Name - Harshvir Singh

Roll no. - 10 2117056 (3CS2)

Assignment-6

#1

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$x_1, x_2, \ldots, x_n \Rightarrow$ sample size n

$$L(x_1, x_2, \ldots x_n) = f(x_1) \cdot f(x_2) \ldots f(x_n)$$

$$\Rightarrow \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) \cdots$$

taking $\ln$ on both sides

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma}\right)$$

taking partial derivative w.r.t. μ :-

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(\frac{2(x_i-\mu)}{2\sigma^2}\right) = 0$$

$$\sum_{i=1}^{n} (x_i - \mu) = 0$$

$$n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

Hence $\theta_1 = \bar{x}$ is therefore sample mean

Taking derivative w.r.t. σ^2

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{2\sigma^4} = 0$$

$$\Rightarrow -n + \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n}(x_i - \mu)^2\right)$$

Hence $\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$

$$\boxed{\frac{1}{n}\sum(x_i - \mu)^2}$$

— sample variance — 102110 56 — 30 5 8

#2

Binomial distribution $\to$ $^nC_{x_i}\,\theta^{x_i}(1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {^nC_{x_i}}\,\theta^{x_i}(1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1}^{n}\left(\ln({^nC_{x_i}}) + \log\theta^{x_i} + \ln(1-\theta)^{n-x_i}\right)$$

$$\log L = \sum_{i=1}^{n}\ln({^nC_{x_i}}) + \ln\theta\,\sum_{i=1}^{n}(x_i) + \ln(1-\theta)\sum_{i=1}^{n}(n-x_i)$$

differentiate w.r.t. θ

$$\frac{d\,(y(L))}{d\theta} = 0$$

$$\frac{1}{\theta}\sum x_i - \frac{1}{1-\theta}\sum(n-x_i) = 0$$

$$\frac{1}{\theta}\sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta}\sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)}\sum x_i = \frac{n^2}{1-\theta}$$

$$\theta = \frac{\sum x_i}{n^2}$$