Case Study # 2: Sally's Personal Training Business (60pts)

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Sally owns and operates a business that sells personal training sessions. She wants to understand customer demand in more detail, so she can improve her ability to forecast demand and analyze the impacts of a variety of pricing strategies. Her customers obtain utility from training sessions (T) and videos (V). She is therefore analyzing customer decisions using budget lines and indifference curves. She has two types of customers Type A and Type B.

In this case study you will: Use graphs and equations to explore the budget line and indifference curve graph:

- Using two specific utility functions, graph indifference curves and budget lines.
- Show the connection between the budget line/ indifference curve diagram, compute the impact of the price change on revenue, and draw a conclusion about the elasticity of demand.
- Show the impact of a price change, and derive an estimated demand curve from this information.

Skills needed to complete this case study:

1. Enter data, enter formulas, and create charts in Excel
2. Use basic algebra

Sally has two types of customers Type A and Type B. The following table shows the utility functions of the two types of customers:

	Type A	Type B
Utility	$U = 12*T*V$	$U = T^2*V$
MRS = amount of V the customer will give up in order to obtain 1 more T	V / T	2V / T

1. (2pts) To get the equations of the indifference curves for each type of customer, solve each of the Utility functions above for V. Show your work.

Type A: $U = 12*T*V$
This can be written as, $12*T*V = U$
Divide both side by T and 12,
$\therefore V = U/(12*T)$

Type B: $U = T^2*V$
This can be written as $T^2*V = U$
Divide both sides by T^2,
$\therefore V = U/(T*T)$

Type A Customer: **$V = U/(12*T)$**

Type B Customer: **$V = U/(T*T)$**

2. (6pts) Use Excel to graph the indifference curve for both types of customers for U = 20. To do this, complete the following steps:

    a. You want to create a graph with V on the vertical axis and T on the horizontal axis. That means that you will need a column of numbers representing possible values of T. In Excel, label the first column (A) with the name "T". Enter the number 0 in the cell in the second row of this column and 1 in the third row. Enter the formula: =0.5+a3 in the fourth row and click enter. Copy this formula and paste down to row 11. This will give you a column with the numbers 1-5.

    b. The indifference curve that you want to graph for Type A people will be a curve showing all possible combinations of T & V that give U=20. Plug 20 into your formula in question #1. Label the second column (B): V_A_20. In box B3 (note that you will leave B2 blank), enter the formula using $a3 for T (the $ sign in front of the a3 will help with question #3 below). Copy that cell and paste down through cell #11.

    c. Likewise, the indifference curve for Type B people will be a curve showing all possible combinations of T & V that give U=20. Plug 20 into your formula in question #1. Label the 3rd column (C): V_B_20. Enter the formula in box C3 (again using $a3 for T). Copy in paste to fill the rest of the column.

    d. (4pts) Graph the two indifference curves. To do this, highlight the area that includes columns A, B, and C for rows 1-11. Choose scatter-plot with smooth lines. Format your graph so that it has a title and each of the axes are labeled. To do this, click on design on the tool bar. On the left, click on “quick layout” and then click on “Layout 1.” Type in your axes labels and your graph title.

    e. (2pts) How much of V is each type of consumer willing to give up in order to move from 2 training sessions to 3? Plot these points on your graph above. Move the cursor over the graph to one of the points and click on it. Click a second time so that only that one point is highlighted. Right click and click on “format data point.” Click on the paint can, and then marker. Click on “marker options” and click “automatic” and “close.” Repeat for the remaining 3 points. Calculate the trade-off. Which type of consumer is willing to give up more V, in order to obtain the third training session?

Type A: The points (T, V) are (2,0.833) & (3,0.556). Therefore, if T changes from 2 to 3, V changes from 0.833 to 0.556.

Change in V = (0.556 – 0.833) = **-0.277**

Therefore, in order to move from 2 training sessions to 3, **Customer Type A is willing to give up 0.277 of Videos.**

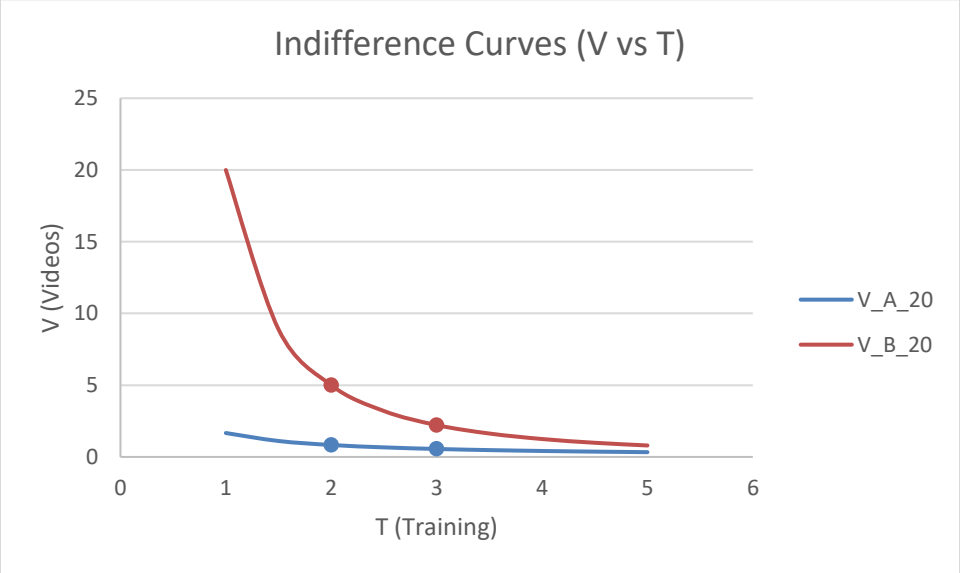
Type B: The points (T, V) are (2,5) & (3,2.22). Therefore, if T changes from 2 to 3, V changes from 5 to 2.22.

Change in V = (2.22 – 5.0) = **-2.78**

Therefore, in order to move from 2 training sessions to 3, **Customer Type B is willing to give up 2.78 of Videos.**

Customer Type B is willing to give up more V than Customer Type A to get 3rd training session.

Cut and paste your graph here:

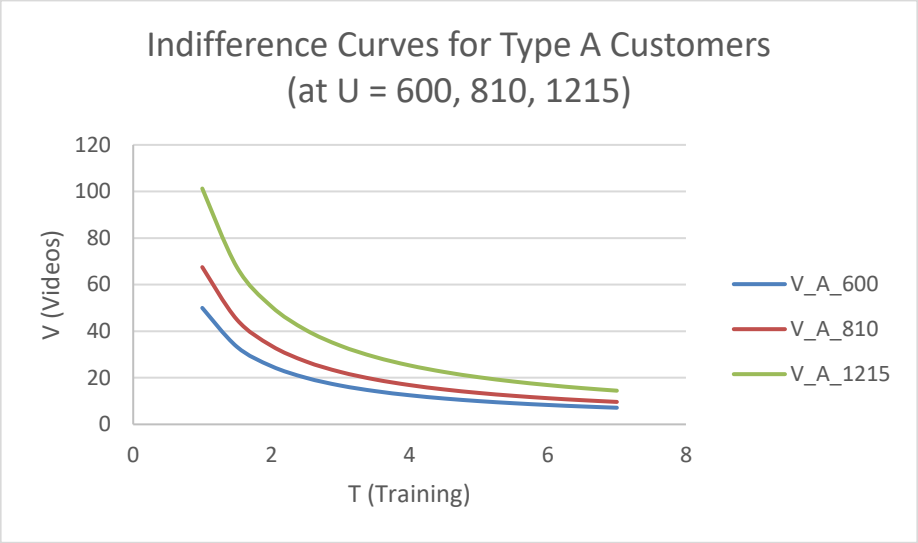


3. (6pts) Use Excel to plot an indifference map for each of Type A and Type B customers. Be sure that the formulas you entered for question 2 used $a3 (not just a). This will allow you to copy and paste these columns while still retaining column A in your formulas.
    a. (3pts) Type A customers:
        i. Extend columns A, B, and C another 4 rows, so that T goes from 1-7 rather than 1-5. [Simply highlight the last row, click copy, highlight rows 12-15 and click paste.]
        ii. Copy columns A and B into columns F and G.
        iii. Adjust the formula in column G. You want to compute the V that will yield utility equal to 600. Refer to the formula calculated in question 1. What component of the formula must be changed? Change the heading for this column to V_A_600.

Value of U in the formula **V = U/(12*T)** should be changed from **20 to 600**.

        iv. Copy column G into columns H and I and adjust the formulas in columns H and I to represent the V yielding utility = 810 (column H) and utility =1215 (column I). Change each heading to V_A_810 and V_A_1215.

v. Graph the three indifference curves (using smooth lines) for Type A consumers. Label axes and create a title. Cut and paste graph here:



b. (3pts) Type B customers:

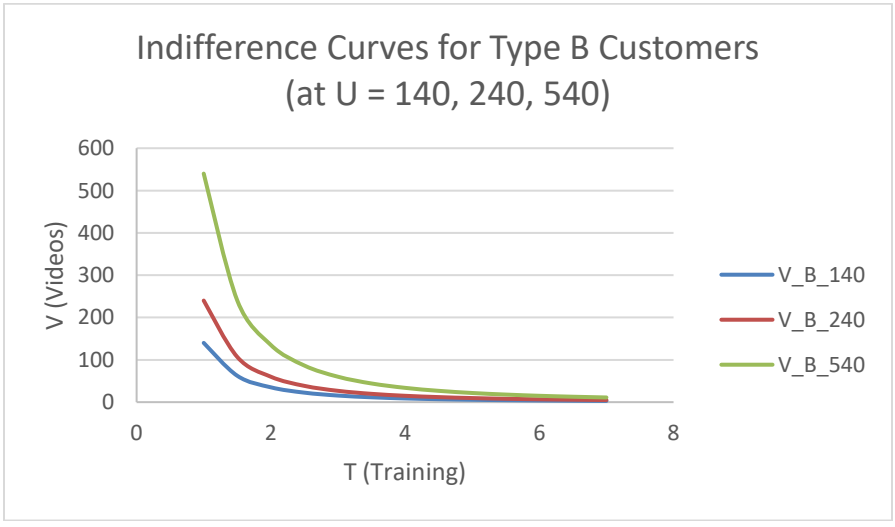
i. Copy columns A and C into columns M and N.

ii. Adjust the formula in column N. You want to compute the V that will yield utility equal to 140. Refer to the formula calculated in question 1. What component of the formula must be changed? Change the heading for this column to V_B_140.

Value of U in the formula **V = U/(T*T)** should be changed from **20 to 140**.

iii. Copy column N into columns O and P and adjust the formulas in columns O and P to represent the V yielding utility = 240 (column O) and utility =540 (column P). Change each heading to V_B_240 and V_B_540.

iv. Graph the three indifference curves (using smooth lines) for Type B consumers. Label axes and create a title. Cut and paste graph here:



4. (20pts) Use Excel to generate two new graphs that show indifference curves and a budget line with a point of tangency for Type A and Type B consumers. **Complete the following steps.**

    a. Sally's current price of training sessions is $20, and the current price of a video is $4. Consider a consumer that has $180 to spend.
        i. (1pt) Write out her budget line in the form: $M = P_T T + P_V V$
        Budget Line: **$180 = 20T + 4V**
        ii. (1pt) Rearrange this formula into slope-intercept form with V as a function of T (you are going to graph this line with V on the vertical axis).
        20T + 4V = 180
        4V = 180 – 20T
        V = (180 - 20T) / 4
        ∴ **V = 45 – 5T**

    b. For type A consumers, put the equation of the budget line in column J (use $a). Column J will show the quantity of V that the consumer can purchase with $180, given that he is buying 0, 1, 2, ... units of T. Label this column "Budget 1"

    c. For type B consumers, copy and paste Budget 1 to column Q.

    d. Find utility maximizing point for Type A consumers.
        i. (3.5pts) Calculate it algebraically. Use the MRS information in the table at the beginning of the case study and the budget line information above to calculate how many training sessions and how many videos Type A consumers purchase when the maximize utility. **Show your work.**

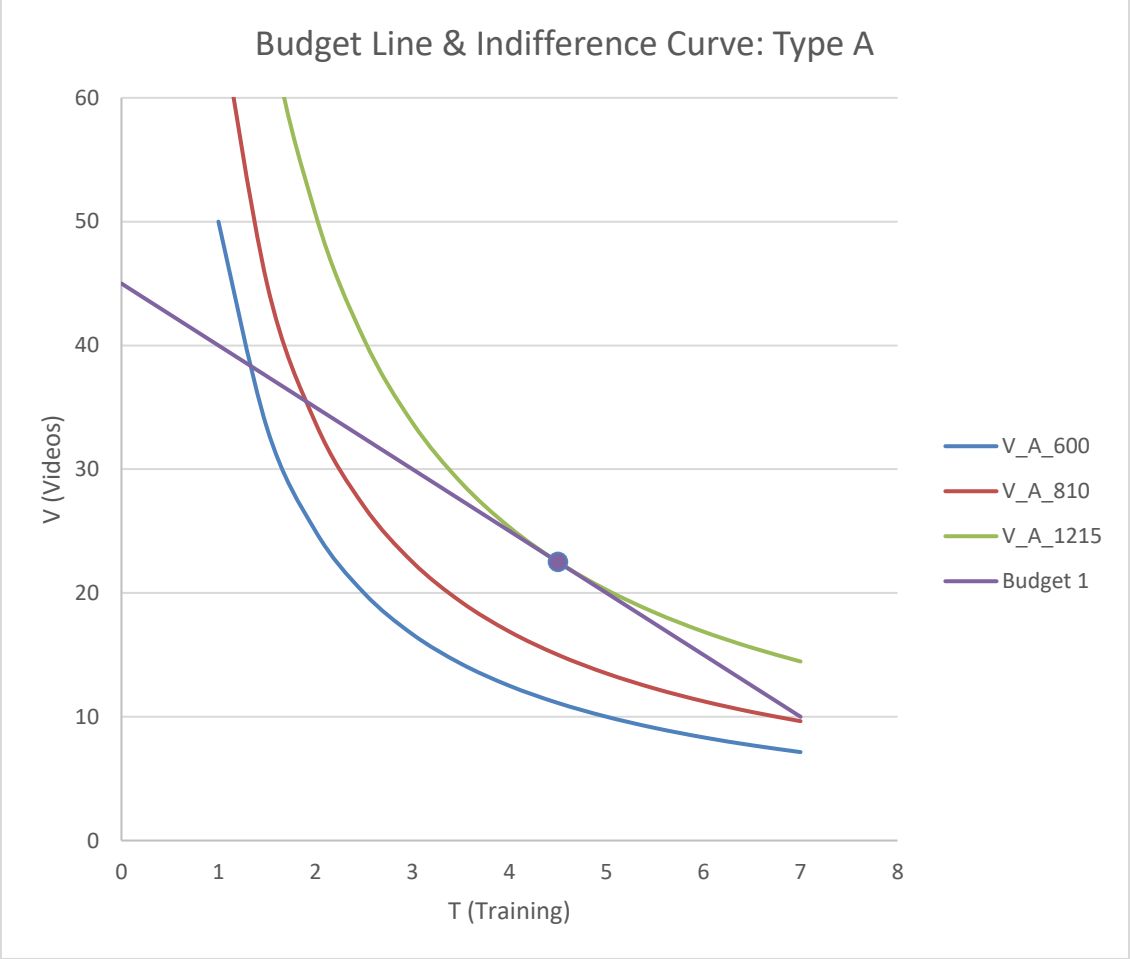
Budget Line: V = 45 – 5T
From this budget line, we know that: **P_V = $1** and **$P_T$ = $5**.
Utility for Type A customers: U = 12TV, MRS_{TV} = V/T.
We know that: $MRS_{TV} = \frac{MU_T}{MU_V} = \frac{V}{T}$.
From tangency condition, we know that: $\frac{MU_T}{MU_V} = \frac{P_T}{P_V} = \frac{5}{1}$.
Therefore, $\boldsymbol{\frac{MU_T}{MU_V} = \frac{P_T}{P_V} = \frac{V}{T} = \frac{5}{1}}$.
And therefore, **V = 5T** (Tangency condition).

If we substitute V = 5T in budget line, we get,
5T = 45 – 5T
10T = 45
T = 4.5

Putting T = 4.5 in tangency condition, we get,
V = 5*(4.5)
V = 22.5

∴ At maximum utility, for Type A, Training Sessions T = 4.5, Videos V = 22.5.

ii. (3.5pts) Graph the three indifference curves and the budget line for Type A consumers (use smooth lines) and indicate the tangency point.
   1. Label axes and create a title.
   2. Right click on the numbers on the vertical axis. From the drop down menu, select "format axis." Change "maximum" from "auto" to "fixed" and type in 60 and click close.
   3. You may also lengthen the graph by dragging the bottom of the box down.
   4. Click on tangency point twice, right click and click "format data point." Click "marker options" and choose "built-in." From the drop-down menu, choose the circle and increase its size to 7. Click close.
   5. Cut and paste your graph here:



iii. (1pt) What level of utility do Type A consumers receive?
U = 12TV
U = 12*4.5*22.5
∴ **U = 1215**

e. Find utility maximizing point for Type B consumers.

i. (3.5pts) Calculate it algebraically. Use the MRS information in the table at the beginning of the case study and the budget line information above to calculate how many training sessions and how many videos Type B consumers purchase when the maximize utility. **Show your work.**

Budget Line: V = 45 – 5T
From this budget line, we know that: **P_V = \$1** and **P_T = \$5**.
Utility for Type B customers: $U = T^2V$, $MRS_{TV} = 2V/T$.
We know that: $MRS_{TV} = \frac{MU_T}{MU_V} = \frac{2V}{T}$.
From tangency condition, we know that: $\frac{MU_T}{MU_V} = \frac{P_T}{P_V} = \frac{5}{1}$.
Therefore, $\boldsymbol{\frac{MU_T}{MU_V} = \frac{P_T}{P_V} = \frac{2V}{T} = \frac{5}{1}}$.
And therefore, 2V = 5T
Or **V = 2.5T** (Tangency condition).

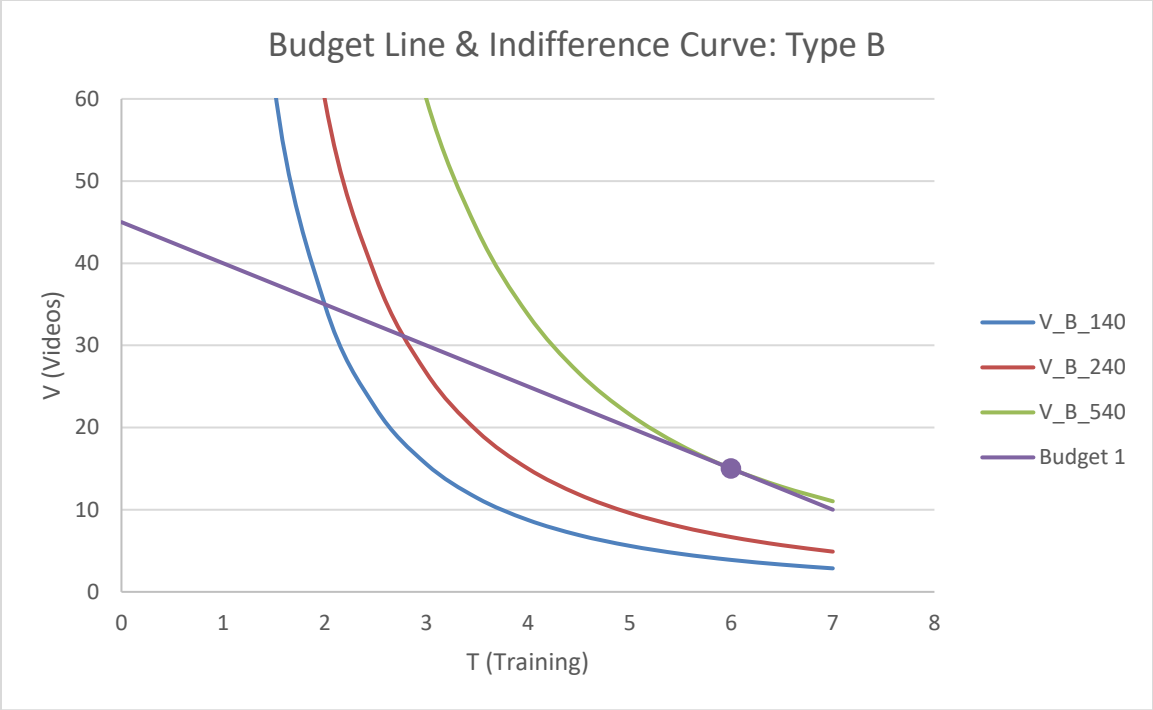
If we substitute V = 2.5T in budget line, we get,
2.5T = 45 – 5T
7.5T = 45
T = 6.0

Putting T = 6 in tangency condition, we get,
V = 2.5*(6)
V = 15.0

∴At maximum utility, for Type B, Training Sessions T = 6, Videos V = 15.

ii. (3.5pts) Graph the three indifference curves and the budget line for Type B consumers and indicate the tangency point.

1. Label axes and create a title.
2. Right click on the numbers on the vertical axis. From the drop-down menu, select "format axis." Change "maximum" from "auto" to "fixed" and type in 60 and click close.
3. You may also lengthen the graph by dragging the bottom of the box down.
4. Click on tangency point twice, right click and click "format data point." Click "marker options" and choose "built-in." From the drop-down menu, choose the circle and increase its size to 7. Click close.
5. Cut and paste your graph here:



iii. (1pt) What level of utility do Type B consumers receive?

$U = T^2V$

$U = 6*6*15$

$\therefore$ **U = 540**

f. (2pts) Think about the reason that the two types of consumers buy different combinations of T and V. They both spend the same amount of money ($180). Refer to your answer to question 2e. Explain why Type B consumers buy more training sessions than Type A consumers.

Consumers derive different levels of utility from different combinations of T & V. Thus, to maximize utility for themselves, customers Type A use the bundle (4.5, 22.5) and Type B use bundle (6,15) as (T, V).

Now, for customer type A $MRS_{TV} = \frac{V}{T}$

and for customer type B $MRS_{TV} = \frac{2V}{T}$

We see that, $\mathbf{MRS_{TV}(B) > MRS_{TV}(A)}$ that means Type B customers are willing to give up more V than Type A customers for one more unit of T.

This is exactly what we derived from question 2e, Type A customers were willing to give up 0.277 videos to get the 3rd training session. And, Type B customers were willing to give up 2.78 videos to get the 3rd training session.

Thus, within their limited budget, Type B consumers value Training Sessions more than Videos compared to Type A consumers.

5. (16pts) Sally is considering increasing the price per training session from $20 to $30. Identify the changes that would occur.
   a. (2pts) Calculate the new budget line (in slope-intercept form). Has income (M) changed? **Show work**.

Budget Line: $180 = 30T + 4V30T + 4V = 180

4V = 180 – 30T
V = (180 - 30T) / 4
∴ **V = 45 – 7.5T**
The income has **NOT** changed. Income M is still $180.

   b. (4pts) Calculate the new utility maximizing combination of V and T that type A consumers will buy. Plug this into the utility function to determine the new utility level. Are these customers better off or worse off than in part 4? **Show work**.

Budget Line: V = 45 – 7.5T
From this budget line, we know that: $\mathbf{P_V = \$1}$ and $\mathbf{P_T = \$7.5}$.
Utility for Type A customers: U = 12TV, MRS_{TV} = V/T.
We know that: $MRS_{TV} = \frac{MU_T}{MU_V} = \frac{V}{T}$.
From tangency condition, we know that: $\frac{MU_T}{MU_V} = \frac{P_T}{P_V} = \frac{7.5}{1}$.
Therefore, $\boldsymbol{\frac{MU_T}{MU_V} = \frac{P_T}{P_V} = \frac{V}{T} = \frac{7.5}{1}}$.
And therefore, **V = 7.5T** (Tangency condition).

If we substitute V = 7.5T in budget line, we get,
7.5T = 45 – 7.5T
15T = 45
T = 3

Putting T = 3 in tangency condition, we get,
V = 7.5*(3)
V = 22.5

∴ At maximum utility, for Type A, Training Sessions T = 3, Videos V = 22.5.

New U = 12TV
U = 12*3*22.5
∴ **U = 810**

Customers are worse off in part 5 than in part 4, because maximum utility in part 4 was U = 1215, and after price change, maximum utility decreased to U = 810.

c. (4pts) Calculate the new utility maximizing combination of V and T that type B consumers will buy. Plug this into the utility function to determine the new utility level. Are these customers better off or worse off than in part 4? **Show work**.

Budget Line: $V = 45 - 7.5T$
From this budget line, we know that: $\mathbf{P_V = \$1}$ and $\mathbf{P_T = \$7.5}$.
Utility for Type B customers: $U = T^2V$, $MRS_{TV} = 2V/T$.
Therefore, $\frac{MU_T}{MU_V} = \frac{P_T}{P_V} = \frac{2V}{T} = \frac{7.5}{1}$.
And therefore, $2V = 7.5T$
Or **$V = 3.75T$** (Tangency condition).

If we substitute $V = 3.75T$ in budget line, we get,
$3.75T = 45 - 7.5T$
$11.25T = 45$
$T = 4$

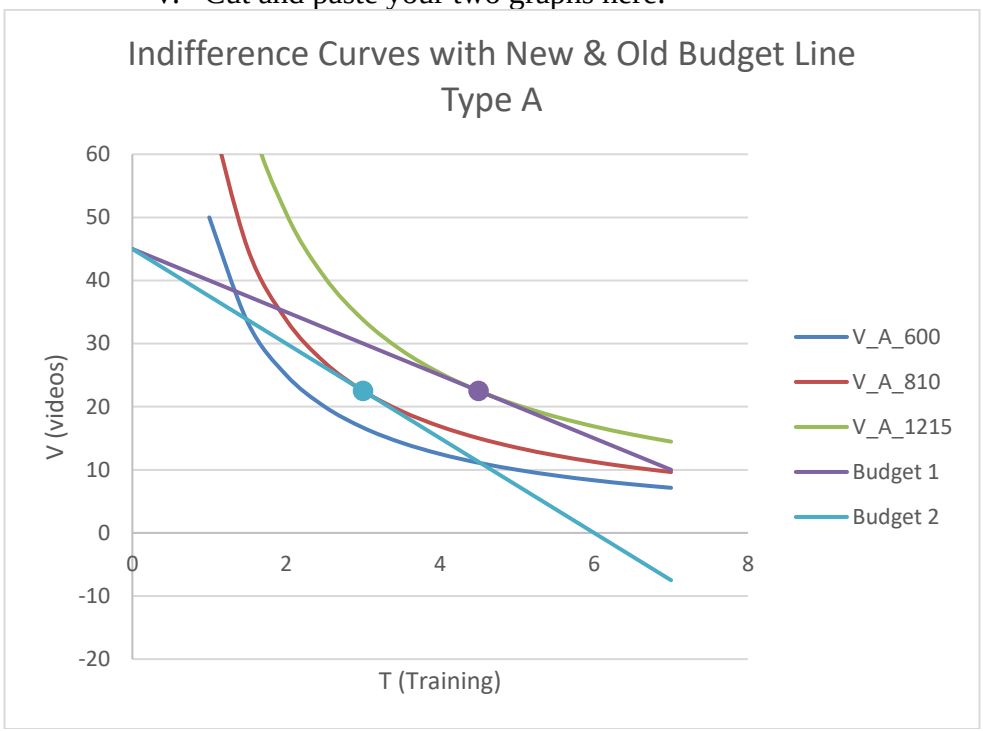
Putting $T = 4$ in tangency condition, we get,
$V = 3.75*(4)$
$V = 15$
∴ At maximum utility, for Type B, Training Sessions T = 4, Videos V = 15.

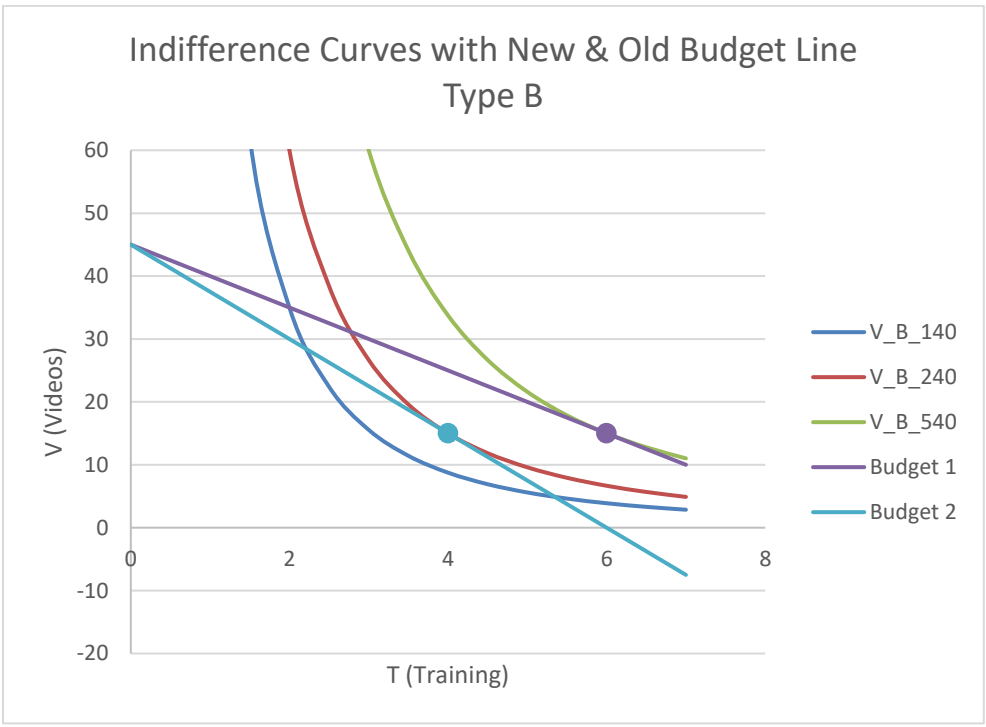
New $U = T^2V$
$U = 4*4*15$
∴ **$U = 240$**
Customers are worse off in part 5 than in part 4, because maximum utility in part 4 was U = 540, and after price change, maximum utility decreased to U = 240.

d. In Excel, in the column to the right of budget 1 (columns K and R), create a column for the new budget line for each type of buyer (it is the same budget line for both types, but you need 2 columns for graphing). Label this column "budget 2."

e. (4pts) Graph the three indifference curves and the two budget lines for each type of consumer. Indicate the old and the new tangency points.
   i. Label and title each graph.
   ii. Right click on numbers on the vertical axis. From the drop-down menu, select "format axis." Change "maximum" from "auto" to "fixed" and type in 60 and click close.
   iii. You may also wish to lengthen your graph by dragging the bottom of the box down to see the tangency point more clearly.
   iv. On each graph, indicate both the old and the new tangency points. (Click on tangency point twice, right click and click "format data point." Click "marker options" and choose "built-in." From the drop-down menu, choose the circle and increase its size to 7. Click close.)

v. Cut and paste your two graphs here:





f. (2pts) Describe the impact of the price change on each type of consumer.

When the price of training sessions increases from $20 to $30, type A customers will still consume the same amount of videos but **consume 1.5 (= 4.5 - 3) less training sessions**. Type B customers will still consume the same amount of videos but **consume 2 (= 6 – 4) less training sessions**. As we see in previous part, **utility will go down for both type A & B customers because they are getting the same amount of videos and less training sessions.** Since the price of training sessions went up, customers cannot consume the same amount of training sessions unless there is a proportional increase in our budget.

Therefore, with the same budget, consumption of T decreases.

6. (5pts) Use a hand-drawn graph (label axes) to show how the Type B customer's demand for training sessions can be derived from her budget/indifference curve.
   a. (1pt) In the top graph, draw the budget line from question 4 (with a P=$20) and a tangent indifference curve for Type B. (Roughly hand draw from your excel graph). Label each curve and each axis on the top graph.

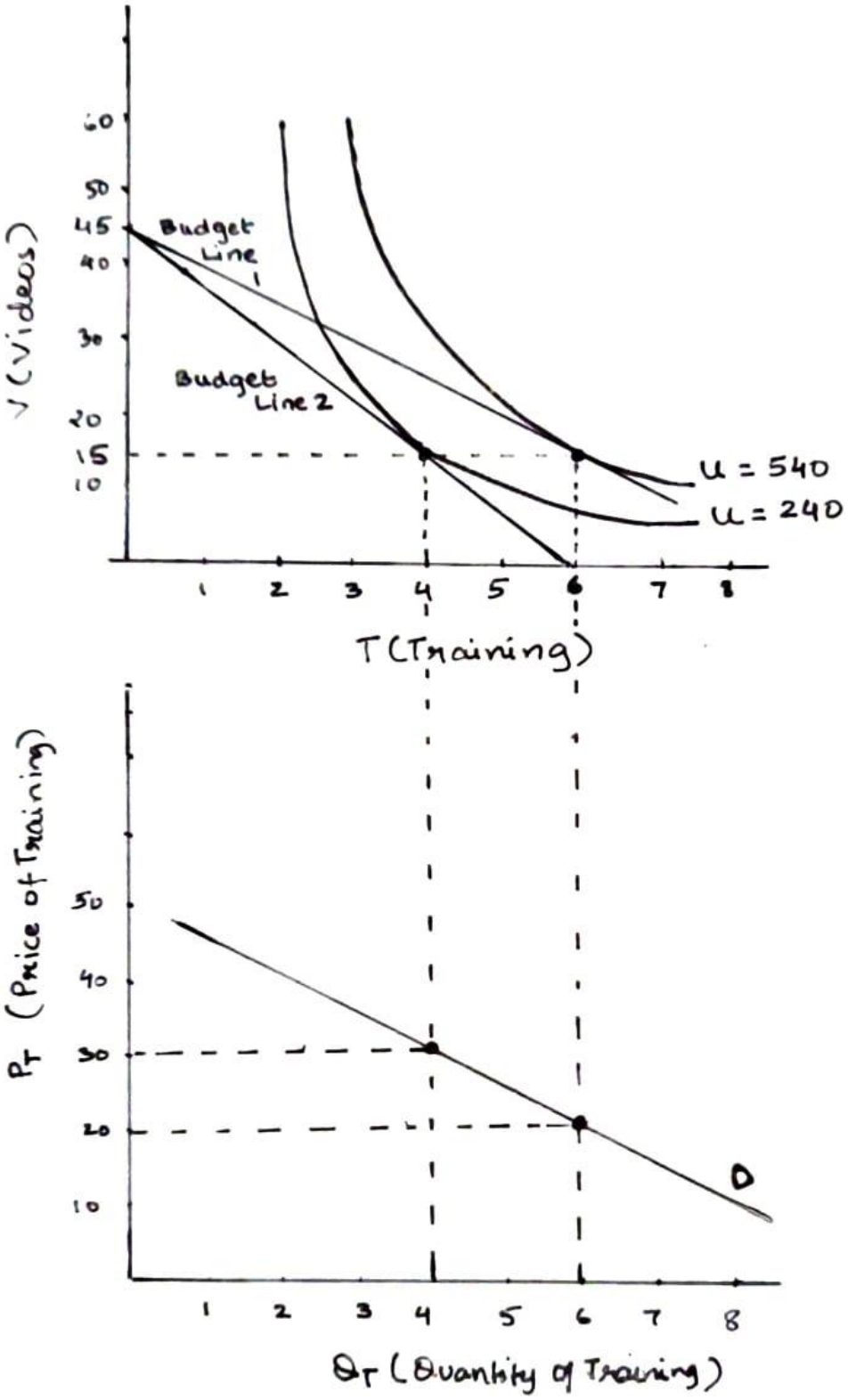
   b. (0.5pts) In the bottom graph, you will derive the demand curve. Think about what labels will need to go on each axis. Label each axis on the bottom graph.

   c. (1pt) Starting with a price of $20 and your point of tangency in the top graph, find the first point on the demand curve. Plot and label that point in your bottom graph.

   d. (1pt) Now allow the price to increase to $30. Draw the new budget line from question 5 and a tangent indifference curve (again, roughly draw your graph from excel). Label each curve.

   e. (1pt) Use this new point of tangency to plot and label a second point on the demand curve.

   f. (0.5pts) Combine these points and this is the demand curve for Type B customers. Label this curve D.


V (Videos)
60
50
45
40
30
20
15
10
Budget Line 1
Budget Line 2
U = 540
U = 240
1
2
3
4
5
6
7
8
T (Training)
P_T (Price of Training)
50
40
30
20
10
D
1
2
3
4
5
6
7
8
Q_T (Quantity of Training)

7. (5pts) Use your Excel results to fill in the following table.

Customer type	Price	Quantity of T demanded	Revenue
Type A	20	**4.5**	**(20*4.5) = 90**
	30	**3**	**(30*3) = 90**
Type B	20	**6**	**(20*6) = 120**
	30	**4**	**(30*4) = 120**

Consider the impact of the price increase on revenue. What can you conclude about elasticity of demand for each type of customer? Explain.

Price change seems to have no effect on the revenue for both type A & B. Thus theoretically, elasticity should be unit.

For elasticity calculations, we use the formula, $Elasticity\ E = \frac{\%\Delta Q}{\%\Delta P}$

Since, we don't have an equation for demand line, and it is a discrete case, we use midpoint formula for calculations.

Type A:

$$E_A = \frac{\%\Delta Q}{\%\Delta P} = \frac{\frac{3-4.5}{(3+4.5)/2}}{\frac{30-20}{(30+20)/2}} = \frac{-0.4}{0.4} = -1.0$$

Type B:

$$E_B = \frac{\%\Delta Q}{\%\Delta P} = \frac{\frac{4-6}{(4+6)/2}}{\frac{30-20}{(30+20)/2}} = \frac{-0.4}{0.4} = -1.0$$

Since, both $|E_A|$ **&** $|E_B|$ **are equal to 1, both Customer Type A & B are unit elastic**. This means that **a change in price will not affect the total revenue**. This is exactly what we derive earlier in this question. We also know that when customers are unit elastic, revenue is maximum. Thus, Maximum revenue for Type A is 90, and for Type B is 120.