Case Study #3: Production/Cost and Perfect Competition (70 pts total)

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Please write answers in this Word document in red. Please be sure any graphs are imbedded in the document for the appropriate question.

General Case Information:

- This case study focuses on a perfectly competitive industry
- Each competitive firm in this industry has a Cobb-Douglas production function: $q = 0.02K^{0.5}L^{0.5}$
- These firms combine capital and labor to produce output
- The price of capital and labor are: r = .05 and w = 45

In this case study, you will: use graphs and equations to analyze competitive firm decisions, the interaction between those decisions and the competitive market determination of price.

Skills needed to complete this case study:

1. Enter data, enter formulas, and create charts in Excel
2. Use basic algebra

Steps to complete Case Study #3 (Part 1 Production/Costs, 35 points):

1. (5pts) Find equations of the firm's Isoquant and Isocost lines.
   a. (2pts) Equation of the Isoquant: Use algebra to solve the production function for K as a function of q and L. **Show your work**.
      $q = 0.02\ K^{0.5}\ L^{0.5}$
      Rearranging the production function,
      $K^{0.5} = \frac{q}{(0.02 * L^{0.5})}$

      $K = (\frac{q}{(0.02 * L^{0.5})})^2$

      $\mathbf{K = 2500\frac{q^2}{L}}$

   b. (3pts) Equation of the Isocost: Write the cost function in general form. Remember that cost is equal to the sum of the expenditures to purchase capital plus the expenditures to purchase labor. Each of these expenditures is equal to the price of the input multiplied by the quantity of the input. Use the letter r to denote the price of capital, and w to denote the price of labor.

      Isocost: TC = wL + rK

Use algebra to solve the cost function for K as a function of TC and L. This is the equation of your isocost line.

$K = \frac{TC - wL}{r}$

$\mathbf{K = \frac{TC}{r} - \frac{wL}{r}}$

Now, assume r = .05 and w = 45. Plug in these values. What is the slope of this line?

$K = \frac{TC}{r} - \frac{wL}{r}$

$K = \frac{TC}{0.05} - \frac{45L}{0.05}$

K = 20TC – 900L

∴ Slope = -900

2. (4pts) Calculate the cost-minimizing combination of L and K for q=6. There are 3 steps to this process, show your work for each step.
   a. (2pts) Step 1: First, find $MRTS_{LK}$. Find the tangency condition to get K in terms of L.

      $MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{w}{r} = \frac{\alpha K}{\beta L}$

      $\alpha = 0.5, \beta = 0.5$

      $MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{45}{0.05} = \frac{0.5K}{0.5L}$

      $900 = \frac{K}{L}$

      ∴ K = 900L

   b. (1pt) Step 2: Plug into the production function for q=6 and solve for L.

      $q = 0.02\ K^{0.5}\ L^{0.5}$

      $6 = 0.02\ (900L)^{0.5}\ L^{0.5}$

      $300 = (900)^{0.5}\ L$

      $300 = 30L$

      ∴ L = 10

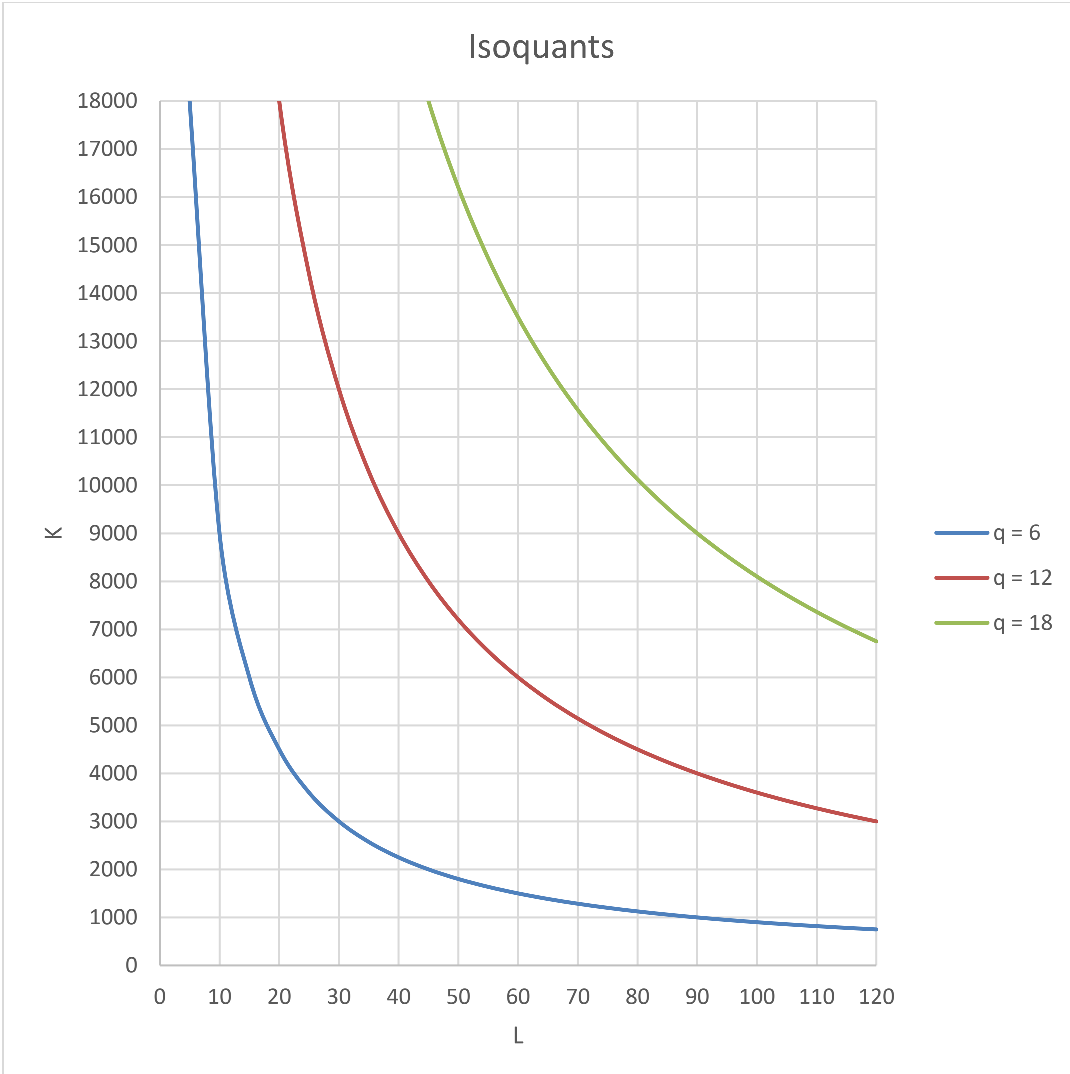
   c. (1pt) Step 3: Plug into the equation found in step 1 to find K.

      K = 900L

      K = 900(10)

      ∴ K = 9000

3. (7pts) Use Excel to create and graph isoquant curves:
    - Open up Excel and right click on the worksheet tab in the lower right corner that says, “sheet1.” From the drop down menu, choose “rename” and name it “production.”
    - Use column A to store possible values for L in 5-unit increments. Use the first row to label the column. Put a 5 in the second row. Put the formula “=a2+5” in the third row. Copy this formula in rows 4-25.
    - Use column B to store the isoquant for 6 units of output. This is the amount of K that is needed to combine with each possible value of L to produce 6 units of output. Label this column “q=6.” Use the equation from question 1, with q = 6. Note: use $a2 in your formula so that you can easily cut and paste.
    - Use column C to store the isoquant for 12 units of output. This is the amount of K that is needed to combine with each possible value of L to produce 12 units of output. Label this column “q=12.” Use the equation from questions 1, with q = 12.
    - Use column D to store the isoquant for 18 units of output. This is the amount of K that is needed to combine with each possible value of L to produce 18 units of output. Label this column “q=18.” Use the equation from question 1, with q = 18.
    - (6pts) Use scatter-plot to graph the isoquants. To do this, highlight the area that includes columns A, B, C and D for rows 1-25. Choose scatter-plot with smooth lines. Format your graph so that it has a title and each of the axes are labeled. To do this, click on the graph and the “chart design” tool bar will appear. Look for “Chart Layouts” or “Quick layout” on the left, and click on “Layout 1.” Type in your axis labels and your graph title as “Isoquants”.
    - Adjust the vertical axis to have a maximum of 18,000 and fix the major unit to 1000. To do this, click on the values on the horizontal axis to isolate them, then right click and select “format axis”. Then, for maximum select “fixed” and enter 18,000, and similarly select the major unit. Adjust the horizontal axis to have a maximum of 120 and fix the major unit to 10. Right click on the horizontal axis again and click “add major grid lines.” Format the graph to have a title and label for each axis. Drag the bottom down a bit to make the graph easier to see. Cut and paste the graph here:



- (1pt) Use the worksheet information to complete the following table:

Q	K	L
6	9000	**10**
12	9000	**40**
18	9000	**90**

4. (15pts) Follow the steps below to find the short run cost curves associated with our Cobb-Douglas production function when K is fixed at 9000 units. Assume r = .05 and w = 45.
   a. (1.5pts) Calculate your SR production function (round any numbers to 4 decimal places).
      $q = 0.02\ K^{0.5}\ L^{0.5}$
      $q = 0.02\ (9000)^{0.5}\ L^{0.5}$
      $\mathbf{q = 1.8974\ L^{0.5}}$

b. (1.5pts) Calculate your FC.
   FC = rK
   FC = 0.05(9000)
   FC = 450

c. (2pts) Derive the equation for your VC (as a function of q)
   VC = wL
   We need to find L,
   where, **L** $= \left(\frac{q}{(0.02 * K^{0.5})}\right)^2 = \left(\frac{q}{(0.02 * 9000^{0.5})}\right)^2$ = **0.2778q^2**

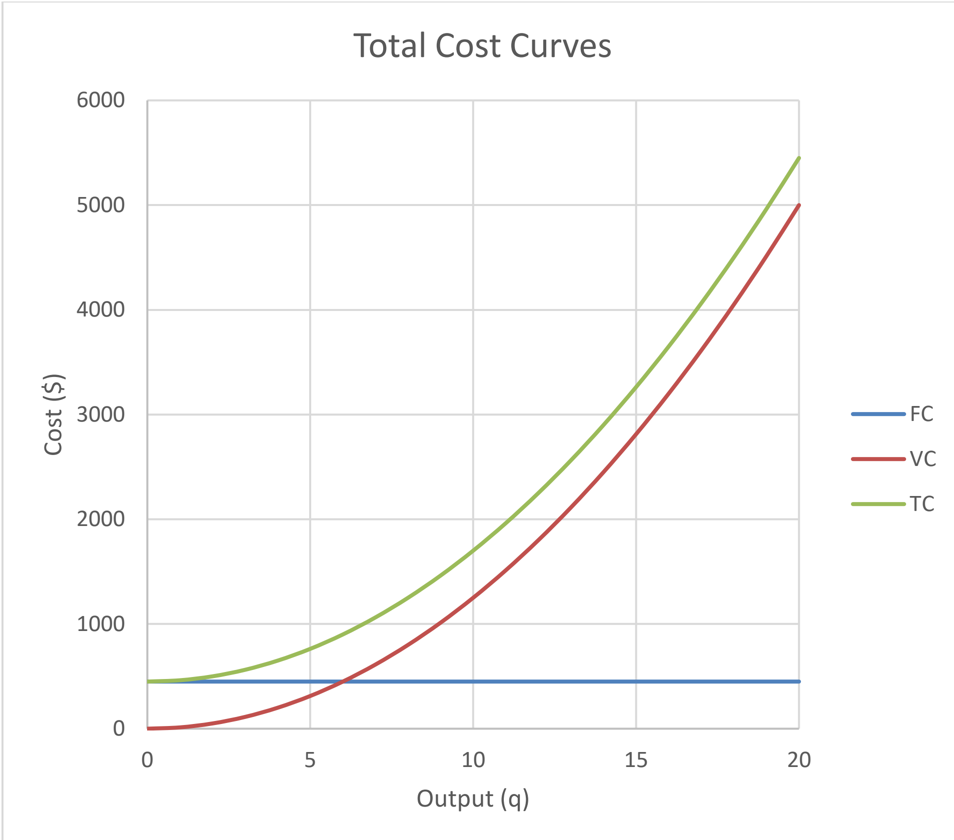
   VC = 45 * (0.2778q^2)
   VC = 12.50 q^2

d. (1pt) Derive the equation for your TC (as a function of q)
   TC = FC + VC
   TC = 450 + 12.50 q^2

e. Graph the total cost curves:

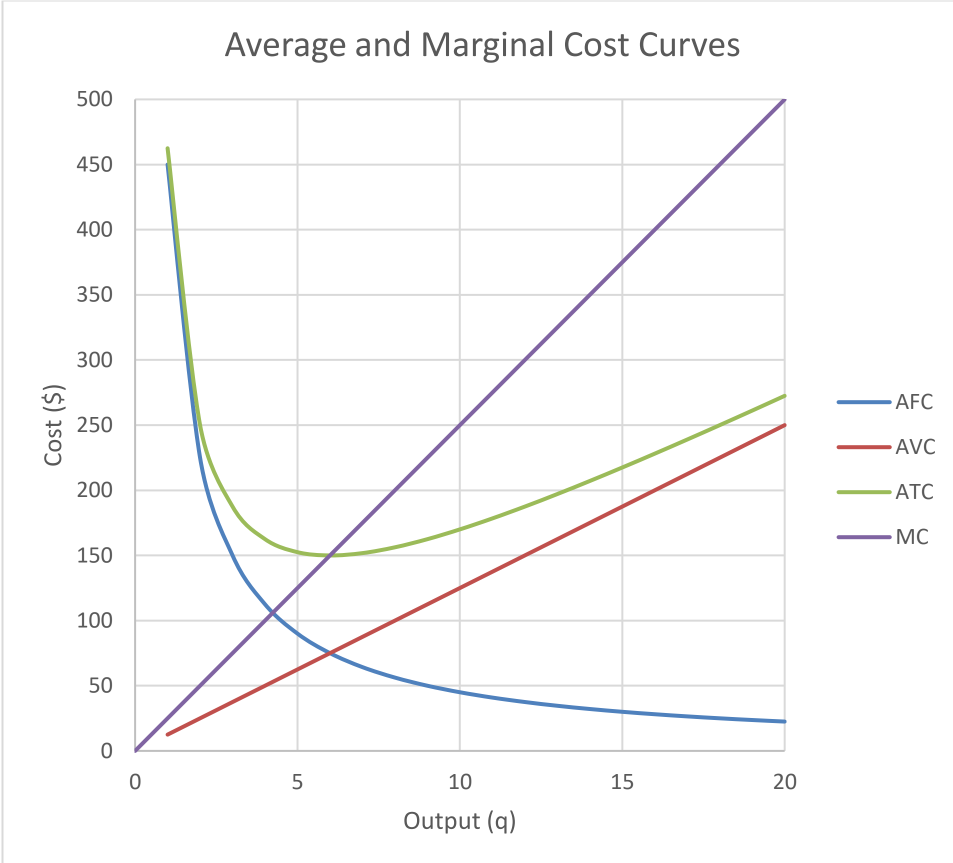
   - In your same Excel file, open a new worksheet (sheet 2) for cost information and rename it "cost" – do this by right clicking on the tab.
     Note the difference between your production worksheet, in which the first column stored possible values of L, and this new cost worksheet in which the first column will store possible values of q. The variable represented in the first column will be graphed on the horizontal axis of the scatter-plot. (For the isoquant diagram, L is shown on the horizontal axis.) The new cost worksheet will be used to graph cost functions, with output (q) on the horizontal axis, and costs on the vertical axis.

   - Use column A to store possible values of q from 0 through 20. (Label this column q)
   - Use column B to store Total Fixed Cost (FC). Use calculations from part b above.
   - Use column C to store Total Variable Cost (VC). Use equation from part c above.
   - Use column D to store Total Cost (TC). Use equation from part d above. (Note: you could also simply add columns B and C)
   - (4pts) Use a smooth scatter-plot to graph TC, FC and VC. Title the graph "Total Cost Curves" and label the axes. Format the graph so the max on the horizontal axis is 20 and there are major grid lines both ways. Drag the bottom down a bit to make the graph easier to see. Cut and paste your graph here:



f. Graph these average and marginal cost curves:
   - Copy and paste column A into column E (you will now graph average and marginal curves.
   - Generate AFC in column F by dividing FC/q
   - Generate AVC in column G by dividing VC/q
   - Generate ATC in column H by dividing TC/q
   - Generate MC in column I. MC is the slope of the TC or VC curves. In this case, it would require calculus. Thus, MC = 25q.

   - (5pts) Use scatter-plot to generate a second graph to show ATC, AFC, AVC and MC. Title the graph "Average and Marginal Cost Curves" and label the axes. Format the graph so the max on the horizontal axis is 20, the max on the vertical axis is 500, and there are major grid lines both ways. Drag the bottom down a bit to make the graph easier to see. Cut and paste your graph here:



5. (4pts) Graph the cost-minimizing point with the isoquant and isocost curves.
    a. Copy your first worksheet that you named “production” into sheet3. Do this by right clicking on “production” and choosing “move or copy.” A box will appear. Click the box at the bottom that says, “create a copy.” In the box above, click on “sheet3” then click ok. This will create a copy of your production worksheet in sheet3. It will be named “production (2)” – rename it “cost-min.”
    b. In this new worksheet, delete your graph and delete the isoquants for q=12 and q=18.
    c. Insert a new first row (below the titles) (right click on the “2” and choose “insert” – a row will be inserted. In the L column, add L=0 in the new cell. Do not put anything in the new q cell.
    d. In cell A4, it currently says, “=a3+5” – change this to “=a3+1” and copy it down to row 28. Now you have labor from 0 to 30. Add in the remaining 2 rows for q (just copy and paste).
    e. From 1b, recall the equation of your isocost line: **K = 20TC – 900L**
    f. In column C, put in the isocost associated with a total cost of $600. Use $a2 so that you can cut and paste easily. Label the column TC=600
    g. Copy and paste column C into columns D and E. Adjust them so that column D is associated with TC=900 and column E is associated with TC=1200. Label each column as such.
    h. (4pts) Use scatter-plot to generate a graph of the isoquant and 3 isocost curves and indicate the point of tangency. Title the graph “Cost Minimization” and label the

axes. Format the graph so the max on the horizontal axis is 30, the max on the vertical axis is 25,000, the min on the vertical axis is 0, and there are major grid lines both ways. Drag the bottom down a bit to make the graph easier to see. Click on tangency point twice, right click and click “format data point.” Click “fill” (paint can), “marker”, “marker options”, and choose “built-in.” From the drop-down menu, choose the circle and increase its size to 7. Click close.

What are the cost minimizing values of K and L for q = 6? What is the minimum Total Cost to produce q = 6?

$q = 0.02\ K^{0.5}\ L^{0.5}$
$6 = 0.02\ K^{0.5}\ L^{0.5}$ ………(i)

$\text{MRTS}_{\text{LK}} = \frac{MP_L}{MP_K} = \frac{w}{r} = \frac{\alpha K}{\beta L}$

$\alpha = 0.5,\ \beta = 0.5$
$\text{MRTS}_{\text{LK}} = \frac{MP_L}{MP_K} = \frac{45}{0.05} = \frac{0.5K}{0.5L}$

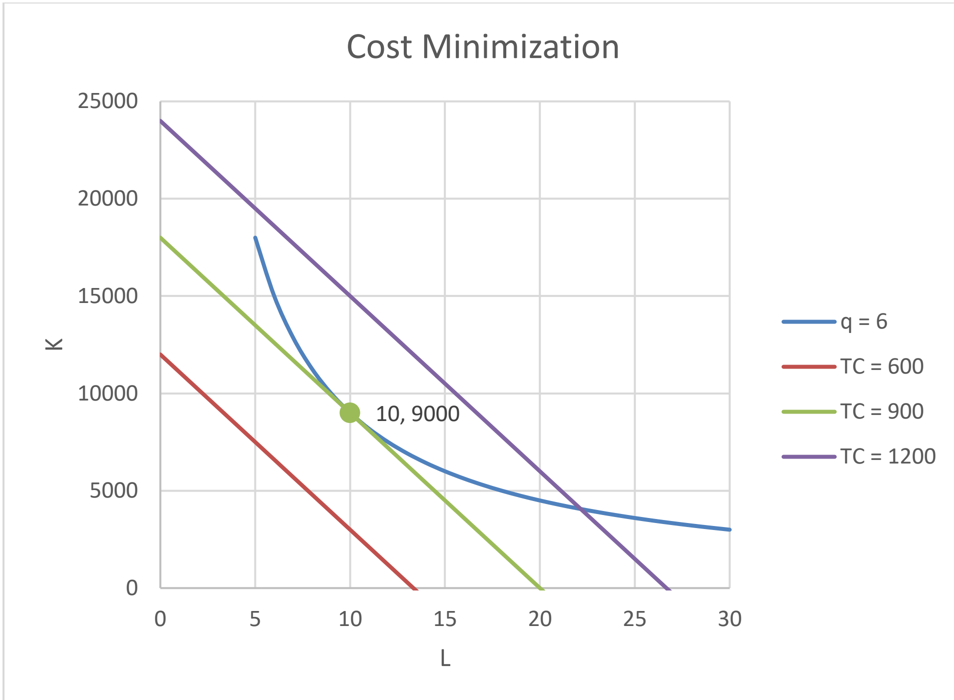
$900 = \frac{K}{L}$

$K = 900\ L$ ………(ii)
Replacing K in equation (i)
$6 = 0.02 * (900L)^{0.5} * L^{0.5}$
$6 = 0.02 * (900)^{0.5} * L$
$6 = 0.02 * 30 * L$
$\therefore L = 6 / (0.02*30)$
L = 10

Put L in equation (ii)
K = 900(10)
K = 9000

Find total cost using the values of L & K,
TC = wL + rK
TC = 45(10) + 0.05(9000)
TC = 900

Tangency points or cost minimizing values (L, K) = (10, 9000)
Minimum Cost to produce q = 6 is 900. This is verified from the graph given below:
Cut and paste your graph here:



Part 2: Perfect Competition (35 pts)

6. (9pts) Find equilibrium P & Q in the perfectly competitive market. Demand is represented by the equation: Q = 1500 – 2P. The perfectly competitive firms are assumed to be identical. The quantity supplied by each individual firm is represented by the firm's MC curve. In order to graph market supply and market demand, however, we need to focus on market quantity, rather than individual firm quantity. The market quantity is equal to $Q = nq$; where q is the individual firm's quantity and n is the number of firms in the industry.
   a. (2pt) What is the equation for the individual firm's SR supply curve? Recall your graphs from question 4. Notice that MC and AVC are both linear. Recall that MC = 25q. Use **both** profit maximizing rules to find your individual SR supply function (get q as a function of P).
      Profit maximizing condition 1: **P = MC**
      P = 25q
      ∴ q = P/25 …(for all P > 0, q > 0)

      Profit maximizing condition 2: **MC = AVC**
      We calculated VC in Q4c as $12.5q^2$.
      AVC = VC/q = $12.5q^2/q$ = 12.5q

      Now, MC = AVC
      25q = 12.5q

      q = 0
      P = 0

b. (1pt) Suppose there are 100 firms in this industry. What is the equation of the SR market supply curve?

$q = P/25$

$Q/n = P/25$

$Q = nP/25$

Given, $n = 100$

$Q = 100 * (P/25)$

$Q_S = 4P$

c. (2pts) Solve for the short-run market equilibrium (find equilibrium P & Q using algebra).

$Q_S = Q_D$

$4P = 1500 – 2P$

$6P = 1500$

$P = 1500/6$

$P = 250$

Substituting $P = 250$ in Supply equation,

$Q = 4P$

$Q = 4(250)$

$Q = 1000$

Short-run market equilibrium (Q, P) = (1000, 250)

d. (2pts) Calculate the inverse demand curve. Calculate the inverse supply curve. Show your work. You will need these inverse functions in order to graph S and D.

Demand Curve: $Q = 1500 - 2P$

$2P = 1500 – Q$

Therefore, Inverse Demand Curve is:

$P = 750 – 0.5Q$

Supply Curve: $Q = 4P$

$4P = Q$

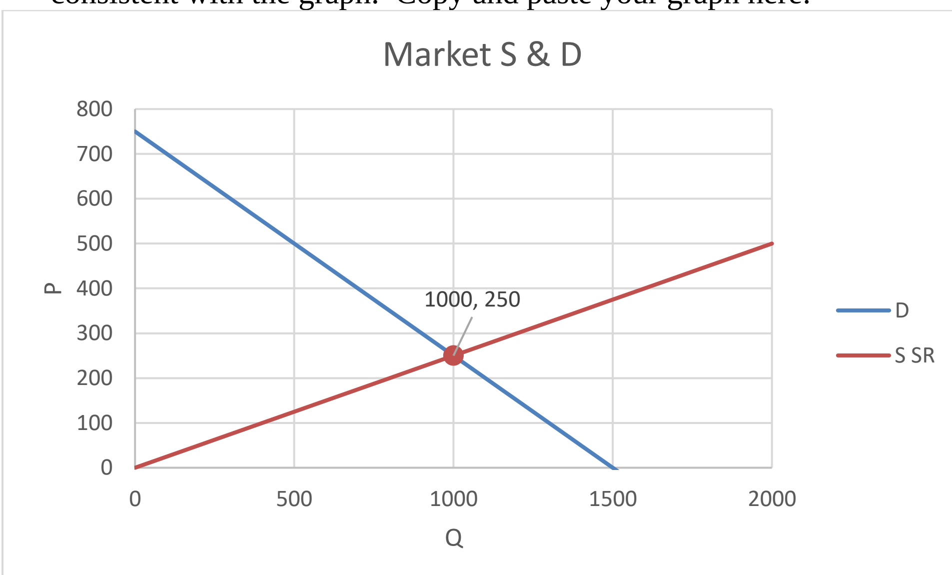
Therefore, Inverse Supply Curve is:

$P = Q/4$

e. In Excel, copy your "cost" worksheet into a new worksheet in the same excel fil. Do this by right clicking on the name "cost" and choosing "move or copy." A box will appear. Click the box at the bottom that says, "create a copy." In the box above, click on a new sheet then click ok. This will create a copy of your cost worksheet. It will be named "cost(2)" – rename it "perfect comp"

f. (2pts) Graph Supply and Demand curves. Generate new columns in Excel to represent demand and supply. This will require some strategic thinking. You will want to generate a graph with market quantity on the horizontal axis. That means that you will need to generate a column of numbers to represent possible values of market quantity.

- Let column K represent market quantity, $Q = 100q$.
- Store values for market demand prices in column L. Use the inverse demand curve from 6d. Label this column "D".
- Store values of market supply in column M, labeling it "S SR" using inverse supply equation. Notice that the numbers in column M are equal to the numbers representing MC in column I.
- Using columns K, L and M, Create S and D graph. Title the graph "Market S & D" and label the axes. Format the graph so the max on the horizontal axis is 2000 and the min on the vertical axis is 0. Make sure there are major grid lines going both directions. Verify that the computed equilibrium P & Q are consistent with the graph. Copy and paste your graph here:



We see that the equilibrium points calculated in part c are consistent with graph. From our calculations in Q6c we see that SR equilibrium points are Q = 1000, P = 250. This is exactly what we see in the graph above.

7. (9pts) Calculate profit for a single perfectly competitive firm in this market.

a. (2pts) Given the equilibrium market price found in question 6, use the profit-maximizing rules to determine the optimal level of output for the individual firm to produce (q^*). **Show work**.

Profit Maximizing Rule: P = MC

P = 25q

Price at equilibrium, P = 250

$250 = 25q^*$

$q^* = 250/25$

$q^* = 10$

b. (1pt) Calculate the total revenue that this firm earns from selling q^*.

TR = P * q^*

TR = 250 * 10

TR = 2500

c. (2pts) Calculate the total cost that this firm incurs in the short run by producing q^*. Recall your SR cost functions found in Case Study 2. How much of this cost is fixed cost and how much is variable cost?

TC = VC + FC

TC = 450 + 12.50 q^2

Therefore, **FC = 450**

VC = 12.5 q^2

VC = 12.5 * 10^2

VC = 1250

TC = 1250 + 450

TC = 1700

d. (1pt) Calculate this firm's profit from producing and selling q^*.

Profit = (P – ATC) q^*

$= \left(250 - \frac{1700}{10}\right) * 10$

= 2500 – 1700

Profit = 800

Another way,

Profit = TR – TC

= TR – (VC + FC)

= 2500 – (1250 + 450)

Profit = 800

e. (1pt) Fill out the table below with your answers:

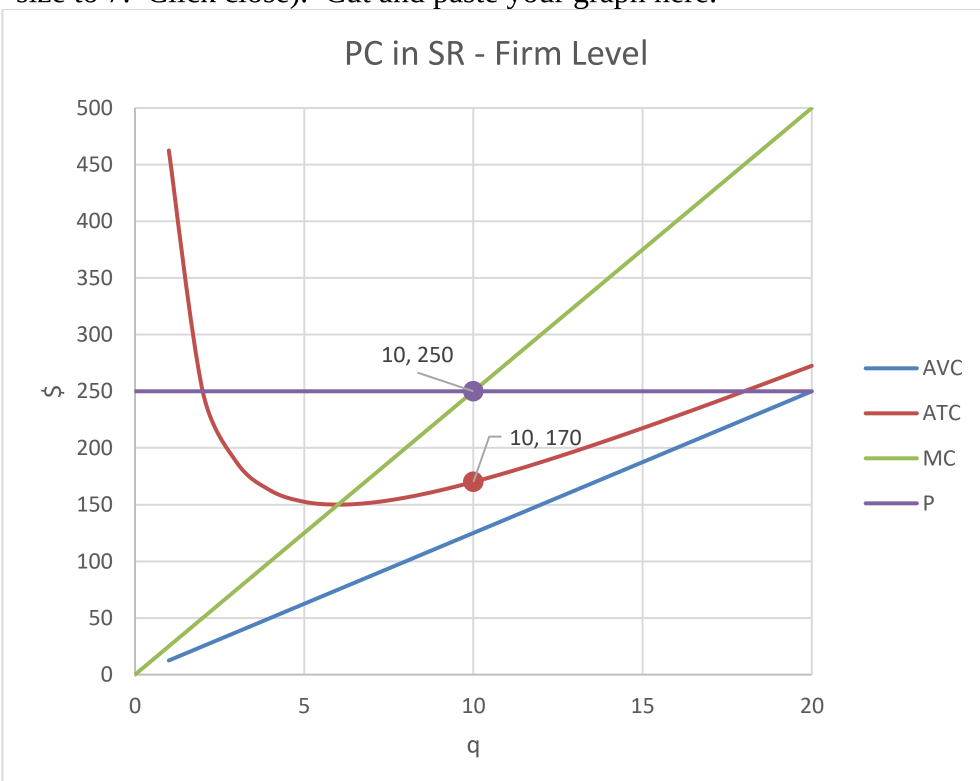
Total Revenue	**2500**
Total Cost	**1700**
FC	450
VC	1250
Profit	**800**

f. (2pts) What will happen in the long run? Explain.

- Long run costs have no fixed factors of production, while short run costs have fixed factors and variables that impact production. In other words, in the long run, there are no fixed costs.
- In the long run, more firms will enter the market if there is profit. As new firms enter, supply will increase thus shifting the supply curve to the right. As a result of this, to maintain equilibrium, price of the products will decrease. Also, cost will go up as a result of more intensive competition. The firms can keep entering the market until the decreased price is equal to average cost, such that economic profit for all the firms is zero.

8. (5pts) Generate a graph to show the optimal quantity that will be produced by each competitive firm, and the resulting profit. This graph will include 4 curves:
   - The short-run equilibrium price (To generate this horizontal line, use column J to store the value of the equilibrium P. Because this line is horizontal, all of the numbers stored in this column are identical.)
   - ATC
   - AVC
   - MC

   a. (3pts) Use scatter-plot to generate this graph using columns E-J (you can delete data from AFC column if you wish). Title the graph "PC in SR – firm level" and label the axes. Format the graph so the max on the horizontal axis is 20, the max on the vertical axis is 500, and there are major grid lines both ways. Drag the bottom down a bit to make the graph easier to see. Create dots on the MC and the ATC curves at the profit-maximizing level of q (Click on each point twice, right click and click "format data point." Click "fill", "marker options", "marker", and choose "built-in." From the drop-down menu, choose the circle and increase its size to 7. Click close). Cut and paste your graph here:



   b. (1pt) On the graph above, shade in the area of profit (do this by hand).

   c. (1pt) Is your graph consistent with your profit computation? Explain.

   Yes, the graph is consistent with the profit computation. As we see from the graph,

   Profit $\pi = (P - ATC)*q^*$

   $(250-170) * 10 = 80*10$

   $\therefore \pi = 800$

   Which is consistent with out computed profits in the previous part.

9. (12pts) You work for a firm that produces an input that is used by these competitive firms. Your marketing vice president has asked you to provide an analysis to support the marketing department's strategic planning committee. They understand that the industry is not currently in long-run equilibrium, and they have asked you to help them estimate the output that will be produced and the number of firms that will exist when the industry reaches long-run equilibrium. They assume that all firms have the same production function and cost structure. This will require several steps:
   a. (2pts) For each firm, the LR equilibrium occurs when profit equals zero. This means that P=ATC. The quantity rule requires that P=MC, thus, at the LR equilibrium, P=MC=ATC. Using this information, calculate q^* and P that satisfy both conditions.
      Using, P = MC = ATC
      MC = ATC
      $25q = \frac{450 + 12.50\,q2}{q}$
      $25q^2 = 450 + 12.50q^2$
      $12.5\ q^2 = 450$
      $q^2 = 450/12.5$
      $q^2 = 36$
      ∴ q = 6

      Substituting q = 6 in (P = MC)
      P = 25q
      P = 25 * 6
      ∴ P = 150
   b. (1pt) In the market, quantity demanded (Q_D) must be at this price. In other words, this must be a market equilibrium price. Calculate the equilibrium market quantity (Q^*).
      Demand: Q = 1500 – 2P
      $Q^* = 1500 - 2(150)$

      ∴ Q^* = 1200

      Long-run market equilibrium (Q, P) = (1200, 150)
   c. (1pt) Use the market Q and the individual firm q to find the number of firms now operating in the market. (Remember Q = nq).
      Q = nq
      n = Q/q
      n = 1200/6

      ∴ n = 200
   d. (2pts) What is the equation of the new market supply curve? Find the inverse market supply curve. This is the LR equilibrium market supply.
      $Q_S = nq$ ….(i)
      P = 25q ….(ii)

      Rearranging equation (ii), we get

q = P/25

Substituting q in equation (i)

$Q_S = n * \frac{P}{25}$

$Q_S = 200 * \frac{P}{25}$

$\therefore Q_S = 8P$

Market Supply Curve: Q_S = 8P

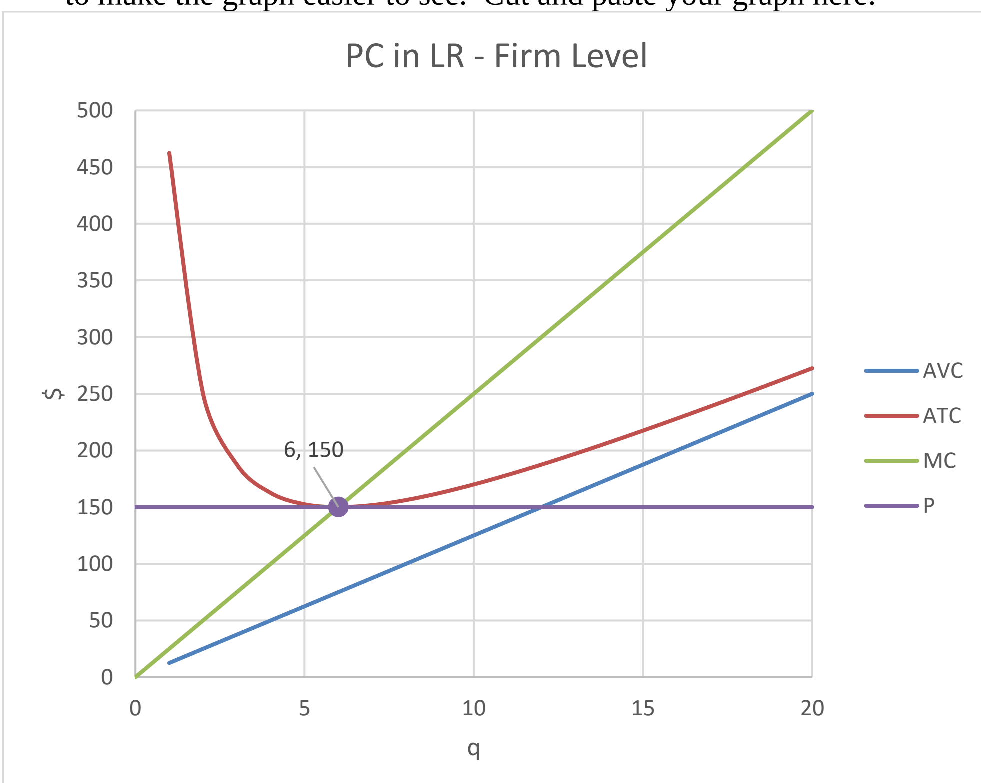
$8P = Q_S$

$\therefore P = Q_S/8$

Inverse Market Supply Curve: P = Q_S/8

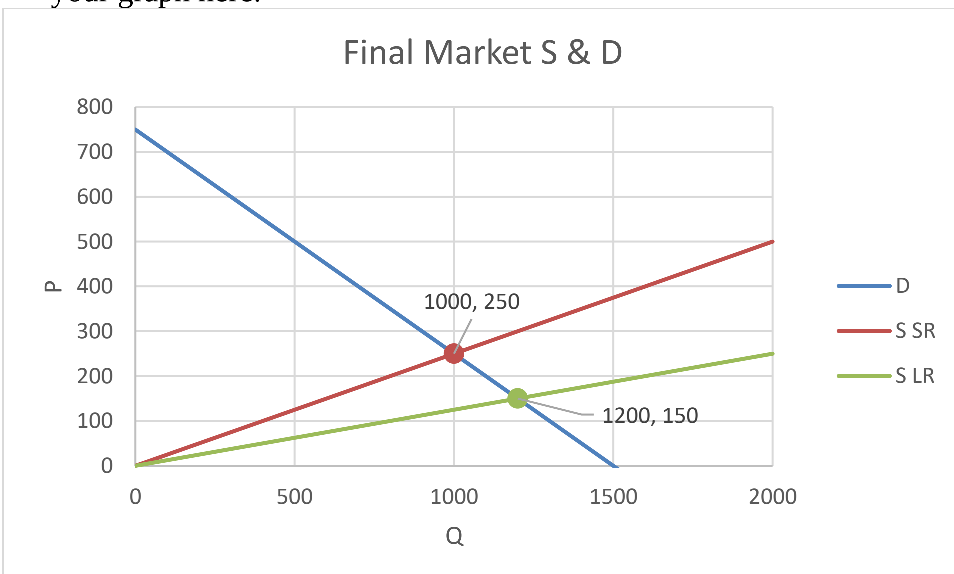
e. Draw the pair of graphs that depict long run competitive equilibrium. To create graphs, complete the following steps:

- Return to your “perfect comp” worksheet
- Replace the price in column J with the new price you calculated above.
- (2pts) Use scatter-plot to create a graph that shows the LR equilibrium for the firm. Only include ATC, MC and your new P. Title the graph “PC in LR – Firm Level” and label the axes. Format the graph so the max on the horizontal axis is 20, the max on the vertical axis is 500, and there are major grid lines both ways and minor vertical grid lines. Drag the bottom down a bit to make the graph easier to see. Cut and paste your graph here:



- Verify that the long run equilibrium is consistent with what you calculated.
  From the graph, we see that our equilibrium values are consistent with calculated values.
  We calculated equilibrium points for LR as q = 6 and P = 150, which is exactly what we see in the graph above.

- In column N, enter the formula for the new inverse market Supply.
- (2pts) Use scatter-plot to create a graph that shows market Demand and both the initial market Supply and the long-run equilibrium market Supply. Title the graph "Final Market S & D" and label the axes. Format the graph so the min on the vertical axis is 0, the max on the horizontal axis is 2000 and there are major grid lines both ways and minor vertical grid lines. Copy and paste your graph here:



- Verify that the short-run and long-run equilibrium prices and quantities are consistent with your algebraic solution values.
  From our calculations in Q6 we see that SR Equilibrium occurs at Q = 1000, P = 250. And LR Equilibrium occurs at Q = 1200, P = 150. This is exactly what we see in the graph above.

- (2pts) Complete the following summary table:

	Short Run Equilibrium	Long Run Equilibrium
Output of each individual firm	**10**	**6**
Industry output	**1000**	**1200**
Number of firms	**100**	**200**
Profit for each firm	**800**	**0**