MANAGEMENT SCIENCE
Vol. 15, No. 8, April, 1969
Printed in U.S.A.

EXTENSIONS OF BOWMAN'S THEORY ON MANAGERIAL DECISION-MAKING*†

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Bowman has developed a theory which claims that a manager makes good decisions on the average but that he may exhibit a high variance in his behavior. This paper proposes three general criteria which must be satisfied for this theory to have validity. These tests are then used to evaluate a production planning model for an electronics firm where the rules are based on "average" past behavior. Several interesting results emerge when comparing the production plans based on an "average" rule with the actual plans for the company. When the manager has limited information regarding future sales, as when he develops his initial production estimates, then a plan based on average behavior performs considerably better than actual initial plans. However, when environmental cues provide reliable information on future sales of specific items (as when revisions are made) then actual decisions are clearly superior to those suggested by an average rule. This case study offers insight into the potential usefulness as well as the limitations of Bowman's hypothesis. Suggestions for future research in this general area conclude the discussion.

Since Bowman [1] proposed his theory of managerial decision-making, several empirical studies have provided evidence as to the applicability of this approach in the context of an industrial firm. (See Gordon [2], Hurst [5], Jones [7].) This paper represents another step in this direction via a case study on planning production in an electronics firm. My primary interest, however, is with the broader aspects of the theory. I will thus propose three criteria for Bowman's hypothesis and will stress both the potential usefulness and limitations of this approach for improving the process of decision-making in a firm. Without the study of production planning at Recordette, however, I would never have been in a position to investigate these latter questions.

The paper can be conveniently divided into three parts. The first section briefly describes Bowman's approach via a production scheduling example and formulates the three general criteria which must be satisfied before the theory has any validity. In the second and third sections, I consider the production planning problem at the Recordette Company, present four models for comparison and evaluate the results. The final portion of the paper offers conclusions on the basis of this case study and raises some questions for future research in this area.

Bowman's Theory of Decision-Making

Description of the Approach

Consider a manager who has to make a decision on a problem which has confronted him many times before (e.g., planning production). He normally will consider only a

* Received August 1967; revised August 1968.

† This paper is based on my doctoral dissertation for the Department of Economics at M.I.T I would like to express my appreciation to Professors Edward Bowman, Edwin Kuh and Robert Solow for their helpful comments at various stages of the work. The Recordette Company (fictitious name) kindly let me spend as much time with company personnel as I desired and utilize any production or cost data of interest. Without the financial help from a Ford Dissertation Fellowship it would have been impossible for me to spend the concentrated period of time at Recordette needed to structure their production planning problem. Helpful comments and suggestions on a preliminary version of this paper were provided by Harry Roberts and students in my Production Seminar at the University of Chicago, Graduate School of Business.

limited number of factors which he has found from past experience to be important guides. Thus his average behavior is likely to be geared to a decision rule based on these key variables even though he may not act with this relation consciously in mind. Bowman's theory claims that the decision-maker tends to be erratic in his actual behavior thus causing unnecessarily high costs to the firm. The variability in behavior arises from selective cues in the organizational environment which are likely to bombard any person making a decision. For example, one type of cue may filter into the production scheduler from sales department personnel pressuring for larger runs to meet a potential order; another may come from top management when they announce that current inventory is too high. If these external stimuli cause the manager to over-respond at times and under-respond at others then a correctly specified decision rule based on his average behavior should lower costs when compared with the actual decisions.

These ideas can be explored further by looking at the problem of scheduling production. After having observed the decision-maker's behavior over a long period of time we may conclude that he considers the following variables in planning production for item i (P_t^i):

$$P_t^i = f[S_t^i, S_{t+1}^i, \cdots, + S_{t+n}^i, (I_N^i - I_{t-1}^i)]$$

where

P_t^i = production of item i in period t,
S_j^i = sales forecast for item i in period j ($j = t, \cdots, t+n$)
I_N^i = normal inventory for item i,
I_{t-1}^i = actual inventory for item i at end of period $t-1$.

No manager would like to be referred to as a computer (otherwise he may be replaced by one) and we are not suggesting that the production scheduler is one. It would be extremely unlikely for him to sit down at his desk and figure out production of each item by a rule such as

$$(1) \quad P_t^i = a_1 S_t^i + \cdots + a_{n+1} S_{t+n}^i + a_{n+2}(I_N^i - I_{t-1}^i) + \epsilon_t^i$$

where the a_j are known coefficients and the error term ϵ_t^i is zero for each item, if (1) is an exact relation. The important point to note for purposes of understanding Bowman's theory is that a decision-maker's behavior may imply such a rule. Presumably when the sales forecasts and inventory figures come to his attention, the manager implicitly weights these data and determines production accordingly. His over- and under-responding regarding certain items are due to environmental factors denoted now by nonzero values of ϵ_j^i in (1).

If the manager considers each of the sales and inventory variables in a linear fashion, then an equation of the form given in (1) would be appropriate. Even if the relationship between the variables was more complicated than a linear one, it would be possible to develop some form of average rule. (See Hurst and McNamara [5].) For purposes of this discussion we will assume that a linear relationship holds. By taking actual past production decisions and statistically regressing these data on the appropriate values of the independent variables we can obtain least square estimates for the a_i, $i = 1, \cdots, n + 2$, which, in effect, specify the average weights implicitly assigned by the manager to each of the factors he considers to be of importance. The statistical rule with its mean estimates of the coefficients should yield lower costs than the current schedule if Bowman's hypothesis, that the variance of actions results in high costs to the firm, is correct.

This heuristic approach to decision-making has a behaviorist flavor in the sense that the manager's *actual* past decisions are used as a basis for determining the coefficients of the independent variables. It parts company with this type of work and comes closer to an operations research approach in its normative suggestion that significant improvement in costs will be forthcoming by eliminating the erratic fluctuations inherent in managerial action. The theory does not suggest that the rule based on average decisions will be the best that one can develop within the context of the organizational constraints. For example, the manager may consistantly underestimate or overestimate his decision variables thus exhibiting a bias in his behavior. The average rule clearly will be most beneficial to a firm when the costs associated with these biases are low in relation to the costs associated with variance.

Criteria and Implicit Assumptions

Several general criteria must be satisfied in order to successfully program the manager's decisions with our rule based on average behavior.

Consistency over time. Implicit in this approach is the assumption that the past is a good description of the future. Expressed in statistical terms, structural changes in the system are not expected. Two types of structural changes must be considered. By far the most serious kind is when there is a shift in the variables important to the decision-maker. For example, increased competition may now lead the manager to consider the variable $\Delta S_{t-1} = S_{t-1} - S_{t-2}$ (i.e., change in sales between the two previous periods) in production planning though he had never done so in the past. In this case, data on past decisions would not be helpful in developing a meaningful rule for the future. For repetitive decisions where there exists a well-established method of procedure it is highly unlikely that such changes in the system will occur, however. If such shifts are observed due either to a new manager who considers other factors or an organizational realignment, then Bowman's approach has little applicability.

A much more likely case of a structural disturbance is the gradual revision of coefficients of existing variables due to changes in the environment or in organizational policy. For example, management may come to the conclusion that the inventory/sales ratio on equipment should be modified because of excessively high overall storage costs. A change of this type would obviously affect the value of the inventory coefficient in the final equation. Fortunately these kinds of situations can be handled quite easily within the context of this theory. One obvious way would be to include only the most recent decisions made by management (e.g., the past year), updating the equation when new information becomes available. A less extreme way of handling the case of changing coefficients over time is to use some form of exponential smoothing where a system of decreasing weights is employed, the most recent management decision receiving the highest value.

Economic significance. This criterion refers to the signs and numerical values of the coefficients in the regression equations. Using the production planning example again, we would expect to obtain a negative sign in front of the inventory variable indicating that level of production moves inversely with amount of stock on hand. Clearly the sales variable should have a positive coefficient. If realistic meaning cannot be attached to the coefficients, then the chances are something is wrong with the model rather than the firm's behavior.

Statistical significance. If the variables in the decision rule are truly important to the manager in influencing his decisions, then its standard error will be low in relation to the regression coefficient. Several other properties are desirable if the least squares estimates are to have properties of unbiasedness and efficiency. If one is working with time series data he would want to make sure that there was little serial correlation between the error terms. For decision rules with more than one independent variable it would be desirable to have a low correlation between the independent variables (i.e., low multicollinearity). If both of these criteria are satisfied then the estimates of the decision rule will be unbiased. To obtain efficient estimates the error term must have a constant variance (i.e., $E(e_t^i)^2 = \sigma^2$), referred to as homoscedasticity. Tests for each of these properties are discussed in Johnston [6], and are illustrated in Appendix A for the production planning model developed for Recordette.

The coefficient of determination (R^2) provides only a rough guide as to the potential success of the model. This value will depend on how much variance the manager actually exhibits in his behavior as well as how accurately his decision-making process has been captured via certain quantifiable variables. Certainly one would hope for an R^2 somewhat greater than zero; a value of $R^2 = 1$, however, would not be desirable in the context of this theory since it would signify that the manager is a human computer exhibiting no random variation in his behavior at all. It would then only be possible to replicate the decision making process via the regression rule rather than to improve upon it. One cannot specify an optimal value of R^2 to strive for in developing a decision rule. Theoretically it is always possible to construct two rules containing different variables which would have the same R^2 but would lead to different decisions and hence differing operating costs.

The ultimate test as to how well the model performs lies with the error variance. If deviations from the regression equation are primarily the result of over and under-responses to environmental cues as the theory would predict, than a rule based on average behavior will be less costly than actual decisions; if the apparent noise is due to positive environmental cues which cannot be captured via quantifiable variables then actual behavior will outperform the computerized regression rule.

Production Planning in an Electronics Firm

Understanding the Decision

The Recordette Company is a medium-sized electronics firm in the Boston area which develops and assembles a number of different medical and industrial recording devices. I have studied the production planning decision for a number of these different recording instruments made at one point in time. Since the problem involves cross-sectional data, there is less chance of structural changes than if a rule had been developed using production plans for one item over time. The schedule is initially drawn up on a quarterly basis four months before the start of the production period, thus leaving enough time to order parts used in assembling the equipment. Some revisions, particularly cut-backs, can be made up until one month before the quarter begins. By looking at each phase of the planning process in more detail, we can better understand the reasons for a potentially high variance in behavior.

Initial Production Plan. Two types of crises are responsible for a large proportion of the initial decisions which deviate significantly from projections based on past data. I have chosen to label these phenomena as the "potential shortage crisis" and "storage crisis."

Potential Shortage Crisis. Sales personnel's exuberance about equipment they are actively promoting often convinces the production manager that these items will sell much better in the future than they have up to now. Lot sizes will thus be increased to avoid an impending "shortage crisis." Action of this nature is most likely to be taken on items where the sales department predicts a rosy future and where inventory on hand is quite low in relation to sales. The decision on these items closely corresponds to the case of over-responding to both internal (inventory/sales ratio) and external (sales potential) factors.

Storage Crisis. If finished goods inventory for a particular item is very high because expected orders have not materialized, a storage crisis has resulted. The production manager frequently reacts to this situation by over-responding to the current high inventory picture and planning a very low level of production for the quarter scheduled to begin in four months. He frequently finds that this proposed lot size is too low so that he must make costly upward revisions in the plan to avoid impending shortages.

These two extreme crises can cause unnecessarily high costs to the firm. Less pronounced fluctuations due to environmental cues may also adversely affect actual behavior. For example, while I was at the firm the sales department advertised a special trade-in on a used recording device for a new machine but neglected to inform the production planning department of their campaign. The promotion successfully induced a rash of new orders which caught production unaware. It was therefore necessary to undertaken costly revisions in the schedule to avoid impending shortages. Another environmental factor which has affected orders and production in the past is the quoted delivery date. Sales may promise immediate shipment to a customer without actually knowing the current status of inventory and scheduled production. As the company is loathe to incur shortages, these sales actions (which at times will induce a number of unexpected orders) may necessitate revisions in the schedule.

The most common cause of slightly erratic behavior is the policy of scheduling only round lot sizes. For most items lost sizes of 25 are scheduled; highly expensive items are produced in lots of 10. Planning and scheduling contends that the uniformity of lots simplifies the paper work in ordering parts and later drawing them from the stockroom. In reality, with a computer available to process the data, it would make no difference what figures were used.[1] A more valid reason for this round lot size procedure may be a desire to reach rapid agreement on planned figures at the joint meetings between sales and production. It would be much harder to make decisions if a wide range of choices were open for consideration. At the same time this policy offers the planning department some excuse for high inventory in the cage. They can contend to top management that "we decided to produce fifty units in order to avoid shortages which might have occurred if a lot of only twenty-five was scheduled. These were the only two options available to us."

The model to be developed will not yield round figures and, except where suggested production is less than minimum lot size, no effort will be made to obtain such uniformity. This is a perfect illustration of the basic difference between Bowman's approach and the behaviorists' techniques. The latter group would be most interested in describing the actual rounding process of lot sizes. The rule based on average behavior attempts to eliminate error variance partly caused by this round lot size constraint; the descriptive phase lies solely in isolating the quantitative factors which influence the decision. It is thus possible to demonstrate to the manager that his actions may imply unnecessarily high costs. If he can be convinced of the advantage of variable lot sizes, he may decide to plan with a little more precision than in the

[1] It is quite likely that before the purchase of a computer in 1960 manual ordering of parts may have been simplified by round figures.

past. Only by formulating a normative model for comparison with current practice can we indicate the efficacy of actual behavior.

Minimal Lot Sizes. If our initial equation yields suggested production less than the minimal lot size during the quarter, then an adjustment will have to be made for this instrument simply because set-up time (i.e., drawing parts from the stockroom) is too expensive to justify smaller batches. Rather than setting an arbitrary figure (e.g. thirteen units) which would divide these marginal items into the zero or twenty-five bracket, a "yes-no" type of rule can be devised. Using only data where the manager scheduled zero or the minimal lot size, a regression can be run with the dependent variable respectively taking on either the value zero or one. By considering the independent variable(s) important in influencing this decision, we can obtain a rule which describes average behavior. This equation will be used to determine whether or not the minimal lot size is produced.

Revised Production Plan. Between the time when the initial plan is formulated and actual production begins several changes can take place.

(a) Some units scheduled for the current quarter may be moved into the following one. The most logical reason for making such a switch is that actual sales have not measured up to expectations and therefore an excessive inventory would exist if this change was not made.

(b) Lot sizes for certain instruments may be increased because sales have exceeded their forecasts.

(c) Some units may be pulled into the production schedule for the last month of the current quarter because of impending shortages.

Although revisions in the schedule can be made throughout the lead-time period, most of the changes are undertaken one month before the quarter is scheduled to begin. To standarize the up-dating procedure without greatly distorting reality, this will be the only time when a revision is permitted.

Variables for Consideration

This section will briefly discuss quantifiable factors which I found to be important in the production planning process. After having attended some of the meetings I was most struck by the infrequent use of past data in making decisions. The only available figures were the latest inventory totals for each instrument and monthly sales during the last half year. It is difficult for an outside observer to determine quantifiable factors influencing the final decision; after carefully listening to comments during the sessions several variables were considered in our regression models. I will list only those which were found to be statistically significant. Thus, I thought that a variable indicating the change in sales between the past two quarters might have been important to the production manager when developing the initial plan. This factor turned out to be statistically insignificant in all the regressions which were run and thus was discarded.

To define these variables explicitly let "t" represent the month when the initial plan is formulated and "v" be the month when revisions are permitted. Superscript "i" refers to instrument i on the schedule. "τ" will represent the quarter for which the plan is being made and $\tau + 1$ the following quarter.

Initial Planning Schedule. Only two quantifiable factors had any importance in determining the initial plan.

(*a*) *Sales During the Past Three Months.* $\sum_{k=1}^{3} S_{t-k}^{i}$. Although sales figures are available on a monthly basis, the planning division preferred to consider quarterly totals since the production plan is initially geared to a three-month period. The aggregation process eliminates a large percentage of short-run monthly movements due to random fluctuations in demand so a truer picture of instrument orders is gained.[3]

(*b*) *Expected Inventory at Beginning of Quarter* τ (I_τ^i). This factor is crucial in determining future production for instrument i. In order to obtain a reasonable estimate for the expected value of inventory, the following data have been utilized.

(i) Inventory on hand at beginning of month t when the initial plan is formulated (I_t^i).

(ii) Production scheduled from month t until the start of quarter ($\sum_{m=0}^{3} P_{t+m}^i$).

(iii) Expected sales during this four month period ($4\sum_{k=1}^{3} S_{t-k}^i/3$). Therefore we see that expected inventory $I_\tau^i = I_t^i + \sum_{m=0}^{3} P_{t+m}^i - 4\sum_{k=1}^{3} S_{t-k}^i/3$.

Minimal Lot Size Model. In obtaining "yes-no" rule for minimum lot size the only quantifiable factor found to be of importance was the expected inventory/sales ratio at the start of quarter τ for each instrument i in this marginal category ($I_\tau^i/\sum_{k=1}^{3} S_{t-k}^i$). Other environmental cues may come into play here but unfortunately it was impossible to express these influences in numerical terms.

Revised Production Plan. The rule for revising the initial plan during month v was based on only one quantifiable factor, expected inventory at the beginning of quarter $\tau + 1$ ($I_{\tau+1}^i$). This figure represents the expected amount on hand at the end of the production period if the initial plan is actually carried out. $I_{\tau+1}^i$ is obtained by the same procedure as I_τ^i except that naturally all figures are updated to represent v rather than t. We therefore see that $I_{\tau+1}^i = I_v^i + \sum_{m=0}^{3} P_{v+m}^i - 4\sum_{k=1}^{3} S_{v-k}^i/3$ where $\sum_{m=0}^{3} P_{v+m}^i$ denotes the scheduled production for this period according to the initial plan. As in the minimal lot size determination, there may be nonquantifiable factors which influence the final decision on whether or not to revise the plan but these vary with each piece of equipment and thus could not be included in any mathematical equation based on cross-sectional data.

Figure 1 depicts the various stages of the planning process and the quantifiable variables which are important to the production manager in making his planning decisions for some item i during the first quarter (January 1–March 31).

Models for Comparison

Several models can be developed to examine the relevance of decision rules based on "average" behavior for production planning at Recordette.

Model 1. A regression rule for the initial plan can be a proxy for the actual production schedule. By precluding revisions over time we are implying that actual changes by the firm were due to variance in behavior on the part of the manager when he initially planned production. If this decision rule outperforms actual company behavior, it will do so under the most rigid possible assumptions.

Model 2. It is possible to compare the results of Model 1 with a production schedule identical to the *actual* initial plan. We will then be able to isolate the effects on costs of erratic behavior four months before the plan is put into effect, since the option of later corrective action through revisions has been ruled out.

[3] The sales department at Recordette felt there was no seasonal pattern for any of their items, particularly in recent years. Before the introduction of the industrial division there used to be some decline in sales during the summer as many doctors took their vacations then. But today with most recorders having both medical and industrial applications, and the great popularity of off-season vacations, the earlier pattern has disappeared. If, in fact, seasonal patterns existed, the decision rule based on average behavior would be biased.

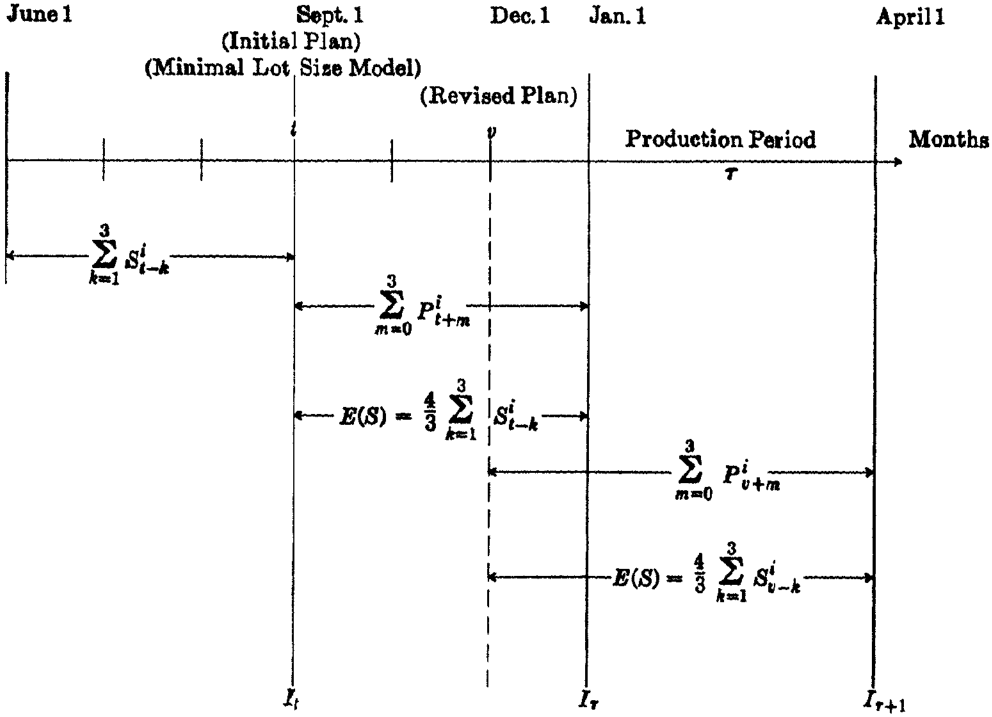

FIGURE 1. Variables important for planning production for item i

TABLE 1

	Initial Plan	Initial and Revised Plans
Average Behavior	Model 1	Model 3
Actual Behavior	Model 2	Model 4

Model 3. A model based on average behavior but geared to more accurately replicating the planning process by including rules for initial planning production as well as for revising the schedule before the quarter begins. If some unexpected change in sales behavior occurred during the four months lead time and was the basis of a scheduled change, then this revision would be incorporated in Model 3 but not in Model 1.

Model 4. In reality this is not a model at all for it will detail *actual* production for each item. If these decisions outperform Model 3 then there is evidence that systematic deviations for certain items (washed out by an average rule) in reality turn out to have beneficial effects.

Table 1 diagrams the appropriate setting for each of the four models.

The actual period of analysis will be one year. Data from three quarters (3rd(61)–1st(62)) will be utilized to obtain coefficients for the various rules; behavior for the company will then be programmed for the next four quarters (2nd(62)–1st(63)) using these equations.

Evaluation of the Models

We will now briefly discuss the assumptions which will be made to evaluate the performance of the four models.

Orders Received vs. Actual Sales. To accurately evaluate each model we would need records of orders received for each instrument as well as the requested delivery date. Old computer tab runs do list each "order received" for a Recordette instrument, but unfortunately no mention is made of the desired delivery date so the figures are quite meaningless. A further difficulty is the failure of the data to distinguish between items requiring special modifications (which are then made by the engineering group) and those in the regular production class. For these reasons I have decided to utilize actual quarterly sales data as a proxy for orders received during each three month period. Because shortages are deliberately held to a very low level this approximation seems justified. It is a rarity for standard equipment to be delivered later than thirty days after it was requested; in fact, company personnel pride themselves on their near-perfect record on satisfying shipping dates. Confirming evidence for this point can be gleaned from finished goods inventory figures which show very few items with a zero stock balance at the end of any month and almost never a depletion condition two months in a row for the same instrument.

Appropriate Inventory Figures. Total finished goods inventory is located either in the company cage or at one of their branch offices. During each quarter some of the instruments are shipped to Recordette representatives throughout the country at their request although this action sometimes induces shortages in the cage. The company firmly believes that branch offices should receive requested items for demonstration purposes since it appears to play a vital role in finalizing customer orders. Despite this external leakage Recordette still prefers to plan on the basis of "cage" figures because these are the only reliable guides for planning future production needs. Branch office demands vary considerably from month to month both in quantity and type of equipment so that past behavior does not provide much help to the decision-maker. In order to obtain a true picture of possible shortages we will adjust inventory figures downward at the end of each quarter by subtracting branch office shipments for each item during the three month period. Naturally, actual behavior (Model 4) will be free from stock-outs since quarterly sales are being used as a measure of orders received.

Costing the Models

If one wanted specific cost comparisons between the four models then point estimates would have to be made for storage and shortage costs. This may be a bit difficult particularly in attempting to evaluate the cost of a shortage with all its intangible elements such as loss of good will. On the other side of the coin, it is always possible to compare the performance of two models under a wide variety of cost conditions. Thus for multi-item production planning it may actually turn out that the model yields lower storage costs and shortage costs on an aggregate basis than another no matter what dollar figures one uses. One generally would not expect to find such clearcut results. Normally a certain model will outperform another one over one range of storage cost figures and then be inferior to it over another range of values. A diagram covering a large spectrum of costs, enables the manager to determine which model appears most appropriate for his situation. His choice may be relatively easy if one model is clearly superior over the range of cost figures which he considers to be meaningful.

Both types of analysis will be made for Recordette. Below I have indicated the specific figures chosen as proxies for storage and shortage costs.

Storage Costs. Management claims that storage costs per year are approximately 22 per cent of inventory value of equipment on hand. This figure covers a number of expenses, the most important of which are

(i) possible obsolescence of equipment due to technological improvements,
(ii) opportunity cost of funds tied up in inventory.

Although this percentage figure may appear fairly high, Recordette executives claim that it is representative of other firms in the electronics industry.

Shortage Costs. The concept of a shortage for our models differs from the normal definition in inventory theory. In our case, the firm can revise their initial plans due to unexpected orders but must incur expediting costs for the extra parts. It is difficult to determine a meaningful figure for this procedure; when there are extra parts in the stockroom to handle this increase in production there may be no extra cost at all.

The main cost to the company occurs when there is a delay in receipt of outside parts so that production cannot meet certain due dates on orders. The sales department feels it has been able to satisfy most of these customers by sending the merchandise via air freight rather than by truck as they normally do. Shipments going as far as the West Coast can be saved almost two weeks of travel time if they were sent by plane rather than truck. By computing the difference in the transportation charges between these two types of carriers we can crudely obtain one estimate of shortage cost to the company.

The traffic department furnished me with comparative rates for the alternative means of transportation to various destinations in the United States, and I computed the average differential based on the overall distribution of company products. Approximately thirty per cent of Recordette equipment is shipped as far as the West Coast, while orders related to the U. S. space program in Florida account for another twenty per cent of deliveries. The remaining half is generally requested by customers in the mid-West states and the Eastern seaboard. Recordette thus estimates that, on the average, an order is shipped about 1,500 miles from Boston. The price differential between the two carriers for this distance is around $7.50/hundred pounds. Utilizing the shipping weights for each individual item and taking note of specific shortages we thus have a means of approximating the respective stockout costs for each quarterly plan.[3]

An alternative procedure would be to base shortage cost on the dollar value of items rather than on their weight. This measure would be appropriate if there was some chance that an order would be cancelled because the customer could not be satisfied on time. This situation is unlikely to occur at Recordette since the firm is producing items which are somewhat differentiated from those of its competitors. A customer would normally be willing to wait an extra few weeks for receipt of the goods, particularly if he has had good service from Recordette in the past. We could also base this cost on the number of items which were short, disregarding their weight or dollar value. The purposes of these latter comparisons would be to gain further insight into the relative differences between the models.

Construction and Evaluation of the Model

Developing an Initial Planning Rule

The decision rules will be developed using cross-sectional data on the production plans for a number of different items. As production was based on past quarterly sales

[3] The minimum shipping weight to any destination is 100 pounds. All the pre-amplifiers weigh considerably less than this figure but these items are almost always sent to a customer with a recorder which is considerably in excess of the minimum figure.

and the value of expected inventory, a regression equation of the form

$$P_\tau{}^i = a_1 + a_2 \sum_k S_{t-k}^i + a_3 I_\tau{}^i + \epsilon_t{}^i \tag{2}$$

seemed logical. Projected lot sizes, however, vary from zero items (infrequently demanded products) to 500 (for the most active instrument) and certain statistical problems were bound to occur. The value of $\epsilon_t{}^i$ is likely to be directly proportional to the lot size of item i, so that heteroscedasticity (nonconstant variance of $\epsilon_t{}^i$) will lead to inefficient estimates of the parameters. Economic considerations would also lead one to believe that decisions on the size of the lot would be specifically determined by factors such as sales and inventory rather than through some arbitrary value given by the constant term a_1.[4]

Both these problems can be alleviated by obtaining least squares estimates from a deflated equation such as

$$P_\tau{}^i / \sum_k S_{t-k}^i = b_1 + b_2 I_\tau{}^i / \sum_k S_{t-k}^i + \nu_t{}^i \tag{3}$$

where $\sum_k S_{t-k}^i$ is used as a common deflator to eliminate the size factor. Once least squares estimates for b_1 and b_2 are obtained, then both sides of the equation can be multiplied by $\sum_k S_{t-k}^i$ to form the desired relationship between $P_\tau{}^i$ and sales and inventory:

$$P_\tau{}^i = b_1 \sum_k S_{t-k}^i + b_2 I_\tau{}^i + \nu_\tau{}^i (\sum_k S_{t-k}^i) \tag{4}$$

The constant term in (3) should thus be interpreted as an estimate of the parameter describing the importance of past sales on planning behavior.

One further adjustment was considered necessary upon observing that there were some extreme values of the ratios $P_t{}^i / \sum_k S_{t-k}^i$ and $I_\tau{}^i / \sum_k S_{t-k}^i$ which could be linked either to a shortage or storage crises. A potential shortage crisis would be reflected in a relatively high $P_t{}^i / \sum_k S_{t-k}^i$ ratio for item i accompanied by a large expected $I_\tau / \sum_k S_{t-k}^i$ ratio. Likewise, an extremely large $I_\tau / \sum_k S_{t-k}^i$ would indicate that planning behavior had led to a storage crisis for instrument i.[5] If these extreme ratio values were incorporated in the regression data, estimates of the coefficients would describe unusual rather than representative behavior. They were therefore eliminated in developing least-squares estimates of the parameters. Representative behavior refers to planning action which can be justified from an economic point of view. In this sense we are *prescribing* rather than describing "average" decisions for the firm. Naturally the decision rule we obtain on the basis of the first three quarters data will be used to plan production for all items, including those which were omitted in the computations. If our theory has validity then planning based on "average" parameters should result in lower costs than actual behavior. On the other hand, if the manager gains foresight

[4] It is possible to suppress the constant term in regression analysis by using only raw moments to compute least squares values. By thus assuming that the means of all variables are equal to zero, the expected value of the disturbance term is no longer guaranteed to equal zero.

[5] If both $P_t{}^i / \sum_k S_{t-k}^i$ and $I_\tau{}^i / \sum_k S_{t-k}^i$ have values greater than 1.5, we will assume that unnecessary production is planned for the next quarter (shortage crisis). An expected inventory/sales ratio as large as 1.5 at the beginning of quarter τ should more than cover customer orders during the planned period even if $P_t^i / \sum_k S_{t-k}^i = 0$, let alone a value as high as 1.5.

When $I_\tau{}^i / \sum_k S_{t-k}^i > 4$ this will be considered a storage crisis for item i since enough inventory will be on hand at the beginning of quarter τ to cover at least one year's sales, unless a drastic change in the demand pattern occurs. The set-up cost associated with any item is implicitly assumed to be low enough so that it would not be necessary to produce four times as many units as would be sold in a quarter.

from special environmental cues which a mathematical rule based on past quantifiable data cannot incorporate, then his seemingly unusual action on certain items may prove to be more appropriate than the estimates obtained from our model.

Once the form of the model has been specified it must be subjected to a critical evaluation based on the three criteria discussed in the first section. Appendix A presents a discussion of the appropriate techniques and the results of the evaluation for the initial plan.

Developing a Minimal Lot Size Model

For marginal items, where the planning department must determine whether to produce zero units or the minimal lot size, special nonquantifiable factors for the instruments in question often influence final action. The number of outstanding quotations in customer hands as well as potential government contracts which would require certain Recordette equipment may be important in determining whether to produce zero or 25 units.

In attempting to develop a "yes-no" type rule for all these marginal items, the inventory/sales ratio at the beginning of the quarter ($I_\tau^i / \sum_k S_{t-k}^i$) was the only quantifiable factor which seemed to be important to the planners. It is the value of the ratio which is critical for decisions rather than the separate sales and inventory figures, as in the over-all initial plan. Our model was of the form

$$Y^i = f(I_\tau^i / \textstyle\sum_k S_{t-k}^i) \tag{5}$$

where $Y^i = 1$ or 0 depending on whether or not the minimal lot size was planned for item i. A description of the development of the rule is provided in Appendix B.

Revised Production Plan

In building a model to revise production from the initial plan one month before the quarter begins, two rules must be delineated:

A Modified "Yes-No" Scheme. Each instrument assumes the value 1, -1, 0 depending on whether actual production was respectively increased, decreased or remained the same as in the initial plan. This rule resembles the minimal lot size model except that the dependent variable, V^i, can now take on one of three values. The only quantifiable factor which influenced revisions was the expected inventory on hand at the start of the quarter ($I_{\tau+1}$) based on the initial plan for quarter τ. The following simple relation was therefore examined:

$$V^i = f(I_{\tau+1}^i) \tag{6}$$

Magnitude of the Change. An equation must specify the magnitude of the change whenever a revision is suggested by (6). In order to be consistent with (5) the following model was utilized

$$\Delta P_\tau^i = f(I_{\tau+1}^i) \tag{7}$$

where ΔP_τ^i represents the actual magnitude of the production change and $I_{\tau+1}^i$ denotes expected inventory at the start of $\tau + 1$ if the change had not been made. Results of both these rules are detailed in Appendix B.

Obtaining Final Equations for the Model

To develop a decision-rule for testing this theory of managerial action, data from the third quarter (1961) through the first quarter (1962) are pooled together. The resulting regression equations will then be used to program behavior for the next four quarters. The homogeneity of the coefficients between initial plans, as detailed in Appendix A, justifies pooling the first three quarters' data. On the other hand, there was no uniformity in the least squares estimates for either the minimum lot size model or revised production equations as shown in Appendix B. In both these cases I attempted to isolate critical quantifiable factors, but as expected, did not accurately specify behavior with them. For these models the grouping procedure can only be defended on the grounds that some form of average action is being described. Equations for the final model are presented in Table 2.

The effects of each phase of the model on planning behavior from the second quarter (1962) through the first quarter (1963) will now be examined more thoroughly. As the decision rules are based on earlier data, these four periods provide a meaningful statistical test of the "average" equations. Cost comparisons between the four models will provide a method for judging the validity of Bowman's concepts for this particular problem.

Cost Comparisons of Models

Summary figures comparing the performance of each of the models are presented in Table 3.

Clearly, Model 4 yields the minimum costs no matter what storage and shortage figures one uses—its average value of inventory over all four quarters is lowest and it has zero shortages (by definition). The initial implication of these results is that actual behavior in the firm (initial plus revised plans) is superior to rules based on average behavior. The four-way comparison presented in Table 2 is somewhat unfair however. Models 1 and 2 assume that the initial plan is the final one while Models 3 and 4 permit revisions.

Let us look at the results on the basis of these two groupings. The inferior performance of Model 3 in relation to Model 4 suggests that revisions based on average managerial behavior are more costly than the actual changes made. Recordette appears to

TABLE 2

Final Equations for Planning Production

(Based on Combined Data from 3rd Quarter (1961) through First Quarter (1962) Plans)

Initial Planning Equation

$$P_r^i = \underset{(.0653)}{1.0266} \sum_k S_{t-k}^i \quad \underset{(.0488)}{-.3170}\, I_r^i$$

Minimal Lot Size Rule

$$Y^i = \underset{(.0866)}{.6954} \quad \underset{(.0453)}{-.1990}\, \frac{I_r^i}{\sum_k S_{t-k}^i}$$

If $Y^i < .5$ Production of item $i = 0$.

If $Y^i \geq .5$ Production of item $i =$ min. lot size.

Revised Production Plan

(a) *"Yes-No" Rule*

$$V_i = \underset{(.0401)}{.2479} \quad \underset{(.0006)}{-.0034}\, I_{r+1}^i$$

If $V^i > .30$ then production is increased ($\Delta P_r^i > 0$).

If $V^i < -.02$ then production is decreased ($\Delta P_r^i < 0$).

If $-.02 \leq V^i \leq .30$ then "no change" in production ($\Delta P_r^i = 0$)

(b) *Magnitude of Production Change* (for items where $\Delta P_r^i \neq 0$ in "Yes-No" Rule)

$$\Delta P_r^i = \underset{(8.98)}{38.28} \quad \underset{(.0996)}{-.4985}\, I_{r+1}^i$$

have revised its initial plan because of reliable information regarding particular items rather than on the basis of a generalized quantifiable factor such as the inventory/sales ratio. We will discuss the implications of this point further in the concluding section.

A comparison of Models 1 and 2 yields very definite evidence in support of Bowman's theory. Both rules cause approximately the same number of shortages while the average inventory value over the four quarters is considerably lower in Model 1. Interrelated factors account for the success of Model 1. The regression rule describing initial plans was statistically and economically significant as well as consistent over time so there was reason to believe that it accurately depicted the manager's average behavior. Furthermore, planning and scheduling was forced to rely almost entirely on past behavior (e.g., sales during the past three months) and known schedules (e.g., sales during the next four months) in formulating initial plans. The erratic behavior can thus be attributed to departmental constraints reflected in the policy of only scheduling round lot sizes and to influences from other departments (e.g. pressure from sales personnel).

Table 4 presents storage and shortage data for Models 1 and 2 which will be utilized to compare their performance under several different assumptions.

If shortages are evaluated on the basis of aggregate weight or dollar value, then Model 1 will always outperform Model 2, no matter what the shortage/storage cost ratio is. The only time that Model 2 has any chance of yielding superior results is when there is a fixed shortage cost per item regardless of its value or weight. The relevant definitions for obtaining the appropriate shortage/shortage cost ratio (w) are listed in the last column of Table 4. These concepts are depicted graphically in Figure 2, using a wide range of values for w. Other things being equal, the longer the shortage lasts

TABLE 3

Average Inventory Value and Number of Shortages in All Four Models
[*2nd Quarter* (1962)–*1st Quarter* (1963)]

Model	Average Inventory Value	Number of Shortages
1	$568,210	1,393
2	$761,570	1,379
3	$613,840	137
4	$525,570	—

TABLE 4

Comparison of Initial Plans Based on Average Behavior (Model 1) and Actual Behavior (Model 2)
[*2nd Quarter* (1962)–*1st Quarter* (1963)]

	Model 1	Model 2	Definition of Storage or Shortage Cost to be used in Figure 2
Storage Figures:			
Average Inventory Value	$568,210	$761,570	Storage Cost per dollar
Shortage Figures:			
Weight of Shortages (in 100 lbs.)	388	649	Shortage Cost per 100 lbs.
Inventory:			
Value of Shortages (in $)	$103,060	$107,250	Shortage Cost per dollar
Number of Shortages	1,393	1,379	Shortage Cost per item

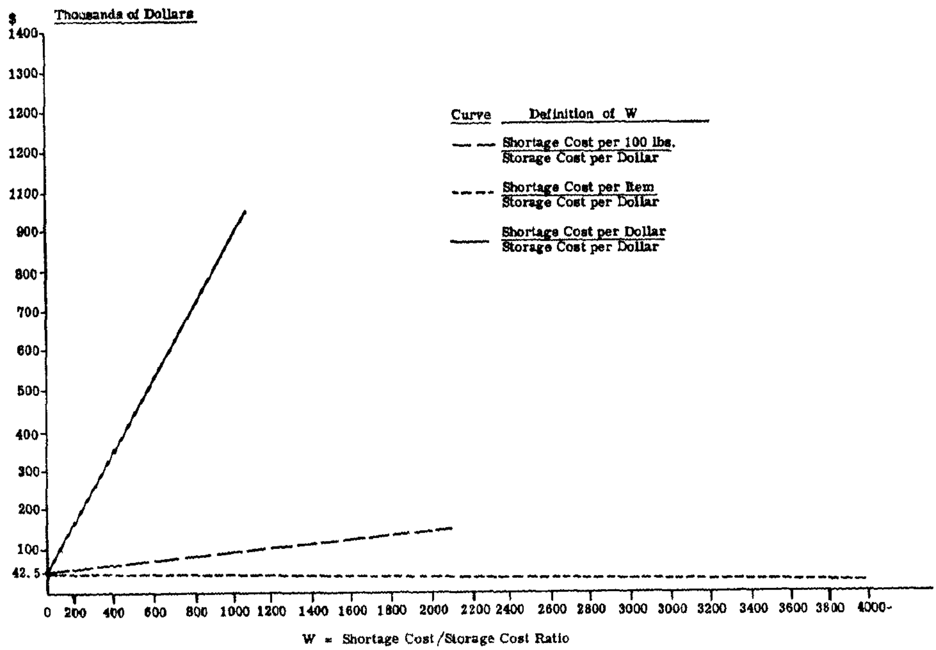

FIGURE 2. Cost improvement of model 1 over model 2 for different measures of shortage cost

the higher the value assigned to w. The analysis in Figure 2 uses aggregate figures because it was impossible to obtain data on the distribution times regarding the length of shortages for individual items. Performance characteristics of individual items are provided in Kunreuther [9]. The total cost figures (y-axis) were obtained by fixing the storage cost per dollar at .22 and determining the appropriate shortage costs through the values of w.

The solid line in Figure 2 compares the performance of the two models when shortages are evaluated on the basis of weight. For example, assume that shortage costs were evaluated using the figure of \$7.50 per 100 lbs. introduced in the section *Costing the Models*. Model 1 would yield costs of \$127,900 compared to the cost of \$172,400 from Model 2. This savings of \$44,000 could be obtained from Figure 2 by finding the appropriate value of w, in this case $7.50/.22 = 34.1$. The long-dashed line presents cost comparisons when shortages are measured on the basis of aggregate dollar value of items which could not be delivered to the customer. Only when shortages are evaluated on the basis of cost per item (the small-dashed line) can Model 2 yield lower total costs Model 1. For this to happen, however, the shortage/shortage cost ratio must be more than 13,800.

The savings presented above are based on the assumption that final plans are synonymous with the initial ones. We know, of course, that this is not the case in practice. The preliminary plans, however, do imply definite commitments for parts both from vendors and the machine shop. Therefore, a better initial schedule will reduce the costs associated with revisions over time.

Examining Implied Workforce

Recordette has tabulated so-called "standard hours" required to assemble one unit of item i so we have been able to calculate the implied aggregate workforce on the basis

TABLE 5

Comparisons of Implied Workforce for Initial Plans Based on Average (Model 1) and Actual (Model 3) Behavior

Quarter	Number of Employees	
	Model 1	Model 2
Second (1962)	233	219
Third (1962)	190	214
Fourth (1962)	241	202
First (1963)	152	223

of planned production. These totals for each quarter are presented in Table 5 based on initial behavior specified by Models 1 and 2.

In our computations we have utilized management's assumption that an employee is only 70 per cent efficient (i.e., only 28 of the 40 hours in the week are actualy spent in assembling equipment).[6] For the third quarter (1962) we have also taken into account the two week shutdown, though the workforce for both models may still be underestimated because senior employees are entitled to three weeks leave.

Naturally, these figures only indicate the proposed workforce if the initial plan is put into effect without any revisions. In this regard, however, the estimates for the first quarter (1963) are worth comparing. Model 1 actually suggested a substantial decrease in employment from their fourth quarter crew while Model 2 proposed an increase. In point of fact, Recordette had its first lay-off in eight years during this three month period when over 50 workers were released. It is interesting to note our "average" regression rule correctly anticipated this decrease in workforce by using just two quantifiable variables—sales during the past three months and expected inventory at the end of 1962.

Conclusions and Suggestions for Further Research

Bowman's theory has certain advantages over the traditional operations-research type approaches; the case study at Recordette has also indicated some of its limitations. This section will briefly investigate both sides of the coin and then suggest areas for future research.

Advantages of Approach

The primary advantage of this approach is that cost data are not required to derive a decision rule as they are in most OR models. The coefficients are obtained by using past data which were available to the individual himself at the time he made his decision. Thus, the manager's past behavior is implicitly utilized in developing the model. This simplifies the implementation phase of the study since an individual would tend to be more receptive to a rule based on his own actions than to a purely theoretical construct.

The theory seems to have validity in situations where past data provides the most reliable guide for the future. The initial plan (P_τ^i) formulated at Recordette almost four months in advance of quarter τ fits into the above category. Thus at the time of the

[6] Sick leave, coffee breaks, early morning and late afternoon inefficiency account for this reduced figure.

original plan, variance in managerial behavior apparently reflected hunches rather than reliable information regarding future sales. For example, the prediction of a shortage crisis for instrument i may have been based on interest by doctors in this device when it was displayed at a regional medical show. Increased demand for the item, if forthcoming at all, will develop only gradually rather than rapidly as forecasted by the sales department.

The results generated by the "average" initial planning rule (model 1) can make the manager more aware of the cost implications of his past behavior. In this way he may become less erratic in the future without even examining decisions from the regression equation. In particular, the comparative results from Models 1 and 2 enable the manager to observe how disastrous storage and shortage crises can be on inventory costs and to also note the limitations of scheduling in round lot sizes. At Recordette these effects of initial planning may otherwise be overlooked by the manager because he is permitted to revise production during the next few months. The only published inventory figures on specific instruments are related to finished goods, so the manager may conveniently forget that downward revisions in the initial plan caused increases in the in-process inventory.

It may be particularly helpful to develop rules describing average behavior in situations where the individual does not reveal his decision making process. From discussions with representatives of a number of firms, I have learned that this type of "seat of the pants" activity can pose very serious problems. Even if the individual is involved in routine activities, he may still be difficult to replace because no one is exactly sure how he made his day to day decisions. The development of an average rule would provide some guidelines for future actions and enable a more orderly transition to take place after his absence.

Limitations of the Theory

This theory of decision-making falls down if the selective cues from the environment represent reliable information about the future rather than just guesses. The variance in managerial action will now be beneficial rather than costly to the firm. These situations are most likely to arise when company personnel have access to specific data regarding the future which could not be extrapolated from the past. At Recordette for example, changes are often made in initial plans after the sales department has received news of large future orders for certain instruments (e.g., through a government contract) which could never have been anticipated from past quarterly sales records. For this reason the overall regression model which included a revision rule (Model 3) fared much worse costwise than actual behavior (Model 4).

The experience with Recordette's planning operation has reinforced my opposition to the exclusive use of a computer in determining certain decisions. Before a plan is finalized, an automatic rule may prove valuable as a guide to the manager but it should not be blindly followed. It is quite conceivable, for example, that certain *initial actions* which were classified as "crises observations" and thus excluded from the regression, in reality represented definite managerial knowledge of future orders which should *not* have been ignored. The most refined forecasting schemes based on past data (as at Recordette) can only have limited success in their estimates of the future. If the lead time is short enough, as when revisions are made in the plans, the selective environmental information should be extremely helpful to a firm in its operations. A computer rule utilizing quantifiable variables based on past behavior cannot possibly react to these specific external cues.

Another danger of implementing a computerized decision scheme in a firm is its inability to discover any structural changes in the system. Thus, if the sales patterns were altered in the future so that ΔS_t^i now had value in determining initial production plans, the automatic rule would still ignore this variable. It would continue to plan production on the basis of $\sum_k S_{t-k}^i$ and I_r^i. A reasonably rational manager having some freedom to make his own planning decisions could be expected to detect the new importance of the "change in sales" variable by observing the data. If the decision-maker then decided to utilize ΔS_t^i as a variable, new coefficients for the "average" initial planning rule would then have to be computed on the basis of these latest actions. Thus by permitting the manager some flexibility in his actions the firm can discover structural changes in the system and can update the regression rule(s) accordingly.

Suggestions For Future Research

One interesting bit of information that could be gleaned from this type of approach is the specific cost ratio(s) implied by the manager's decisions. Thus, we should be able to obtain a ratio of storage/shortage costs from the "average" coefficients describing the planning equation. In order to determine this ratio some assumption has to be made regarding the shape of the relevant cost curves. If we hold that they are quadratic then the problem is just the mirror image of the one attacked by Holt *et al.* [4]. By assuming quadratic cost curves with known parameters they were able to develop linear decision rules describing the importance of expected inventory, workforce and predicted future sales in "optimally" determining production. Our problem is to estimate cost parameters from "average" type rules based on managerial behavior. By imputing costs from our regression rules these values would be based on actual managerial decisions rather than on "guesstimates." Better still, we can ask the manager whether he agrees with the figures implied by his behavior and, if not, what values would be more appropriate.

The approach taken by Bowman combines clinical judgments with statistical variables in a somewhat different form from the models discussed in Pankoff and Roberts [11], where they treat the actual behavior as one of the independent variables in a regression equation. In our case, the clinical judgment (e.g., production plans) becomes the dependent variable in the equation and is thus used to judge the importance of quantifiable independent variables (e.s. sales, inventory) on the decision-making process. The aim of the regression rule is to describe average behavior rather than to forecast future behavior.

We could incorporate the element of prediction by broadening the concepts of Bowman's theory somewhat. Suppose we felt that the decision maker utilized a biased sales forecasts which was then reflected in his production plans. Rather than developing a rule based on average behavior, we could first regress predicted sales on actual sales to determine whether there was a systematic bias. By adjusting the sales forecast for this bias, we could develop a regression rule for planning production. Results from this model should lead to lower costs than if the original sales forecasts had been utilized in the rule.

A great deal more testing and analysis has to be done in this clinical-statistical area. By incorporating actual decisions or judgments into a more formal analysis, we may help satisfy some of the needs of the decision-makers who would like to improve their behavior subject to the constraints of the real-world.

TABLE 6

Quarterly Regressions and Covariance Analysis of Initial Planning Model—$P_r^i / \sum_k S_{t-k}^i = f(I_r^i / \sum_k S_{t-k}^i)$, $P_r^i / \sum_k S_{t-k}^i = b_1 + b_2 I_r^i / \sum_k S_{t-k}^i$

Quarter	b_1	b_2
Third (1961)	.9799 (.1029)	−.3293 (.0765)
Fourth (1961)	.9953 (.1250)	−.3017 (.0847)
First (1962)	1.0899 (.1140)	−.3104 (.0950)
Second (1962)	1.2054 (.0976)	−.1888 (.0816)
Third (1962)	1.1193 (.1151)	−.2196 (.1128)
Fourth (1962)	1.0368 (.0900)	−.2395 (.0690)
First (1963)	1.1378 (.0927)	−.3100 (.0653)
Grand Regression (3rd (61)–1st (63))	1.0918 (.0398)	−.2821 (.0309)

F Ratios Based on Covariance Analysis

F Test	Degrees of Freedom*	F Ratio	Significance Level
F_1	(12,337)	1.1462	.3216
F_2	(6,337)	.3923	.8838
F_3	(5,337)	1.8452	.1035
F_4	(1,337)	2.1752	.1411

* The degrees of freedom in the numerator is always listed first.

Appendix A. Techniques for Evaluating the Initial Plan

Although the rule for planning behavior will be based on data from the first three quarters' it is still appropriate to use data from the seven initial plans to examine how well the model satisfies the three criteria discussed in the first section. This Appendix will detail the results of these analyses:

Consistency Over Time

The best method for examining the consistency of estimated cross-sectional parameters over time is through covariance analysis.[7] The basic ingredients for this statistical analysis are presented in Table 6—the cross-sectional regression equations for the seven initial plans based on (3) as well as the grand regression whose coefficients are determined by pooling the data for all seven quarters together. Results of the statistical tests are concisely expressed at the bottom of the table by several F ratios whose meaning and interpretation will now be discussed briefly. The F_1 test measures whether a significantly better statistical fit is obtained by *not* pooling data for separate equations

[7] For a statistical discussion of covariance methodology see Mood, [10]. An excellent treatment of the use of this technique for analyzing cross-sectional data can be found in the book by Kuh. [8].

into one grand regression. For our purposes this test will tell us whether any significant difference exists in the manager's average behavior for separate quarterly production plans using a decision rule based on (3). In this case the null hypothesis was accepted at the extremely high value of .32, thus indicating significant homogeneity.[8] In general if the null hypothesis is accepted by an F_1 procedure at such a high value, there is no need to separately investigate slope coefficients (by an F_2 test) and the intercept term (by $F_3 - F_4$ ratios) because statistical confirmation of their uniformity is practically guaranteed. However, since we are interested in the exact significance level of the inventory and sales coefficients between quarters, further analysis is necessary.

A glance at the equations in Table 5 reveals the similarity of average planning behavior with respect to anticipated inventory on hand at the beginning of quarter τ for instrument i (I_τ^i). The F_2 test provides statistical confirmation of this phenomenon by claiming homogeneity between quarters at the extremely high level of .88. Results of the $F_3 - F_4$ procedure offer statistical evidence that the manager tended to give similar weights to past quarterly sales $(\sum_k S_{t-k}^i)$ in planning production for each of the seven periods. The F_3 test was accepted at the .10 level, while F_4 showed that the estimated sales coefficients for the seven quarters were considered to be homogeneous at the .14 significance level.

Economic Significance

Even if the least-squares estimates for the coefficients of the independent variables happened to be consistent between quarters, we would still be loathe to utilize the equation as a decision-rule unless a meaningful economic interpretation could be given to their values. The general results seem satisfactory for this model since it is logical to expect the initial quarterly plan for an item to be approximately the same as sales during the past three months (assuming that this is an accurate measure of future demand) minus some fraction of anticipated inventory on hand when production begins.

Since there is a lower bound for values of the dependent variable (i.e., P_τ^i must be $\geqq 0$), the regression coefficients in the equation would not be very accurate if they yielded negative production for some of the instruments on the schedule. By excluding items with extremely high $I_\tau^i / \sum_k S_{t-k}^i$ because they fell in the storage crisis category, this difficulty was effectively overcome without having to resort to any special statistical methods.[9] After examining values of the residuals for the seven separate regressions only five out of the 351 data points were found to have an estimate of P_τ^i less than zero. The largest negative value of these observations was only $P_\tau^i = -2$, a negligible difference from zero.

Statistical Significance

By examining the ratio of each estimated coefficient with its standard error (i.e., the t-values) statistical evidence is obtained on the accuracy of the model. The large

[8] All statistical tests are geared to accepting the null hypothesis that there is no difference between observations. Unless overwhelming evidence is found indicating a real difference between equations or data points, homogeneity will be affirmed on the assumption that existing variation in the system is random and due to chance occurrences. Since our objective is to gather evidence *confirming* consistency in managerial planning behavior we have reported the exact significance level (e.g., .32 for the F_1 test) for every F ratio computed. It is much more meaningful to assert that the seven quarterly plans do not differ significantly at the .32 level than just to note that the null hypothesis was not rejected at either the one per cent or five per cent significance level.

[9] Tobin has developed an estimation procedure for the case of either an upper or lower bound in the dependent variable. The method is a combination of probit and multiple regression analysis. See James Tobin [12].

TABLE 7

Testing Homoscedasticity for Residuals of Undeflated Equations— $p_r^i = f(\sum_k S_{t-k}^i, I_r^i)$

(Blocked Sections Based on Homogeneity at 5% Significance Level)

Value of $\Sigma_k S_{t-k}^i$	No. of Observations	Variance of ϵ_i
2–6	19	38.8
42–46	17	132.9
12–16	25	156.8
47–51	15	153.8
7–11	22	228.0
22–26	27	266.6
17–21	44	310.4
32–36	24	327.4
37–41	15	442.8
27–31	23	466.8
57–61	14	526.1
67–71	9	836.7
52–56	16	1099.9
72–99	17	1235.5
62–66	10	1631.9
400–534	11	2624.7
100–199	22	2702.4
300–399	14	2800.6
200–299	7	5908.9

TABLE 8

Testing Homoscedasticity for Residuals of Deflated Equations— $p_r^i/\sum_k S_{t-k}^i = f(I_r^i/\sum_k S_{t-k}^i)$

(Blocked Sections Based on Homogeneity of 5% Significance Level)

Value of $\Sigma_k S_{t-k}^i$	No. of Observations	Variance of ϵ_i
400–534	11	.0275
300–399	14	.0649
42–46	17	.0963
100–199	22	.1026
2–6	19	.1248
47–51	15	.1281
72–99	17	.1364
67–71	9	.1375
57–61	14	.1711
22–26	27	.1946
200–299	7	.1839
37–41	15	.2626
32–36	24	.2859
52–56	16	.3223
12–16	25	.3586
62–66	10	.4737
27–31	23	.4824
42–46	44	.6970
7–11	22	.8107

t-values for the two estimated coefficients are clearly discernible for each quarterly regression rule shown in Table 6.[10] In order for the estimates of the parameters to be efficient the error term is required to have a constant variance [i.e., $E(\epsilon_i^2) = \sigma^2$] referred to as homoscedasticity. Since both undeflated and deflated regression equations were run for all seven quarters it is interesting to compare their error terms. Ideally we would want to have a few observations for each specific value of $\sum_k S_{t-k}^i$ to test the homoscedasticity assumption, but due to data limitations we were forced to compare residual variances on the basis of sales classes.

It is desirable to use equal sales intervals in grouping items but this was impossible since the data were nicely clustered for low-activity items (quarterly demand $\leqq$ 70 units) and spread thinly through the wide range of high demand items (quarter sales > 70). Separate groups based on an interval width of five were thus formed for instruments with $\sum_k S_{t-k} \leqq 71$; much wider classes were used for higher demand items so as to have at least seven observations in a group.[11]

One method of determining the degree of homoscedasticity is to examine simple F ratios for classes after ranking them according to the value of their variances. By grouping together those categories whose variance are considered homogeneous at the .05 significance level we have some measure of the amount of heteroscedasticity. Tables 7 and 8 present the results of these computations for the undeflated and deflated equations respectively. Although the variance of the disturbance term is not constant after deflation, there is a great deal more uniformity between groups than for the undeflated model as shown by the blocked sections in each table.

Appendix B. Developing Rules for Minimal Lot Sizes and Revising Production

Minimal Lot Size Model

A statistical regression approach has been utilized to obtain "average" parameter estimates for the $I_r^i / \sum_k S_{t-k}^i$ variable in the minimal lot size equation. As the objective of the "yes-no" rule is to classify items into one of two groups there may be a tendency to interpret it as a linear discriminant function. The equation is still a regression function in which the data points assume only specialized values for the dependent variable. A discriminant function, on the other hand, possesses *no* dependent variable but instead uses a two-way classification of the data to determine the coefficients. A geometrical and mathematical description of linear discriminant functions is presented in Hoel, [3].

We are not specifically concerned with the desired statistical properties of the error term (as in the overall initial plan formulation);[12] rather interest is centered on developing a rule that depicts "average" behavior with regard to these special items. The only data used to estimate the least squares coefficients were those instruments whose planned production based on the initial rule fell below their minimal lot size. Thus any instrument where the regression equation suggested a quantity greater than the minimal lot size was not included even though planning may have decided to produce zero or just 25 units of this item. Furthermore, those instruments where the "average" rule

[10] One should not forget, however, that in obtaining the regression estimates all "crises observations" were excluded. The discussion in this section thus applies to an analysis of representative planning behavior.

[11] The specific classes based on $\sum_k S_{t-k}^i$ were: 2-6, 7-11, 12-16, $\cdots$ 67-71, 72-99, 100-199, 200-299, 300-399, 400-534.

[12] If we were concerned with the distribution of the error term, then we would have to take into account the nonnormality of disturbances implied by the two-valued dependent variable.

TABLE 9

"Yes-No" Rule for Minimal Lot Size Production

$Y^i = f(I_r^i / \sum_k S_{t-k}^i)$
$Y^i = a + b\, I_r^i$
$Y^i = 0$ if production of item i is zero.
$Y^i = 1$ if production of item = min. lot size.

Quarter	a	b
Third (1961)	.7281	−.2177
	(.1497)	(.0729)
Fourth (1961)	.6514	−.1894
	(.1616)	(.0824)
First (1962)	.7024	−.1812
	(.1647)	(.0905)
Second (1962)	1.0524	−.2518
	(.2876)	(.1543)
Third (1962)	.9418	−.2375
	(.1505)	(.0877)
Fourth (1962)	.7694	−.2108
	(.1798)	(.0811)
First (1963)	.9114	−.2768
	(.1772)	(.0715)

yielded a value between 0 and 25 were incorporated even though actual plans may have called for a lot size in excess of the minimum (e.g., 50 units) for this particular piece of equipment.

Results of the seven quarterly regressions based on (5) are reported in Table 9. Some variability is evident in the estimates of the inventory/sales parameter between plans, although its t-value is larger than two in all periods except the second quarter (1962).

In developing coefficients for this type of rule we did not expect to obtain consistent results over time (although we would gladly have accepted them) because the main determinants of these decisions were nonquantifiable factors. We at least managed to produce meaningful average coefficients for the inventory/sales variable with the expected negative sign for every quarterly regression.

The following arbitrary criterion will be utilized for planning production of marginal items:

(1) Zero production will occur for instrument i if the value of $Y^i < .5$.

(2) The minimal lot size will be planned if $Y^i \geqq .5$.

Using the equations for the third quarter (1961) through first quarter (1962) as a basis for this criterion, the rule is effectively claiming that production will be zero if $I_r^i / \sum_k S_{t-k}^i > 1$. This rule makes economic sense, for the firm would not want to plan a large quantity if expected inventory can cover projected future demand. If subsequent trends are not reflected by earlier patterns, however, then the "average" rule may lead to unsatisfactory results.

Revised Production Plan

The Modified "Yes-No" Model. The least squares equations based on (6) are enumerated in Table 10. Although all the inventory coefficients have negative signs and t-values greater than 2.5, the parameter estimates show large variation from quarter to quarter, although no distinct pattern over time is evident. The only significant irregularity is the very low constant term in the first quarter (1963) equation. This value was due to the unusual number of downward revisions of planned production made at the end of November.

Magnitude of the Change. In determining the magnitude of the production change, the only observations utilized for the regressions were those items where a revision actually occurred.

Table 11 presents the results of these regressions based on (7). Variability exists between quarterly plans though this heterogeneity is partly due to the small number of

TABLE 10

"Yes-No" Rule for Revised Production Plan

$V^i = f(I^i_{\tau+1})$
$V^i = a + b\, I^i_{\tau+1}$
$V^i = 1$ if production of item i is increased,
$V^i = -1$ if production of item i is decreased,
$V^i = 0$ if production of item i is unchanged.

Quarter	a	b
Third (1961)	.1468	−.0022
	(.0605)	(.0008)
Fourth (1961)	.3336	−.0029
	(.0652)	(.0010)
First (1962)	.2963	−.0059
	(.0802)	(.0015)
Second (1962)	.1361	−.0053
	(.0078)	(.0012)
Third (1962)	.2924	−.0076
	(.0899)	(.0017)
Fourth (1962)	.1699	−.0040
	(.0599)	(.0008)
First (1963)	.0565	−.0033
	(.0655)	(.0013)

TABLE 11

Rule for Determining Magnitude of Revisions in Production $(\Delta P_\tau{}^i)$, $\Delta P_\tau{}^i = f(I^i_{\tau+1})$

$\Delta P_\tau{}^i = a + b\, I^i_{\tau+1}$

Quarter	a	b
Third (1961)	40.94	−.2459
	(25.39)	(.2194)
Fourth (1961)	30.59	−.7600
	(9.10)	(.0967)
First (1962)	36.53	−.4698
	(16.52)	(.2322)
Second (1962)	18.62	−.9657
	(8.94)	(.1216)
Third (1962)	29.57	−.6562
	(9.90)	(.1315)
Fourth (1962)	12.80	−.5850
	(11.46)	(.1157)
First (1963)	44.19	−1.4063
	(18.51)	(.2742)

observations for each period, as revisions are relatively infrequent.[13] All the t-values for the inventory parameter estimates are larger than 2 except for the third quarter (1961) where only eight data points were available.

Criteria for "Yes-No" Rule. The criteria for specifying a revision in production must be formulated so as to be consistent with the ΔP_τ rule (7). We would not want (6) to suggest a decrease in production, only to find that $\Delta P_\tau > 0$ according to (7).

The criteria were also motivated by economic considerations. In particular, the upper bound reflects the firm's basic policy of keeping shortages to a minimum—if there is an expected backlog at the end of the quarter in excess of fifteen units, then production will be revised upwards by a much larger figure than the prospective shortage implies. In this sense the revision rule incorporates an over-response mechanism. Production will not be revised downward unless $I^{i}_{\tau+1}$ is greater than the value of 78. This represents Recordette's general reluctance to ever revise production downward unless some severe decreases in demand or misjudgments in the initial plan occur.

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[13] In the regressions specified by (6) the following number of data points were used for each of the seven quarters being examined: 8, 14, 17, 18, 19, 12, 14.