Part 1

$\mu = 0$

initial $x(t) = x_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$

1st derivative $x'(t) = -x_0 \sqrt{\frac{k}{m}} \sin\left(\sqrt{\frac{k}{m}}\, t\right)$

2nd derivative $x''(t) = -x_0 \frac{k}{m} \cos\left(\sqrt{\frac{k}{m}}\, t\right)$

plug equation into formula $x''(t) + \frac{k}{m} x(t) = -x_0 \frac{k}{m} \cos\left(\sqrt{\frac{k}{m}}\, t\right) + \frac{k}{m} x_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right) = 0$

satisfy initial condition $x(0) = x_0 \cos\left(\sqrt{\frac{k}{m}}\, 0\right) = x_0 * 1 = x_0$

$x(0) = x_0$

Part 3

$$x''(t) = \frac{x(t + dt) - 2x(t) + x(t - dt)}{dt^2}$$

Part 4

$$x''(t) + \frac{k}{m} x(t) = 0$$

$$\frac{x(t + dt) - 2x(t) + x(t - dt)}{dt^2} + \frac{k}{m} x(t) = 0$$

$$\frac{x(t + dt) - 2x(t) + x(t - dt)}{dt^2} = -\frac{k}{m} x(t)$$

$$x(t + dt) - 2x(t) + x(t - dt) = -\frac{k}{m} x(t)\, dt^2$$

$$x(t + dt) + x(t - dt) = -\frac{k}{m} x(t)\, dt^2 + 2x(t)$$

$$x(t + dt) = -\frac{k}{m} x(t)\, dt^2 + 2x(t) - x(t - dt)$$