EXPERIMENT NO: 9

Aim: To study PERT.

Theory:

The **program** (or **project**) **evaluation and review technique**, commonly abbreviated **PERT**, is a statistical tool, used in project management, which was designed to analyze and represent the tasks involved in completing a given project. The PERT method is used to determine how long a project should take to complete, and which steps in the project planning are most critical -- that is, which steps would act as bottlenecks if delayed.

To apply the method, certain information is needed in advance: it must be possible to analyze the project into activities, the dependency relationships between the activities must be known, and the time taken for each activity must be known or at least estimable. Some cases are too simple for PERT -- cases where we've performed very similar projects many times before, and know that the current project will take the same time as they did, plus or minus a few per cent. Other cases are too complex -- for example, the Manhattan project, where the individual activities making up the project are themselves so novel that we don't know whether they're feasible at all, let alone how long they can be expected to take.

Where we have the necessary information, it can be represented as a project network. Consider, for example, building a house. This involves a number of activities -- building the walls, putting on the roof, painting the outside, painting the inside -- which can be represented as edges in a network. The nodes of the network represent states, for example, the state of having the roof on. We show the dependence of one state on another by the existence of an edge joining the states. In some cases, this edge may represent a real activity -- to reach the state of ready-to-start-painting from the state interior-plumbing-completed, we have to perform the activity **install wall board**. In other cases, one state must precede another, even though no activity links them -- for example, the state rough-exterior-plumbing-completed must be reached before the state ready-to-begin-exterior-painting. This can be indicated graphically by a dotted edge connecting the two states, representing a dummy activity.

Deterministic PERT:

Now, we have the network and we have estimates of the time for each activity. To find out how long the project as a whole will take, we perform two passes through the network. We begin the forward pass by noting that the earliest time we can reach the first state is time zero. Now we examine the next state, s_j, say. If there are several states s_i preceding state s_j, then the earliest we can reach state s_j is

$\max_i$ {(earliest we can reach state i + time for activity $a_{i,j}$)}

In this way we work through the network, until we've found the earliest time that the final state can be reached. We now adopt that time as the deadline for our project, and perform a backward pass through the network, asking what the latest time is that we can reach a particular state if the deadline is not going to be delayed. At the end of this second pass, we have a pair of times for each state, giving the earliest and the latest times at which we can reach the state. The difference between these times is called the slack for the state. Those states having zero slack are said to lie on the critical path; delay of any state on the critical path will lead to a delay in meeting the overall deadline.

Probabilistic PERT:

In many cases, we will not know accurately how long each activity will take. A variant of PERT, the PERT 3-Estimate Approach, can be used in these cases. We assume that three estimates are available for the time taken by each activity: an optimistic approach, a, a realistic approach, m, and a pessimistic approach b. We further assume that the probability distribution for the time taken by each activity is a Beta distribution. (The Beta distribution has a long right tail, reflecting the fact that there's more ways for an activity to be delayed than for it to be sped up.) From these three estimates, we can calculate the mean and variance of the corresponding Beta distribution as follows:

$t=((2m+(a+b)/2)/3)$

$sigma^2=((b-a)/6)^2$

We need one further assumption: that the times taken for each activity are independent variables. We can then apply the Central Limit Theorem to argue that the sum of the activity times on the critical path is itself a random variable with a normal distribution, its mean and variance the sum of the means and variances of the activities on the critical path. Now, knowing this mean and variance, we can calculate the probability that the time for the total project will be less than a pre-defined deadline.

Terminology:

- **PERT event:** A point that marks the start or completion of one or more activities. It consumes no time and uses no resources. When it marks the completion of one or more activities, it is not "reached" (does not occur) until all of the activities leading to that event have been completed.
- **Predecessor Event:** An event that immediately precedes some other event without any other events intervening. An event can have multiple predecessor events and can be the predecessor of multiple events.
- **Successor Event:** An event that immediately follows some other event without any other intervening events. An event can have multiple successor events and can be the successor of multiple events.
- **PERT activity:** The actual performance of a task which consumes time and requires resources (such as labor, materials, space, machinery). It can be understood as representing the time, effort, and resources required to move from one event to another. A PERT activity cannot be performed until the predecessor event has occurred.
- **PERT sub-activity:** A PERT activity can be further decomposed into a set of sub-activities. For example, activity A1 can be decomposed into A1.1, A1.2 and A1.3 for example. Sub-activities have all the properties of activities, in particular a sub-activity

has predecessor or successor events just like an activity. A sub-activity can be decomposed again into finer-grained sub-activities.

- **Optimistic Time (O):** The minimum possible time required to accomplish a task, assuming everything proceeds better than is normally expected
- **Pessimistic Time (P):** The maximum possible time required to accomplish a task, assuming everything goes wrong (but excluding major catastrophes).
- **Most Likely Time (M):** The best estimate of the time required to accomplish a task, assuming everything proceeds as normal.
- **Expected Time (T_E):** The best estimate of the time required to accomplish a task, accounting for the fact that things don't always proceed as normal (the implication being that the expected time is the average time the task would require if the task were repeated on a number of occasions over an extended period of time).

  $\mathbf{T_E = (O + 4M + P) \div 6}$

- **Float or Slack:** It is a measure of the excess time and resources available to complete a task. It is the amount of time that a project task can be delayed without causing a delay in any subsequent tasks (free float) or the whole project (total float). Positive slack would indicate ahead of schedule; negative slack would indicate behind schedule; and zero slack would indicate on schedule.
- **Critical Path:** The longest possible continuous pathway taken from the initial event to the terminal event. It determines the total calendar time required for the project; and, therefore, any time delays along the critical path will delay the reaching of the terminal event by at least the same amount.
- **Critical Activity:** An activity that has total float equal to zero. An activity with zero float is not necessarily on the critical path since its path may not be the longest.
- **Lead Time:** The time by which a predecessor event must be completed in order to allow sufficient time for the activities that must elapse before a specific PERT event reaches completion.
- **Lag Time:** The earliest time by which a successor event can follow a specific PERT event.
- **Fast Tracking:** Performing more critical activities in parallel
- **Crashing Critical Path:** Shortening duration of critical activities

Example:

In the following example there are seven tasks, labeled A through G. Some tasks can be done concurrently (A and B) while others cannot be done until their predecessor task is complete (C cannot begin until A is complete). Additionally, each task has three time estimates: the optimistic time estimate (O), the most likely or normal time estimate (M), and the pessimistic time estimate (P). The expected time (T_E) is computed using the formula (O + 4M + P) ÷ 6.

Activity	Predecessor	Time estimates			Expected time
		Opt. (*O*)	Normal (*M*)	Pess. (*P*)	
A	—	2	4	6	4.00
B	—	3	5	9	5.33
C	*A*	4	5	7	5.17
D	*A*	4	6	10	6.33
E	*B*, *C*	4	5	7	5.17
F	*D*	3	4	8	4.50
G	*E*	3	5	8	5.17

Advantages:

- PERT chart explicitly defines and makes visible dependencies (precedence relationships) between the work breakdown structure (commonly WBS) elements.
- PERT facilitates identification of the critical path and makes this visible.
- PERT facilitates identification of early start, late start, and slack for each activity.
- PERT provides for potentially reduced project duration due to better understanding of dependencies leading to improved overlapping of activities and tasks where feasible.
- The large amount of project data can be organized & presented in diagram for use in decision making.
- PERT can provide a probability of completing before a given time.

Disadvantages:

- There can be potentially hundreds or thousands of activities and individual dependency relationships.
- PERT is not easily scalable for smaller projects.
- The network charts tend to be large and unwieldy requiring several pages to print and requiring specially sized paper.
- The lack of a timeframe on most PERT/CPM charts makes it harder to show status although colours can help (e.g., specific colour for completed nodes).

Conclusion:

Thus PERT helps in project scheduling which gives probabilistic idea of completing the project before schedule.