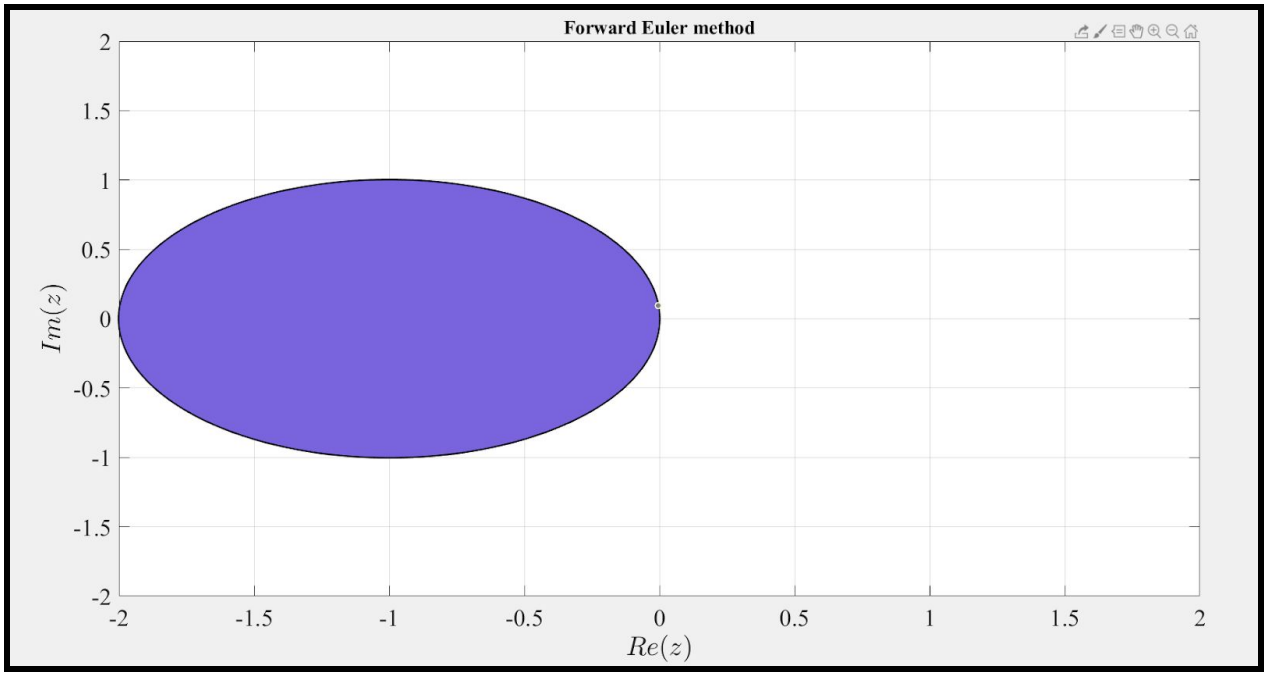
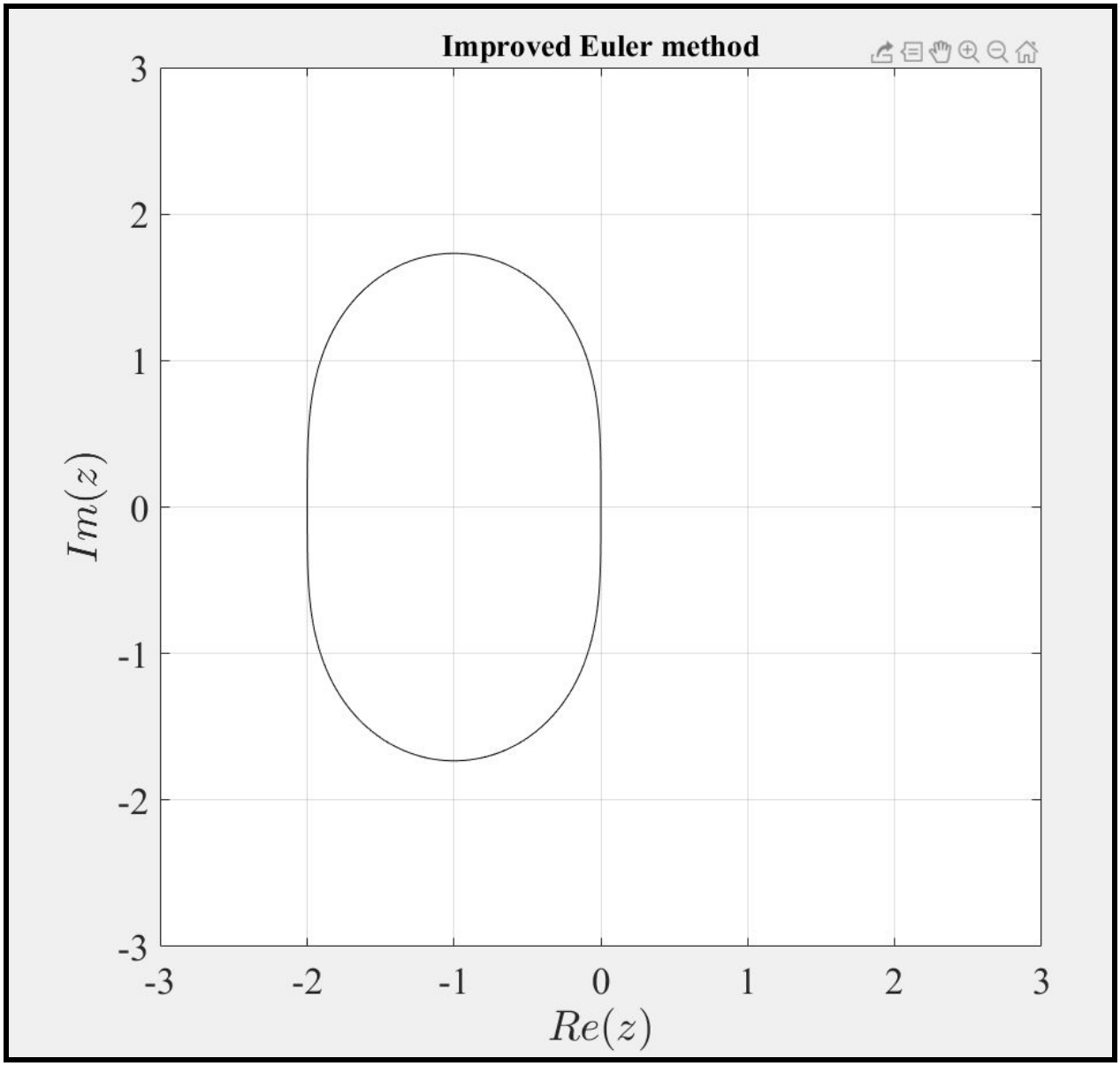
Assignment -2 Report

20th August 2019 || HARSH KUMAR (2016MT10629)

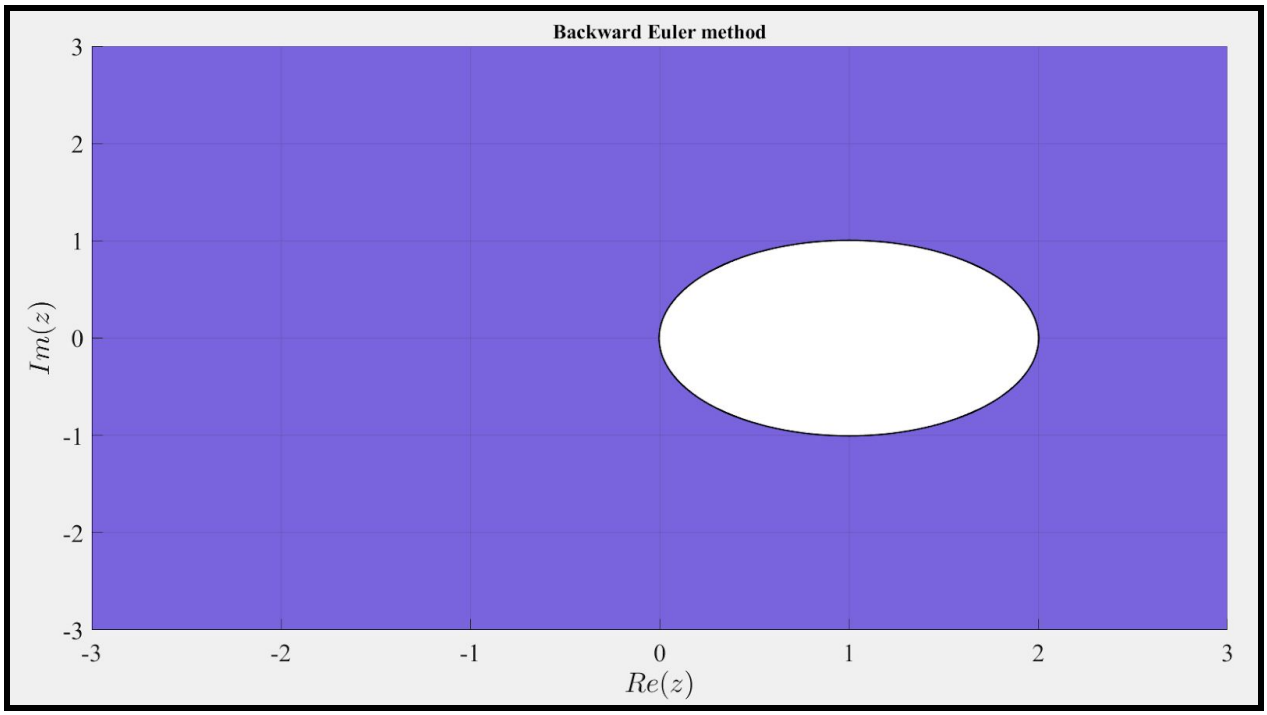
EULER METHOD



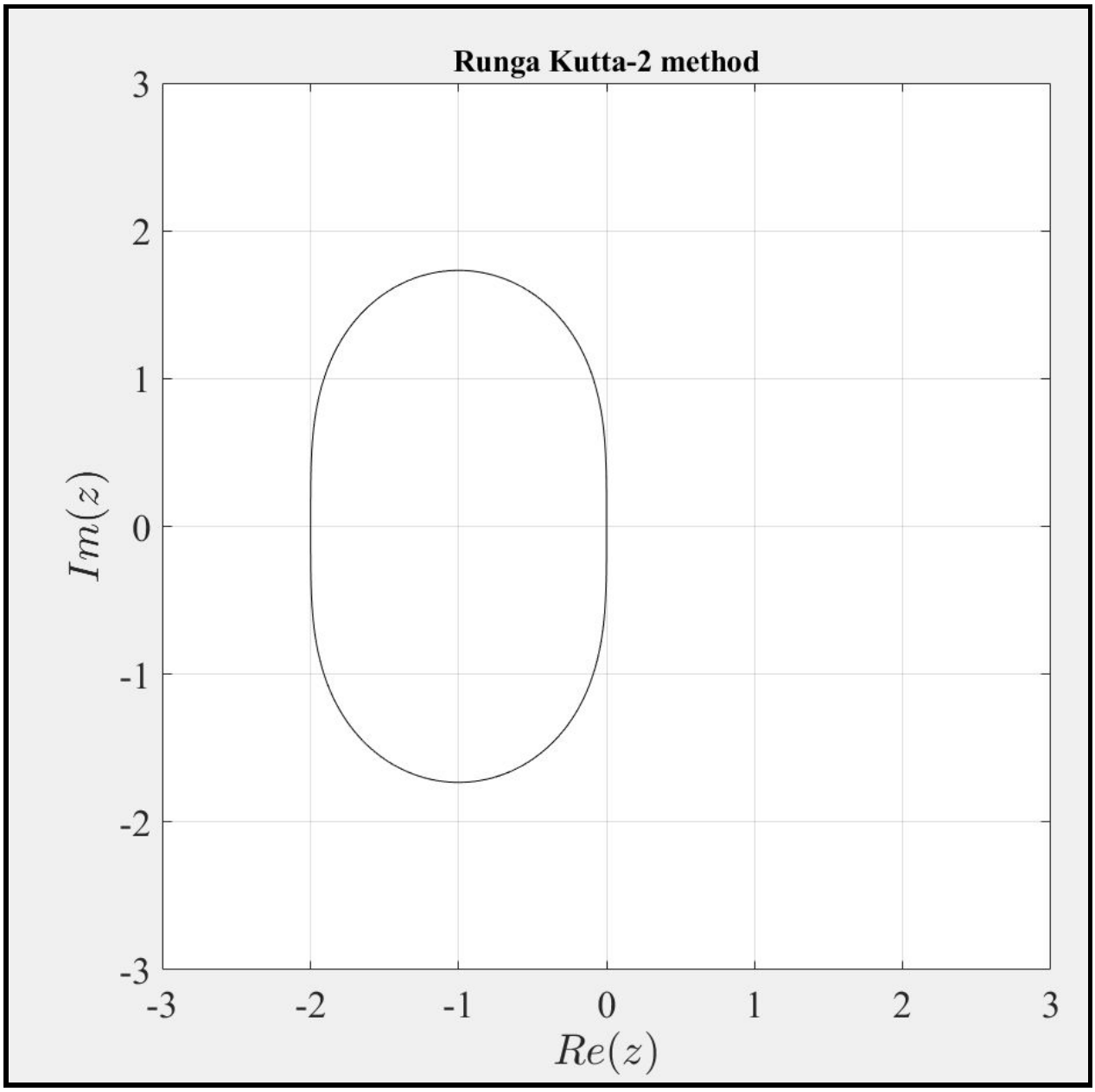
IMPROVED EULER METHOD



BACKWARD EULER METHOD



RK -2 METHOD



1. Euler Method:

$$y_{n+1} = y_n + h f(x_n, y_n) \qquad f(x,y) = \lambda y \quad y(0) = 1$$

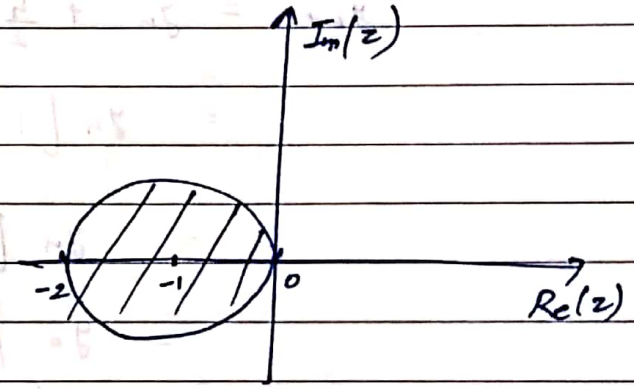
$\lambda \leftarrow$ complex Number

$$y_{n+1} = y_n + h(\lambda y_n)$$
$$= y_n [1 + \lambda h]$$
$$y_{n+1} = y_0 (1 + \lambda h)^n$$

Stab. region $|1 + \lambda h| < 1$ take $\lambda h = z$

$|1 + z| < 1$

Graph →



2. Backward Euler

$$y_{n+1} = y_n + h f(x_{n+1}, y_{n+1}) \qquad f(x,y) = \lambda y$$
$$y(0) = 1$$
$\lambda \leftarrow$ comple

$$y_{n+1} = y_n + h \lambda y_{n+1}$$
$$(1 - \lambda h) y_{n+1} = y_n$$
$$y_{n+1} = \frac{y_0}{(1 - \lambda h)^n}$$

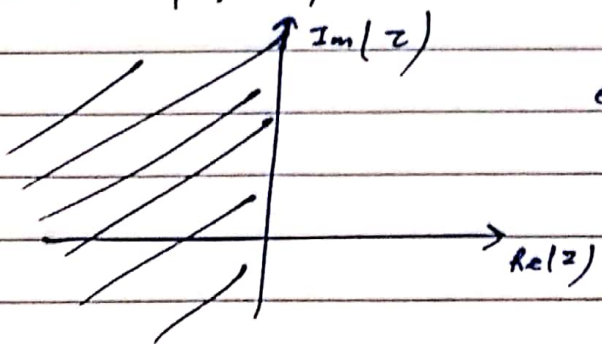
Stab region: $\left| \frac{1}{1 - \lambda h} \right| < 1$ take $\lambda h = z$

$$\Rightarrow \left| \frac{1}{1 - z} \right| < 1$$

we restrict λ to have negative real part

exact sol^b $\rightarrow 0$ as $n \to \infty$

graph →



3. Improved Euler Method (Heun's Method)

$$\tilde{y}_{n+1} = y_n + h f(x_n, y_n) \quad \text{---(1)}$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, \tilde{y}_{n+1})\right]$$

putting $y' = f(x,y) = \lambda y$

$$y_{n+1} = y_n + \frac{h}{2}\left[\lambda y_n + \lambda \tilde{y}_{n+1}\right]$$

from (1)

$$y_{n+1} = y_n + \frac{h}{2}\left[\lambda y_n + \lambda(y_n + h\lambda y_n)\right]$$

$$= y_n\left[1 + \frac{h\lambda}{2} + \frac{\lambda h}{2} + \frac{(h\lambda)^2}{2}\right]$$

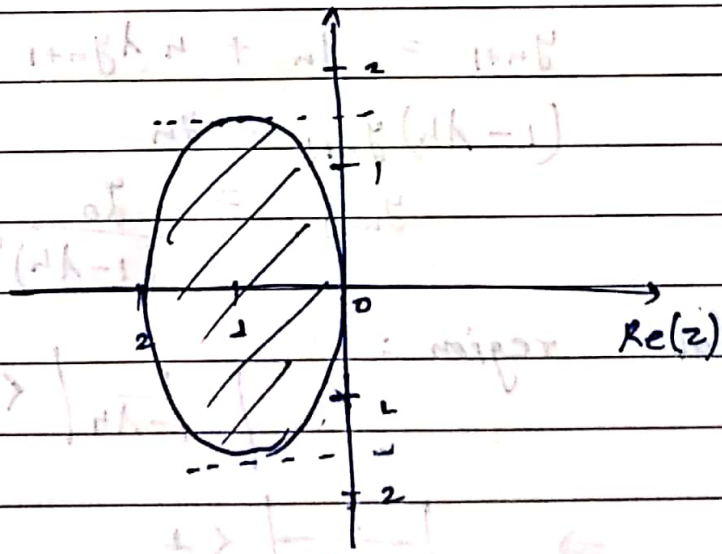
$$= y_n\left[1 + \lambda h + \frac{(\lambda h)^2}{2}\right]$$

$$= y_0\left[1 + \lambda h + \frac{(\lambda h)^2}{2}\right]^h$$

let $z = \lambda h$

$$\left|1 + z + \frac{z^2}{2}\right| < 1$$

graph ⟶



RK-2 Method

$$k_1 = h f(x_n, y_n)$$

$$k_2 = h f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$y_{n+1} = y_n + k_2$$

put $f(x,y) = y' = \lambda y$ $\quad y(0) = 1$

$$y_{n+1} = y_n + h\lambda\left(y_n + \frac{k_1}{2}\right)$$

$$= y_n + h\lambda\left[y_n + \frac{h}{2}\lambda y_n\right]$$

$$= y_n\left[1 + h\lambda + \frac{(h\lambda)^2}{2}\right]$$

$$= y_0\left[1 + h\lambda + \frac{(h\lambda)^2}{2}\right]^n$$

Let $z = h\lambda$.

$$\left|1 + z + \frac{z^2}{2}\right| < 1$$

Graph $\rightarrow$