Harshvardhan Aditya
Enr CSE365

SML Lab Assignment 3

A.1. Let the edge length of square that is removed be $\underline{x}$ inch

$\therefore (12-2x)(18-2x)(x) \Rightarrow$ Volume

For maximum volume

$$\frac{d\,Vol}{dx} = 0$$

$\therefore -2(18-2x)(x) \quad -2(12-2x)(x) + (12-2x)(18-2x) = 0$

$\Rightarrow -36x + 4x^2 - 24x + 4x^2 + 216 - 60x + 4x^2 = 0$

$\Rightarrow 12x^2 - 120x + 216 = 0$

$\Rightarrow x^2 - 10x + 18 = 0$

&

Since we have to maximize volume, the sides cannot be negative $\therefore$ $12 - 2x > 0 \rightarrow 6 > x$

$\therefore$ bounds of x are $0 < x < 6$

Solving $x^2 - 10x + 18 = 0$ we get 2 values

$x \simeq 2.35$

$x \simeq 7.64$

Since x can't be greater than ~~7.64~~ 6 $\therefore$ 2.35 is the only acceptable saddle point.

$f''(x) = 2x - 10$

for local maximum $f''(x^*)$ should be less than 0

$f''(2.35) = 4.7 - 10 = -5.3 \Rightarrow -ve$

$\therefore$ It is a local maxima

$\therefore$ The max. volume is when $x \simeq 2.35$ & that value is $\simeq 228.1615$.

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2. Perimeter of rectangle = 100
let one side be x & adjacent side be y
$\therefore$ Area $= xy$
Given there is a rock wall on one side -
$2x + y = 100$ - constraint.
$y = 100 - 2x$
For maximum area $\frac{d\,Area}{dx} = 0$

$\therefore x(100 - 2x) = area$
$\frac{d\,(x)(100-2x)}{dx} = 0$

$\therefore 100 - 2x - 2x = 0$
$\therefore x = 25$ [saddle point]
For maximum area $Area'' < 0$
$\therefore \frac{d(100 - 4x)}{dx} = -4 < 0$
local
$\therefore 25$ is the point for local maxima
$x = 25$ & $y = 50$
Ans The maximum area is for $x = 25$ ft & $y = 50$ ft & that area is $1250\ ft^2$.

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3. To maximize $f(x, y, z) = xy + yz$

Constraints - $x + 2y = 6$ ①; $x - 3z = 0$ ②

We can write y & z in terms of x -

$y = \frac{6-x}{2}$ (from ①)

$z = \frac{x}{3}$ (from ②)

$\therefore f(x, y, z) = \frac{x(6-x)}{2} + \frac{(6-x)}{2} \times \frac{x}{3}$

For maximization we'll find saddle points -

$f'(x, y, z) = \frac{(6-x)}{2} - \frac{x}{2} + \left(\frac{6-x}{6}\right) - \frac{x}{6}$

$= \frac{18 - 3x - 3x + 6 - x - x}{6}$

$= \frac{24 - 8x}{6} = 0$

$x = 3$ is the saddle point.

$f''(x, y, z) = -\frac{8}{6} \Rightarrow -ve \therefore$ saddle point

is ~~local~~ maxima.

Ans The maximum point is when $x = 3$, $y = \frac{3}{2}$ & $z = 1$

and that value is 6.

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4. $A = \begin{bmatrix} 13 & 5 \\ 5 & 7 \end{bmatrix}$; $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $c = 2$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$f(x) = x^T A x + b^T x + c$

$x^T A = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 13 & 5 \\ 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} (13x_1 + 5x_2) & (5x_1 + 7x_2) \end{bmatrix}$

$x^T A x = \begin{bmatrix} (13x_1 + 5x_2) & (5x_1 + 7x_2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= (13x_1 + 5x_2) \times x_1 + (5x_1 + 7x_2) x_2$

$= 13x_1^2 + 10x_1x_2 + 7x_2^2$

$b^T x = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= x_1 + x_2$

$\therefore f(x) = 13x_1^2 + 10x_1x_2 + 7x_2^2 + x_1 + x_2 + 2$

Constraints:

$g(x) \Rightarrow 2x_1 - 5x_2 = 2$ — ①

$h(x) \Rightarrow x_1 + x_2 = 1$ — ②

Using ① & ② we get

$2x_1 - 5x_2 = 2$

$-2x_1 - 2x_2 = -2$

$\therefore x_2 = 0$

$x_1 = 1$

Ans The minimum value of $f(x)$ is when $x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & that value is 16.

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5. Objective function $f(x, y) = 5x - 3y$

Constraint : $x^2 + y^2 = 136$

Using the constraint -

$y = \pm\sqrt{136 - y^2}$

$f(x, y) = 5x - 3\sqrt{136 - x^2}$

$\frac{df}{dx} = 5 - \frac{3}{2} \times (-2x) \left(\sqrt{136 - x^2}\right)^{-1/2}$

$= 5 + \frac{3x}{\sqrt{136 - x^2}} = 0$

$\frac{3x}{\sqrt{136 - x^2}} = -5$

$\therefore \Rightarrow 9x^2 = 25(136 - x^2)$

$\Rightarrow 9x^2 = 3400 - 25x^2$

$\Rightarrow 34x^2 = 3400$

$\Rightarrow x^2 = 100$

$x = \pm 10$

Double differentiate -

$$\frac{\left(\sqrt{136 - x^2}\right) \times 3 - 3x \times \frac{1}{2} \times \frac{1}{\sqrt{136 - x^2}} \times -2x}{(136 - x^2)}$$

$$= \frac{3\sqrt{136 - x^2} + 3x^2 / \sqrt{136 - x^2}}{(136 - x^2)}$$

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Putting $x = 10$

$$= \frac{3\sqrt{36} + \frac{300}{6}}{36}$$

$$= \frac{18 + 50}{36} = \frac{68}{36} > 0$$

$\therefore$ $x = 10$, maximization condition

$y = -6$

$f(x, y) = 5x - 3y = 68$

Minimization condition,

$x = -10$

$y = 6$

$f(x, y) = 5x - 3y = -68$