# CSC 349-01: Design and Analysis of Algorithms Fall 2021

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## 1 Announcements

None

## 2 Approximation Algorithm for Bin Packing

### 2.1 General Idea for Approximation Algorithms

Main idea is to find fast algoritms that produce feasible solutions that are probably close to optimal.

### 2.2 Pseudocode for Bin Packing

**Input**: $n$ items each with a weight wi 1 and an unlimited supply of unit capacity bins, *b1,b2,....*
**Goal**: An assignment of items to bins such that the bin capacity is not exceeded and the minimum number of bins is used.

Example:

- Suppose we are given 7 items with weights:.3,.2,.2,.2,.2,.4,.5
- *b1* = 1,2,3,4, *b2* = 5,6, and *b3* = 7 is a feasible assignment.
- Is it optimal? Ans: No, we can do better.
- How about **b1** = 2,3,4,6 and *b2* = 1,5,7?

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FirstFit({*w1,w2,w3,...wn*})

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**Input**: $n$ items each with a weight wi 1 and an unlimited supply of unit capacity bins, *b1,b2,....*
**Output**: An assignment of items to bins such that the bin capacity is not exceeded and the minimum number of bins is used.

1. **for** i from 1 to n **do**
2. Suppose the first x bins are partially filled.
3. **for** j from 1 to x **do**
4. **if** Item i fits into bin j **then**
5. Add item i to bin j
6. **if** Item i has not been assigned **then**
7. Start a new bin, bx +1 and add item i to it.

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### 2.3 Things to note about the above Algorithm

- Every item gets assigned a bin.
- The capacity constraint for each bin is met
- **FirstFit** produces a feasible solution

### 2.4 Feasibility of FirstFit({w1,w2,w3,...wn})

**Lemma**: *returns* a proper bin packing (no bin has more than 1 unit of weight assigned to it).

**Proof**: We can see in line 4 of FirstFit({w1,w2,w3,...wn }) that we only add item i to bin j if it fits (if it doesn't exceed the unit capacity of bin j ).

- What is the running time of FirstFit({w1,w2,w3,...wn})? $\mathbf{O(n^2)}$
- FirstFit produces a feasible solution.
- Is it optimal? **NO (example where it wont work is with inputs: .5, .6, .2, .5)**
- How Close to OPT?
- Note: If on completion of **First Fit**, there are *m* bins used, then at least *m-1* bins are more than half filled.
- Thus:

$$OPT \geq \sum_{i=1}^{n} wi > \frac{1}{2}(m-1) \tag{1}$$

$$So, 2OPT > m-1 \tag{2}$$

and OPT and $m$ are both integers so:

$$2OPT > m \geq OPT \tag{3}$$