MODEL TASK 4

WIND DRIVEN CIRCULATION IN A SHALLOW LAKE

Figure 1 is a topographic map of a lake consisting of two basins connected by a sound. The largest depth is 40 m. The bottom matrix is enclosed as a file with 40 rows and 20 columns. (First row corresponds to the values in the south.) Let *H=H(x,y)* be the bottom topography. Let $\zeta = \zeta(x,y,t)$ be the sea surface elevation, *(u=u(x,y,t), v=v(x,y,t))* the current velocity and *(WX,WY)* a constant wind. The linearized shallow water equations on impulse form are:

$$U_t + gH\zeta_x + rU\frac{\sqrt{U^2+V^2}}{H^2} = \lambda WX\sqrt{WX^2+WY^2} + fV \qquad (1)$$

$$V_t + gH\zeta_y + rV\frac{\sqrt{U^2+V^2}}{H^2} = \lambda WY\sqrt{WX^2+WY^2} - fU \qquad (2)$$

$$\zeta_t + U_x + V_y = 0 \qquad (3)$$

where the friction coefficient *r* = 0.003 and the wind stress factor $\lambda = 3.2\,x10^{-6}$.

Here $U = \int_{-H}^{0} udz$ and $V = \int_{-H}^{0} vdz$ are the depth integrated transport components (units m^2s^{-1}).

We shall derive a numerical model of the lake based on the equations 1-3 and the map in Figure 1. The lake is at 60N.

Define a 20x40 matrix for the Richardson scheme (staggered in both space and time). Let the point (1,1) be in southwest. The grid distance is 10 km. The time integration is explicit, choose a time step in an integer number of minutes that fulfils the CFL condition.

In the first case there is a fresh breeze from the west at 10 m/s. In a second case we have a fresh breeze from the south at the same speed. By calculating the net transport northward through the sound, and plot the time series for that, you can see how the circulation gradually becomes stationary. Find the stationary state of the current and the surface elevation in both cases, and discuss the results; try to explain why the current goes with the wind where it is shallow, and contrary where it is deep, and what happens with the gravity-inertial waves.

Keep a record of the model's mass balance to check for bugs. This you can do by summing the water level over all grid cells and see if it keeps near zero.

The Richardson scheme (see Figure 2) is a suitable scheme for our problem because it is staggered in both space and time. Space staggering gives a simple treatment of pressure and divergence terms, but a more complicated treatment of the Coriolis and friction terms, because we have to interpolate. Interpolation in time (as in for example the Crank-Nicholson scheme) is avoided because of the time staggering. The time staggering has the same properties as the forward-backward scheme (see p.143 in Haltiner and Williams). In this task we choose to take the transport equations (1) and (2) forward, and the equation for the sea level (3) backward. The space staggering is given by an Arakawa C-grid with the component *U* displaced half a grid length to the east, and the component *V* half a grid length to the south of the sea level location.

You may use Figure 2 as a help to formulate the interpolation and difference operators needed during your discretisation work. A valuable detail: Put the impulse factor that gives the direction of the friction force at the new time level n+1. That will assure that the friction always is directed against the current.

For example: $U_{j,l}^{n+1} = \ldots - \Delta t\ r\ U_{j,l}^{n+1} \sqrt{\left(U_{j,l}^{n}\right)^2 + \left(\overline{\overline{V_{j,l}^{n}}}\right)^2}\ (\overline{H}_{j,l}^{x})^{-2}$, where the *V*- and *H*-values are interpolated to the *U*-point.

Finally, find the flushing time of the northern basin in each of the two cases by calculating the northward transport in the sound at l=20.

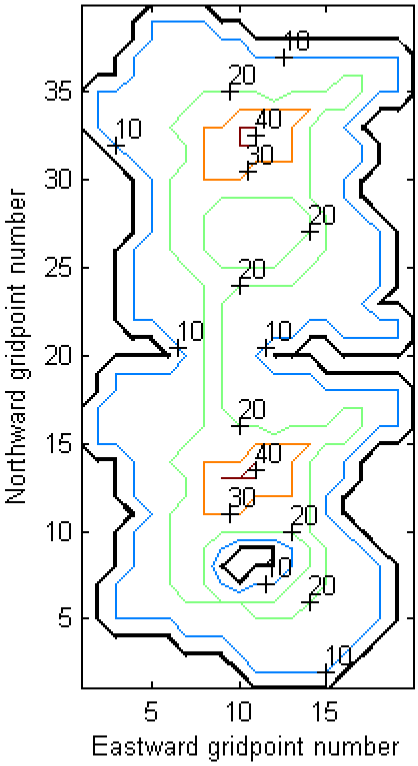
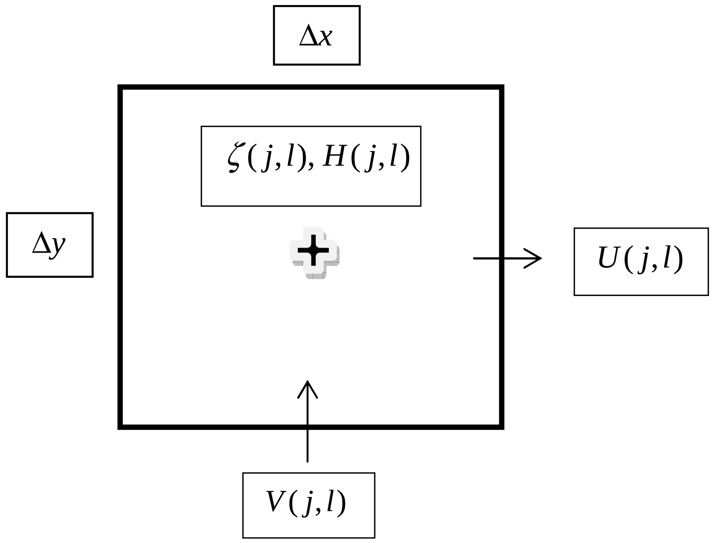
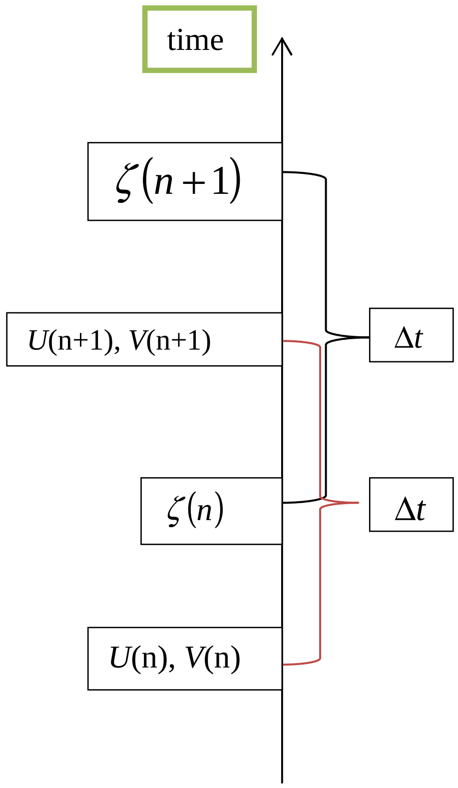


Figure 1. The bottom topography of the lake, the fat curves denote the land contours. Depths are given in meter.



a)



b)

Figure 2. a) Staggered space grid: Depth and sea level are located at the centre of the cell (marked by the +-sign), the component U is displaced half a grid length to the east, and the component V half a grid length to the south of the sea level location. b) Time staggering: the sea level is delayed by half a time step with respect to the transport.