Interfaces

HP Transforms Product Portfolio Management with Operations Research

Julie Ward, Bin Zhang, Shailendra Jain, Chris Fry, Thomas Olavson, Holger Mishal, Jason Amaral, Dirk Beyer, Ann Brecht, Brian Cargille, Russ Chadinha, Kathy Chou, Gavin DeNyse, Qi Feng, Cookie Padovani, Sesh Raj, Kurt Sunderbruch, Robert Tarjan, Krishna Venkatraman, Joseph Woods, Jing Zhou,

Please scroll down for article—it is on subsequent pages

Q R

THE FRANZ EDELMAN AWARD
Achievement in Operations Research

HP Transforms Product Portfolio Management with Operations Research

Julie Ward,[a] Bin Zhang,[a] Shailendra Jain,[a] Chris Fry,[b] Thomas Olavson,[c] Holger Mishal,[d] Jason Amaral,[e]
Dirk Beyer,[f] Ann Brecht,[g] Brian Cargille,[h] Russ Chadinha,[i] Kathy Chou,[j] Gavin DeNyse,[k]
Qi Feng,[l] Cookie Padovani,[m] Sesh Raj,[n] Kurt Sunderbruch,[o] Robert Tarjan,[p] Krishna Venkatraman,[q]
Joseph Woods,[r] Jing Zhou[s]

[a]Hewlett-Packard Labs, Palo Alto, California 94304 {jward@hp.com, bin.zhang2@hp.com, shelen.jain@hp.com} [b]Strategic Management Solutions, Redwood Shores, California 94065, chris@strategicmgmtsolutions.com [c]Hewlett-Packard SPaM, Palo Alto, California 94304, thomas.olavson@hp.com [d]Hewlett-Packard Personal Systems Group, Cupertino, California 95014, holger.mishal@hp.com [e]Emeraldwise LLC, Woodside, California 94062, jason@emeraldwise.com [f]M-Factor, Inc., San Mateo, California 94404, dirk.beyer@m-factor.com [g]Hewlett-Packard Technology Solutions Group, Fort Collins, Colorado 80528, ann.brecht@hp.com [h]Hewlett-Packard SPaM, Palo Alto, California 94304, brian.cargille@hp.com [i]Hewlett-Packard Personal Systems Group, Houston, Texas 77070, russ.chadinha@hp.com [j]Hewlett-Packard Personal Systems Group, Cupertino, California 95014, kathryn.chou@hp.com [k]Hewlett-Packard SPaM, Portland, Oregon 97221, gavin.denyse@hp.com [l]McCombs School of Business, University of Texas at Austin, Austin, Texas 78712, annabelle.feng@mccombs.utexas.edu [m]Hewlett-Packard Technology Solutions Group, Roseville, California 95747, cookie.padovani@hp.com [n]DSApps, Inc., Sunnyvale, California 94087, sesh@dsapps.com [o]Hewlett-Packard Technology Solutions Group, Roseville, California 95747, kurt.sunderbruch@hp.com [p]Hewlett-Packard Labs, Palo Alto, California 94304, and Department of Computer Science, Princeton University, Princeton, New Jersey 08540, robert.tarjan@hp.com [q]Intuit, Mountain View, California 94043, krishna_venkatraman@intuit.com [r]Hewlett-Packard Technology Solutions Group, Roseville, California 95747, joe.woods@hp.com [s]Belk College of Business, University of North Carolina at Charlotte, Charlotte, North Carolina 28223, jzhou7@uncc.edu

Hewlett-Packard (HP) offers many innovative products to meet diverse customer needs. The breadth of its product offering has helped the company achieve unparalleled market reach; however, it has come with significant costs and challenges. By offering multiple similar products, a manufacturer increases its overall demand volatility, reduces forecast accuracy, and can adversely affect revenue and costs across the entire product life cycle. At HP, these impacts included increases in inventory-driven costs and order-cycle time; liabilities to channel partners; and costs of operations, research and development, marketing, and administration. Furthermore, complexity in HP's product lines confused customers, sales representatives, and channel partners, sometimes driving business to competitors. HP developed two powerful operations research-based solutions for managing product variety. The first, a framework for screening new products, uses custom-built return-on-investment (ROI) calculators to evaluate each proposed new product before introduction; those that do not meet a threshold ROI level are targeted for exclusion from the proposed lineup. The second, HP's Revenue Coverage Optimization (RCO) tool, which is based on a fast, new maximum-flow algorithm, is used to manage product variety after introduction. By identifying a core portfolio of products that are important to order coverage, RCO enables HP businesses to increase operational focus on their most critical products. These tools have enabled HP to increase its profits across business units by more than $500 million since 2005. Moreover, HP has streamlined its product offerings, improved execution, achieved faster delivery, lowered overhead, and increased customer satisfaction and market share.

Key words: cost analysis; stochastic inventory analysis; flow algorithms; product portfolio management; inventory management; regression; statistics; binary programming; Lagrangian relaxation; parametric maximum flow.

Hewlett-Packard (HP) serves more than one billion customers (consumers, small-to-medium businesses, and large-enterprise customers in virtually every industry) in more than 170 countries on six continents. Its four business units offer products spanning enterprise storage and servers, personal systems, imaging and printing, software, services, and corporate investments. With 2008 revenues of $118 billion, HP is the global market share leader in PCs, printers, and servers. It ships 48 million PCs annually

and over one million printers weekly. One of every three servers shipped worldwide is an HP product. Its product lineup includes significant variety in nearly every offering, boasting more than 2,000 laser printer stock-keeping units (SKUs), more than 15,000 server and storage SKUs, and over eight million possible configure-to-order combinations in its notebook and desktop product lines.

Although HP's product offerings drove sales and market share, the variety of its product portfolio caused significant organizational complexity, created major operational and performance challenges, and caused HP to fall behind its competitors on a number of metrics. Although revenues grew each year, unplanned increases in operating costs eroded its profits, partly because of complexities such as increases in inventory-driven costs, product design costs, channel liabilities, and rework. As variety increased, forecasting accuracy decreased, resulting in excesses of some products and shortages of others. In 2002, HP's inventory turns were lower than many of its competitors, and shortages and excesses were rampant. Despite high inventory levels, major deals were lost because products were not available to meet demand. By 2004, HP's order-cycle time (OCT) was unpredictable, and its average OCT was nearly twice that of its leading competitor, adversely affecting customer satisfaction and making it difficult to win accounts although product quality was high. Across all HP businesses, ineffective management of complexity was diminishing the benefits of HP's broad, diverse product portfolio.

The challenge was not simply that the variety was difficult to manage; more fundamentally, measuring the true costs and benefits of variety was difficult. Many product line complexity costs are hidden, i.e., not captured in standard accounting systems and difficult to measure systematically and fairly. To estimate the impact on overhead costs of adding a single SKU or product feature to the product line seemed almost impossible. In many cases, new products strained existing resources but did not give rise to new direct overhead costs. It was only in aggregate that we began to see the cost impacts. Estimating the impact of variety on inventory-driven costs required sophisticated statistics and stochastic modeling capabilities that the teams (i.e., marketing and product management) who managed product portfolios did not have.

Because the cost impacts of variety were so difficult to measure, debates over new-product introductions were often very one-sided. Marketing teams could argue that new products would generate incremental revenue; however, making a counterargument that the introductions would impact cost, let alone objectively weighing these costs and benefits against each other, was difficult. Without clear standards for evaluating product proposals in a balanced way, HP was unable to implement an effective process for systematically managing product proposals.

Once products were launched into the portfolio, it was difficult to measure and manage their impact. Few standards existed for how and when to remove a product from the portfolio. Rationalization decisions were often made based on a product's individual revenue; however, this metric neglects key elements of the product's importance, such as the value of a low-revenue product in fulfilling a high-revenue order. Moreover, the cost structure and impact of variety differed dramatically from business to business within HP. Some businesses, such as high-end imaging and printing products and business-critical servers, faced high variety-driven costs associated with creating, developing, testing, and launching new SKUs. However, their processes for reviewing and approving new SKU introductions did not incorporate a comprehensive, quantitative cost assessment. SKU introduction decisions were often based on a business case from marketing that focuses on the benefits rather than a balanced and data-driven view of incremental costs and benefits, making it easy for variety to proliferate.

Other businesses, such as HP's Personal Systems Group (PSG), which sells configurable PC products, had comparatively low per-SKU costs but high costs for simultaneously managing inventory and availability on many underlying parts. In PSG, most orders do not ship until every product is available; a stockout of a single product can delay an entire order. Because of difficulties in maintaining adequate availability across its vast product line, PSG's average OCT was not competitive, and the lack of predictability and long lead times frustrated customers.

Over the past five years, HP has made managing product variety a strategic business priority. It has developed and implemented two operations research

(OR)-based solutions that have helped HP dramatically improve its performance, resulting in bottom-line profit improvements of more than $500 million. In this paper, we present these two methodologies, describe the details of their application in PSG, and present the substantial quantitative and qualitative benefits obtained through the broad use of these tools across many HP businesses.

Solutions

The first solution, a process for screening new-product proposals before introduction, is driven by OR-based supply chain analytics for measuring the true cost impact and projected return on investment (ROI) of proposed products. We evaluate the projected *complexity-adjusted ROI* for each proposed new product, prior to its creation, using a complexity ROI calculator; this calculator is developed for each business through a one-time analysis of the up-front and ongoing cost impacts of introducing and managing products. Products that do not meet a threshold ROI level are targeted for exclusion from the proposed lineup (Cargille et al. 2005, Olavson and Fry 2006, Cargille and Melia 2007).

The second solution, the Revenue Coverage Optimization (RCO) tool, is used to manage product variety after introduction. RCO analyzes order history to rank products along the efficient frontier of portfolio size and *order coverage*—defined as the portion of the number, revenue, or margin of orders that can be completely fulfilled by products in the portfolio. By helping to identify a core portfolio of products that are important to order coverage, the RCO results enable HP businesses to increase operational focus on their most critical products and make data-driven rationalization decisions.

Because these two tools address different aspects of managing product variety, their use varies according to each business' requirements. Businesses that incur significant one-time costs for each new-product launch might emphasize a screening process that uses complexity ROI calculators to quantitatively evaluate proposed new variety. Businesses with highly configurable product lines emphasize RCO to help them identify the product offering "sweet spot" that covers the majority of orders; thus, they can achieve operational efficiencies through improved focus on these

Prior to introduction: Complexity ROI screening	After introduction: Portfolio management with RCO
• Identify and estimate cost impacts of variety, modeling relationships using activity-based costing, stochastic inventory modeling, and other quantitative methods, as needed. • Codify relationships into a complexity ROI calculator. • Screen new SKU or feature proposals for projected ROI using complexity ROI calculator.	• Use RCO to segment portfolio into core high-contribution offering and lower-contribution extended offering. • Construct differentiated service offerings to improve performance of core offering and reduce overhead associated with serving extended offering. • Target select elements of extended offering for elimination or rationalization, as appropriate.

Figure 1: HP follows a two-phased approach that involves up-front screening of new-product proposals for ROI and ongoing evaluation and reprioritization of the product portfolio using RCO. Both phases rely heavily on OR to provide a data-driven framework for decision making.

products. Some businesses, such as PSG, use both heavily.

Figure 1 shows HP's overall portfolio management approach, highlighting the role of these OR solutions at each phase.

New-Product ROI Screening Framework

HP screens proposals for new SKUs, features, product bundles, and platforms prior to investing in them. The screening process begins with a detailed analysis of the cost structure and drivers in each business and product line—cost relationships that are generally obscure and not captured in accounting systems. A team of OR professionals spends one to three months developing a model of how business costs respond to increases in product variety. It captures the cost relationships and codifies them into a set of guidelines and an ROI calculator that the business can deploy to evaluate new-product proposals. As costs change, the business can update the calculator's parameters, enabling it to evaluate proposals on an ongoing basis.

We identify the major cost drivers that product variety impacts. First, we examine the complete life cycle, from conception through postlife support (i.e., support after the product has been removed from the

Cost type	Nature of relationship	Cost categories
Variable complexity costs	Volume-driven	• Material costs: volume discounts • Variability-driven costs: excess costs (financing, storage, depreciation, obsolescence, fire sales) and shortage costs (material price premiums, expediting, lost sales because of shortages)
Fixed complexity costs	Variety-driven	• Resource costs: R&D, testing, product management, etc. • External cash outlays: tooling, costs to contract manufacturer • Indirect impacts of variety: manufacturing switching costs, warranty-program expenses, quality impacts, returns costs

Figure 2: HP systematically assesses the cost impacts of product variety using a framework that captures major cost impacts along the full profit & loss statement and throughout the entire life cycle of each product.

portfolio). Second, we examine the entire business cost structure, including fixed and variable costs. We use the cost categories (Figure 2) as a guide to ensure that we have considered all important cost elements.

In our analysis, we distinguish between fixed and variable complexity costs. We define fixed complexity costs as those that scale with the number of products offered; therefore, the key cost driver is the number of SKUs or variants sold, regardless of each variant's unit volume. These are the costs, such as research and development (R&D) or product marketing resources, that are required to bring new products to market and support them over their life cycles. To estimate variety-driven cost impacts, we use a mix of activity-based costing, regression analysis, and other techniques. For example, as variety increases, customers have more options and are more likely to return or exchange products. To quantify this, we use regression analysis to evaluate the impact of SKU variety on product returns cost, allowing us to estimate a rough relationship between the numbers of products offered and the cost of returns.

We define variable complexity costs as those where the unit variable cost of a SKU or part increases because of insufficient volume to reach an operationally efficient scale; the unit volume for a given SKU or variant is the key driver. Examples include material costs, which are higher for lower-volume parts because of inadequate negotiating leverage for the buyer or economies of scale for the supplier, and costs associated with demand variability, which result in significantly more excess costs (devaluation, excess, and obsolescence) and shortage costs (freight expediting, supplier price premiums, lost sales). Figure 3 shows an approach we use to estimate how excess and shortage costs related to demand variability scale up and down with volume.

To evaluate whether or not to introduce a new-product variant, we balance the complexity costs against its projected marketing and sales benefits. We recommend screening out low-*value* products, which are not necessarily the same as low-*volume* products. Screening products by volume overlooks the significant differences in complexity cost among different product types. Volume thresholds also miss that some high-volume SKUs could drive very little incremental revenue and margin if they have a close substitute. Volume thresholds and rules of thumb can be useful but only if they adjust for cannibalization effects and complexity cost differences between different SKU types.

Our solution is to screen based on complexity-adjusted margin: the incremental margin (with cannibalization effects subjectively estimated by marketing) less the incremental complexity costs. To keep the solution simple, we start with an exhaustive consideration of all possible complexity costs and pare down that list to include only the most significant categories—for example, those that drive roughly 80 percent of the complexity costs. We then capture complexity cost guidelines for those costs in easy-to-use spreadsheet calculators. Because the key complexity cost drivers vary across businesses, we develop a different calculator for each business while leveraging common frameworks and techniques for complexity cost modeling.

In some cases, constrained resources (e.g., R&D) might be a significant portion of the complexity costs;
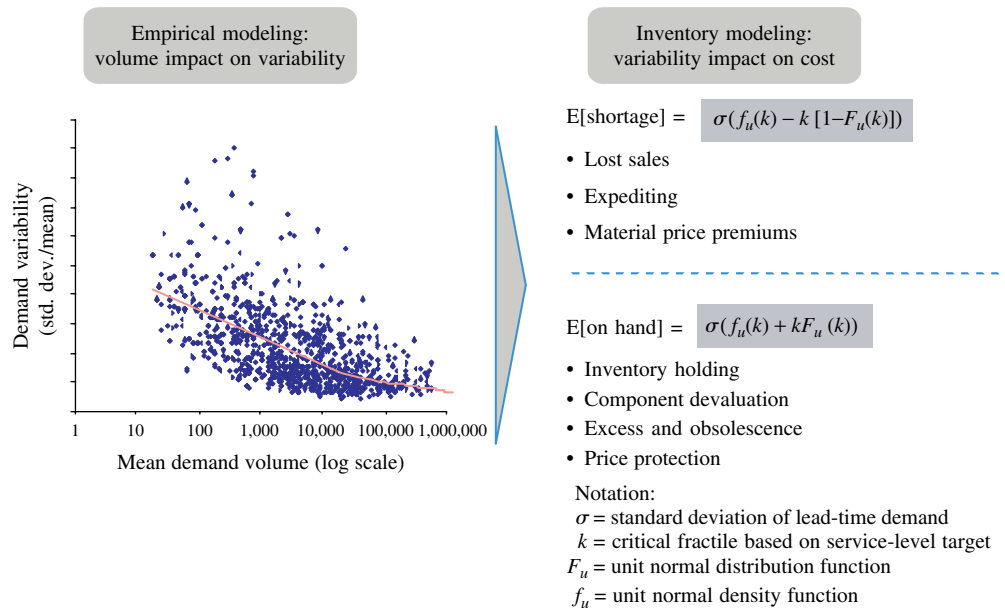
Empirical modeling: volume impact on variability

Inventory modeling: variability impact on cost

$E[\text{shortage}] = \sigma(f_u(k) - k\,[1 - F_u(k)])$

- Lost sales
- Expediting
- Material price premiums

$E[\text{on hand}] = \sigma(f_u(k) + kF_u(k))$

- Inventory holding
- Component devaluation
- Excess and obsolescence
- Price protection

Notation:
σ = standard deviation of lead-time demand
k = critical fractile based on service-level target
F_u = unit normal distribution function
f_u = unit normal density function

Figure 3: HP models the relationship between product volumes and demand variability empirically as an input to measuring the impact of product variety on costs associated with demand variability. Greater product variety and less-concentrated demand result in higher variability, which in turn results in higher inventory-driven costs and shortage-related costs. Once the relationship is modeled at this level, calculations of inventory-driven costs and shortage costs can be embedded into calculators to automate the estimation of complexity cost impacts because of different volume assumptions.

thus, we calculate a complexity ROI using our earlier distinctions between fixed and variable complexity costs:

Complexity ROI

$$= \frac{(\text{Incremental margin} - \text{Variable complexity costs})}{(\text{Fixed complexity costs})}.$$

Typically, HP businesses set their ROI hurdle fairly high (6:1 or greater) to ensure that the investment in introducing new variants is justified vis-à-vis other investments that HP could make with the same resources. Often we limit the costs that we include in the denominator to only those that represent the specific resource or resources that are constrained, moving some (non-resource-based) fixed costs to the numerator when we compute the ROI.

Once the cost structure has been identified and major costs quantified, a cross-functional team, including supply chain, finance, and marketing representatives, validates the modeled cost relationships. A cross-functional core team and sponsorship

team must go through the complexity cost modeling journey together, to gain input and commitment from all functions on the cost guidelines that the ROI calculator will use going forward. The sponsorship team should include executives from the major functions impacted by the costs and benefits of variety: supply chain, R&D, and marketing. To the extent possible, the team conducts the validation by comparing predicted results with actual data. However, because the relationships often do not show up directly in data, it is more important to obtain buy-in from the organizations that the calculator represents a *reasonable model* of costs than to prove conclusively that the modeled relationships are an exact predictor.

Figure 4 shows an example of the main interface of a complexity ROI calculator used in HP's LaserJet business. For ease of use and diffusion across a large number of users, we keep the calculator interface intuitive and graphical, despite considerable behind-the-scenes sophistication and modeling used to assess and capture the relationships that drive the calculations. The tool should require only inputs (e.g.,

LaserJet variety cost-benefit calculator

Model Number/Name of Proposed Model:	ABC123
Platform:	XXX
Description:	
Product category:	SF mono ▼
Is a new accessory required?	No ▼
Is the model being added as part of NPI?	NPI ▼
What kind of accessory is being added?	Class I ▼
Is new documentation required?	No ▼
Which supply chain?	Low-Touch ▼

Model Statistics

Projected lifetime (months)	15
Monthly volume	5,900
SKUs added with Model	31
List price	$349
Hardware net revenue/unit	$329
Contrib. margin/unit	$79

Engine Requirements

Engines	Percentage of volume	Monthly volume	Total volume	Avg net rev/unit
110V NW BL	40	2,360	1,680	$596
220V NW BL	40	2,360	1,680	$596
110V NW BL MIJ	10	590	420	$596
220V NW BL MIJ	10	590	420	$596

Incremental volume

Percent incremental volume	45
Total cannibalized units per month	3,245

Forecast

	Monthly	Lifetime	Incremental
Volume (units)	5,900	88,500	39,825
Hardware revenue	$1,941,100	$29,116,500	$13,102,425
Contrib. margin	$466,100	$6,991,500	$3,146,175

Cannibalization

	Related product number 1		Related product number 2	
Name:	ABC	Name:	DEF	
% of cannib	80	% of cannib	20	
Cannib'd units	2,596	Cannib'd units	649	

Related products

	Planned	Adjusted	Planned	Adjusted
Projected lifetime (months)	17	17.0	14	14.0
Monthly volume	5,000	2,404	2,000	1,351
List price	$369	$369	$299	$299
Hardware net revenue/unit	$349	$349	$279	$279
IFS2 margin/unit	$89	$89	$59	$59

Estimated unaccounted costs of adding model

Opportunity costs:

	Cost impact
Lost sales due to stockouts	$1.7K–$3.7K
Total	**$2K–$4K**

COGS + Contra-revenue impacts:

	Cost impact
Inventory holding, storage, and financing	$17.5K–$37.6K
Excess and obsolescence/fire sales	$3.6K–$7.7K
Expediting	$1.1K–$2.3K
Price protection	$3.9K–$8.3K
Spare parts inventory-driven costs	$0.7K–$1.5K
Material cost volume discounts	$0.0K–$0.0K
MOH (switching and setup)	$12.6K–$49.3K
Refurb and returns	$12.5K–$18.8K
Warranty	$3.0K–$10.2K
Total	**$55K–$136K**

Operating-expense impacts:

	Cost impact
Sales and Mktg OH	$10.0K–$18.8K
Product data mgmt	$58.6K–$118.4K
Commodity mgmt	$3.8K–$6.3K
Mfg program mgmt	$5.0K–$10.0K
Planning and Forecasting	$96.5K–$157.1K
Product completion center	$3.4K–$9.6K
Sr. mgmt attention for escalations	$0.0K–$0.0K
Test center	$0.0K–$0.0K
Other un accounted costs	$0.0K $0.0K
Total	**$177K–$320K**

Grand total

$234K–$459K

ROI:

	ROI Assessment
Incremental margin	**$2.53M–$2.53M**
Adj. incremental margin	**$2.47M–$2.39M**
Fixed cost	**$177K–$320K**
ROI	13.94:1–7.46:1

Cutoff values

Yellow zone ROI	1:1
Green zone ROI	7:1

Minimum percentage incremental to qualify	
…for green zone	45
…for yellow zone	19

Figure 4: Users enter information on new-model proposals into the calculator. The projected ROI for the model is shown, and detail on the cost impacts is included. Color coding is used to indicate whether an SKU achieves "green zone," "yellow zone," or "red zone" ROI thresholds.

projected volume and product type) that are readily available from marketing when making the case for a new SKU. More complex modeling (Figure 3) goes into making and calibrating the tool but is not part of the user interface.

Our complexity-analysis framework has proven to be flexible and effective. Wherever possible, we try to leverage the ROI approach and cost classification (Figures 2 and 3) and reuse components of calculator spreadsheets. However, because the key cost drivers

and the impacts of product variety can vary significantly across business environments, a customized solution is typically required. The average cost to conduct a complexity-analysis project (i.e., the time required by the OR team and sponsoring organization) for one HP business is approximately $90,000.

The Revenue Coverage Optimization (RCO) Tool

After products or features have been launched into HP's product portfolio, some costs of variety become sunk, and the variety management focus shifts from screening new products to maximizing value from the active portfolio. As products begin to sell in the marketplace, transaction-level sales data become available, enabling more sophisticated analysis. RCO was designed to help HP understand the revenue trade-offs in managing product variety when a history of customer order data is available. Prior to implementing RCO, HP's prevailing method of product portfolio management had been to judge products by their individual revenue or volume contributions in recent order history. However, researchers and business managers knew that when determining the importance of products in businesses with configurable products, examining each product in isolation would not suffice. A product that generates relatively little revenue on its own, such as a power supply, might be a critical component in high-revenue orders and essential to order fulfillment. To capture the interrelationship among products through orders, HP developed a new metric, *order coverage*, which represents the percentage of a given set of past orders that could be completely fulfilled from the portfolio. Similarly, *revenue (margin) coverage* of a portfolio is the revenue (margin) of its covered orders as a percentage of the total revenue from the data set. The concept of coverage provides a meaningful way to measure each product's overall impact on a business. RCO is a deterministic optimization tool that finds the smallest portfolio of products that covers any given percentage of historical order revenue. It answers questions such as, "If I pick only 100 products, which ones should I choose to maximize revenue from orders containing only these products?" More generally, given a set of historical orders, RCO computes a nested series of

product portfolios along the efficient frontier of order-revenue coverage and portfolio size.

The black curve in Figure 5 illustrates this efficient frontier. In this example, 80 percent of order revenue can be covered with less than 27 percent of the total product portfolio if we select those products according to RCO's recommendations. One can use this tool to select the portfolio along the efficient frontier that offers the best trade-off—relative to business objectives—between revenue coverage and portfolio size. The strong Pareto effect in the RCO curve presents an opportunity to improve on-time delivery performance. A small investment in improved availability of the top few products will significantly reduce average OCT.

The portfolios corresponding to points along the efficient frontier are nested; the portfolio with 95 percent revenue coverage contains the one with 90 percent coverage. Thus, RCO provides a product ranking that yields a continuum of portfolio choices that are easily modified to adjust to changes in desired coverage level.

The problem of generating a single portfolio on the efficient frontier is known as a selection problem. Its canonical formulation is an integer program-
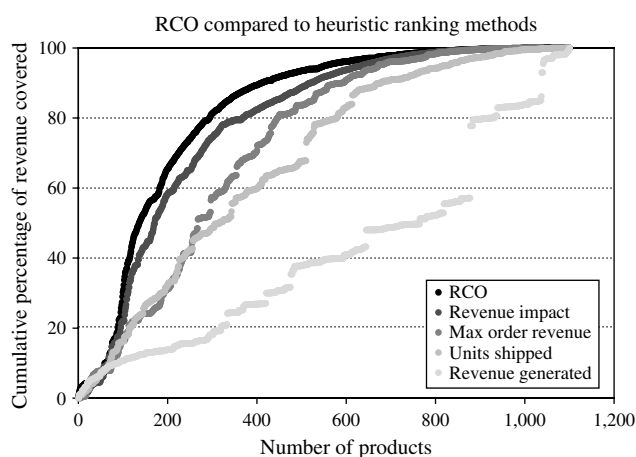
Figure 5: This chart shows revenue coverage versus portfolio size achieved by RCO (black) and four other product-ranking methods, applied to the same historical data. The four other curves, in decreasingly saturated grays, are based on ranking by the following product metrics: *revenue impact* (the total revenue of orders containing the product), maximum revenue of orders containing the product, number of units shipped, and individual product revenue.

ming problem that, for HP's order-history data sets, is too big to solve by standard methods. We found that the problem of generating a series of solutions along the efficient frontier can be posed as a parametric maximum-flow problem in a bipartite network (Balinksi 1970, Rhys 1970). The team developed a new, efficient, and exact algorithm to solve the parametric maximum-flow problem (Tarjan et al. 2006; Zhang et al. 2004; 2005a, b). This algorithm, called simultaneous parametric maximum flow (SPMF), is several times faster than best-known prior solution techniques for the same problem on the large real-world data sets that we faced (Babenko et al. 2007). It is also much easier to implement than previous algorithms. In the new algorithm, the parametric maximum-flow problem A is converted to a special nonparametric maximum-flow problem B. Solving B gives the chain of nested solutions to problem A at all break points of the parameter. The special nonparametric maximum-flow problem B is solved by a new flow-balancing method, which redistributes the flows over a number of arcs either around a closed loop or among all the arcs incident to a vertex. This flow-balancing method differs from two main types of maximum-flow algorithms in the literature—the augmenting path method (Ford and Fulkerson 1956) and the preflow-push-relabel method (Goldberg and Tarjan 1986, 1988). Tarjan et al. (2006) later generalized the flow-balancing method to general nonparametric maximum-flow problems. Appendix B shows details of the portfolio-selection problem, its equivalence to a parametric maximum-flow problem, and the SPMF algorithm.

RCO compares favorably to other heuristics for ranking products (Figure 5). The gray curves show the cumulative revenue coverage achieved by four heuristic product rankings, in comparison to the coverage achieved by RCO. The best alternative to RCO is one that ranks each product according to its *revenue impact*, a metric our team devised to represent the total revenue of orders in which the product appears. The revenue-impact heuristic comes closest to RCO's coverage curve, because it is best among the heuristics at capturing product interdependencies. Still, in our empirical tests, we found that the revenue-impact ranking provides notably less revenue coverage than RCO's ranking. Given that RCO runs in less

than two minutes for typical data sets, HP had no reason to settle for inferior coverage.

Although we have emphasized the objective of maximizing historical revenue coverage subject to a constraint on portfolio size, RCO is flexible enough to allow a much wider range of objectives, such as coverage of order margin, number of orders, or any other metric associated with individual orders. It can easily accommodate up-front strategic constraints on product inclusion or exclusion and can be applied at any level of the product hierarchy, from SKUs down to components.

The SPMF algorithm has applications well beyond product portfolio management, such as in the selection of parts and tools for repair kits, terminal selection in transportation networks, and database-record segmentation. Each problem can be naturally formulated as a parametric maximum-flow problem in a bipartite network. The team's extension of SPMF to nonparametric max flows in general networks has an even broader range of applications, e.g., in airline scheduling, open-pit mining, graph partitioning in social networks, baseball elimination, staff scheduling, and homeland security.

In practice, RCO is used to enhance and facilitate human judgment in managing product variety. Portfolio design depends critically on knowledge of strategic new-product introductions and planned obsolescence, which historical order data do not reveal.

HP businesses typically use the previous three months of orders as input data to RCO, because this duration provides a representative set of orders. Significantly longer horizons might place too much weight on products that are obsolete or nearing end of life. When analysis on longer horizons is desired, RCO allows weighting of orders in the objective, thus placing more emphasis on covering the most recent orders in a given time window.

SPMF was implemented in C++. A graphical user interface (GUI) in a Web browser and RCO output visualization in Excel were integrated with SPMF and the corporate financial database for RCO deployment in HP businesses. RCO's approximately $1.1 million development cost includes researchers' time to develop and implement the algorithm and contractor time to build the GUI.

As we deployed RCO, we validated the work on multiple fronts to ensure that all stakeholders would feel confident in the results. We verified the algorithm's correctness in three ways. First, we proved its convergence to correct results theoretically and by matching its output with those of a well-established commercial optimization solver (CPLEX) applied to an equivalent problem formulation. Second, with help from domain experts in product marketing, we carefully reviewed the tool's input and cross-validated the results with these experts' intuition and through comparison to other metrics. Third, we validated the model. Although the model's objective of computing the efficient frontier of coverage and portfolio size had been defined jointly by the business stakeholders and OR professionals on the team, the best evidence of its validity is that the business results match the model's predictions: improved focus on the top-ranked products yields significant overall operational benefits. The next section highlights these results.

Case Example: Portfolio Management in PSG

PSG, a $42 billion business, includes HP's commercial and consumer desktop and notebook PC businesses, as well as its workstations, handheld computing, and digital entertainment product lines.

PSG experienced many of the challenges discussed above as its variety grew over the past decade. To increase market reach and maximize competitiveness in the markets in which it competes, it offered many products and options, allowing customers to select from many technology platforms and product form factors. Within each platform, customers could select from an array of processors, drives, memory configurations, and accessories; using HP's configure-to-order process, these components could be combined to generate millions of desktop and notebook PC order combinations. Adding to the complexity, each configuration option could include different subassemblies and components, which might be sourced from multiple suppliers.

Managing each component required testing, forecasting, supplier management, inventory, end-of-life and warranty support, production planning, and a host of other processes and outlays. If one component was unavailable, production and order shipments might be delayed and customers dissatisfied. Some products were sold often and in high volumes; others were involved in only a handful of orders and were of little strategic importance to the business. However, differentiating between these products in a systematic and data-driven way was difficult; thus, some components might be overstocked and some understocked. These availability issues led to unpredictable and uncompetitive OCTs and frustrated customers. Last, the inconsistencies between offerings in the Americas, Europe, and Asia, despite product similarities, gave rise to even greater variety and costs for PSG.

HP's approach, which it began in its notebook division and extended later to desktops, started as an engagement with its Strategic Planning and Modeling (SPaM) team to review the divisions' cost structures and develop an appropriate complexity ROI calculator; an engagement with HP Labs to integrate the RCO algorithm into its business planning systems followed.

Initial complexity cost analysis for notebook PCs indicated relatively low up-front variety-driven fixed costs and relatively high volume-driven variable costs. Therefore, PSG's emphasis was on managing the portfolio to steer demand from low-volume features to medium- and high-volume features, rather than on up-front screening of new entrants. PSG deployed complexity ROI calculators for notebooks and desktop PCs to allow it to quantitatively evaluate feature decisions, and receive guidance on minimum incremental margins necessary for features to be viable. It then used RCO to prioritize features in its offerings. Today, these tools continue to provide critical input into three major PSG programs:

• **Worldwide Recommended Offering (RO) program for notebooks and desktops**. This program uses RCO to identify the most critical features in each region; it designates them as the "core offering," which includes about 20 percent of the feature portfolio and covers 80 percent of all orders. It classifies all other features as the "extended offering." PSG provides different service levels for the two classes of features. It stocks core features in higher inventory levels; thus, they have short lead times. It stocks extended features at lower levels or not at all; thus,

they have longer lead times. By reallocating its inventory investment, PSG has reduced its average OCT by four days on core notebook features and two days on core desktop features since 2006. These operational efficiency gains are self-reinforcing. Demand shifts to the core because customers who choose core features are rewarded with rapid delivery. As demand concentrates on fewer features, demand variability declines, leading to additional cycle-time improvements. In the program's first six months, the number of features required to cover 80 percent of orders dropped by one-fifth; the average revenue contribution of core features doubled and that of extended features stayed flat. RO-based demand steering has improved forecast accuracy and availability, lowered inventory expenses, and improved consistency and predictability for both customers and suppliers. To date, PSG has implemented the notebook RO program in the United States and the Asia/Pacific region, and the desktop RO program in the United States and Europe/Middle East region. PSG estimates that each day of OCT improvement saves it at least $38 million annually in inventory-driven costs, operational expenses, and financing and saves a $12 million one-time gross margin increase through cash-flow benefits. PSG management estimates that it realized savings of $130 million in Europe, Middle East, and Africa (EMEA) and the United States in the first year of RO implementation and $100 million annually thereafter. Rollouts to other PSG product lines are likely to generate comparable benefits.

• **Global Product Offering program.** This initiative is a set of products made available worldwide to HP's largest global customers. Each customer has a preferred set of standard products and wants those products offered worldwide with consistent price, components, and life cycle. Since early 2008, RCO has been used to design this offering, replacing the previously used manual "best guess" process. The global product offering adoption rate has increased from 18 to 85 percent. Moreover, revenue from PSG's global customers grew 23 percent in 2008, partly because of a better-designed global product offering. PSG conservatively estimates a $130 million annual revenue increase because of using RCO for this initiative; global customer escalations, because of inconsistent worldwide product offerings, have also decreased significantly.

• **Feature Screening program.** Using the complexity cost assessment and the complexity ROI calculator tools developed for notebooks and desktops, PSG can perform data-driven evaluation of product proposals and eliminate low-ROI products or features before launching them into the portfolio, thus avoiding a range of unrecoverable costs. To date, these programs have generated over $100 million in margin improvements and continue to generate over $40 million per year for PSG.

The initiatives have generated hundreds of millions of dollars in impact in PSG and dramatically changed how its management and operations teams work. Marketing and supply chain teams can now have informed, fact-based discussions around trade-offs in the portfolio and, for the first time, jointly discuss the concept of the sweet spot.

Many organizational and informational hurdles hindered implementation of the initiatives. The primary challenge, in PSG and across HP, was to shift the mindset from revenue-focused management to margin-focused management. The decisions that drove explosions in product variety were made to chase revenue opportunities without a clear understanding of their cost and margin impacts. Margin-focused management required cross-functional teams to bridge the organizational divide between supply chain, marketing, and R&D to bring together a complete picture of the costs and benefits of variety. To exacerbate matters, many incentives both in the sales organization and for executive management were based on revenue results rather than profitability results. The efforts supported by HP's OR teams provided data-driven and unbiased insights and tools to bridge the organizational divide. We also helped put in place explicit complexity metrics, which eventually became part of the scorecards used to evaluate management performance.

A second challenge was around disseminating and gaining agreement on the many process changes that came out of the programs in PSG, such as new rules for customer communication, inventory management, and supplier management.

Last, the data-driven processes that we put in place also required new information links and improved management of product and part information within PSG. Building and supporting the tools driving the solution implementation required support from HP's information technology organization.

Impact

Across all its businesses, HP has achieved, and continues to enjoy, substantial benefits from the implementation of the portfolio management techniques described in this paper. These include direct financial benefits, operational improvements, and a range of important "soft" benefits. For example, using a complexity ROI calculator custom-built with executives and program managers from their business, the company's trademark LaserJet business reduced SKU counts by 40 percent between 2006 and 2009, dramatically streamlining the offering and generating annual net profits of approximately $20 million.

HP's enterprise server business, Business Critical Systems (BCS), runs RCO quarterly to evaluate its existing product portfolio and make product rationalization decisions. RCO enabled the elimination of more than 3,300 products from BCS' portfolio of over 10,000 products from late 2004 through 2008. BCS supply chain managers estimate that this reduction has resulted in at least $11 million in administrative cost savings, excluding any inventory cost or other operational expense. They have also used RCO to help them prioritize the order in which products should be brought into compliance with new European environmental standards. Moreover, BCS employs RCO for improved, data-driven design of options for new-product platforms based on the order history of previous-generation platforms.

The use of the ROI screening framework and RCO in HP's portfolio management programs has yielded company-wide profit improvements, conservatively estimated, of over $500 million between 2005 and 2008, and continues to generate benefits of about $180 million per year, with the potential for greater benefits through expanded deployment of these methods.

The application of these tools has also yielded important qualitative benefits for HP.

• Improved customer satisfaction: PSG customers appreciate the overall cycle-time reduction and the more predictable product availability that has resulted from the RO program. These changes have improved customer loyalty, market share, and competitive positioning for PSG. Moreover, the global product offering allows PSG's large global customers to satisfy their product requirements in multiple countries, leading to increased demand, higher customer satisfaction, and markedly fewer global customer escalations to HP's top management.

• Product line complexity reduction: Elimination of thousands of low-value products from HP's line-ups, both through prelaunch screening and postlaunch rationalization, reduced complexity and drove cost reductions across many areas, such as product development, qualification, testing, forecasting, planning, order management, manufacturing, data management, marketing, and supplier management.

• Reduced confusion among customers and sales representatives: Significant SKU reductions throughout the company have lessened the confusion that excess product variety caused among customers and HP sales professionals.

• Partner and supplier benefits: HP's suppliers and channel partners benefit from improved forecasting accuracy, as well as reduced up-front tooling and qualification costs achieved through these portfolio management techniques.

• Increased organizational effectiveness: The use of the OR-based tools to manage product variety has heightened awareness of the cost of complexity throughout HP businesses, brought about better organizational discipline in SKU introduction, improved collaboration between product marketing and supply chain teams, and resulted in a significant reduction in costly manual errors that arose from overloading the forecasting and planning teams.

• Improved visibility of OR within HP: The success of RCO and the complexity ROI calculators has improved organizational understanding, up to the senior executive level, of how OR can provide operational efficiencies to increase revenue and profit. The high visibility of RCO and the complexity ROI framework benefits has led to several new deployments of these tools.

• Portability: HP's approaches to managing product variety are applicable to many businesses inside and outside of HP. In configurable product businesses, where fulfillment of an order depends on availability of several products, RCO can help achieve operational efficiencies by identifying the sweet spot in the offering. The complexity ROI framework has been and continues to be applied successfully outside of HP. The consulting authors have worked with companies in a variety of industries to implement similar approaches in their businesses.

OR at HP

HP has been a leader in applying OR to its important business problems for decades. In addition to its many OR professionals throughout the company, HP has two major centers of excellence in OR: SPaM and HP Labs. SPaM is an analytics team that works through an internal consulting model to support operational innovation and operations strategy at HP while delivering immediate impact through business projects. It combines the talents of OR PhDs with top-tier management-consulting experience (Olavson and Cargille 2008). SPaM sometimes partners with OR academics or analytical consultants at specialized firms such as Strategic Management Solutions, an important contributor to the development of the variety-management ROI screening framework that SPaM uses. HP Labs is HP's central R&D organization chartered to conduct high-impact scientific research to address the most important opportunities that HP and its customers will face in the next decade. Within HP Labs, researchers with PhDs in OR, mathematics, statistics, economics, and computer science deliver sophisticated analytical tools to HP's businesses and advance the state of the art in business process research (Jain 2008). SPaM and HP Labs, in collaboration with the authors of this paper, developed the approaches to product portfolio management described herein.

Conclusions

HP's approach to managing its product portfolio is a real-world example of applying OR to dramatically improve business performance. Although grounded in science, our solutions are highly tailored to the true nature of the business problems facing HP. They represent a powerful, comprehensive, and flexible approach to managing product variety. It has proven successful at HP and is extendable to businesses in other industries.

Appendix A. A Method for Calculating Variability-Driven Costs

We define variability-driven costs as costs resulting from physical mismatches of demand and available supply, including both costs of shortage (expediting, lost sales, and material price premiums) and inventory holding and excess (excess and obsolescence, component devaluation, product discounting, or price protection). We also present an extension of the method to allow variability-driven costs to be approximated as a simple function of part or product volume, making it easy to operationalize the method in decision support tools such as the ROI calculators.

The advantage of calculating variability-driven costs directly is that this method can be applied to analyze the impact of process improvements on known cost pools or high-risk events (e.g., new-product introduction or product end of life). Most importantly, the method provides an alternative for quantifying the financial benefits of any process change that pools demand risk to reduce demand variability, such as reducing part variety or consolidating inventory stocking locations. The typical rough-cut approach estimates how much an inventory buffer can be reduced and applies an average inventory-driven cost rate against this reduction to calculate a cost savings. Although useful in some contexts, this rough-cut method also has serious shortcomings. First, it addresses only the costs of excess and not the costs of shortage. Second, the calculated risk-pooling benefits approach zero as the planned inventory buffer approaches zero; however, some of the highest sources of variability-driven cost concern events in which the buffer is relatively small, e.g., end-of-life planning, buffer planning in advance of announced component price drops, or large demand spikes because of big account orders. By contrast, our method does not have these shortcomings.

The foundation for calculating variability-driven costs is a closed-form calculation of expected stockouts and expected on-hand units. We use the following definitions:

$$X: \text{ random demand over replenishment lead time (lead-time demand).}$$
$$\mu = \mathrm{E}(X): \text{ expected lead-time demand.}$$
$$\sigma = \mathrm{stdev}(X): \text{ standard deviation of lead-time demand.}$$
$$S = \mathrm{E}(X) + b: \text{ order-up-to point of inventory in a periodic-review system.}$$
$$b: \text{ target safety stock buffer quantity in units, where } b = S - E(X).$$

k: the fractile on the unit normal associated with the buffer quantity: $k = (S - \mu)/\sigma = b/\sigma$.

$F_u(k)$, $f_u(k)$: distribution and density function for the unit normal distribution.

Fundamentally, we are interested in evaluating two quantities: E[stockout(S)], the expected stockout if the order-up-to point is S, and E[on-hand inventory(S)], the expected on-hand inventory units if the order-up-to point is S. Special properties of the normal and Gamma distributions allow closed-form solutions to these expressions. We illustrate for the normal case below. First, observe that stockout(S) − on-hand inventory(S) = $X - S$, from which we have

$$\text{E[on-hand inventory } (S)] = S - \text{E}(X) + \text{E[stockout}(S)]$$
$$= b + \text{E[stockout}(S)]. \quad (1)$$

In the case of normally distributed demand, we can calculate the expected stockout by scaling the solution derived in Silver et al. (1998, Equation B.7) for the unit normal by the factor σ:

$$\text{E[stockout}(S)] = \sigma(f_u(k) - k[1 - F_u(k)]). \quad (2)$$

Stockouts are typically measured in percentage terms as a fill rate, calculated as

$$\text{fill rate}(S) = 1 - \text{E[stockout}(S)]/\mu.$$

Combining Equations (1) and (2) yields an expression for expected on-hand inventory:

$$\text{E[on-hand inventory}(S)] = \sigma(f_u(k) + kF_u(k)). \quad (3)$$

From Equations (2) and (3), we can calculate how the expected excess and shortage rates (units as a percent of mean demand) vary with changes in lead-time demand variability. If we assume that historical total variability-driven cost pools (e.g., total excess and obsolescence costs) scale in proportion to the shortage and excess rates, then we have a model for how each cost pool can be impacted based on the lead time and buffer stock relevant to that cost pool. For example, for end-of-life discounting costs, the target buffer is zero, and the lead time for critical components could be longer than usual as suppliers demand end-of-life buys at longer lead times.

An extension of the method allows the variability-driven cost per unit to be modeled as a function of average part volume. This allows us to quantify how the variability-driven cost varies with volume. For example, consider the guideline "an LCD panel selling less than 10 K units per month has a \$10/unit higher variability-driven cost than a panel selling more than 50 K units per month." This is easy to understand and easy to factor into the decision process of whether to offer an additional panel in the product line. Figure A.1 summarizes the analysis process.

To derive the cost per unit as a function of volume relationship, it is also necessary to model a statistical relationship between part volume and variability—low-volume parts have a higher coefficient of variation (CoV, or the ratio of standard deviation to average demand). We conducted a statistical analysis and concluded that a statistical estimator for a part's demand variability derived from such an aggregate volume-variability relationship (using data from many different parts) carries as much predictive value for the part's true variability going forward as did an estimator derived from a limited history of actual data on that specific part. Next, we collect data on the distribution of part volumes across the portfolio so that the following equality holds

$$\sum_{i=1}^{n} v_i c(v_i) = C_{\text{portfolio}},$$

where we sum across all the n parts, each with average volume v_i and variability-driven cost per unit $c(v_i)$, to arrive at the total portfolio variability-driven
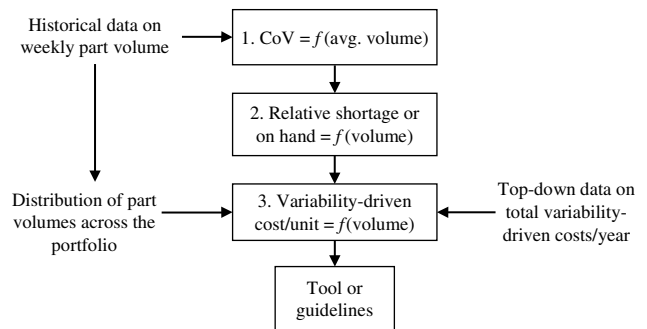

Figure A.1: The flowchart shows an overview of an approach for estimating variability-driven cost per unit.

cost $C_{\text{portfolio}}$. Although we know the part volumes and the total cost, we do not know $c(v)$. However, we can calculate a relative cost curve, $r(v)$, using the on-hand inventory and shortage-cost modeling, and we can link the two through a constant d for which we will solve

$$c(v_i) = dr(v_i) \rightarrow \sum_{i=1}^{n} v_i dr(v_i) = C_{\text{portfolio}} \rightarrow d = \frac{C_{\text{portfolio}}}{\sum v_i r(v_i)}.$$

We now have a simple link between part volume and cost; we can use it in the complexity cost calculator to compare the costs of multiple small-volume SKUs versus pooling the volume in a single SKU.

Appendix B. RCO Problem Formulation and SPMF Algorithm

In this appendix, we describe the technical details of the RCO problem formulation and its equivalence to a parametric maximum-flow problem in a bipartite network, and we provide an overview of a new algorithm for parametric maximum flow.

The problem of finding a product portfolio of size of at most n that maximizes the revenue of orders covered can be formulated as the integer program IP(n):

$$\text{Maximize} \sum_{o} r_o y_o \quad \text{subject to}$$

$$y_o \leq x_p \text{ for each product-order combination } (o, p), \quad (4)$$

$$\sum_{p} x_p \leq n, \quad (5)$$

$$x_p \in \{0, 1\}, \quad y_o \in \{0, 1\}, \quad (6)$$

where r_o is the revenue of order o, and binary decision variables x_p and y_o represent whether product p is included in the portfolio and whether order o is covered by the portfolio, respectively.

Because typical data sets have hundreds of thousands of product-order combinations, IP(n) can take many hours to solve or might even exceed memory limitations. However, it does have a nice structural property—the constraints (4) are totally unimodular. We exploit this property by creating the following Lagrangian relaxation LR(λ):

$$\text{Maximize} \sum_{o} r_o y_o - \lambda \sum_{p} x_p \quad \text{subject to}$$

$y_o \leq x_p$ for each product-order combination (o, p),
$x_p \in [0, 1], \quad y_o \in [0, 1].$

Because LR(λ) is totally unimodular, it has an integer optimal solution. Moreover, if a set of covered orders and selected products, (O, P), is optimal for LR(λ), then it is optimal for IP($|P|$). Thus, solving LR(λ) for a series of values of λ generates a series of solutions to IP(n) for several values of n. Solutions generated by this method are nested—the optimal set of covered orders for a given λ_0 is a subset of the optimal covered orders for smaller $\lambda \leq \lambda_0$. Moreover, these solutions lie along the efficient frontier of revenue coverage versus portfolio size. This series does not provide an integer solution for every possible value of n; solutions below the concave envelope of the efficient frontier are skipped. However, in practice the number of distinct solutions is typically about 85 percent of the total product count. A wise selection of values of λ produces quite a dense curve of solutions. To obtain a complete product ranking, we use product revenue impact as a heuristic to break ties among products, because this metric proved to provide the best coverage among the heuristics we tried.

Our original implementation of RCO used a commercial LP solver (CPLEX) to solve the series of problems, LR(λ). However, for typical data sets with millions of order line items, each such problem took several minutes to solve. To solve it for many values of λ to create a dense efficient frontier took many hours. We needed a more efficient approach.

The key to a more efficient approach is the equivalence between the problem, LR(λ), and the problem of finding a minimum cut in a particular bipartite network (Balinksi 1970). To see how LR(λ) can be viewed as a minimum-cut problem, consider the network in Figure B.1, with a source node s at the far left and a sink node t at the far right. Adjacent to the source node is a set of nodes, each corresponding to one product. Adjacent to the sink node is a set of nodes, each corresponding to one order. The capacity of the links adjacent to s is λ. The capacity of the link from the node for order i to t is the revenue of order i. The capacity of links between product nodes and order nodes is infinite.
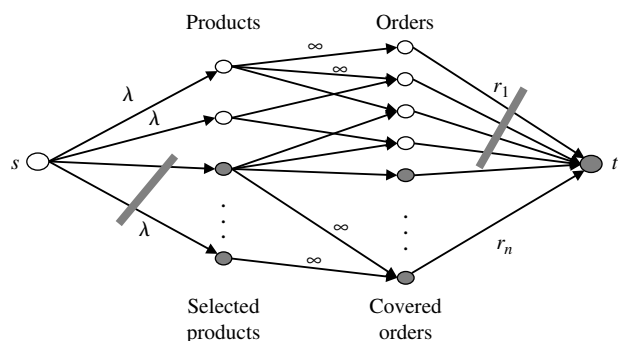
Figure B.1: The diagram illustrates a bipartite minimum-cut/maximum-flow problem that corresponds to the Lagrangian relaxation LR(λ).

For this network, the set T in a minimum cut corresponds to the products selected and orders covered by an optimal solution to LR(λ). To see why, first observe that because the links from product nodes to order nodes have infinite capacity, they will not be included in a finite capacity cut. Therefore, for any order node in the T set of a finite capacity cut, each product in the order must also have its node in T. Therefore, a finite capacity cut corresponds to a feasible solution to LR(λ). Moreover, the value of an $s - t$ cut is $\sum_o r_o(1 - y_o) + \lambda \sum_p x_p$. Minimizing this quantity is equivalent to maximizing $\sum_o r_o y_o - \lambda \sum_p x_p$; therefore, a minimum cut is an optimal solution to LR(λ).

It is a well-known result of Ford and Fulkerson (1956) that the value of a maximal flow equals the value of a minimum cut. Moreover, the minimum cut can be obtained by finding a maximal flow.

Because we seek a solution to LR(λ) for multiple values of λ, it is a parametric maximum-flow problem because the arc capacities depend on a parameter. Several known algorithms for parametric maximum flow exist, including those in Gallo et al. (1989) for general networks and Ahuja et al. (1994) for bipartite networks. In most prior algorithms for parametric maximum flow, a series of maximum-flow problems is solved, and each problem's solution is used to speed up the solution to the next one. By comparison, the algorithm presented here simultaneously finds the maximum flow in the network for all *break points* of the parameter value. The value of the maximum flow from s to t is a piecewise-linear function of λ. A break point of the parameter value is where the derivative of the piecewise-linear function changes.

Parametric Bipartite Maximum-Flow Algorithm

The new parametric bipartite maximum-flow algorithm takes advantage of the special structure of the capacity constraints that Figure B.1 shows.

The logic behind the algorithm is as follows. First assume that $\lambda = \infty$. Then, the only constraints on flows result from the capacity limitations on arcs incident to t. Finding flow assignments that saturate all capacitated links, resulting in a maximum total flow, is easy.

The next step is to find such a maximum-flow assignment that distributes flows as evenly as possible across all arcs leaving s. The property "evenly as possible" means that it is impossible to rebalance flows between any pair of arcs in such a way that the absolute difference between these two flows decreases. Note that even in this most even maximum-flow assignment, not all flows will be the same.

Now, with the most even assignment discussed above, impose capacity constraints of $\lambda < \infty$ on the arcs leaving s. If the flow assignment for a given one of these arcs exceeds λ, reduce the flow on this arc to λ and propagate the flow reduction appropriately through the rest of the graph. Because the original flow assignment was most evenly balanced, the total flow lost to the capacity constraint is minimal and the total flow remaining is maximal for the given parameter λ.

More formally, the algorithm works as follows:

Step 1. For a graph as in Figure B.1 with $\lambda = \infty$, select an initial flow assignment that saturates the arcs incident to t. This is most easily done backwards, starting from t and choosing an arbitrary path for a flow of size r_i from t through o_i to s.

Step 2. Rebalance the flow assignment iteratively to obtain a "most evenly balanced" flow assignment. Let $f(a \rightarrow b)$ denote the flow along the link from node a to node b. The rule for redistributing the flows is as follows. Pick i and j for which there exists an order node o_k and arcs $p_i \rightarrow o_k$ and $p_j \rightarrow o_k$ such that $f(s \rightarrow p_i) < f(s \rightarrow p_j)$ and $f(p_j \rightarrow o_k) > 0$. Then, reduce $f(s \rightarrow p_j)$ and $f(p_j \rightarrow o_k)$ by $\min\{(f(s \rightarrow p_j) - f(s \rightarrow p_i))/2, \ f(p_j \rightarrow o_k)\}$ and increase $f(s \rightarrow p_i)$ and $f(p_i \rightarrow o_k)$ by the same amount. Repeat Step 2 until no such rebalancing can be found.

The procedure in Step 2 converges, as Zhang et al. (2004, 2005a) prove. The limit is a flow assignment that is most evenly balanced. In addition, because

total flow is never reduced, the resulting flow assignment is a maximum flow for the graph with $\lambda = \infty$.

Step 3. To find a maximum-flow assignment for a given value of λ, replace flows exceeding λ on arcs leaving the source s by λ and reduce subsequent flows appropriately to reconcile flow conservation. The resulting flow assignment is a maximum flow for λ.

Zhang et al. (2005a) provide more details and a rigorous mathematical treatment of the problem. Zhang et al. (2004) show that the algorithm generalizes to the case in which arc capacities are a more general function of a single parameter.

Because our application requires only knowledge of the minimum cut, one only needs to identify those arcs that exceed the capacity limit of λ after Step 2. Those arcs will be part of the minimum cut; the ones leaving s with flows less than λ will not. To find the remaining arcs that are part of the minimum cut, one has only to identify which order nodes connect to s through one of the arcs with flows less than λ, and cut through those nodes' arcs to t.

We can show that the t-partition of the minimum cut with respect to λ contains products whose flows from the source equal λ and the orders containing only those products. These products constitute the optimal portfolio for parameter λ. Note that Steps 1 and 2 are independent of λ. The result of Step 2 allows us to immediately determine the optimal portfolio for any value of λ.

Because the flows are balanced between two arcs, $s \to p_i$ and $s \to p_j$, in the algorithm described above, we call it an arc-balancing method. Arc-balancing SPMF reduced the time for finding the entire efficient frontier from hours to a few minutes.

We developed a second version of the SPMF algorithm based on the idea of redistributing the flows going into a node o in a single step; for all pairs $p_i \to o$ and $p_j \to o$, flows $f(s \to p_j)$ and $f(p_j \to o)$ are most evenly balanced. This method of redistributing flows around a vertex o is the vertex-balancing method (Zhang et al. 2005b). Vertex-balancing SPMF further reduces the time for finding the entire efficient frontier to seconds for typical problems.

References

Ahuja, R. K., J. B. Orlin, C. Stein, R. E. Tarjan. 1994. Improved algorithms for bipartite network flow. *SIAM J. Comput.* **23**(5) 906–933.

Babenko, M., J. Derryberry, A. Goldberg, R. Tarjan, Y. Zhou. 2007. Experimental evaluation of parametric max-flow algorithms. C. Demetrescu, ed. *Proc. WEA 2007, Lecture Notes in Computer Science*, Vol. 4525. Springer, Heidelberg, Germany, 256–269.

Balinksi, M. L. 1970. On a selection problem. *Management Sci.* **17**(3) 230–231.

Cargille, B., J. Melia. 2007. Less is more, later is better: Keeping sales high and costs low at Hewlett-Packard. *APICS* (October 7) 40–43.

Cargille, B., C. Fry, A. Raphel. 2005. Managing product line complexity. *OR/MS Today* **32**(3). http://www.lionhrtpub.com/orms/orms-6-05/frcomplexity.html.

Ford, L. R., D. R. Fulkerson. 1956. Maximum flow through a network. *Canadian J. Math.* **8** 339–404.

Gallo, G., M. D. Grigoriadis, R. E. Tarjan. 1989. A fast parametric maximum flow algorithm and applications. *SIAM J. Comput.* **18**(1) 30–55.

Goldberg, A. V., R. E. Tarjan. 1986. A new approach to the maximum flow problem. *Proc. 18th Annual ACM Sympos. Theory Comput., Berkeley, CA*, ACM, New York, 136–146.

Goldberg, A. V., R. E. Tarjan. 1988. A new approach to the maximum flow problem. *J. ACM* **35**(4) 921–940.

Jain, S. 2008. Decision sciences: A story of excellence at Hewlett-Packard. *OR/MS Today* **35**(2). http://www.lionhrtpub.com/orms/orms-4-08/frroundtable.html.

Olavson, T., B. Cargille. 2008. OR inside. *OR/MS Today* **35**(5). http://www.lionhrtpub.com/orms/orms-10-08/franalytics.html.

Olavson, T., C. Fry. 2006. Understanding the dynamics of value driven variety management. *MIT Sloan Management Rev.* **48**(1) 63–69.

Rhys, J. M. W. 1970. A selection problem of shared fixed costs and network flows. *Management Sci.* **17**(3) 200–207.

Silver, E. A., D. F. Pyke, R. Peterson. 1998. *Inventory Management and Production Planning and Scheduling*. John Wiley & Sons, New York.

Tarjan, R., J. Ward, B. Zhang, Y. Zhou, J. Mao. 2006. Balancing applied to maximum network flow problems. *Proc. 14th Conf. Ann. Eur. Sympos. ESA, Lecture Notes in Computer Science*, Vol. 4168. Springer, Heidelberg, Germany, 612–623.

Zhang, B., J. Ward, Q. Feng. 2004. A simultaneous parametric maximum flow algorithm for finding the complete chain of solutions. HP Technical Report HPL-2004-189, Palo Alto, CA.

Zhang B., J. Ward, Q. Feng. 2005a. Simultaneous parametric maximum flow algorithm for the selection model. HP Technical Report HPL-2005-91, Palo Alto, CA.

Zhang B., J. Ward, Q. Feng. 2005b. Simultaneous parametric maximum flow algorithm with vertex balancing. HP Technical Report HPL-2005-121, Palo Alto, CA.