8)$~~

and as we know that

$h(0) = h(8)$, $h(1) = h(7)$, $h(2) = h(6)$ &

$h(3) = h(5)$

$$\Rightarrow H(z) = h(0)\left(1 + z^{-8}\right) + h(1)\left(z^{-1} + z^{-7}\right) + h(2)\left(z^{-2} + z^{-6}\right) + h(3)\left(z^{-3} + z^{-5}\right) + h(4) z^{-4}.$$

Taking common z^{-4} from $H(z)$, and then taking magnitude:

$$|H(z)| = \left|\left[h(0)\{z^{-4}+z^{4}\} + h(1)\{z^{3}+z^{-3}\} + h(2)\{z^{2}+z^{-2}\} + h(3)\{z^{1}+z^{-1}\} + h(4)\right]\right| \quad [\because |z^{-4}|=1]$$

$= \cancel{2h(0)\cos(4\omega)}$

now, replace $z = e^{j\omega}$.

$$\Rightarrow |H'(e^{j\omega})| = \left|h(0)\{e^{4j\omega}+e^{-4j\omega}\} + h(1)\{e^{3j\omega}+e^{-3j\omega}\} + h(2)\{e^{2j\omega}+e^{-2j\omega}\} + h(3)\{e^{j\omega}+e^{-j\omega}\} + h(4)\right|$$

$$= |2h(0)\cos(4\omega) + 2h(1)\cos(3\omega) + 2h(2)\cos(2\omega) + 2h(3)\cos(\omega) + h(4)|$$

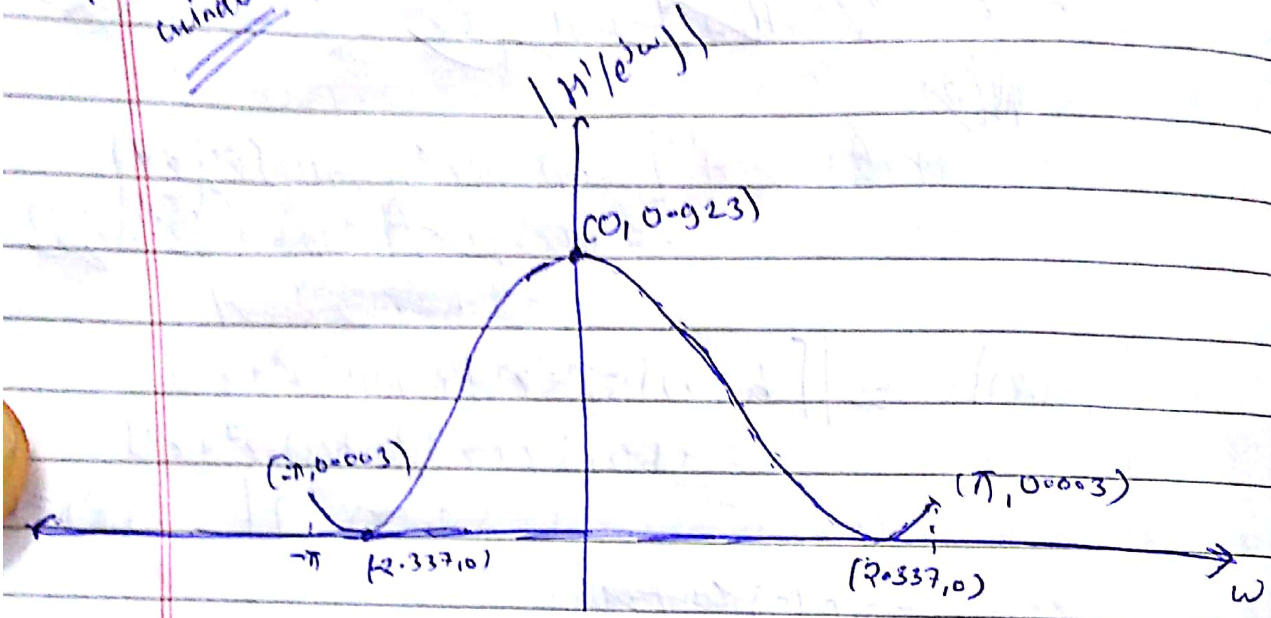
$$= |0.00436\cos(3\omega) + 0.1447\cos(2\omega) + 0.4557\cos(\omega) + 0.3183|$$

$\cancel{H(e^{j\omega}) = 0.3183 + 0.1449\cos(2\omega) + 0.6}$

$$\boxed{|H'(e^{j\omega})| = |0.3183 + 0.4557\cos(\omega) + 0.1449\cos(2\omega) + 0.00436\cos(3\omega)|}$$

$$|H'(e^{j\omega})| = |0.3183 + 0.4557\cos(\omega) + 0.1447\cos(2\omega) \cancel{+0.00...} + 0.00436\cos(3\omega)|$$

with window



And now, without window,

$$h(n) = h_d(n)$$

$$h_d(n) = \{-0.06022,\ 0.0149,\ 0.1447,\ 0.267,\ 0.3183,\ 0.267,\ 0.1447,\ 0.0149,\ -0.060\}$$

Now, we will find $H(z)$, then we will put $z = e^{j\omega}$ to find the freq. response.

$$H_d^{\prime}(z) = \sum_{n=0}^{8} h_d(n) z^{-n}$$

$$= h_d(0) + h_d(1) z^{-1} + h_d(2) z^{-2} + h_d(3) z^{-3} + \cancel{h_d(4)} + h_d(4) z^{-4} + h_d(5) z^{-5} + h_d(6) z^{-6} + h_d(7) z^{-7} + h_d(8) z^{-8}$$

And as we know, $hd(n)$ is Symmetric about $n=4$.

$$\Rightarrow \underline{hd(n) = hd(8-n)}, \quad n \in [0,8].$$

and also taking common z^{-4} from $Hd(z)$, we get

$$Hd(z) = z^{-4} \Big[h(0)[z^{4} + z^{-4}] + h(1)[z^{3}+z^{-3}] + h(2)[z^{2}+z^{-2}] + h(3)[z^{1}+z^{-1}] + h(4) \Big].$$

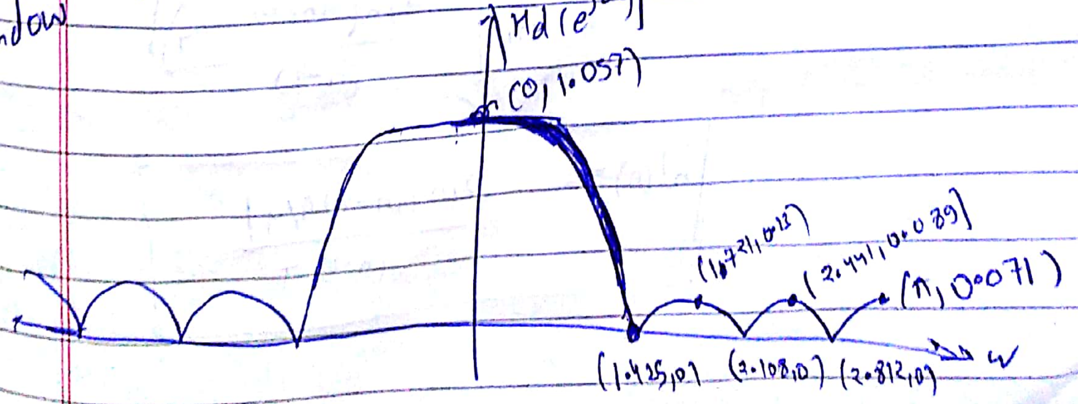
Now, taking the magnitude and Putting $z = e^{j\omega}$, we get.

$$|Hd(e^{j\omega})| = \Big| h(4) + h(3)[e^{j\omega} + e^{-j\omega}] + h(2)[e^{2j\omega} + e^{-2j\omega}] + h(1)[e^{3j\omega} + e^{-3j\omega}] + h(0)[e^{4j\omega} + e^{-4j\omega}] \Big|$$

$$= \Big| 0.3183 + 0.267[e^{j\omega} + e^{-j\omega}] + 0.1477[e^{2j\omega} + e^{-2j\omega}] + 0.0149[e^{3j\omega} + e^{-3j\omega}] - 0.06022[e^{4j\omega} + e^{-4j\omega}] \Big|$$

without window

$$\boxed{Hd(e^{j\omega}) = \Big| 0.3183 + 0.534\cos(\omega) + 0.2957\cos(2\omega) + 0.0298\cos(3\omega) - 0.13044\cos(4\omega) \Big|}$$



Q2)

$$H_d(e^{j\omega}) = \begin{cases} e^{-2j\omega} & -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0 & \text{else} \end{cases}$$

$$w(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{else} \end{cases}$$

~~As length window~~ ow

As length of window funct^n. is 5, so, we take $\underline{N=5}$.

Firstly, we will find $hd(n)$, as

$$hd(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j\omega(n-2)}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-2)}}{j(n-2)}\Bigg|_{-\pi/4}^{\pi/4}\right]$$

$$= \frac{1}{2\pi}\left[\frac{\sin((n-2)\pi/4)}{(n-2)}\right]$$

$$\boxed{hd(n) = \frac{\sin((n-2)\pi/4)}{(n-2)\pi}}$$

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~~kd~~ As we can see, $h_d(n)$ is symmetric around $n=2$, and $N=5$,

so, $h_d(0) = h_d(4) = 0.1591$

$h_d(1) = h_d(3) = 0.22507$

$h_d(2) = 1/4.$

Now, $h(n) = h_d(n) \cdot w(n)$.

$\Rightarrow h(0) = h(4) = 0.1591$

$h(1) = h(3) = 0.22507$

$h(2) = 1/4.$

Now, we will find the Z-transform of $H(n)$ and then Put $z = e^{jw}$.

$$\Rightarrow H(z) = \sum_{n=0}^{4} h(n) z^{-n}.$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4}.$$

Now, taking z^{-2} common and as it is symmetric about $n=2$,

$$H(z) = z^{-2}\left[h(0)\left[z^{2} + z^{-2}\right] + h(1)\left[z^{-1} + z^{1}\right] + h(2)\right]$$

Now, Putting $z = e^{jw}$, ~~in~~ we get

$$H(e^{jw}) = e^{-jw \times 2}\left[h(2) + h(0)\left[e^{2jw} + e^{-2jw}\right] + h(1)\left[e^{jw} + e^{-jw}\right]\right]$$

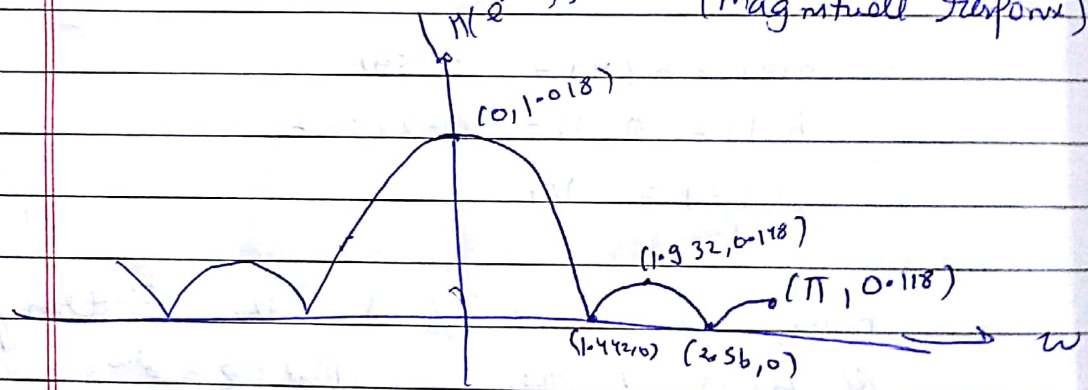
$$H(e^{j\omega}) = e^{-2j\omega}\left[1/4 + 0.1591\{e^{2j\omega} + e^{-2j\omega}\} + 0.3350 7\{e^{j\omega} + e^{-j\omega}\}\right]$$

$$\Rightarrow \boxed{H(e^{j\omega}) = e^{-2j\omega}\left[1/4 + 0.450\cos(\omega) + 0.3182\cos(2\omega)\right]}$$

$$\boxed{|H(e^{j\omega})| = \left|\left[1/4 + 0.450\cos(\omega) + 0.3182\cos(2\omega)\right]\right|}$$

magnitude

$|H(e^{j\omega})|$ ... (magnitude response)



phase response