Linear Regression

Harshvardhan

University of Tennessee

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Case Study:
Health Insurance Premiums

Case Study

What drives Health Insurance Premiums?

Health Insurance Premiums in Massachusetts: What Drives Costs?

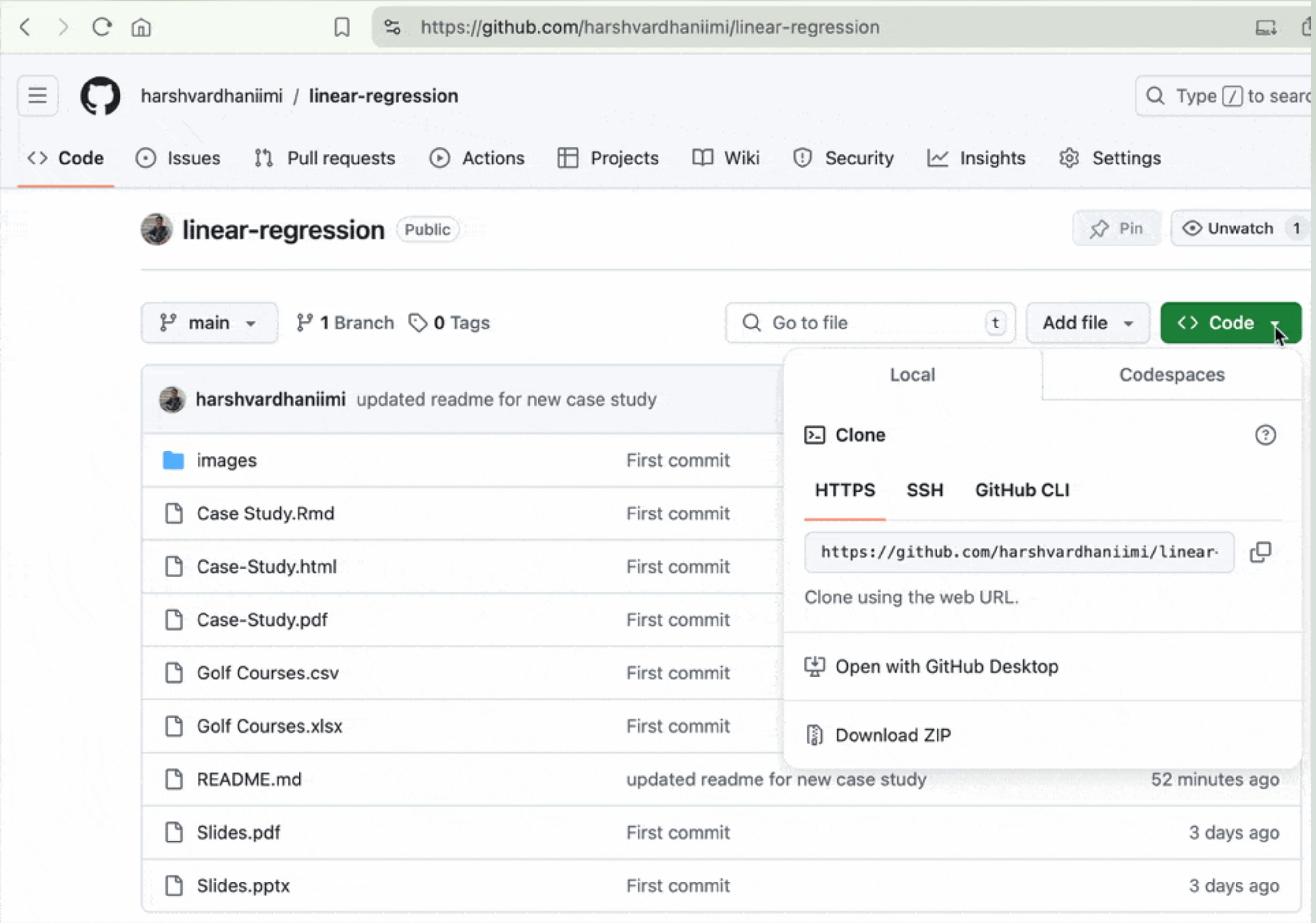
- In 2022, per capita health care spending was $10,264, which was a 5.8% increase from 2021.
- The average cost of an individual health insurance plan in Massachusetts is $721.19 per person.

What drives insurance premium in general?

1.

2.

3.



Health Insurance Data

We will use data from Kaggle for a fictional case study.

Download files from Github:

github.com/harshvardhaniimi/linear-regression

Massachusetts Department of Public Health (MDPH)

Identifying key factors driving health insurance charges.

- Goals
  1. Identify actionable policy insights for MDPH
  2. Empower insurers to structure premiums based on evidence rather than assumptions
- Variables
  1. Sex, Smoking Habits, Region
  2. Age, BMI, Children
  3. Insurance Charges

NOTE: Fictional Case Study

Linear Regression

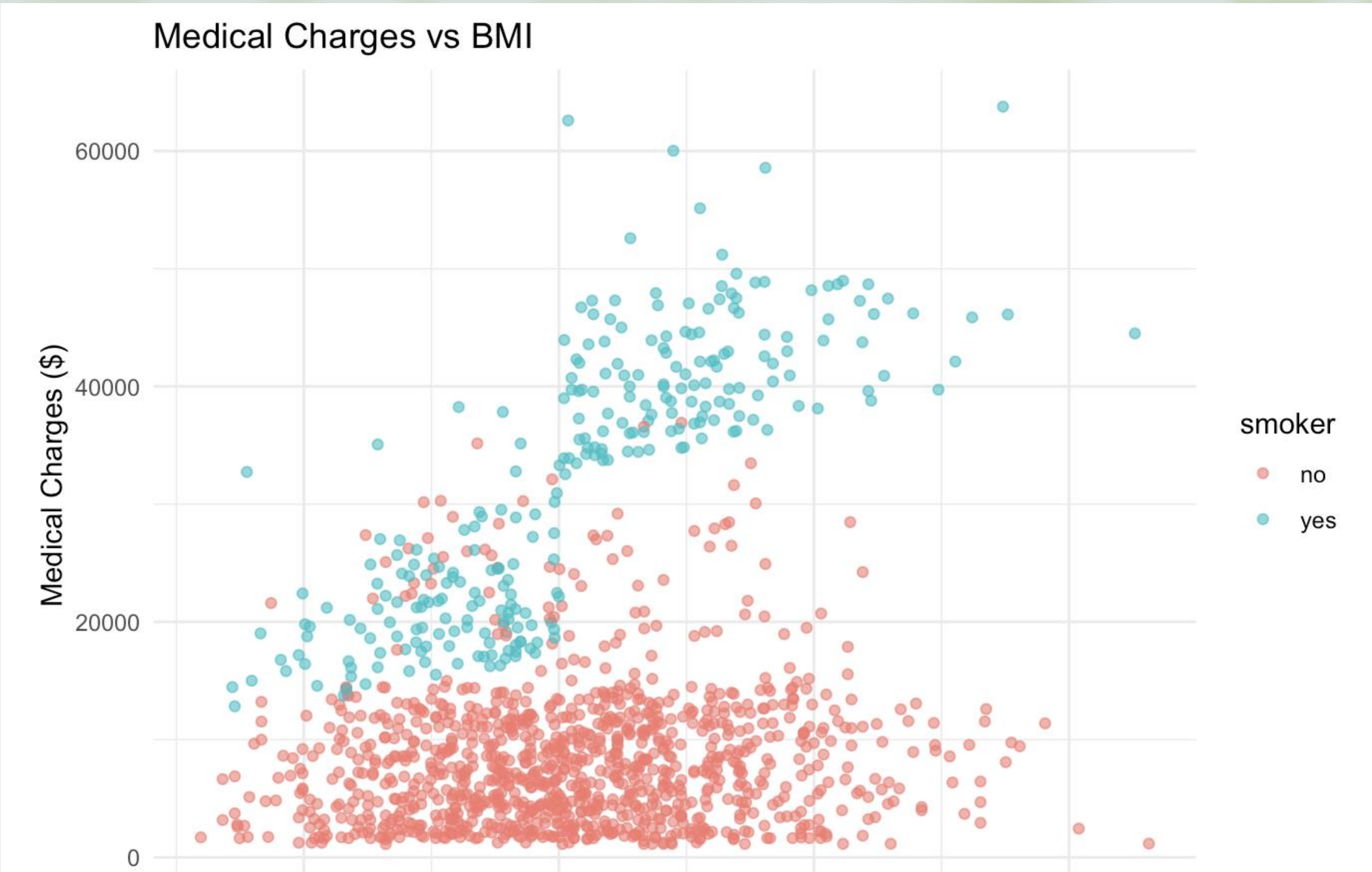
Going from correlation to regression

What is Linear Regression?

- Linear regression is a *model* that estimates *linear relationship* between a dependent variable and one or more independent variable(s)
- It is an attempt to find the best fit line between independent and dependent variables
- **Strengths:**
  - Explainability and interpretability
  - Simple, quick and easy
  - Statistical basis for usage and interpretation
- **Weaknesses:**
  - Assumes linear relationship – simple model is too simple
  - Sensitive to outliers
  - Assumptions – we will talk about them
  - **No causation implied, only a sophisticated form of correlation**

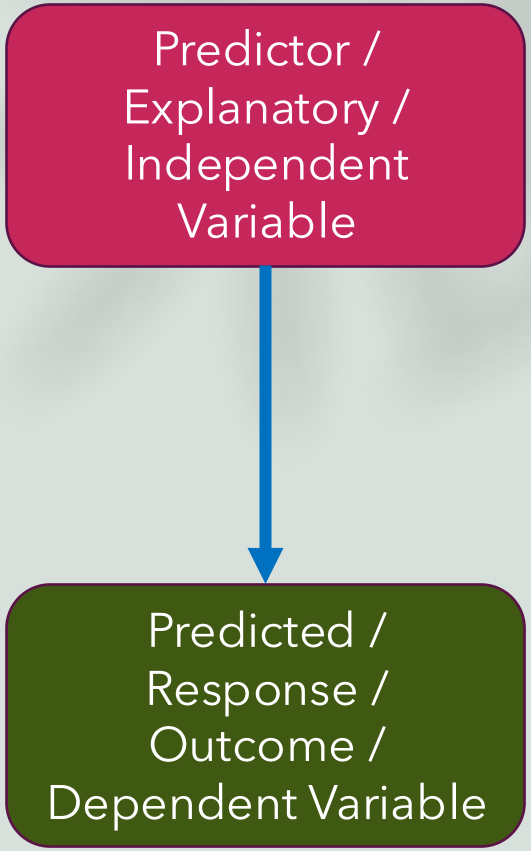
Scatterplot and Correlation

- Note for:
  - Direction
  - Strength
  - Outliers
- Linear regression is square of correlation between Y and X
- In multilinear regression, its squared correlation between $\hat{Y}$ and Y (Why?)
- https://digitalfirst.bfwpub.com/stats_applet/stats_applet_5_correg.html



Key Components

- **Dependent Variable:**
  - Variable to be predicted or explained
  - Also known as **Response** or **Outcome** variables.
  - Usually written as y_i
  - Example: *Insurance Premium*
- **Independent Variable(s):**
  - Variables used to predict or explain dependent variable
  - Also known as **Predictor** or **Explanatory** variables
  - Usually written as x_i
  - Example: *BMI, Smoking Habits, # of Dependents, Age, etc.*



Linear Regression Mathematics

Linear Model, Coefficients and Predicted Responses

Mathematics of Linear Regression

- Simple Linear Regression (single predictor)

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

- Multiple Linear Regression (*p* predictors)

$$y_i = \beta_0 + \beta_1 x_i^1 + \beta_2 x_i^2 + \beta_3 x_i^3 + \cdots + \beta_p x_i^p + \epsilon_i$$
$$Y = X\beta + \mathrm{E}$$

Calculating Coefficients

Simple Linear Regression

$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$

- $\beta_1 = {}^{Cov(x,y)}/_{Var(x)}$

  $$= \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

- $\beta_0 = \bar{y} \ - \beta\bar{x}$
- β_1 measures how much y changes w.r.t. x, standardized by variability in x

Multiple Linear Regression

$Y = X\beta + \mathrm{E}$

- $\beta = (X'X)^{-1}X'Y$
- where X is data matrix, Y is response vector

Predicted Response: $\hat{Y}$

- $\hat{Y}$ is the estimated or predicted value of the dependent variable Y based on the estimated linear regression model
- $\hat{y}_i = \widehat{\beta_0} + \widehat{\beta_1} x_i$ – Notice there is no residual term here. Why?
- **Interpretation:** $\hat{y}_i$ is the best guess we have for y_i given information about (x_i, y_i)
- **Residual:** $\hat{\epsilon}_i = y_i - \hat{y}_i$
- **Example:** Predicted insurance premium based on lifestyle and other indicators

Interpretation, Assumptions and Diagnostics

How reliable are our conclusions?

Significance and Strength of Relationship

```
fit = lm(charges ~ bmi, data = df)
summary(fit)
```

```
##
## Call:
## lm(formula = charges ~ bmi, data = df)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -20956  -8118  -3757   4722  49442
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1192.94    1664.80   0.717    0.474
## bmi           393.87      53.25   7.397 2.46e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11870 on 1336 degrees of freedom
## Multiple R-squared:  0.03934,	Adjusted R-squared:  0.03862
## F-statistic: 54.71 on 1 and 1336 DF,  p-value: 2.459e-13
```

- **P-value** measures significance of a relationship, typical threshold is 0.05
- **R-squared** measures proportion of variance in Y explained by Xs
  - R2 = 0 means no relationship
  - R2 = 1 implies perfect relationship
  - Also called "goodness of fit"
- **Adjusted R-squared** accounts for number of variables
- **F-statistic** tests whether the regression model provides a better fit than a model with no predictors (i.e. simple mean)
  - Higher is better (and will have low p-value)
- **Residual Standard Error** measures average distance between $\widehat{\boldsymbol{Y}}$ and $\boldsymbol{Y}$

R-Squared: Goodness of Fit

- R2 measures *the proportion of variance in dependent variable Y explained by independent variables X* in a linear regression model
  - R2 = 0.75 means 75% of variance in Y is explained by X
- R lies between 0 (nothing can be explained) and 1 (everything explained)
- Higher R2 generally implies better model
  - How high? Depends on field – economics or psychology (0.3 is okay), physics (even 0.9 may not be enough)
- Limitations:
  - R2 doesn't indicate if the model is appropriate
  - Doesn't measure predictive performance for out of sample ("test") data
  - Doesn't imply causality
  - Will increase if # of independent variables increase
- Adjusted RSquared

Interpretation

```
fit = lm(charges ~ bmi, data = df)
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Based on the results:

1. Dependent variable (Y) = ________________

2. Independent variable (X) = _______________

3. Linear model, mathematically:

Interpretation

```
fit = lm(charges ~ bmi, data = df)
summary(fit)
```

```
##
## Call:
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```

Is regression model significant?

- F-statistic =
- R2 =
- Adj R2 =

R2 Interpretation:

Is the relationship between BMI and insurance premium significant?

- P-value =

Slope Interpretation:

Intercept Interpretation:

Multiple Linear Regression

- We can choose to include more than one variables in regression. Let's see an example.

```
Call:
lm(formula = charges ~ age + sex + bmi + children + smoker +
    region, data = df)

Residuals:
     Min       1Q   Median       3Q      Max
-11304.9  -2848.1   -982.1   1393.9  29992.8

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)     -11938.5      987.8 -12.086  < 2e-16 ***
age                256.9       11.9  21.587  < 2e-16 ***
sexmale           -131.3      332.9  -0.394 0.693348
bmi                339.2       28.6  11.860  < 2e-16 ***
children           475.5      137.8   3.451 0.000577 ***
smokeryes        23848.5      413.1  57.723  < 2e-16 ***
regionnorthwest   -353.0      476.3  -0.741 0.458769
regionsoutheast  -1035.0      478.7  -2.162 0.030782 *
regionsouthwest   -960.0      477.9  -2.009 0.044765 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6062 on 1329 degrees of freedom
Multiple R-squared:  0.7509, Adjusted R-squared:  0.7494
F-statistic: 500.8 on 8 and 1329 DF,  p-value: < 2.2e-16
```

Interpret R2:

Estimate effect of 'age':

Estimate effect of 'smoking':

Confidence Intervals of Estimated Coefficients

confint (model) function in R can give us 95% confidence intervals for all intercepts

```r
confint(mlr, level = 0.95)
```

```
##                         2.5 %       97.5 %
## (Intercept)     -13910.3546 -10070.18519
## age                233.6457    280.30145
## bmi                282.6391    394.69014
## children           204.3551    744.77779
## smokeryes        23028.3414  24644.25958
## regionnorthwest  -1286.2110    581.84680
## regionsoutheast  -1973.1303    -95.58998
## regionsouthwest  -1896.6557    -22.09365
```

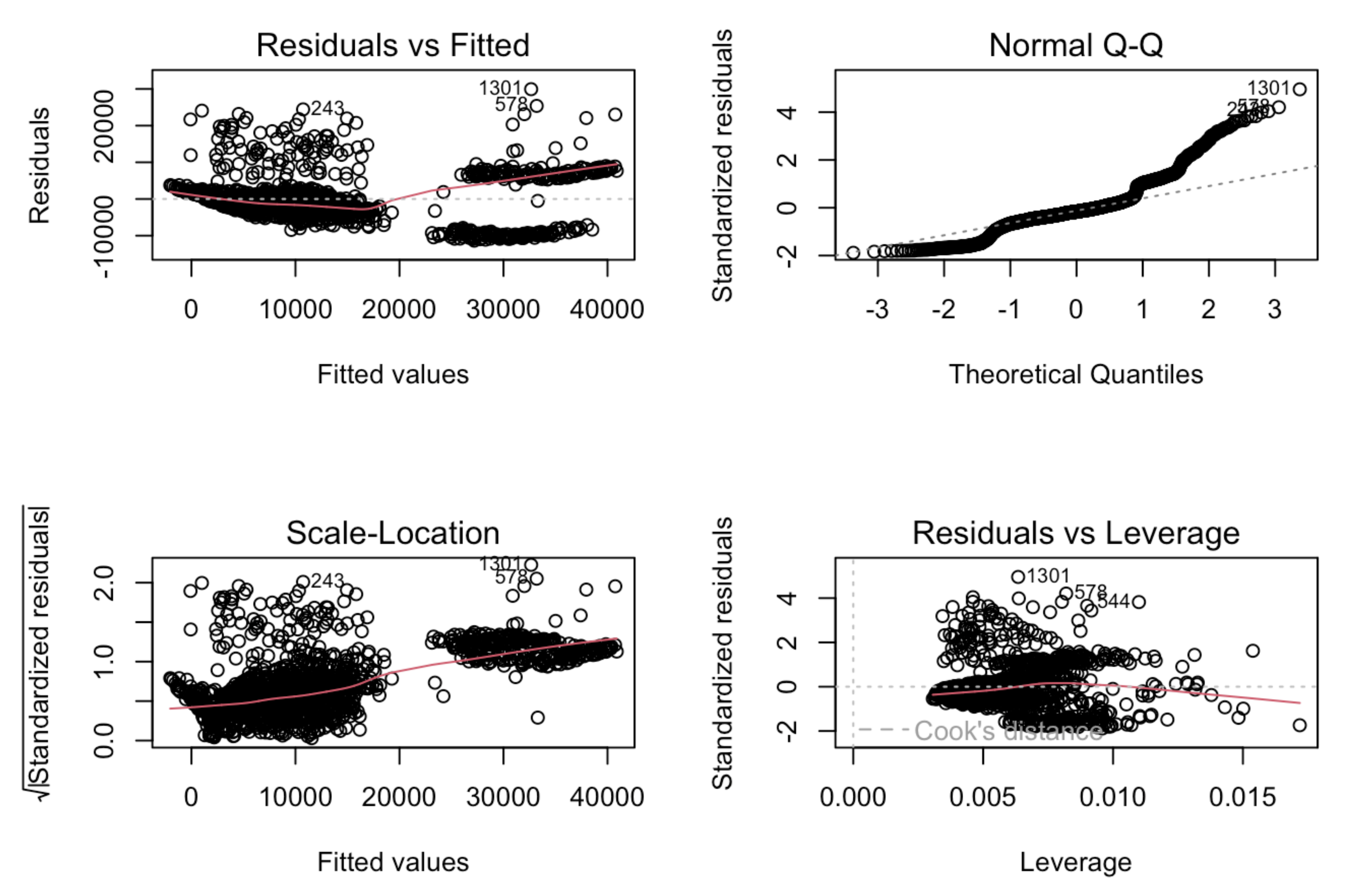
Assumptions in Linear Regression (LINE)

- Like all statistical models, Linear Regression works under certain assumptions:
  1. **L**inearity – assumed linear relationship between X and Y
  2. **I**ndependence – residuals (errors) are independent of each other
  3. **N**ormal distribution of residuals $N(0, \sigma^2)$
  4. **E**qual variance across values of X (homoskedasticity)

Model Diagnostics

- R provides four diagnostic plots:
  1. Residual vs Fitted plot (Linearity assumption)
     - Good if horizontal line shows with no distinct patterns
     - plot(model, which = 1)
  2. Normal Q-Q plot (Normality assumption)
     - Good if residuals follow diagonal dotted line
     - plot(model, which = 2)
  3. Scale-Location plot (Equal variance or Homoskedasticity assumption)
     - Good if horizontal line with equally spread points
     - plot(model, which = 3)
  4. Residuals vs Leverage (Detecting outliers)
     - Good if few points stand out
     - plot(model, which = 4)

Model Diagnostics Case



In-class Quiz

- Let’s see how much we understood from today’s class so far
- Visit https://play.blooket.com/ and enter code XXXXXX

Business Insights

Real-World Implications and Actionable Insights

Impact of Smoking Habits, BMI and Others

Impact of Smoking on Charges
From our MLR results, we found that being a smoker is associated with an increase of approximately $23,836 in annual medical charges, holding other variables constant.

Effect of BMI on Costs
For every one-unit increase in BMI, medical charges increase by around $338.66, controlling for other factors like age and smoking.

Explaining Variability in Medical Charges
Smoking habits, BMI, age, number of dependents, and regional differences together explain 75% of the variability in health insurance premiums.

Recommended Policy Interventions

Expand the **Massachusetts Tobacco Cessation and Prevention Program (MTCP)** with targeted campaigns for high-risk populations – regions, incentives like tax benefits, etc.

Higher BMI leads to higher charges! Consider **subsidizing gym memberships, nutritional counseling, and fitness programs**, particularly for lower-income groups that face barriers to access

Extra premium for children's insurance causes higher childcare costs for parents. Consider subsidizing children's insurance!

Concluding Remarks

What we learned today?

- Linear regression helps us identify relationship and patterns between *independent* and *dependent* variables
- Smoking and BMI have high impact!
- Model coefficients tell us the impact of an individual variable
- RSquared tells us "goodness of fit" of a model
- Assumptions of LR should be verified before interpretation
- Residuals analysis can tell us more about the data than we think

What to remember?

- Regression's strength is interpretability
- Regression coefficient being significant doesn't imply causality
- Assumption of "linear model" might be too simplistic for pure prediction
- **LINE Assumptions:** Linearity, Independence, Normality, Equal Variance
- **There is more to regression! We will cover additional topics in coming classes**

Next Up…

- **Interaction between Variables**
  - Identifying relationships that have combined/conditional effects, not additive effects
  - Effect of exercise on weight might depend on diet type
- **Ranking Importance of Independent Variables**
  - Scaling X by its mean and standard deviation
- **Linear regression for optimization**
  - https://blog.harsh17.in/using-linear-regression-to-find-optimal-value/
- **Variable selection:** Lasso and ridge regressions

Questions?