

$$|aux, \psi\rangle = |0\rangle \otimes |\psi\rangle = |0, \psi\rangle = \phi$$

$$H(aux) \rightarrow \text{changes } |0\rangle \text{ to } \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Samahard gate

$$\text{changes } |1\rangle \text{ to } \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$H(\phi) = \frac{|0, \psi\rangle + |1, \psi\rangle}{\sqrt{2}}$$

$$\text{Controlled } u([aux], \psi) \rightarrow \text{changes } |0, \psi\rangle \text{ to } |0, \psi\rangle$$

$$\text{changes } |1, \psi\rangle \text{ to } |1, u(\psi)\rangle$$

$$\text{Controlled } u(H(\phi)) = \frac{|0, \psi\rangle + |1, u(\psi)\rangle}{\sqrt{2}}$$

If $|\psi\rangle$ be an eigenstate of u with eigenvalue $e^{i\phi}$
 then $u(\psi) = e^{i\phi}(\psi)$

$$\therefore \text{Controlled } u(H(\phi)) = \frac{|0, \psi\rangle + |1, e^{i\phi}\psi\rangle}{\sqrt{2}}$$

$$H(\text{Controlled } u(H(\phi))) =$$

$$\frac{1}{2} \left[(|0, \psi\rangle + |1, \psi\rangle) + e^{i\phi} (|0, \psi\rangle - |1, \psi\rangle) \right]$$

$$= \frac{1}{2} \left[|0, \psi\rangle (1 + e^{i\phi}) + |1, \psi\rangle (1 - e^{i\phi}) \right]$$

$$= \frac{1}{2} \left[|0, \psi\rangle (1 + \cos\phi + i\sin\phi) + |1, \psi\rangle (1 - \cos\phi - i\sin\phi) \right]$$

$$= \frac{1}{2} \left[|0, \Psi\rangle \left(2 \cos^2 \frac{\theta}{2} + i 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right) + |1, \Psi\rangle \left(2 \sin^2 \frac{\theta}{2} - i 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right) \right]$$

$$= \frac{1}{2} \left[|0, \Psi\rangle \left(2 \cos \frac{\theta}{2} \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right) \right) + |1, \Psi\rangle \left(2 \sin \frac{\theta}{2} \left(\sin \frac{\theta}{2} - i \cos \frac{\theta}{2} \right) \right) \right]$$

$$= \frac{1}{2} \left[|0, \Psi\rangle \left(2 \cos \frac{\theta}{2} \cdot e^{i\theta/2} \right) + |1, \Psi\rangle \left(i 2 \sin \frac{\theta}{2} \cdot e^{-i\theta/2} \right) \right]$$

$$= |0, \Psi\rangle \left(\cos \frac{\theta}{2} \cdot e^{i\theta/2} \right) + |1, \Psi\rangle \left(i \sin \frac{\theta}{2} \cdot e^{-i\theta/2} \right)$$

$$\begin{aligned} \text{Get measurement} &= M(\text{aux}) \\ &= M(H(\text{controlled } \mu(H(b)))) \end{aligned}$$

$$P(\text{measure } |0, \Psi\rangle) = \text{Re} |(\cos \theta/2 \cdot e^{i\theta/2})| = \cos^2 \theta/2$$

$$P(\text{measure } |1, \Psi\rangle) = \text{Re} |(\sin \theta/2 \cdot e^{-i\theta/2})| = \sin^2 \theta/2$$

Reset (aux)

↳ must needed step to avoid any quantum interferences and reduce load on the quantum comp.

(kind of like resetting heap memory in C)

Return measurement == zero? 0/1;

Returns 0 with prob. $\cos^2 \theta/2$

Returns 1 with prob. $\sin^2 \theta/2$

NOTE and FIND out: what was the purpose of Ψ if never used :).