

The Failure of Practical Intuition: How Forward-Coverage Inventory Targets Cause the Landslide Effect

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Seasonal demand for products is common at many companies including Kraft Foods, Case New Holland, and Elmer's Products. This study documents how these, and many other companies, experience bloated inventories as they transition from a low season to a high season and a severe drop in service levels as they transition from a high season to a low season. Kraft has termed this latter phenomenon the "landslide effect." In this study, we present real examples of the landslide effect and attribute its root cause to a common industry practice employing forward days of coverage when setting inventory targets. While inventory textbooks and academic articles prescribe correct ways to set inventory targets, forward coverage is the dominant method employed in practice. We investigate the magnitude and drivers of the landslide effect through both an analytical model and a case study. We find that the effect increases with seasonality, lead time, and demand uncertainty and can lower service by an average of ten points at a representative company. While the logic is initially counterintuitive to many practitioners, companies can avoid the landslide effect by using demand forecasts over the preceding lead time to calculate safety stock targets.

Key words: seasonal demand; days of supply; safety stock targets; production and inventory management; industry practice

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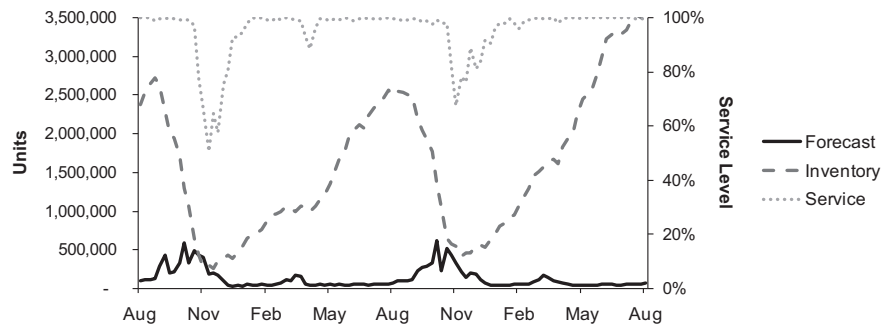
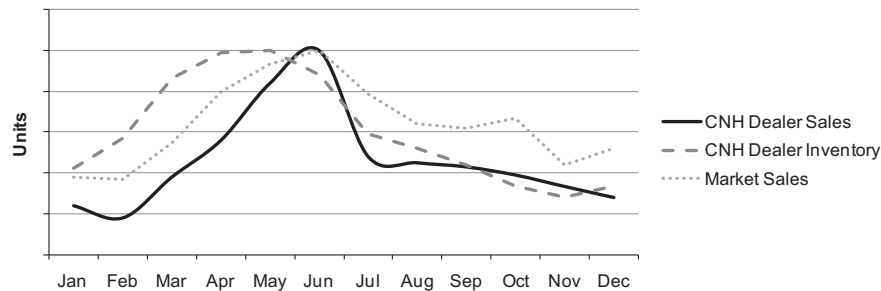
1. Introduction

Many products experience seasonal demand. More often than not, the seasonal pattern repeats on an annual basis. For example, Microsoft experiences about two-thirds of the annual demand for its Xbox video game consoles in the 13 weeks before Christmas. Kraft Foods sells roughly twice as many hot dogs per week during the summer as it does during the rest of the year. Elmer's Products does almost 75% of its annual school glue volume during the 4-month back-to-school season from May to August. Monthly or quarterly patterns are common as well. For example, Dell experiences end-of-month and end-of-quarter peaks in demand as high as 25% for its enterprise products due to corporate buying patterns. Dell also experiences a demand spike in July due to school and government buying behavior and in December for many of its consumer products.

Inventory planning in the face of seasonal demand is challenging. Many companies experience problems as they transition between seasons. They have too much inventory as they transition from a low to high

season and experience a severe drop in service levels as they transition from a high season to a low season. Figure 1 shows 2 years of historical forecasts, inventory, and service levels for the stuffing product category at Kraft Foods. Demand for stuffing peaks from late September through mid-December, including the Thanksgiving holiday. Inventory levels, however, begin to drop precipitously just before the high season begins. As a result, service levels also drop, falling almost 50 points during the latter portion of the high season. Kraft has observed this same pattern in other seasonal product categories such as coffee, hot dogs, and barbeque sauce. Kraft termed this the "landslide effect" since the forward-facing slope of the seasonal inventory mountain has disappeared, as if in a landslide.

Case New Holland (CNH) experienced the same phenomenon with its tractor business. Figure 2 shows a year of historical sales and inventory data for CNH utility tractors alongside industry sales as a whole. CNH sales and dealer inventory employ the same scale but market sales are scaled to fit on the graph. The industry's peak season runs April through July.

Figure 1 An Example of the Landslide Effect at Kraft Foods**Figure 2** An Example of the Landslide Effect at CNH

While still in the low season, the inventory in the dealer network increases far beyond what is required. Inventory reaches its apex in the first half of the high season and begins to drop well before the end of the peak season. As a result, CNH's sales fall off a cliff in July while the rest of the market continues to enjoy high sales. This is another example of the so-called landslide effect.

In this study, we explain how a forward Days of Supply (DOS) inventory target causes the landslide effect and how the correct application of a nonstationary inventory model avoids it. The next section provides a brief survey of the related literature. Section 3 describes the approach used by most companies to plan inventories with seasonal demand. Section 4 reviews the correct mathematics for calculating inventory targets when demand is nonstationary. In section 5, we point out the disconnect between current practice and the correct mathematics and explore the drivers and magnitude of the landslide effect through both an analytical model and a case study. We conclude the paper with suggestions for how to avoid the landslide effect and ideas for further study.

2. Related Literature

Our work is similar in intent to previous studies on the bullwhip effect (Lee et al. 1997) and spiral-down effect (Cooper et al. 2006). As in these studies, we document a problematic phenomenon that occurs in practice and explain its causes with relatively simple

math models. The domain of the landslide effect is inventory planning for seasonal, nonperishable products. We characterize previous work in this area by its treatment of demand as deterministic or stochastic and its focus on single or multiple products.

For a single product with stochastic, nonstationary demand, Karlin (1960a) proves that a critical number (i.e., base stock) policy is optimal assuming no fixed ordering costs. Under this policy, an order is placed in each period to bring the inventory position up to the base stock level. Because demand is nonstationary, the base stock level may change from period to period and there is no explicit procedure for calculating the base stock levels in the general case. Morton and Pentico (1995) develop and test a number of heuristics based on the myopic policy which minimizes costs for the current period. Such a policy is optimal when demand is increasing over time and provides an upper bound on the optimal order quantity in general. Karlin (1960b) and Zipkin (1989) present algorithms to calculate the optimal base stock levels for the special case of cyclic demand (i.e., demand densities with a repeating or seasonal pattern). Zipkin (1989) observes that the optimal base stock levels follow the same pattern as the myopic values but exhibit a smoothing effect (i.e., larger values are reduced to anticipate smaller ones in the future). Kapuscinski and Tayur (1998) and Aviv and Federgruen (1997) include capacity constraints and show that a cyclic base stock policy is still optimal. Metters (1997) also considers limited capacity and develops and tests a

heuristic to calculate base stock levels. Capacity constraints add the complication of pre-build stock as it may be necessary to build ahead during periods of low demand in order to meet demand during high periods.

Inventory and production-planning models for multiple products typically assume shared and limited production capacity. Problems with deterministic, seasonal demand fall under the category of aggregate planning. Typical aggregate planning models utilize linear or mixed-integer programming to determine production levels in each period and treat safety stock as an input parameter, if at all. Nam and Logendran (1992) provide a survey of the extensive literature in this area while Bradley and Arntzen (1999) provide an example of the implementation of such a model. Problems with stochastic, seasonal demand, and multiple products have received far less attention in the literature. Due to the complexity of the optimal policy for even a two-product problem, Ketzenberg et al. (2006) propose a heuristic to determine production quantities that starts by solving N unconstrained single-product problems. Their heuristic then utilizes a marginal analysis to either allocate limited capacity or anticipate future shortages depending on whether the single-product targets can be reached in the current period. Graves and Willems (2008) consider inventory planning for multiple products with stochastic, nonstationary demand in a supply chain (i.e., multi-echelon) setting. Their model, however, assumes no constraints on production capacity.

A sizable literature considers safety stock sizing in MRP systems. They range from analytical works like Buzacott and Shanthikumar (1994) that consider safety stock vs. safety time to simulation-based approaches like Zhao et al. (2001) and Dellaert and Jeunet (2005) that consider the impact of safety stock levels and lot sizes on total system cost. Guide and Srivastava (2000) provide a detailed literature review of this problem area.

To the best of our knowledge, ours is the first work to identify and explain the landslide effect. As will become apparent in future sections, our modeling approach differs from those described above in at least two key ways that make the cause of the landslide effect more evident. First, instead of assuming a known shortage cost, we calculate base stock levels in order to meet a user-specified service level target. In our experience, this is common practice in industry. Few companies are able to quantify the cost of a shortage with any confidence and most managers are more comfortable planning in terms of service. In fact, companies often negotiate service level targets with their supply chain partners and face financial penalties if they miss these targets. The targets typically remain

steady over the year. The second key difference in our approach is that we focus on safety stock as our planning parameter. The theoretical models described above focus on base stock and treat safety stock as an afterthought, if at all, while the simulation-based papers determine safety stock but focus on macro issues like total system cost. Safety stock is the primary inventory input for most commercial production-planning systems. It is critical to understand the timing with which safety stock values change in relation to base stock levels in order to understand the landslide effect.

3. Current Practice

In our experience working with a number of companies, the most common planning approach for items with seasonal demand is a Days of Supply (DOS) inventory target. Companies that employed a DOS target prior to our involvement include, but are not limited to, Dell, Intel, Microsoft, and Stanley Black & Decker. As the name implies, under a DOS approach the desired safety stock level is expressed in days. In each planning period, the target safety stock in units is found by summing demand over the specified number of days. This approach has the intuitively pleasing property that it self-adjusts with demand. As demand grows or shrinks so does the safety stock target. It can also be defended mathematically. If the relative uncertainty (coefficient of variation) of demand remains constant and only the mean fluctuates by season, then the safety stock that results from classic textbook equations like $z\sigma\sqrt{T}$ will be the same for each season when expressed in DOS.

In order to convert a DOS inventory target into units, one must specify the method used to determine each day of supply. One could use the forecast for the current period, the forecasts for upcoming periods, actual demand from the most recent periods, or some combination of these. While we have encountered different approaches at different companies, the dominant approach is commonly referred to as forward-coverage. Under a forward-coverage approach, a DOS safety stock target is converted to units by summing the daily forecasts over that number of days into the future, starting with the next day. For example, if the DOS target is 20 days, then the safety stock target for day 30 will be the sum of the forecasts for days 31 through 50. This approach is intuitively pleasing since it reflects the length of time, on average, that the safety stock will cover forecasted demand if no subsequent replenishments arrive. Alternatively, it can be viewed as the average time it will take to burn off the safety stock.

A forward-coverage DOS safety stock planning approach is supported and even recommended by all

of the planning software vendors with whom we have worked. SAP offers a Safety Days Supply approach as one of the two standard methods in its Supply Chain Management applications (SAP Library 2006). Manugistics uses days of coverage in three of the five safety stock rules in its NetWORKS planning applications (Manugistics Inc. 2002). Both SAP and Manugistics use the forward-coverage method as their default approach for DOS calculations. We have seen forward-coverage DOS implemented at dozens of companies, with specific examples in this study from CNH, Elmer's Products, and Kraft.

Table 1 demonstrates this typical planning approach for a product transitioning from its high season to its low season. The planning system operates in weekly buckets, with 5 days per week in this example and an assumed replenishment lead time of 3 weeks. The user specifies the safety stock target in DOS (15 in this case). The system calculates the safety stock target in units by summing the forecast for the next 15 days (or 3 weeks), starting with the next week. For example, the safety stock target for week two (500) is the sum of the forecasts for weeks three (200), four (200), and five (100). For weeks beyond the end of the horizon, we use the last week's forecast. The system then plans replenishment orders as necessary so that the projected supply on-hand at the end of each week does not fall below the safety stock target. For example, the planned-order release in week one will be received and used to meet demand in week four. This planned-order release for week one is only 100 units because we expect to enter week four with 400 units of safety stock from week three and want to end week four with 300 units of safety stock. Since we expect demand to be 200 in week four, an order of 100 will bring our starting inventory to 500 and ending inventory to 300.

As Table 1 shows, the safety stock targets adjust in anticipation of future demand under a forward-coverage planning approach. In this example, the safety stock targets begin to drop with 3 weeks remaining in the high season and reach their low point in the last week of the high season. The declining safety stock targets drive replenishment decisions a lead time earlier.

Table 1 An Example of the Forward-Coverage Approach for Planning Safety Stock Units

Season Week	High				Low			
	1	2	3	4	5	6	7	8
Forecast	200	200	200	200	100	100	100	100
Safety stock (DOS)	15	15	15	15	15	15	15	15
Safety stock (Units)	600	500	400	300	300	300	300	300
Planned-order release	100	100	100	100	100	100	100	100
Planned-order receipt	200	100	100	100	100	100	100	100

4. Nonstationary Inventory Mathematics

The practical planning approach described above is known as an adaptive base stock inventory policy in the academic literature. Plans are reviewed periodically (most often weekly), and replenishment orders are calculated to bring the inventory position (on-hand plus on-order) up to a target level. The target level, or base stock, is a function of the forecasts and safety stock targets. Since the forecasts and safety stock targets may change from one time period to the next, the base stock level will adapt to changes in demand.

In this section, we describe the correct mathematics for calculating safety stock levels under an adaptive base stock policy with service level targets. Our results are not new but are presented in a form that makes the cause of the landslide effect more clear. We borrow much of our presentation from Neale and Willems (2009). For ease of exposition we assume a review period of one time unit and let T represent the deterministic replenishment lead time. We let $d(a, b)$ denote the demand over the time interval $(a, b]$ and assume that demand in period t has mean $\mu(t)$ and standard deviation $\sigma(t)$. The goal in setting inventory targets is to achieve a service level of $\alpha(t)$ in period t . The service level is defined as the probability of fulfilling all demand in the period.

We begin by deriving the inventory on-hand at time t . At time t , the order placed in period $t - T$ will have just arrived. This order, when placed in period $t - T$, brought the inventory position in period $t - T$ up to the adaptive base stock level $B(t - T)$. Since all orders prior to period $t - T$ have also arrived by period t , the inventory position $B(t - T)$ represents the total supply to inventory by period t net of all demand up to and including period $t - T$. The total demand filled from inventory since period $t - T$ is $d(t - T, t)$. Combining these observations, we can express the inventory on-hand in period t , $I(t)$, as the difference between the total supply received and total demand filled:

$$I(t) = B(t - T) - d(t - T, t). \quad (1)$$

In order to avoid a stock-out in period t , we require $I(t) \geq 0$. From Equation (1), we see that for this to occur with the desired probability, $\alpha(t)$, we need:

$$\Pr\{d(t - T, t) \leq B(t - T)\} = \alpha(t).$$

If we assume independence over time and that lead time demand can be approximated by a Normal distribution, then we get the following equation for the adaptive base stock level in period $t - T$:

$$B(t - T) = \sum_{i=1}^T \mu(t - T + i) + \Phi^{-1}(\alpha(t)) \sqrt{\sum_{i=1}^T \sigma^2(t - T + i)}, \quad (2)$$

where $\Phi^{-1}(x)$ represents the inverse of the standard Normal cumulative distribution function. We observe that the adaptive base stock level in period $t - T$ must be large enough to cover demand over the upcoming lead time (periods $t - T + 1$ to t).

The safety stock level, $SS(t)$, is defined as the expected amount of inventory on-hand at time t . It represents the extra “buffer” inventory that is required on top of forecasted demand in order to achieve the service level. Taking the expected value of Equation (1) and applying Equation (2) we get the following equation for the safety stock level:

$$SS(t) = \Phi^{-1}(\alpha(t)) \sqrt{\sum_{i=1}^T \sigma^2(t - T + i)}. \quad (3)$$

The key observation from this result is that the variance of demand in periods $t - T + 1$ through t drives the safety stock at time t . That is to say the safety stock level should be a function of demand parameters over the *preceding* replenishment lead time. This differs from the base stock level which changes in anticipation of *upcoming* demand. The difference is because the base stock level triggers a replenishment order that does not result in safety stock *on-hand* until T periods later. In other words, we plan for the safety stock in advance, but it does not materialize until the end of the replenishment lead time. Since commercial production-planning systems use safety stock on-hand as their input parameter, it is important to get this timing correct. We note that this basic relationship holds even if we extend the model to include a nonunit review period, stochastic lead time, or percentage fill rate measure of service.

While this result may seem obvious to academics who are familiar with nonstationary inventory models, it is initially counterintuitive for most practitioners. Practitioners find it odd that safety stock is calculated as a function of the demand that comes before it, not after it. What they fail to appreciate is that at the time the order is placed to produce the safety stock for a period, the demand that will occur over the lead time before that period is still unknown. This unknown demand determines the amount of supply carried into the period and hence the expected inventory on-hand or safety stock for that period.

To summarize, the order to produce the safety stock for period t must be placed at time $t - T$. As shown

above, this safety stock for period t should be a function of the demand parameters from time $t - T$ to t (i.e., the preceding lead time). Since, at time $t - T$, we do not know the actual demand from $t - T$ to t , we use the forecast for these periods to estimate the mean demand and historical data to estimate the demand uncertainty. It is better to use forecasts for the mean demand than recent actuals since demand is nonstationary (typically seasonal) and often predictably different than recent actuals. This approach is in contrast to a forward-coverage approach that calculates the safety stock for period t as a function of the forecasts for period $t + 1$ and beyond.

In our presentation above, we made a couple of simplifying assumptions that deserve further explanation. First, Equation (1) assumes that it is always possible to place an order at time $t - T$ to bring the inventory position to $B(t - T)$. If we allow returns (i.e., negative orders), then this is true. When returns are not possible, Equation (1) may understate the inventory on-hand. The inventory targets in Equations (2) and (3), however, still apply. Second, we assume that lead time demand can be approximated by a Normal distribution. When demand is highly uncertain, this suggests that demand can be negative. However, since service level targets typically exceed 95%, our equations are based on the positive tail of this distribution. In both cases, we made these assumptions to keep the model simple and focus our attention on the fundamental causes of the landslide effect.

In Table 2, we apply the correct nonstationary mathematics to the simple planning example from Table 1. We assume a lead time of 3 weeks, weekly coefficient of variation of demand of 74.5%, and service level target of 99%. We selected these parameters to produce the same safety stock target of 15 DOS for each season that was used in Table 1. The targets slightly exceed 15 DOS during the transition between seasons due to the manner in which heterogeneous variances combine. However, the key point in this case is that the safety stock units in each week are calculated via Equation (3) and consequently are a function of the forecasts over the preceding replenishment lead time of 3 weeks (including the current week). For example, the forecasts in weeks three (200), four (200), and five (100) drive the safety stock in week five (520). We use the first week’s forecast for all weeks prior to the start of the horizon. Note that the safety stock units (and consequently planned-order receipts) remain steady through the end of the high season and only begin to drop as they enter the low season. Within the low season the safety stock units decrease gradually until the high season demand is no longer found in the preceding replenishment lead time.

Table 2 Applying the Correct Nonstationary Math to the Example from Table 1

Season Week	High				Low			
	1	2	3	4	5	6	7	8
Forecast	200	200	200	200	100	100	100	100
Safety stock (DOS)	15.0	15.0	15.0	15.0	15.6	15.9	15.0	15.0
Safety stock (Units)	600	600	600	600	520	424	300	300
Planned-order release	200	20	4	0	76	100	100	100
Planned-order receipt	200	200	200	200	20	4	0	76

5. Understanding the Landslide Effect

Comparing the safety stock units for the stylized example in Tables 1 and 2, the cause of the landslide effect becomes clear. As our nonstationary model established, safety stock targets should be a function of demand forecasts over the preceding replenishment lead time (backward-looking) when trying to maintain a service level target in every time period. However, under a forward-coverage DOS approach, safety stock targets are calculated using upcoming demand forecasts (forward-looking). This fundamental disconnect leads to a mismatch in the timing of supply and demand that causes the landslide effect. The forward-coverage approach lowers the safety stock targets too soon when transitioning from a high season to a low season. It begins to drop the targets while still in the high season as its forward-looking window picks up more and more of the low season demand. This premature drop in safety stock leads to the lower inventories and poor service experienced by companies at the end of their high seasons.

It may in fact be desirable to lower inventories and service levels at the end of a high season in order to reduce the risk of carrying large inventories into a low season. However, such an approach should be planned carefully and systematically. Target service levels should only be reduced when the expected savings in inventory exceed the expected costs of a stock out. The forward-coverage DOS approach does not evaluate any such tradeoff. Instead it reduces inventories haphazardly. Further, most practitioners are not able to quantify the costs of a stock out with any confidence. Instead they typically set a relatively high service level target (95% to 99% is common), often negotiated with their supply chain partners, and try to maintain this target across seasons. Consequently, the landslide effect is definitely undesired behavior.

We note that the opposite situation holds when transitioning from a low season to a high season. A forward-coverage DOS approach will again change its safety stock targets prematurely, this time raising the targets while still in the low season. While less damaging than the landslide effect since in practice the penalties for shortage and surplus are not

symmetric, this reverse landslide effect results in unnecessarily high inventories and cost.

5.1. An Analytical Model

To gain further insight into the drivers and potential magnitude of the landslide effect, we develop an analytical model for a simple case. Consider a product with two seasons ($j = 1, 2$). Let μ_j and σ_j denote the (nonzero) mean and standard deviation of demand for each period in season j . Let $r = \mu_2/\mu_1$ represent the ratio of the mean demand for season two compared to season one. We assume that the coefficient of variation of demand is the same for each season (i.e., $\sigma_1/\mu_1 = \sigma_2/\mu_2 = C$). As in section 4, we assume a unit review period and deterministic lead time $T > 0$. We also assume that demand over lead time follows a Normal distribution. Finally, we assume a stationary service level target α for all periods with $\alpha > 50\%$ (otherwise, safety stock would not be necessary).

Under these assumptions, most companies would calculate a DOS safety stock target using the classic textbook equation $z\sigma\sqrt{T}$, where $z = \Phi^{-1}(\alpha)$. Note that this equation produces the same DOS target for both seasons, namely $D = z\sigma_1\sqrt{T}/\mu_1 = z\sigma_2\sqrt{T}/\mu_2 = zC\sqrt{T} > 0$. A forward-coverage approach would then convert this safety stock target to units in each period by summing the upcoming forecasts. Mathematically, if $SS^F(t)$ represents the safety stock units in period t under a forward-coverage approach, then:

$$SS^F(t) = \sum_{i=1}^D \mu(t+i).$$

For ease of exposition, we assume that D is an integer. Our results extend without modification to the case of a noninteger DOS target.

We are interested in comparing this forward-coverage safety stock value to the correct nonstationary result, $SS^N(t)$, as given by Equation (3). In particular, we will study the ratio:

$$\frac{SS^F(t)}{SS^N(t)} = \frac{\sum_{i=1}^D \mu(t+i)}{z\sqrt{\sum_{i=1}^T \sigma^2(t-T+i)}}. \quad (4)$$

Suppose the change in season occurs immediately after period s (i.e., period s is the last period in season one). Then it can be easily shown that $SS^F(t)/SS^N(t) = 1$ for $t \leq s - D$ or $t \geq s + T$. In other words, the two approaches result in the same safety stock level for periods far enough before or after the change of season. The region of interest for the landslide effect is $s - D + 1 \leq t \leq s + T - 1$ which we will refer to as the transition window. To evaluate Equation (4), it is helpful to divide the transition window into two parts.

Case 1: $s - D + 1 \leq t \leq s$. Let $b = s - t$ represent the number of periods before the final period of season one ($b = 0, 1, \dots, D - 1$). Then Equation (4) becomes

$$\frac{SS^F(t)}{SS^N(t)} = \frac{b\mu_1 + (D - b)\mu_2}{z\sqrt{T\sigma_1^2}}.$$

Substituting $\mu_2 = r\mu_1$ and $D = z\sigma_1\sqrt{T}/\mu_1$ and simplifying, we get

$$\frac{SS^F(t)}{SS^N(t)} = r + \frac{b(1 - r)}{D}. \quad (5)$$

Case 2: $s \leq t \leq s + T - 1$. Let $a = t - s$ represent the number of periods after the final period of season one ($a = 0, 1, \dots, T - 1$). Then Equation (4) becomes

$$\frac{SS^F(t)}{SS^N(t)} = \frac{D\mu_2}{z\sqrt{a\sigma_2^2 + (T - a)\sigma_1^2}}.$$

Substituting $\sigma_1 = \sigma_2/r$ and $D = z\sigma_2\sqrt{T}/\mu_2$ and simplifying, we get

$$\frac{SS^F(t)}{SS^N(t)} = \sqrt{\frac{Tr^2}{a(r^2 - 1) + T}}. \quad (6)$$

We can now use Equations (5) and (6) to prove the following results. All proofs are provided in the Appendix at the end of the paper.

PROPOSITION 1. *For the transition window from $D - 1$ periods before the end of a season until $T - 1$ periods after the end of that season:*

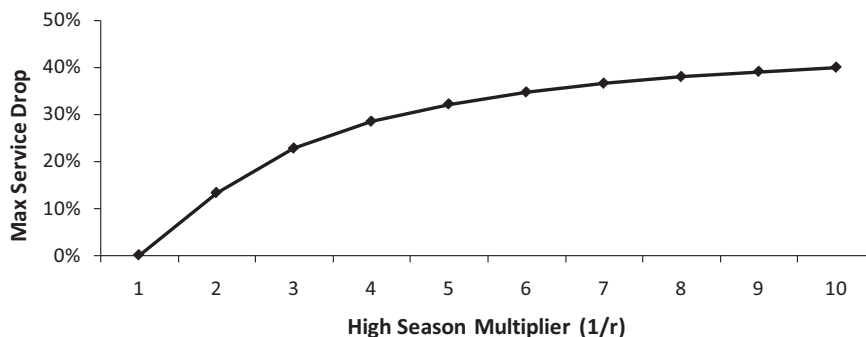
- (a) *If $r < 1$, then $SS^F(t)/SS^N(t) < 1$. In other words, when transitioning from a high season to a low season, the forward-coverage approach results in too little safety stock in each period of the transition window. The greatest error (i.e., smallest ratio) occurs in the final period of the high season at which point $SS^F(t)/SS^N(t) = r$.*
- (b) *If $r > 1$, then $SS^F(t)/SS^N(t) > 1$. In other words, when transitioning from a low season to a high season, the forward-coverage approach results in too much safety stock in each period of the transition window. The greatest error (i.e., largest ratio) occurs in the final period of the low season at which point $SS^F(t)/SS^N(t) = r$.*

Proposition 1 confirms the landslide effect and identifies the last period before the change of season as the most problematic. During this final period, the forward-coverage safety stock is based entirely on demand forecasts from the next season while the correct mathematics depend only on demand forecasts from the previous season. At this time the disconnect is most severe. Since both approaches result in the same DOS target within each season, the ratio between their safety stock units in this period is the same as the ratio between their forecasts (i.e., mean demands). We can also use Proposition 1 to make the following statement about service levels during the landslide effect.

PROPOSITION 2. *The worst case expected service level when transitioning from a high season to a low season via forward-coverage occurs in the final period of the high season and is equal to $\Phi(r \times \Phi^{-1}(\alpha))$.*

Proposition 2 allows us to characterize the maximum service drop that results from the landslide effect, as shown in Figure 3. For our simple demand

Figure 3 Worst Case Service Drop for Varying Degrees of Seasonality ($\alpha = 98\%$)



pattern, a product with a service target of 98% and high season multiplier (μ_1/μ_2) of two would experience a drop in service of almost 15 percentage points at the end of the high season. A product with the same service target but a high season multiplier of 10 would experience a drop in service of approximately 40 percentage points. Clearly, the landslide effect can be significant. The following proposition further describes the conditions under which the effect is most severe.

PROPOSITION 3. *The magnitude of the safety stock and service level errors in each period under forward-coverage is increasing in $r > 1$, decreasing in $r < 1$, and nondecreasing in T and C .*

Proposition 3 helps us understand the drivers of the landslide effect. Products with high seasonality, long lead times, and high demand uncertainty are most at risk. Intuitively, the longer the lead time, the farther forward a forward-coverage approach will look due to increased safety stock when it should be looking farther backward. Similarly, demand uncertainty increases the forward reach of a forward-coverage approach, thus increasing the misalignment. Finally, large or small values of the ratio between demand in each season mean the mismatch in supply and demand is more severe. Interestingly, we cannot make a claim about the impact of the target service level, as a higher target could exacerbate or alleviate the service error depending on the values of the other parameters. While our simple analytical model of the landslide effect does not account for additional real-world complexities such as capacity constraints, stochastic lead times, and nonunit review periods between stages, it does suggest their likely impact. Since each of these factors increases either the length or uncertainty of the total replenishment lead time, the landslide effect is almost surely exacerbated by them.

5.2. A Case Study

Of course, real-world demand patterns are more complicated than the simple case considered in our

analytical model. In order to examine the magnitude of the landslide effect under more realistic conditions, we conduct a simple simulation using real data from Elmer's Products.

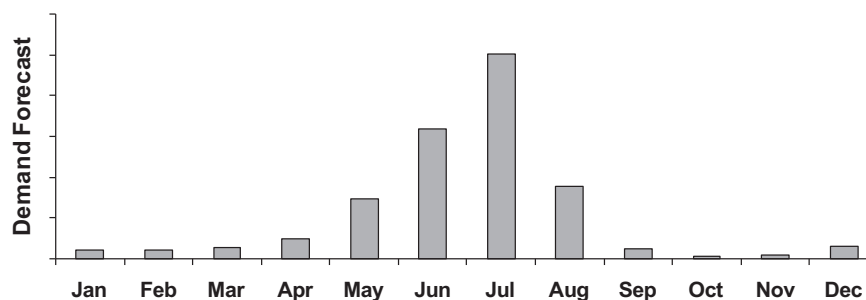
Elmer's Products is a privately held company headquartered in Columbus, Ohio with approximately \$200 million in annual sales. Elmer's is best known for its ubiquitous school glue, but it also sells other types of glues and fillers (e.g., wood glue and caulking material) and various forms of arts and crafts (e.g., finger paints, rubber cement, and glitter). Not surprisingly given the nature of its products, much of Elmer's demand is seasonal. Figure 4 shows the annual forecast for Elmer's highest volume SKU, a 48-pack of four-ounce school glue. Demand for school-related products typically peaks each year during the back-to-school selling season from May to August, as seen in Figure 4.

Elmer's manufactures roughly half of its products at its facilities in New York and Ontario and outsources the rest, primarily to Asia. The company stocks finished product in one of three North American warehouses. Elmer's sells its products through the retail channel, ranging from big box retailers like Wal-Mart, Staples, and Home Depot to hundreds of smaller retailers including mom-and-pop craft shops. Its goal in setting inventory targets is to achieve service levels that are negotiated with its largest retail partners.

Due to the seasonal nature of its demand, Elmer's uses a DOS safety stock approach. Two different safety stock targets are used, one for the back-to-school season and the other for outside of this high season. The separate targets are necessary to capture seasonal differences in demand uncertainty and lead time. Elmer's is in frequent contact with its retail partners during the critical back-to-school season and consequently its forecast accuracy is higher during this season. Conversely, lead times for some outsourced products are longer during the back-to-school season because of limited capacity in the supply chain.

In order to test the impact of the landslide effect at a seasonal company like Elmer's, we simulated the

Figure 4 Demand Forecast for Elmer's School Glue



performance of a forward-coverage DOS approach using data provided by Elmer's. We included roughly 700 finished stock keeping units (SKUs) in our analysis, representing approximately 85% of Elmer's total demand. We did not include SKUs that are made-to-order or had insufficient data (for example, new or soon-to-be discontinued SKUs). We simulated performance over a horizon of 1 year into the future using monthly forecasts from Elmer's. We used two and a half years of sales and forecast history to estimate demand uncertainty by season. For each SKU at each of their three warehouses, we first calculated the appropriate DOS target using the classic $z\sigma\sqrt{T}$ safety stock equation. We calculated a different target for each season taking into account the seasonal differences in demand uncertainty and lead time. For each week in the horizon, we then calculated the corresponding forward-coverage safety stock value and the expected service level it would provide.

The results of our analysis are displayed in Table 3. The service level target for each month is a constant 96% as set by Elmer's. Assuming that historical demand uncertainties are an accurate predictor of future uncertainties, the nonstationary model described in section 4 exactly achieves these targets in every month. As the results in Table 3 clearly demonstrate, a forward-coverage approach does not. Under a forward-coverage approach, the weighted average service level for the year is more than three points lower due to the landslide effect. The forward-coverage service level reaches its low point of 86.5% in August, the final month of the back-to-school season. This represents a dip of almost 10 points in service level. Service is also low in the months around the transition out of the back-to-school season (July and September), as we would expect due to the premature drop in safety stock levels that results from the forward-looking approach. For individual SKUs, the effect can be more severe. Some highly seasonal SKUs like the Office Depot-branded jumbo glue stick experienced an average annual service level below 80% and a minimum service level close to 50% at the end of the high season. We note that the average amount of inventory for the year is basically the same between the forward-coverage approach and nonstationary model. This is because both approaches produce similar inventory targets when given the same input parameters but the targets appear at different times of the year.

In addition to the drop in service when transitioning from the high to low season, we also see evidence of the reverse landslide effect in Table 3. Elmer's experiences smaller surges in demand in March for some of its woodworking products like wood glues and fillers and in December for some of its craft products like glitter glues and rubber cement. In the month before these surges, we see service levels slightly higher than the target of 96%. This is because these are the middle of three sequential months of increasing demand and the forward-looking targets pick up the higher demand and raise inventory levels sooner than they should. We do not see similar behavior in May because Elmer's switches from its low season DOS targets to its back-to-school DOS targets in this month. Since demand uncertainty is generally lower in the back-to-school season, most DOS targets decrease in May, thus muting the reverse landslide effect.

In order to study the sensitivity of the landslide effect at Elmer's to factors such as seasonality, lead time, and uncertainty, we isolated the service level results for certain subsets of the 700 total SKUs. To explore the impact of seasonality, we identified the 140 SKUs with an average demand during the back-to-school season at least twice as large as their average demand outside of this high season. These SKUs represent 20% of the SKU count but over 35% of the total volume in our study. Table 4 reports the service levels achieved by the forward-coverage DOS approach for these highly seasonal SKUs. As expected, both the landslide and reverse landslide effects are more severe. In spite of the 96% target, service dips below 80% in August and September and peaks above 98% in February. This is because the large difference in demand between seasons accentuates the mismatch between supply and demand. Overall, the weighted average service level for the year drops an additional two points for highly seasonal SKUs.

To study the impact of lead time, we identified the 150 SKUs that are outsourced to Asia with lead times greater than or equal to 60 days. These approximately 21% of the SKUs represent just under 15% of the total volume in our study, indicating that higher volume SKUs tend to be manufactured in-house. Table 4 reports the service levels achieved by the forward-coverage DOS approach for these outsourced SKUs. While we again see evidence of the landslide effect, it

Table 3 Service Level Results for Forward-Coverage DOS at Elmer's

Month	Forecast	Service level (%)
January	361,022	92.9
February	455,114	96.9
March	634,266	94.9
April	496,689	94.6
May	644,407	94.1
June	1,078,328	94.1
July	960,097	90.7
August	703,521	86.5
September	501,487	90.9
October	384,123	93.1
November	425,644	96.1
December	549,673	92.4
Average	599,531	92.8

Table 4 Sensitivity Analysis for the Landslide Effect at Elmer's

Month	Seasonal SKUs		Outsourced SKUs		Uncertain SKUs	
	Forecast	Service level (%)	Forecast	Service level (%)	Forecast	Service level (%)
January	60,130	94.4	65,077	90.9	21,172	90.8
February	102,594	98.3	69,871	94.6	31,890	96.0
March	173,164	94.7	99,230	95.0	45,258	94.4
April	130,303	96.5	72,147	94.5	36,095	95.8
May	269,805	96.4	87,030	93.6	54,567	93.6
June	573,781	94.1	130,039	94.0	121,617	90.5
July	571,690	89.6	99,203	90.8	136,023	86.3
August	349,797	78.6	85,259	89.4	95,699	73.0
September	97,526	79.0	80,147	88.8	32,351	81.5
October	49,358	89.1	71,174	92.0	24,108	91.9
November	57,398	96.6	81,789	94.7	29,003	91.2
December	93,966	92.2	107,675	92.9	33,004	88.2
Average	210,793	90.8	87,387	92.7	55,066	87.7

is not any more severe than what we saw in Table 3 for all of Elmer's SKUs. Although we would normally expect longer lead times to exacerbate the landslide effect, we do not see this behavior at Elmer's due to the SKUs that Elmer's has selected to outsource. The outsourced SKUs, such as Krazy Glue and tape, have lower seasonality than a typical Elmer's SKU. As a result, the increase in the landslide effect due to longer lead times is offset by a corresponding decrease due to lower seasonality.

To study the impact of demand uncertainty, we identified the 165 SKUs with a weighted average monthly coefficient of variation of forecast error of at least 80%. These approximately 24% of the SKUs represent only 9% of the total volume in our study, which is not surprising since low volume SKUs are often the most difficult to forecast. Table 4 reports the service levels achieved by the forward-coverage DOS approach for these highly uncertain SKUs. The landslide effect is more severe for these uncertain SKUs than for any other subset of Elmer's SKUs that we have studied. The service level hits a low of 73% in August and begins to drop earlier in the back-to-school season than the other scenarios due to the large forward-looking DOS targets that result from the high demand uncertainty. The severity of the landslide effect is driven by the fact that the uncertain SKUs also tend to be highly seasonal and the combination amplifies the error as predicted by Proposition 3. Consequently, for Elmer's, the SKUs with high demand uncertainty are most at risk with an expected annual service level of only 87.7%. In contrast, the nonstationary model of Section 4 achieves the targeted 96% service level in each month for all cases.

6. Conclusions

In this study, we described a common issue encountered by companies facing seasonal demand. We

presented real examples of the unintended drop in service levels that can occur at the end of a high season and explained why this drop occurs by comparing the pervasive forward DOS planning heuristic to the correct nonstationary inventory math. We showed that the problem is exacerbated by high seasonality, long lead times, and high demand uncertainty and can lower a company's aggregate annual service level by multiple points.

Companies can avoid the landslide effect by using the correct nonstationary math to set safety stock targets in their planning systems. The benefits can be significant. Microsoft, which experiences seasonal demand for its Xbox video game consoles, Zune digital music players, and personal computer hardware, increased service levels by 6–7% while simultaneously decreasing inventory by almost 20% by implementing inventory optimization software with a nonstationary model (Neale and Willems 2009). A simpler solution is also possible. If utilizing a DOS rule-of-thumb, much of the landslide effect can be avoided by using the demand forecasts over the preceding lead time to convert the DOS target into units. Specifically, this requires the following steps:

1. Calculate the average forecast from time $t + 1$ to $t + T$ (where t represents an arbitrary time period and T represents the replenishment lead time).
2. Use this average forecast to convert the DOS target into a safety stock unit target for time $t + T$.
3. Use this safety stock unit target to calculate the planned-order release for time t .

Such an approach will deviate from the optimal targets during the transition window between seasons. However, in our experience these deviations are typically small and certainly less significant than the misalignment caused by utilizing a forward-coverage

approach. This will require a change in mindset for practitioners. However, we note that the forward-coverage approach could still be used for reporting inventory metrics even if this different approach is used for calculating inventory targets.

The ability to understand and avoid the landslide effect would be advanced by further investigation in at least two areas. First, it would be useful to study the drivers and magnitude of the landslide effect using models that specifically account for constraints on capacity. These constraints are common in seasonal businesses since it is typically cost prohibitive to carry peak-demand capacity year-round. Second, it would be helpful to develop methods to assist practitioners in setting appropriate service level targets, particularly when transitioning from a high to a low season. It likely makes sense to lower service near the end of a high season to reduce the risk of carrying large inventories into the low season. But how much should service be lowered and how can this be sold to one's supply chain partners? A smoothing approach based on Zipkin's (1989) characterization of the optimal policy as an exponentially weighted moving average of the myopic critical fractiles may be a good place to start.

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Appendix: Proofs of Propositions

PROOF OF PROPOSITION 1. *Case 1a:* Suppose $r < 1$ and $s - D + 1 \leq t \leq s$. We can rewrite the right-hand side of Equation (5) as $(Dr + b(1 - r))/D$. We note that $Dr + b(1 - r) < Dr + D(1 - r) = D$ since $0 \leq b < D$ and $1 - r > 0$. This establishes that $SS^F(t)/SS^N(t) < D/D = 1$. The first derivative of Equation (5) with respect to b is $(1 - r)/D$. Since $(1 - r)/D > 0$, Equation (5) is increasing in b and assumes its minimum value at $b = 0$, at which point $SS^F(t)/SS^N(t) = r$.

Case 2a: Suppose $r < 1$ and $s \leq t \leq s + T - 1$. We note that $T(r^2 - 1) < a(r^2 - 1)$ since $a < T$ and $r^2 - 1 < 0$. This implies that $Tr^2/(a(r^2 - 1) + T) < 1$ and thus $SS^F(t)/SS^N(t) < \sqrt{1} = 1$. The first derivative of Equation (6) with respect to a is $(1 - r^2)(Tr^2)^{1/2}/2(a(r^2 - 1)T)^{3/2}$. We note that $a(r^2 - 1) \geq -a$ since $r^2 - 1 \geq -1$ and $a \geq 0$. Since $a < T$ implies $-a > -T$, we know that $a(r^2 - 1) > -T$ and the denominator of the derivative is positive. The numerator is also positive since $r < 1$ implies

$1 - r^2 > 0$. Consequently, Equation (6) is increasing in a and assumes its minimum value at $a = 0$, at which point $SS^F(t)/SS^N(t) = r$.

This proves part (a). The proof of part (b) is similar and we omit the details in the interest of space.

PROOF OF PROPOSITION 2. The service level provided by a forward-coverage approach can be calculated as

$$\begin{aligned} \alpha^F(t) &= \Pr \left\{ d(t - T + 1, t) \leq SS^F(t) + \sum_{i=1}^T \mu(t - T + i) \right\} \\ &= \Phi \left(SS^F(t) / \sqrt{\sum_{i=1}^T \sigma^2(t - T + i)} \right) \quad (A1) \\ &= \Phi(z \times SS^F(t)/SS^N(t)) \end{aligned}$$

From part (a) of Proposition 1, the ratio $SS^F(t)/SS^N(t)$ assumes its minimum value of r in the final period of the high season. At this time, the service level will be $\Phi(z \times r) = \Phi(r \times \Phi^{-1}(x))$. \square

PROOF OF PROPOSITION 3. The greater the ratio SS^F/SS^N deviates from one in either direction, the greater the safety stock error. Consequently, we will study the safety stock error term $\varepsilon_{ss} = |SS^F/SS^N - 1|$.

Case 1a: Suppose $r < 1$ and $s - D + 1 \leq t \leq s$. From Theorem 1, we know $SS^F/SS^N < 1$, so $\varepsilon_{ss} = 1 - SS^F/SS^N$. Applying Equation (5) we get $\varepsilon_{ss} = 1 - r - b(1 - r)/D$. We note that for a given $b > 0$, $\delta(\varepsilon_{ss})/\delta D = b(1 - r)/D^2 > 0$ so the error is increasing in $D = zC\sqrt{T}$ and thus increasing in C and T . For $b = 0$, $\varepsilon_{ss} = 1 - r$ and the error does not depend on C or T . For a given $b \geq 0$, $\delta(\varepsilon_{ss})/\delta r = -1 + b/D < 0$ since $b < D$, so the error is decreasing in r .

Case 1b: Suppose $r > 1$ and $s - D + 1 \leq t \leq s$. In this case $SS^F/SS^N > 1$. Again applying Equation (5) we get $\varepsilon_{ss} = r + b(1 - r)/D - 1$. For a given $b > 0$, $\delta(\varepsilon_{ss})/\delta D = b(r - 1)/D^2 > 0$ so the error is again increasing in $D = zC\sqrt{T}$ and thus increasing in C and T . For $b = 0$, $\varepsilon_{ss} = r - 1$ and the error does not depend on C or T . For a given $b \geq 0$, $\delta(\varepsilon_{ss})/\delta r = 1 + b/D > 0$, so the error is increasing in r .

The proof that ε_{ss} is decreasing in $r < 1$, increasing in $r > 1$, and nondecreasing in C and T when $s \leq t \leq s + T - 1$ is similar so we omit the details in the interest of space.

This proves the result for the safety stock error. From Equation (A1), we know that the service level under forward coverage, $\alpha^F(t)$, is an increasing function of the safety stock ratio SS^F/SS^N . Consequently, using the same approach as above, it is not difficult to

show that the service level error term, $\varepsilon_{SL} = |\alpha^F(t)/\alpha - 1|$, is decreasing in $r < 1$, increasing in $r > 1$, and nondecreasing in C and T . Interestingly, while the safety stock ratio and error are nondecreasing in α , the same result does not hold for the service level error due to the presence of α in the denominator of ε_{SL} . \square

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