An Exploration of Cable Suspended Loads with Quadrotors Using IPOPT

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***Abstract*—This paper extends the framework presented in "Scalable Cooperative Transport of Cable-Suspended Loads with UAVs using Distributed Trajectory Optimization" [1] to solve the scenario slack being introduced in the cables, an extension to the work discussed in the paper. We leveraged a hybrid control strategy that allows the quadrotors to dynamically reconfigure to accommodate a quadrotor ceasing to contribute to carrying the suspended load. The results indicate that a hybrid approach enables the desired reconfiguration of the quadrotors while adhering to nonlinear dynamics and nonconvex constraints. However, due to the approximations used to model the rope, additional work is required to ensure this approach is robust enough to allow for slack cables on real hardware. Additionally this work examines the performance of IPOPT in comparison to other solvers such as ALTRO.**

***Index Terms*—Optimal Control, IPOPT, ALATRO, UAVs, Hybrid Approach**

I. Background

The, "Scalable Cooperative Transport of Cable-Suspended Loads with UAVs using Distributed Trajectory Optimization," [1] thoroughly explores the advantages and challenges of employing quadrotors for transporting heavy loads. It emphasizes potential cost reductions in transportation, the enhanced versatility offered by quadrotors, and their ease of deployment compared to alternative methods. The paper presents detailed approaches, both distributed and batch problem-based, for solving the trajectory optimization problem inherent in transporting loads between locations. This formulation considers various factors, including quadrotor dynamics, load dynamics, collision constraints, and obstacle avoidance constraints, ensuring the safe and efficient transport of loads. The authors utilize ALTRO, a solver for constrained trajectory optimization, to solve the optimization problem. Thus, we aim to leverage the framework outlined in this paper to expand its application, particularly in solving trajectories that incorporate slack, a capability that could be pivotal in real-world scenarios requiring flexibility and adaptability.

II. Introduction

As noted in the background the approach used in the Distributed Paper does not account for variations to the problem/dynamics introduced by slack in the suspension cables.

As noted in Jackson et al [1] problem formations tend to make an approximation of the ropes as massless rigid links to avoid the 1-hour program solve time that accurately modeling rope dynamics leads to. Additionally "Cooperative quadrotors carrying a suspended load" outlines an approach that models the spring as a high stiffness spring with a damping coefficient [3]. In this paper, the ropes will also be modeled as massless rigid links with tension and rope distance being enforced as constraints which are then modified when a quadrotor goes slack.

Building upon this existing framework we introduce a hybrid control strategy designed to handle scenarios when slack in the suspension cables occurs potentially due to a UAV becoming inactive or due to constraints in the workspace that make it no longer possible to apply tension. To avoid alternating between two different dynamics functions an active set method [4] was employed to identify the active constraints during the slack period and zero out the constraints that constrain the slack drone's contribution towards supporting the cable-suspended load. Similar to the original paper quaternions are used to allow for aggressive maneuvers [1] . This project seeks to explore how this collaborative task can be solved with agents dropping out and being introduced while still being able to resiliently fulfill the problem objective of transporting the payload. and pass this batch problem that integrates each quadrotor along with the load to solve this nonlinear problem using IPOPT. Due to the absence of a convergence guarantee we used IPOPT to solve smaller problems to seed our reference trajectories to improve the likelihood of finding a solution.

The rest of this paper is organized as follows Section III outlines the problem formulation and the changes we made to model the quaternions via a slack variable along with our introduction of the hybrid approach Section IV outlines a discussion of our simulation results Section IV-B discusses how this project highlighted the limitations of IPOPT for large batch problems. Lastly, we conclude with a discussion and discuss possible extensions to this work Section V

III. Problem Formulation

In this section, we formulate the trajectory optimization problem for the cable-suspended load with N quadrotors. We assume that the suspension cables are attached to the center of mass of each quadrotor and the center of mass of the load. These suspension cables are modeled as massless linkages, meaning they do not directly affect the overall dynamics of the problem. The following selections outline our optimization problem.

A. Quadrotor Dynamics

The quadrotor dynamics are defined below:

$$\dot{x} = \begin{bmatrix} \dot{r} \\ \dot{q} \\ \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} v \\ \frac{1}{2} q \otimes \hat{\omega} \\ g + \frac{1}{m}\left(R(q)F(u) + F_c(u_5, x, x')\right) \\ J^{-1}\left(\tau(u) - \omega \times J\omega\right) \end{bmatrix} \tag{1}$$

where r $\in \mathbb{R}^3$ is the position, q is a unit quaternion, R(q) $\in \mathbb{SO}(3)$ is a quaternion-dependent rotation matrix from the body frame to world frame, v $\in \mathbb{R}^3$ is the linear velocity in the world frame, $\omega \in \mathbb{R}^3$ is the angular velocity in the body frame, x $\in \mathbb{R}^1 3$ is the state vector, u $\in \mathbb{R}^6$ is the control vector with u_5 being the magnitude of the cable force and u_6 being the slack variable used to normalize the quaternion, $x^\ell \in \mathbb{R}^6$ is the state vector of the load, $g \in \mathbb{R}^3$ is the gravity vector, and m $\in \mathbb{R}$ is the mass of the quadrotor, J $\in \mathbb{S}^3$ is the moment of inertia tensor, $\otimes$ is the quaternion multiplication, and $\hat{\omega}$ is a quaternion with zero scalar part, and ω the vector part.

The forces and torques $F, \tau \in \mathbb{R}^3$ on the quadrotors are defined as:

$$F(u) = \begin{bmatrix} 0 \\ 0 \\ k_f(u_1 + u_2 + u_3 + u_4) \end{bmatrix} \tag{2}$$

$$\tau(u) = \begin{bmatrix} k_f d_{motor}(u_2 - u_4) \\ k_f d_{motor}(u_3 - u_1) \\ k_m(u_1 - u_2 + u_3 - u_4) \end{bmatrix} \tag{3}$$

where k_f and k_m are motor constants, d_{motor} is the distance between motors and $u_{1:4}$ are motor thrusts.The forces from the cables will be modeled as (in the world frame):

$$F_c(\gamma, x, x') = \gamma \frac{r^\ell - r}{|r^\ell - r|^2} \tag{4}$$

where $\gamma \in \mathbb{R}$ is the magnitude of the tension in the cable and r^ℓ is the three-dimensional position of the load. γ will be represented as u_5 for each quadrotor.

B. Load

The dynamics of the load being transported by the quadrotors are:

$$\dot{x}^\ell = \begin{bmatrix} \dot{r}^\ell \\ \dot{v}^\ell \end{bmatrix} = \begin{bmatrix} v^\ell \\ g + \frac{1}{m^\ell} F^\ell(x^\ell, u^\ell, x^{1:L}) \end{bmatrix} \tag{5}$$

$$= f^\ell(x^\ell, u^\ell, x^{1:L}) \tag{6}$$

where $r^\ell \in \mathbb{R}^3$ is the load position, $v^\ell \in \mathbb{R}^3$ is the load velocity in the world frame, $m^\ell \in \mathbb{R}$ is the mass of the load, $x^\ell \in \mathbb{R}^6$ is the state vector, $u^\ell \in \mathbb{R}^L$ is the force acting on the load.

C. Hybrid Approach

In the context of a hybrid approach where a quadrotor ceases to exert force on a load introducing slack, we'll define the following sets to manage the coordination sequence:

Let Q_i Represent the time steps when quadrotor i is actively exerting force (active pulling) still with equal tension between the load and the quadrotor.

Let S_i Represent the time steps when quadrotor i is no longer contributing to supporting the load and instead goes slack.

where $Q_i \cap S_i = \emptyset$

By casing on membership in S_i when forming our constraints for each time-step we can vary our constraints accordingly based on whether they need to be enforced when our timestep $t_i \in Q_i$ and when the active set method can be employed and the slack constraint can be introduced.

D. Optimization Problem

The batch problem is created by concatenating the states and controls of all the quadrotors and the load:

$$\mathbf{x} \in \mathbb{R}^{13L+6} = \begin{bmatrix} x^1 \\ \vdots \\ x^L \\ x^\ell \end{bmatrix}, \quad \mathbf{u} \in \mathbb{R}^{6L+L} = \begin{bmatrix} u^1 \\ \vdots \\ u^L \\ u^\ell \end{bmatrix}. \tag{7}$$

where $I_L = \{1, \ldots, L\}$ are the indices of the quadrotors and $I_A = \{1, \ldots, L, \ell\}$ are the indices of all the agents (quadrotor and load). We can formulate the following optimization problem:

Consider the following optimization problem:

$$\text{minimize} \quad J^\ell(\mathbf{X}^\ell, \mathbf{U}^\ell) + \sum_{i=1}^{L} J^i(\mathbf{X}^i, \mathbf{U}^i) \tag{8a}$$

subject to

$$x^i_{k+1} = f^i_k(x^i_k, u^i_k, \Delta t; x^\ell_k), \quad \forall i \in \mathcal{I}_L \tag{8b}$$

$$x^\ell_{k+1} = f^\ell_k(x^\ell_k, u^\ell_k, \Delta t; x^1_k, \ldots, x^L_k) \tag{8c}$$

$$x^i_0 = x(0)^i, \quad \forall i \in \mathcal{I}_A \tag{8d}$$

$$x^\ell_N = x(t_f)^\ell \tag{8e}$$

$$r^i_{\min} \le r^i_k \le r^i_{\max}, \quad \forall i \in \mathcal{I}_A \tag{8f}$$

$$0 \le (u^i_k)_j \le u^i_{\max}, \quad \forall i \in \mathcal{I}_L, j \in \{1, \ldots, 4\} \tag{8g}$$

$$u^\ell_k > 0 \tag{8h}$$

$$(u^i_k)_5 = (u^\ell_k)_i, \quad \forall i \in \mathcal{I}_L \tag{8i}$$

$$\|r^i_k - r^\ell_k\|_2 = d_{\text{cable}}, \quad \forall i \in \mathcal{I}_L \tag{8j}$$

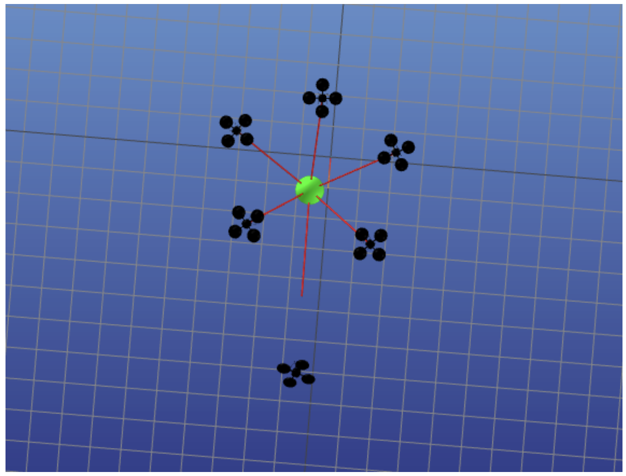

Fig. 1. example of the figure re-configuring with slack in the cable/ the quadrotor disengaging the remaining 5 agents reconfigure into a stable pentagon)

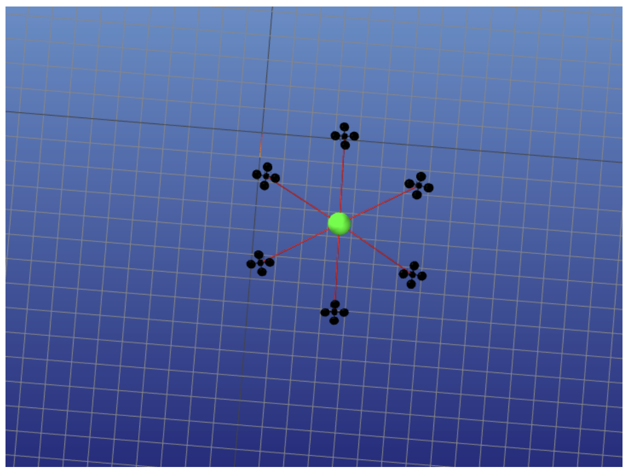

Fig. 2. upon reintroduction of the quadrotor we see the quad rotors return to their original hexagon form

$$2 d_{\text{quad}} - \|p_k^i - p_k^j\|_2 \leq 0, \quad \forall i, j \in \mathcal{I}_L, i \neq j \tag{8k}$$

where $X = [x_0, \ldots, x_N]$ is a state trajectory of length N, $U = [u_0, \ldots, u_{N-1}]$ is a control trajectory of length $N-1$, $\mathbf{X} = \left[X^1, \ldots, X^L, X^\ell\right]$ and $\mathbf{U} = \left[U^1, \ldots, U^L, U^\ell\right]$ are sets of trajectories for the system, $p^i \in \mathbb{R}^2$ is the two-dimensional position of the quadrotor or load (discarding height), $x(0)$ and $x\left(t_f\right)$ are initial and final conditions, d is a scalar dimension (e.g., quadrotor radius), Δt is the time step duration, and all constraints apply at each time steps k.

The constraints are, from top to bottom: discrete quadrotor dynamics (8b) from (1), discrete load dynamics (8c) from (5), initial conditions $8 \mathrm{~d}$, final condition for the load 8e), workspace constraints (i.e. floor and ceiling constraints) 8f), quadrotor motor constraints $8 \mathrm{~g}$), positive cable tension $8 \mathrm{~h}$), equal tension force on quadrotor and load 8i), cable length (8j), collision avoidance (8k). The objective for each agent was a quadratic cost, having the form: $\left(x_k-x_{\text{ref}}\right)^T Q_k\left(x_k-x_{\text{ref}}\right)$ and $\left(u_k-u_{\text{ref}}\right)^T R_k\left(u_k-u_{\text{ref}}\right)$ for the states and controls, respectively, at each time step k.

We achieved improved simulation results by adopting a strategy that involved carefully initializing our solver with valid state and control references. To start, we constructed a state reference trajectory by linearly interpolating between the initial state and the final state. This trajectory served as our reference state, providing the solver with a clear path from start to finish. With this reference trajectory in hand, we proceeded to run the solver, utilizing the initial state derived from our interpolation process. The solver then generated a series of control inputs based on this initial state. These control inputs were crucial, as they not only influenced the trajectory of the simulation but also served as the control reference for solving the problem. By reusing these extracted control inputs as our control reference, we ensured that the solver had a consistent set of inputs to work with throughout the problem-solving process. This approach was intended to guide the solver towards convergence upon a solution, ultimately leading to the output of a trajectory that met our simulation objectives. This is detailed more in section IV

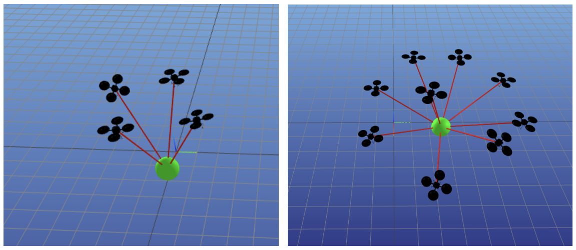

Fig. 3. Example of 4 and 8 agent examples mid trajectory. Similar to the Distributed paper we specified quaternion orientations for our initial and final configurations to incentivize hovering

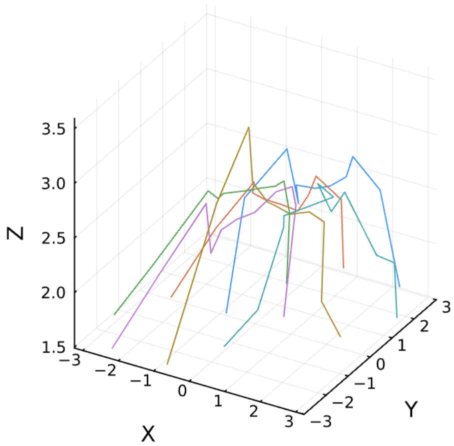

Fig. 4. Position plot for a 6 agent configuration from start to goal. the start was set in quadrant 1 and the goal was set in quadrant 3 resulting in the point-to-point transfer moving diagonally across the plane

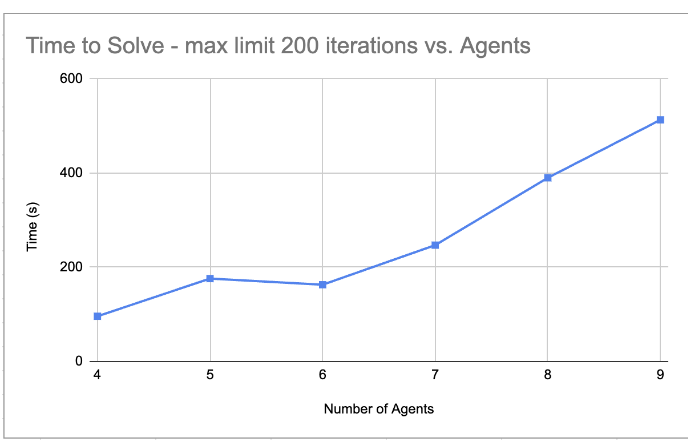

Fig. 5. Enter Caption

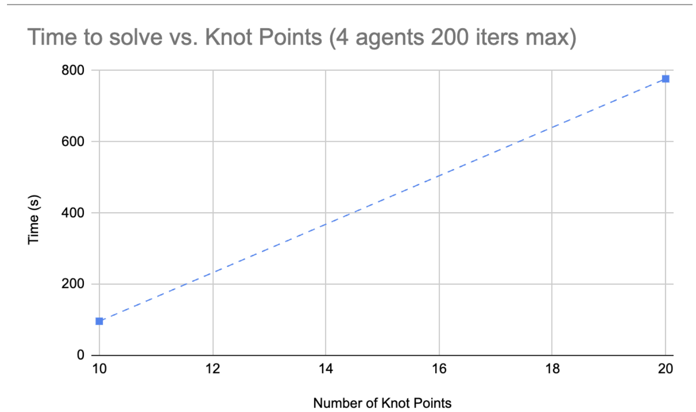

Fig. 6. A chart showcasing the time to solve vs knot points for 4 agents with 200 max iterations.

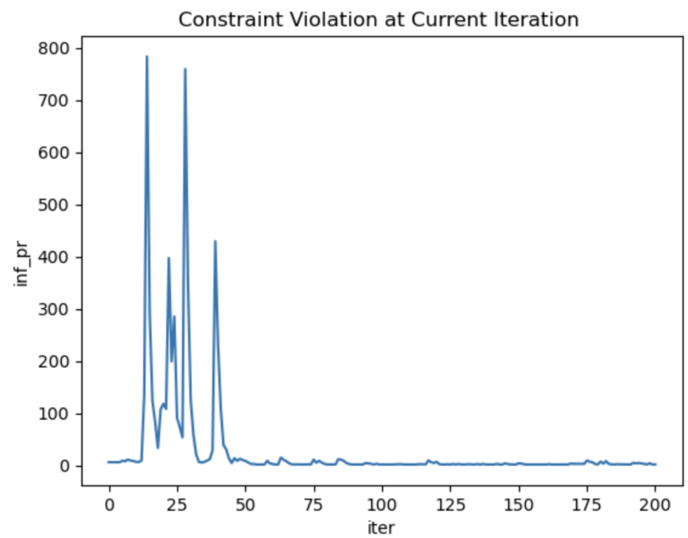

Fig. 7. A chart showcasing the infinite norm of our constraint violations at a specific iteration.

IV. Simulation Results

All trajectory optimization sub-problems were solved using IPOPT and performed on an Apple MacBook using the ARM architecture, M1 Pro processor and 16GB RAM. The code was written in the Julia Programming Language.

Figure 5 outlines the times it took for IPOPT to produce a trajectory and how that varied across the number of agents. Trim conditions were used to seed the initial conditions to prevent IPOPT from converging to a locally infeasible position to mixed success this process was done iteratively with the hover orientations for a linear interpolation between initial and goal with only the dynamics constraints being enforced. Then this trajectory was used to solve the problem with the equality constraints for the rope. Lastly, this trajectory was used to solve the overall problem including the inequality constraints

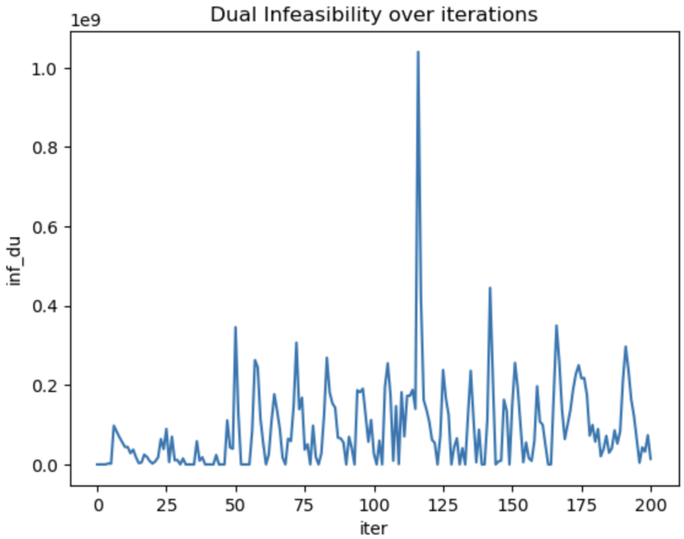

Fig. 8. A chart showcasing the dual in-feasibility across iterations.

of proximity and positive cable tension.

Figure 1 outlines an example of the reconfiguration taking place this was done with an active set method that turned off the necessary tension constraints and a new inequality constraint for the the relaxed rope distance.

4 shows an example of the positions plotted in 3d. The resolution of the graph was somewhat limited due to the runtime of the solver increasing drastically upon increasing the number of knot points. As seen in 6. Doubling the number of knot points led to an 8-fold increase in the time to solve. Attempting the granularity of the problem structured in ALATRO of 100-knot points. The 100-knot point batch problem led to a jacobian with non-zero entries on the order of 54 million which was unsolvable in our implementation of the problem in IPOPT. This limitation of IPOPT will be discussed in the following section.

A. ALTRO vs IPOPT

ALTRO is a trajectory optimization method that combines an augmented Lagrangian (AL) approach with an active-set method to achieve rapid convergence on constraint satisfaction. It performs competitively with DIRCOL methods, such as IPOPT, in terms of computation time and constraint satisfaction on benchmark problems [2].

One of ALTRO's strengths is its ability to be initialized with infeasible state trajectories, which DIRCOL and other methods struggle with. This strength makes it easier to handle complex optimization problems where finding an initial feasible solution is challenging [2]. In constant, IPOPT may struggle with infeasible initializations due to its reliance on interior-point methods that require a feasible starting point. This is a contributing factor as to why our IPOPT isn't converging correctly.

Additionally, ALTRO's handling of constraints, particularly in problems involving obstacle avoidance, appears to be more effective than IPOPT's approach [2]. This is further showcased in the next section where we discuss our lack of convergence.

B. Lack of convergence

While providing trajectories that abide by our expectations of the problem due to the nonconvexity and nonlinearity of the problem the solver did not converge well. While the constraint bounds demonstrated good convergence 7 properties the dual did not converge well 8. Since there is no guarantee of convergence there are a multitude of reasons as to what factors might be impacting convergence. Firstly the problem overall being too large for IPOPT to reliably solve, some numerical instability/inaccuracy in the implementation of the problem formulation, or the initial conditions not being tuned sufficiently well enough such that the solver gets stuck oscillating rather than making forward progress in decreasing dual infeasibility.

V. Conclusion

In conclusion, the paper "Scalable Cooperative Transport of Cable-Suspended Loads with UAVs using Distributed Trajectory Optimization" [1] extensively explores the benefits and challenges associated with using quadrotors for heavy-load transportation. It highlights potential cost reductions, increased versatility, and ease of deployment compared to other methods. The paper presents detailed approaches for solving the trajectory optimization problem, considering factors such as quadrotor and load dynamics, collision constraints, and obstacle avoidance. The authors utilize the ALTRO solver for constrained trajectory optimization.

Building upon this framework, our project introduces a hybrid control strategy to handle scenarios involving slack in the suspension cables. This strategy addresses situations where a UAV becomes inactive or when workspace constraints prevent the application of tension. By identifying active constraints during slack periods and adjusting them accordingly, our approach should ensure the safe and efficient transport of payloads. In terms of IPOPT and solving our convergence issue, several solutions can be considered. One approach is to implement distributed optimization using IPOPT, where each quadrotor's state is solved in a distributed way instead of utilizing a batch approach. The solutions for each quadrotor would then be combined to obtain the final solution, reducing overall problem size and potential numerical instabilities. Another strategy is to extend the implementation of the slack variable in a distributed manner, allowing each quadrotor and its load to dynamically adjust their trajectories based on the slack in the suspension cables. This distributed approach can improve adaptability to environmental changes or drone behavior. Additionally, utilizing ALTRO for trajectory optimization could enhance solve times, as ALTRO has shown promise in handling infeasible initializations and constraints more effectively than IPOPT.

Overall, while IPOPT struggles with infeasible initializations, ALTRO's ability to handle such situations makes it a more promising alternative. Our project highlighted challenges with IPOPT's convergence, possibly due to the problem's overall size, numerical instabilities, or problem formulation. Further research and refinement of the optimization approach are needed to address these challenges and find a more efficient way to utilize IPOPT to solve the trajectory optimization problem for the quadrotor.

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