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1(a) ① $U = A$, $L = I$
② for $k = 1$ to $m-1$
③ for $j = k+1$ to $\min(m, k+l)$
④ $l_{jk} = u_{jk} / u_{kk}$
⑤ $u_{j, k:\min(m, k+u)} -= l_{jk} U_{k, k:\min(m, k+u)}$

outer loop takes m steps inner loop takes l steps for every outer step updating u takes u steps for every inner step

Hence algo has $O(lum)$

Outer loop takes m flops
inner loop takes l flops
⑤ step takes 2u flops

time complexity = 2mlu flops.

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~~Recursive~~

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(2) Let Cholesky factorisation of M is

$$M = L_1 L_1^T$$

and say L_1 can be written as

$$L_1 = \begin{bmatrix} L_2 & x \\ y & q \end{bmatrix}$$

where

$$x, y \in R^m, \quad q \in R, \quad L_2 \in R^{m \times m}$$

$$\begin{bmatrix} A & b \\ b^T & c \end{bmatrix} = \begin{bmatrix} L_2 & x \\ y & q \end{bmatrix} \begin{bmatrix} L_2^T & y^T \\ x^T & q \end{bmatrix}$$

Also since L_1 is lower triangular matrix, So $x = 0$ and L_2 is lower triangular

$$\Rightarrow \begin{bmatrix} A & b \\ b^T & c \end{bmatrix} = \begin{bmatrix} L_2 & 0 \\ y & q \end{bmatrix} \begin{bmatrix} L_2^T & y^T \\ 0 & q \end{bmatrix}$$

$$\begin{bmatrix} A & b \\ b^T & c \end{bmatrix} = \begin{bmatrix} L_2 L_2^T & L_2 y^T \\ (L_2 y^T)^T & y y^T + q^2 \end{bmatrix}$$

$$A = L_2 L_2^T$$

Since Cholesky factorisation is unique for every matrix & L_2 is lower triangular

$$\Rightarrow L_2 = L$$

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Also $b = L_2 y^T$, $c = y y^T + q^2$

$\Rightarrow b = L y^T$

We can get y by backsubstitution method which takes less than $O(m^2)$ time

Once we get y, we can compute the value of c from above equation

$$L_1 = \begin{bmatrix} L_2 & x \\ y & q \end{bmatrix}$$

$$L_1 = \begin{bmatrix} L & 0 \\ y & q \end{bmatrix}$$

y, q can be computed in $O(m^2)$ time hence L_1 ~~can~~ i.e. Cholesky factorisation of M can be computed in $O(m^2)$ time.

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③ (a)

$$x_k \leftarrow \frac{1}{a_{kk}} \left(b_k - \sum_{j \neq k} a_{kj} \, x_j \right)$$

If we write the above equation in matrix form by forming a matrix from $\sum_{j \neq k} a_{kj} \, x_j$ with digonal elements as zero.

$$x^{(k+1)} = D^{-1} \left(b - (L+U) \, x^{(k)} \right)$$

where $A = D + L + U$

D → digonal, L → strictly lower Δ, U → strictly upper Δ

~~This method performs Jacobi iteration.~~

(b) equation p_k udpates var. q_k

$$x_{q_k} \leftarrow \frac{b_{p_k} - \sum_{j \neq q_k} a_{p_k j} \, x_j}{a_{p_k q_k}}$$

P matrix is permutation matrix of $\{p_1 \_\_\_\_ p_m\}$ & similarly Q is permutation of $(q_1 - - - q_m)$

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Consider
$$A' = P A Q^T$$
where
$$A' = D' + L' + U' \quad \text{as in (a)}$$
$$Q x^{n+1} = D'^{-1}\left(P b - (L' + U') Q x^n\right)$$

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5 (a) $\|A - I\|_2 = 0.6$

$\|A - I\|_2 \geq \dfrac{\|(A - I)v\|_2}{\|v\|_2}$ for any v

$\|(A - I)v\|_2 \leq 0.6\|v\|_2$

$\|Av - v\|_2 \leq 0.6\|v\|_2$

Since v is any arbitary vector
So lets take eigen vector of A
(v_1) as v

$\|Av_1 - v_1\|_2 \leq 0.6\|v_1\|_2$

$\|\lambda v_1 - v_1\|_2 \leq 0.6\|v_1\|_2$

$\|(\lambda - 1)v_1\|_2 \leq 0.6\|v_1\|_2$

$|\lambda - 1|(\|v_1\|_2) \leq 0.6\|v_1\|_2$

Since $\|v_1\| > 0$ as A is spd

$|\lambda - 1| \leq 0.6$

$\Rightarrow \lambda \in [0.4, 1.6]$

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$$\frac{\|e_n\|_A}{\|e_0\|_A} \leq 2\left(\frac{\sqrt{K(A)}-1}{\sqrt{K(A)}+1}\right)^n$$

Also $K(A) = \frac{\lambda_{max}}{\lambda_{min}} \leq \frac{1.6}{0.4} = 4$

$$\frac{\|e_n\|_A}{\|e_0\|_A} \leq 2\left(\frac{2-1}{2+1}\right)^n = \frac{2}{3^n}$$

(b) $B = A + \omega\,\omega^T$

$B = A + 49 v_1 v_1^T$

$Bv_1 = Av_1 + 49 v_1 v_1^T v_1$

$Bv_1 = \lambda_1 v_1 + 49 v_1 \|v_1\|^2$

$\lambda_1 = 1$ & $\|v_1\| = 1$

$Bv_1 = 50 v_1$ so $\boxed{\lambda_1' = 50}$

Take other eigen vectors of A, v_i $i \neq 1$

$Bv_i = Av_i + 49 v_i v_1^T v_i$

Since A is spd, so every eigen vectors are orthogonal to each other ie $v_i^T v_j = 0$

$Bv_i = \lambda_i v_i + 0 = \lambda_i v_i$

ie v_i & λ_i are eigen vector & eigen value of B

So we have, ~~eve~~ same eigen vector of A & B

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6 (a)

$$a_{ij} = \begin{cases} \dfrac{-1}{R_{ij}} & i \neq j \\[2ex] \displaystyle\sum_{\substack{\forall k \text{ connected} \\ \text{to } i}} \frac{1}{R_{ik}} & i = j \end{cases}$$

Since $R_{ij} = R_{ji} \Rightarrow a_{ij} = a_{ji}$ Hence A is symmetric

1(b)

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 10 & 5 & 0 & 0 & 0 \\ 0 & 10 & 7 & 0 & 0 \\ 0 & 0 & 10 & 7 & 0 \\ 0 & 0 & 0 & 10 & 13 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.5 & -0.25 & .175 & -.1225 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 10 & 5 & 0 & 0 & 0 \\ 0 & 10 & 7 & 0 & 0 \\ 0 & 0 & 10 & 7 & 0 \\ 0 & 0 & 0 & 10 & 13 \\ 0 & 0 & 0 & 0 & 1.5925 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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Figure 1 : plot of cg residual - error of random1 network

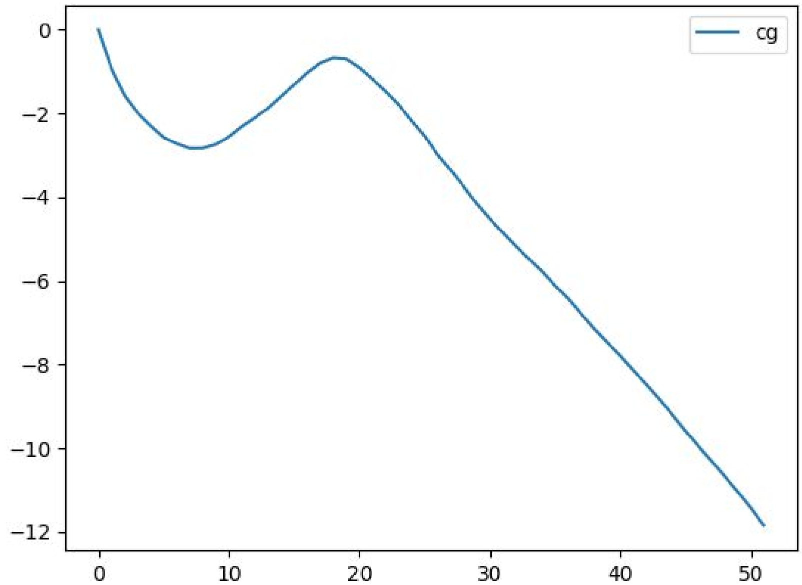

Figure 2 : plot of cg and pcg - residual error of random2 network