# Case Study #4: Monopoly and Price Discrimination (75pts total)

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Fill in Answers below in red. Embed all graphs directly into appropriate question.

**Pricing Software**

You have an idea to develop and patent a new software application. You have the following options:

- Spend $110 to develop and patent the application, and then sell it
- Do not develop the software and do something else.

**In this case study, you will:** use graphs and equations to analyze pricing strategies.

**Skills needed to complete this case study:**

1. Enter data, enter formulas, and create charts in Excel
2. Use basic algebra

**Scenario:**

This monopolist has an idea for a new software application. It could spend $110 to develop and patent the software, and then sell it online. There would be zero variable costs associated with the product. Thus, the firm would have:

- TFC = $110
- MC = 0

This monopolist sells to "type 1" and "type 2" customers that have different observable customer characteristics.

- The equation of demand 1 is Q = 6 – 0.5P
- The equation of demand 2 is Q = 14 – 0.5P

**Steps to complete Case Study #4**

1. Open the data file for Case 4. The worksheet labeled Demand_1 provides information about the Demand from Type 1 customers. Demand_2 provides information about the Demand from Type 2 customers.

2. (44pts) First, assume that the firm can treat each demand separately. This means that they can sell to one group and that group cannot resell to the other group. In this case, we can find the profit-maximizing Q and P for each type and then add up the profit from both to obtain our overall profit.

a. (1.5pts) Regardless of how demand is treated, the firm faces the same cost structure. So, find the following cost information for this firm:
   i. Total cost (TC)
   We know that, TC = FC + VC
   Given, VC = 0, TFC = $110
   Thus,
   TC = 110 + 0
   **TC = $110**
   ii. Average variable cost (AVC)
   AVC = VC/Q
   = 0/Q
   **AVC = 0**
   iii. Average total cost (ATC)
   ATC = TC/Q
   **ATC = 110/Q**

b. (12.5pts) In the data file for case 3, start with the Demand 1 worksheet. Find the profit-max P & Q and profit for Demand 1:
   i. (2pts) Find the equation for marginal revenue (MR).
   Q = 6 – 0.5P
   0.5P = 6 – Q
   **P = 12 – 2Q**

   For Marginal Revenue MR, we double the slope,
   **MR = 12 – 4Q**
   ii. (2.5pts) Calculate the profit-maximizing level of output for this firm and find the price they will charge.
   For Profit maximization,
   MR = MC, given MC = 0
   12 – 4Q = 0
   4Q = 12
   **Q = 3**

   Substituting Q = 3 in (P = 12-2Q)
   P = 12 – 2Q
   P = 12 – 2(3)
   **P = $6**
   iii. (2pts) What is the mark up? Calculate the Lerner Index (P-MC)/P value for this firm. Do they have a lot of market power? Explain.
   Markup = P – MC
   Since, MC = 0
   Markup = P – 0
   **∴ Markup = P**
   The monopoly markup is the difference between price and marginal cost. We know that in a competitive market, price would be equal to marginal cost.

Lerner Index $= \frac{P-MC}{P}$

$= \frac{6-0}{6} = \frac{6}{6}$

**∴Lerner Index, L = 1**

The Lerner index provides a concise measure of monopoly power. The index ranges from a low value of 0 to a high of 1. The higher the value of the Lerner index, the more the firm can charge over its marginal cost, hence, the greater its monopoly power. Since, Lerner index is 1, means very high monopoly power exists.

iv. (1pt) Calculate the total revenue that this firm earns from selling to type 1 customers.

Total Revenue, TR = P*Q

TR = 6*3

**TR = $18**

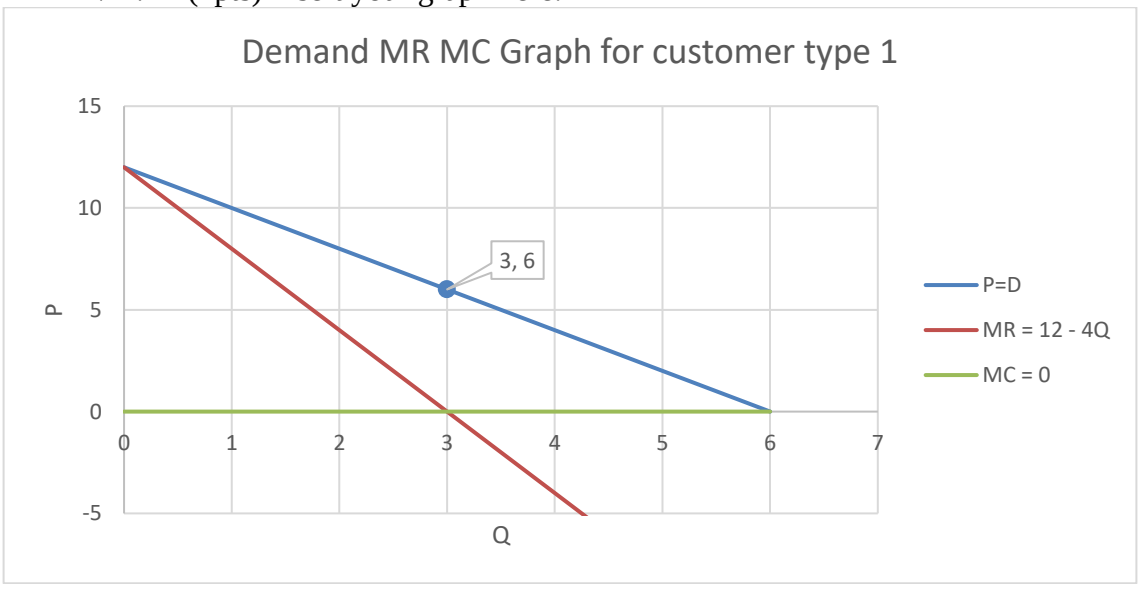
v. In Excel, in column C, put in the equation for MR

vi. In column D, put the MC=0.

vii. (1pt) Graph the demand curve, MR, and MC. Title and label your graph. Set the min on the vertical axis to -5 and the major unit to 5. Does your graph support your calculations above? Explain.

Yes, using demand equation 1, we calculated profit maximizing P = 6 and Q = 3. We see this point lies on our demand curve and MR intersects MC at Q = 3. This is exactly what we see from our calculations.

viii. (4pts) Insert your graph here:



c. (11pts) Consider what is happening with elasticity and total revenue associated with demand 1.

i. (0.5pts) What is the formula for elasticity ($E^d$)?

Elasticity, $\mathbf{E^d = \frac{dQ}{dP} * \frac{P}{Q}}$

Also, $\mathbf{Ed = -\frac{1}{L} = \frac{P}{MC-P}}$

Where, L is Lerner's Index.

ii. (2pts) Calculate the elasticity of demand at the profit-maximizing price.

P = 6

Q = 3

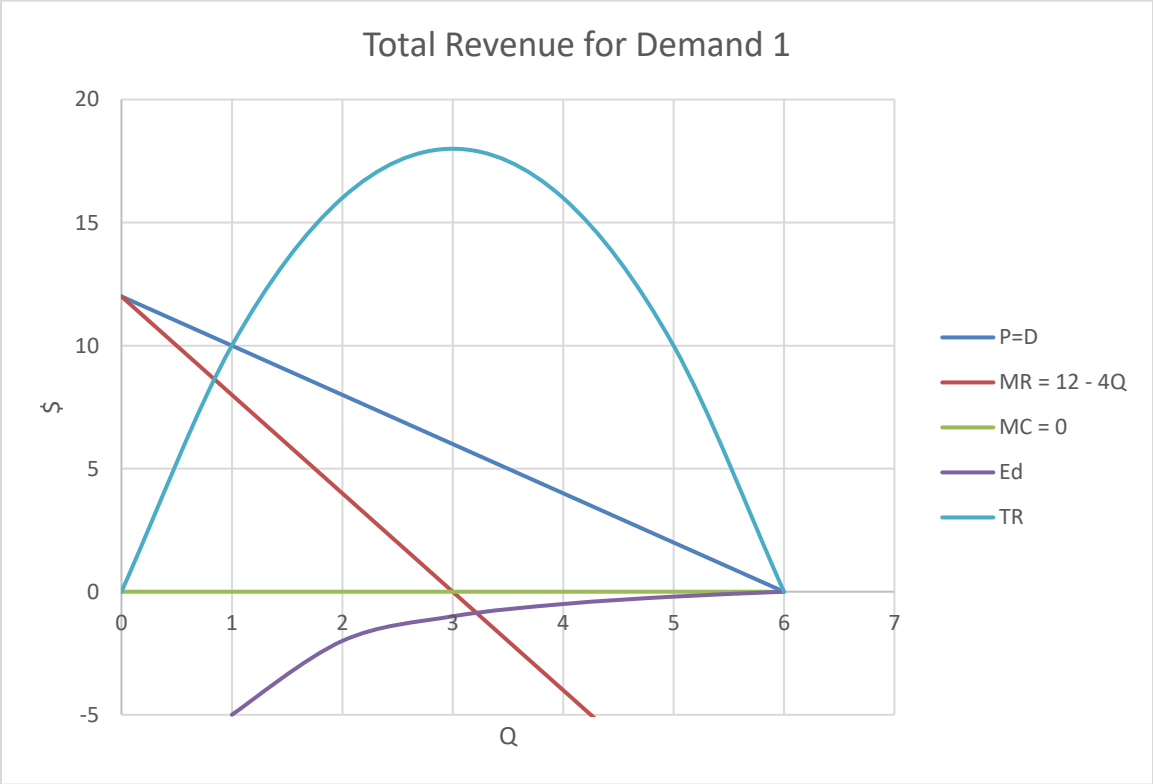
$E^d = \frac{P}{MC-P} = \frac{6}{0-6} = -\frac{6}{6}$

$\mathbf{E^d = -1}$

iii. In Excel, in column E, Create a column that computes point elasticity for each price. Recall that $E^d$ is negative – do not ignore the negative sign in this case.

iv. In column F, compute the total revenue for each price.

v. (2.5pts) Graph the demand, MR, MC, $E^d$, and TR. Title and label your graph. Set the min on the vertical axis to -5 and lengthen the graph to see it better. Insert your graph here:



vi. (5pts) What do you notice about the relationship between TR and $E^d$ and the relationship between MR and $E^d$? What do you notice about the profit-maximizing level of Q and TR? What do you notice about the Lerner Index value calculated above and the $E^d$? Does this make sense given that MC=0? Would it be true if MC>0? Explain.

Relationship between TR & $E^d$:

- If $E^d$ is inelastic ($E^d < 1$), a rise in price increases total revenue.
- If $E^d$ is unit elastic ($E^d = 1$), the total revenue is maximum.
- If $E^d$ is elastic ($E^d > 1$), a rise in price lowers total revenue.

Relationship between MR & $E^d$:

- When marginal revenue is positive, demand is elastic
- When marginal revenue is negative, demand is inelastic
- The output level at which marginal revenue equals zero corresponds to unit elasticity.

About profit maximizing level of Q & TR:

- Profit maximization occurs at MR = MC
- The point at which MR intersects MC gives us profit maximizing level of Q.
- Also, TR is maximized at profit maximizing level of Q.

About Learner Index value & $E^d$:

- Lerner Index value is 1 and elasticity is -1.
- $L = (-1/ E^d) = (-1/-1) = 1$

When MC=0, Lerner index is equal to unity, indicating the presence of monopoly power.

When MC > 0, Lerner index decreases.
$L = (P-MC)/P$
$MC > 0, L < 1$.
The market power in this case will be less compared to the market power when MC = 0.
The ratio P/MC is always greater than one. The higher the P/MC ratio, the more market power the firm possesses. As PED increases in magnitude, the P/MC ratio approaches one, and market power approaches zero.

vii. (1pt) Fill in the following table:

| | Quantity at which each event occurs: |
|---|---|
| Revenue is maximized | **3** |
| elasticity = -1 | **3** |
| Marginal revenue = 0 | **3** |
| Profit is maximized | **3** |

d. (11pts) In the data file for case 3, click on the Demand 2 worksheet. Find the profit-max P & Q and profit for Demand 2:

i. (2pts) Find the equation for marginal revenue (MR). Show all work.

Q = 14 – 0.5P
0.5P = 14 – Q
P = 28 – 2Q

For Marginal Revenue MR, we double the slope,
**MR = 28 – 4Q**

ii. (2pts) Calculate the profit-maximizing level of output for this firm and find the price they will charge. Show all work.
For Profit maximization,
MR = MC
28 – 4Q = 0
4Q = 28
**Q = 7**

Substituting Q = 7 in (P = 28-2Q)
P = 28 – 2Q
P = 28 – 2(7)
**P = $14**

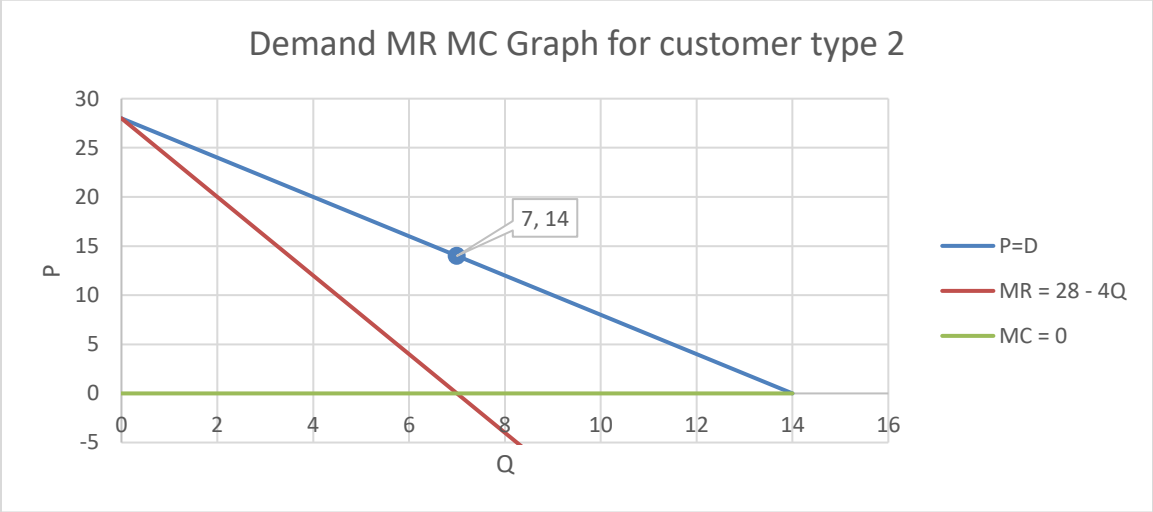
iii. (1pt) What is the mark up? Calculate the Lerner Index (P-MC)/P value for this firm. Do they have a lot of market power? Explain.
Markup = P – MC
Since, MC = 0
Markup = P – 0
**∴ Markup = P**
The monopoly markup is the difference between price and marginal cost. We know that in a competitive market, price would be equal to marginal cost.
Lerner Index $= \frac{P-MC}{P}$
$= \frac{14-0}{14} = \frac{14}{14}$
**∴Lerner Index, L = 1**
The Lerner index provides a concise measure of monopoly power. The index ranges from a low value of 0 to a high of 1. The higher the value of the Lerner index, the more the firm can charge over its marginal cost, hence, the greater its monopoly power. Since, Lerner index is 1, means very high monopoly power exists.

iv. (1pt) Calculate the total revenue that this firm earns from selling to type 2 customers.
Total Revenue, TR = P*Q
TR = 14*7
**TR = $98**

v. In Excel, in column C, put in the equation for MR
vi. In column D, put the MC=0.

vii. (1pt) Graph the demand curve, MR, and MC. Title and label your graph. Set the min on the vertical axis to -5. Does your graph support your calculations above? Explain.
Yes, using demand equation 2, we calculated profit maximizing P = 14 and Q = 7. We see this point lies on our demand curve and MR intersects MC at Q = 7. This is exactly what we see from our calculations.

viii. (4pts) Insert your graph here:

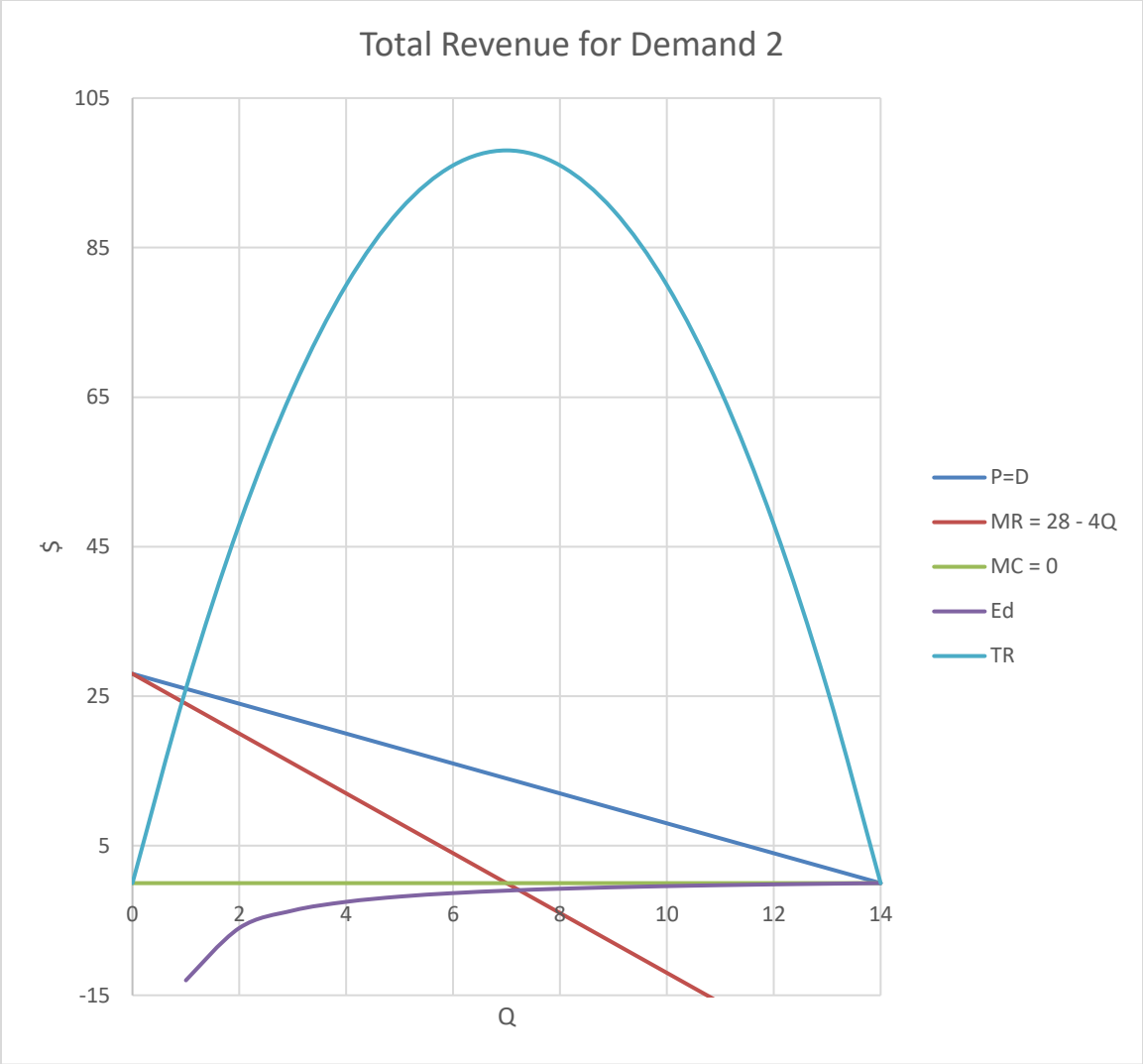



e. (5.5pts) Consider what is happening with elasticity and total revenue associated with demand 2.
i. (2pts) Calculate the elasticity of demand at the profit-maximizing price.
P = 14
Q = 7
$E^d = \frac{P}{MC-P} = \frac{14}{0-14} = -\frac{14}{14}$
**$E^d$ = -1**

ii. In Excel, in column E, Create a column that computes point elasticity for each price. Recall that $E^d$ is negative – do not ignore the negative sign in this case.

iii. In column F, compute the total revenue for each price.

iv. (2.5pts) Graph the demand, MR, MC, $E^d$, and TR. Title and label your graph. Set the min on the vertical axis to -15 and the max on the horizontal axis to 14 and lengthen the graph to see it better. Insert your graph here:




v. Notice the same relationships that were found in 2 d vi, between TR and $E^d$ and MR and $E^d$ and the profit-maximizing level of Q and TR and the Lerner Index value calculated above and the $E^d$.

vi. (1pt) Fill in the following table:

| | Quantity at which each event occurs: |
|---|---|
| Revenue is maximized | **7** |
| elasticity = -1 | **7** |
| Marginal revenue = 0 | **7** |
| Profit is maximized | **7** |

f. (0.5pts) Add up the total revenue earned from both markets.
TR from market 1: $18
TR from market 2: $98
TR from both markets: $TR_1 + TR_2 = 18 + 98 = 116$
**TR = $116**

g. (1pt) Calculate profit using the total revenue calculated in part f.
Profit, $\pi$ = TR – TC
= 116 - 110
**$\pi$ = $16**

h. (1pt) Will the firm spend the $110 to develop and patent the software? Explain.
**Yes, the firm should develop and patent this application.**
In this case, firm is making $16 profit. The value of profit is positive; therefore firm should develop.

3. (13.5pts) Now suppose that the buyers in the high-price market find a way to purchase the software in the lower price market. The firm can no longer separate the two markets. It must therefore charge a single price to all buyers. You need to combine the 2 demand curves. Think about this for a minute. You need the horizontal summation of these two demand curves. That means you need to add up the quantity demanded for each price. Look at your 2 demand curves. For which prices are D1 and D2 combined and for which prices is D2 the only relevant demand? In Excel, click on the "Combined D" worksheet.
   a. In the column marked Q1, cut and paste the quantities from "demand 1" into the spaces that match up with the relevant prices.
   b. In the column marked Q2, cut and paste the quantities from "demand 2" into the spaces that match up with the relevant prices.
   c. In the column marked Qcomb, add Q1 and Q2.
   d. For the prices where D2 is the only relevant demand, the MR is the same as in 2d(i) above. Thus, in the column marked MR2, cut and paste the MR from demand 2 into the spaces that match up with the relevant prices. Use the drop down menu on the "paste" button and click on "paste values" (this will eliminate the formulas behind the values).

   e. For the prices where D1 and D2 are combined, the MR is different. You need the inverse demand equation for the combined portion to find the new MR.
      i. (2pts) Calculate the combined demand equation. Take the demand equations listed in the "scenario" at the top of the case and simply add them together. This gives you $Q_{comb}$ as a function of P. Show your work.
      Demand 1: $Q_1 = 6 – 0.5P$
      Demand 2: is $Q_2 = 14 – 0.5P$
      Combined Demand: $Q = Q_1 + Q_2$
      $Q_{comb} = 6 – 0.5P + 14 – 0.5P$
      $Q_{comb} = 20 – P$

      **Combined Demand Equation: $Q_{comb} = 20 – P$**

ii. (1pt) Now, solve for P as a function of Q.
$Q_{comb} = 20 - P$
$20 - P = Q_{comb}$

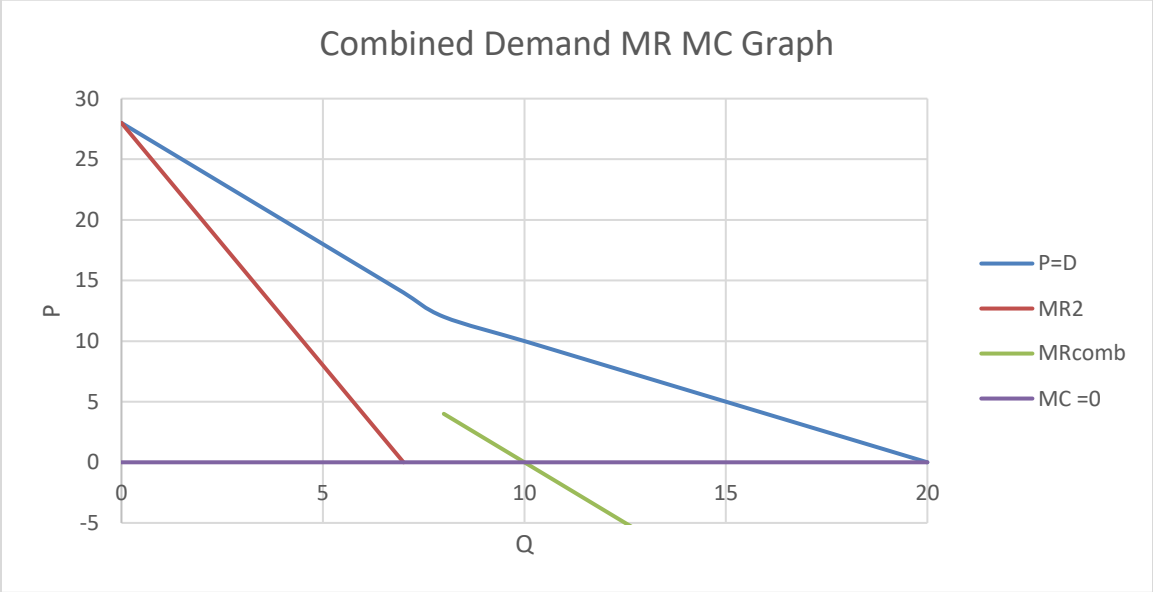
**P = 20 - Q**

iii. (0.5pt) What is the MR for this combined demand function?
To find MR, double the slope
**MR = 20 – 2Q**

iv. In Excel, insert the MR equation into the column marked MRcomb. Only put the values in the spaces that match up with the relevant prices (your MR should be discontinuous).

f. In column G, put in the MC

g. (6pts) Graph the demand (Qcomb and P), both MR curves, and MC. Title and label your graph. Set the min on the vertical axis to -5, set the max on the horizontal axis to 20, add major grid lines to the horizontal axis, and lengthen the graph to see it better. Insert your graph here:




h. (3pts) Looking at the graph above, determine the firm's profit-maximizing level of output and the price will they charge. Notice that there are 2 places where MR=MC. Which one did you choose and why? [Hint: Compare the profit for each choice.]
TC = $110 (given)
**Choice 1: $MR_{Comb}$**

| | | |
|---|---|---|
| $MR_{comb} = MC$ | P = 20 - Q | TR = P*Q |
| 20 - 2Q = 0 | P = 20 – 10 | TR = 10*10 |
| **Q = 10** | **P = $10** | **TR = $100** |

Profit, $\pi_1$ = TR – TC
= 100 - 110
**$\pi_1$ = -$10 (loss)**

**Choice 2: $MR_2$**
$MR_2 = MC$
$28 – 4Q = 0$
$4Q = 28$
**Q = 7**

$P = 28 – 2Q$
$P = 28 – 2(7)$
**P = $14**

$TR = P*Q$
$TR = 14*7$
**TR = $98**

Profit, $\pi_2$ = TR – TC
= 98 - 110
**$\pi_2$ = -$12 (loss)**

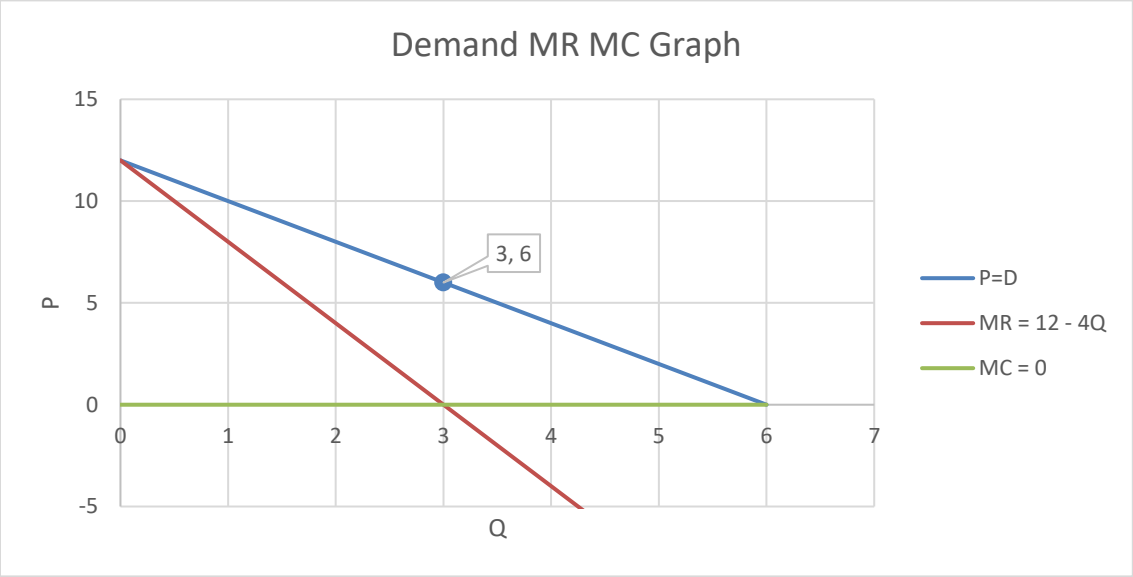
If we notice, $\boldsymbol{\pi_1 > \pi_2}$. Even though we are losing profit in both choices, loss is lesser in choice 1, thus making choice 2 worse.
**Therefore, we choose option 1, i.e., profit maximizing condition where $MR_{comb}$ = MC, thus P = $10, Q = 10.**

i. (1pt) Given the profit you calculated in part 3(h) above, will the firm spend the $110 to develop and patent this application? Explain.
**No, the firm should not develop and patent this application.**
In the previous case, when the market was separate, firm was making $16 profit. In single market case, firm is faced with $10 loss. Therefore, the firm should not develop and patent in this single market case.

4. (1pt) Summarize your results in the table below.

| | Separate markets | | | Single market |
|---|---|---|---|---|
| | D1 | D2 | Total | Dcomb |
| Optimal P | **6** | **14** | ------ | **10** |
| Quantity sold | **3** | **7** | **10** | **10** |
| Total Revenue | **18** | **98** | **116** | **100** |
| Will the firm develop/patent app? | **YES** | | | **NO** |

5. (16.5pts) Consider the welfare effects from price discrimination.
   a. (2pts) Given that the firm price discriminates, use the information found in part 2 above to calculate the consumer surplus for demand 1. Show your work.

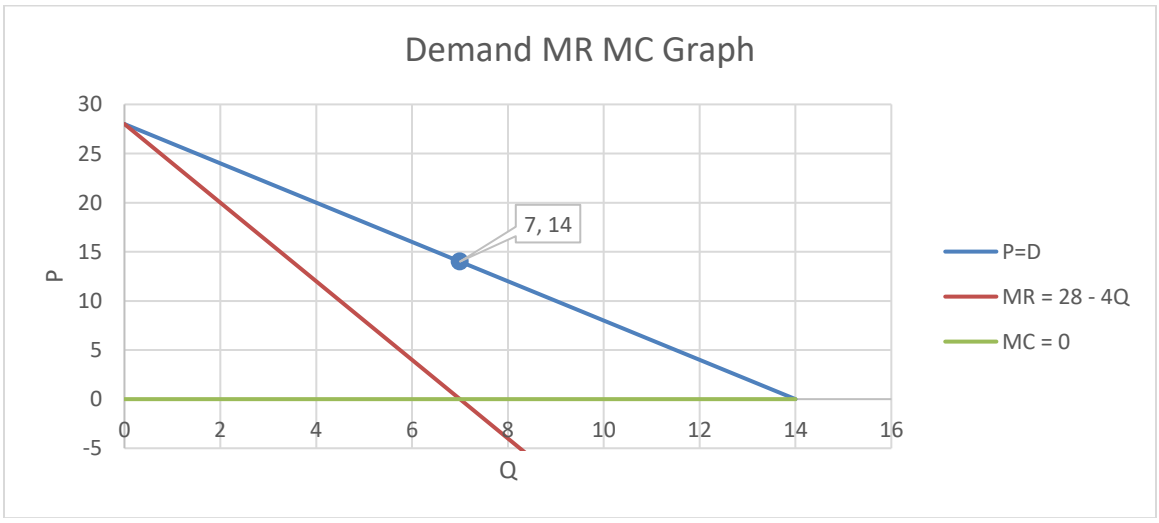



Consumer surplus is the area marked by letters A & B.
For Demand 1:

Consumer Surplus $= \frac{(12-6)*(3-0)}{2} = \frac{6*3}{2} = 9$

**Consumer Surplus$_1$ = 9**

   b. (2pts) Given that the firm price discriminates, use the information found in part 2 above to calculate the consumer surplus for demand 2. Show your work.




Consumer surplus is the area marked by letters A & B.
For Demand 2:

Consumer Surplus $= \frac{(28-14)*(7-0)}{2} = \frac{14*7}{2} = 49$

**Consumer Surplus$_2$ = 49**

c. (0.5pts) Add the results from a and b to find the total consumer surplus with price discrimination.
Total Consumer Surplus = Consumer Surplus 1 + Consumer Surplus 2
= 9 + 49
**∴ Total Consumer Surplus = 58**

d. Now consider the combined demand.

i. (2pts) Use the "optimal P" for the single market and plug it into demand 1 to find the number of units purchased by type 1 consumers. Use this information to calculate the consumer surplus for type 1 consumers with combined demand.
Optimal P for single market = \$10
Demand 1: $Q_1 = 6 – 0.5P$
$Q_1 = 6 – 0.5(10)$
$Q_1 = 6 - 5$
**$Q_1 = 1$**

Consumer Surplus = $\frac{(12-10)*(1-0)}{2} = \frac{2*1}{2} = 1$
**Consumer Surplus$_1$ = 1**

ii. (2pts) Plug this "optimal P" into demand 2 to find the number of units purchased by type 2 consumers. Use this information to calculate the consumer surplus for type 2 consumers with combined demand.
Optimal P for single market = \$10
Demand 2: $Q_2 = 14 – 0.5P$
$Q_2 = 14 – 0.5(10)$
$Q_2 = 14 - 5$
**$Q_2 = 9$**

Consumer Surplus = $\frac{(28-10)*(9-0)}{2} = \frac{18*9}{2} = 81$
**Consumer Surplus$_2$ = 81**

iii. (0.5pts) Add the results from i and ii to find the total consumer surplus without price discrimination.
Total Consumer Surplus = Consumer Surplus 1 + Consumer Surplus 2
= 1 + 81
**∴ Total Consumer Surplus = 82**

e. (1pt) Summarize your results in the table below.

| | Price Discrimination | | No Price Discrimination | |
|---|---|---|---|---|
| | Type 1 | Type 2 | Type 1 | Type 2 |
| Q consumed | **3** | **7** | **1** | **9** |
| P paid | **6** | **14** | **10** | **10** |
| Cons Surplus | **9** | **49** | **1** | **81** |
| Total CS | **58** | | **82** | |

f. Compare the results:

i. (1pt) How do type 1 consumers fare with price discrimination? Explain.

- With no price discrimination, type 1 consumers pay $10 for 1 unit of Q. Whereas, with price discrimination, type 1 consumers only pay $6 for 3 units of Q.
- Also, for type 1 consumers, consumer surplus is more with price discrimination because 9 > 1.
- Therefore, price discrimination benefits type 1 consumers as they consume more quantity (3) for less price ($6) as compared to type 1 consumers without price discrimination who consume less quantity (1) for more price ($10).
- Type 1 consumers will be happy with price discrimination.

ii. (1pt) How do type 2 consumers fare with price discrimination? Explain.

- With no price discrimination, type 2 consumers pay $10 for 9 units of Q. Whereas, with price discrimination, type 2 consumers pay $14 for 7 units of Q.
- Also, for type 2 consumers, consumer surplus is less with price discrimination because 49 < 81.
- Therefore, price discrimination does not benefit type 2 consumers as they consume less quantity (7) for more price ($14) as compared to type 2 consumers without price discrimination who consume more quantity (9) for less price ($10).
- Type 2 consumers will not be happy with price discrimination.

iii. (1pt) Overall how does consumer surplus change with price discrimination? Explain. Calculate the change.

Overall change in CS = $CS_{\text{no price discrimination}} - CS_{\text{price discrimination}}$

= 82 - 58

**∴Overall change in CS = 24**

g. (1.5pts) How much does the total profit increase with price discrimination?

| Price Discrimination | | No Price Discrimination | |
|---|---|---|---|
| Type 1 | Type 2 | Type 1 | Type 2 |
| $TR_1 = P*Q = 6*3$ **$TR_1 = 18$** | $TR_2 = P*Q = 14*7$ **$TR_2 = 98$** | $TR_1 = P*Q = 10*1$ **$TR_1 = 10$** | $TR_2 = P*Q = 10*9$ **$TR_2 = 90$** |
| Total Revenue TR = $TR_1 + TR_2$ = 18+98<br>TR = 116<br>Profit $\pi$ = TR – TC = 116 – 110<br>$\pi_{\text{price discrimination}}$ = $6 | | Total Revenue TR = $TR_1 + TR_2$ = 10+90<br>TR = 100<br>Profit $\pi$ = TR – TC = 100 – 110<br>$\pi_{\text{no price discrimination}}$ = -$10 | |

Total Profit increase, $\pi = \pi_{\text{price discrimination}} - \pi_{\text{no price discrimination}}$

$\pi = 6 - (-10)$

**$\pi$ = $16**

h. (2pts) Using the information from f and g, what is the effect on aggregate surplus? Explain and calculate the change.

Using information from f & g, we calculated the change in total consumer surplus as 24 and increase in profit as $16.

Change in CS = $CS_{\text{price discrimination}} - CS_{\text{no price discrimination}}$

Change is 58-82 = -24

The Consumers are worse off by $24 with price discrimination.

Aggregate surplus = - 24+16

= -8 $

Thus, there is a loss of 8 $, because of price discrimination.