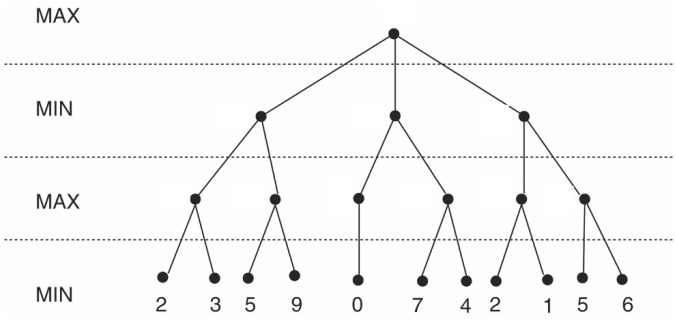
COMP 6721 Applied Artificial Intelligence (Winter 2021)

Worksheet #2: Adversarial Search

Game of Nim. Play a game of Nim against your team mate, starting with 7 tokens (write down the number of tokens at each move).

MiniMax. Let's apply the MiniMax algorithm discussed in the lecture on an example (fixed ply depth of 3):



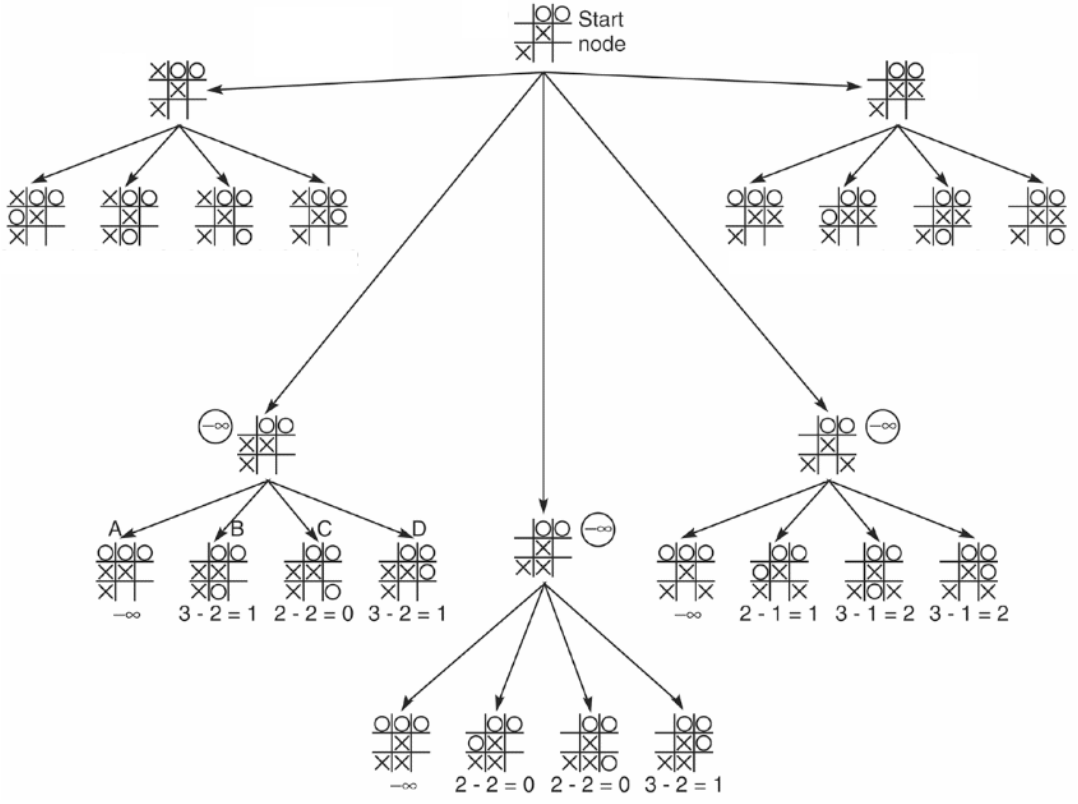
Leaf nodes show the actual heuristic value *e(n)*

- Find the *back-up* heuristic values of all non-leaf (internal) nodes using the MiniMax procedure.
- Highlight the *best next move* for MAX (starting from the root node).

MiniMax Heuristic for Tic-Tac-Toe. Using the heuristic shown below, compute the values of $e(n)$ for the three game states (MAX plays X):

$e(n) =$ { number of rows, columns, and diagonals open for MAX − number of rows, columns, and diagonals open for MIN; $+\infty$, if n is a forced win for MAX; $-\infty$, if n is a forced win for MIN }

Two-ply MiniMax. Compute the missing values using MiniMax in the game tree shown below (same heuristic as above, start node is MAX). What will be MAX's next move?



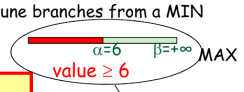
Alpha-Beta Pruning. Apply the Alpha-Beta Pruning algorithm:

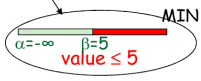
```
01 function alphabeta(node, depth, α, β, maximizingPlayer)
02     if depth = 0 or node is a terminal node
03         return the heuristic value of node
04     if maximizingPlayer
05         v := -∞
06         for each child of node
07             v := max(v, alphabeta(child, depth - 1, α, β, FALSE))
08             α := max(α, v)
09             if β ≤ α
10                 break (* β cut-off *)
11         return v
12     else
13         v := ∞
14         for each child of node
15             v := min(v, alphabeta(child, depth - 1, α, β, TRUE))
16             β := min(β, v)
17             if β ≤ α
18                 break (* α cut-off *)
19         return v
```

Initial call:
`alphabeta(origin, depth, -∞, +∞, TRUE)`

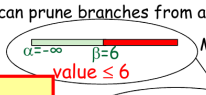
- α : lower bound on the final backed-up value.
- β : upper bound on the final backed-up value.
- Alpha pruning:
  - eg. if MAX node's α = 6, then the search can prune branches from a MIN descendant that has a β <= 6.
  - if child β <= ancestor α → prune

incompatible...
so stop searching the right branch;
the value cannot come from there!

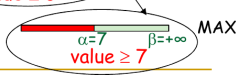




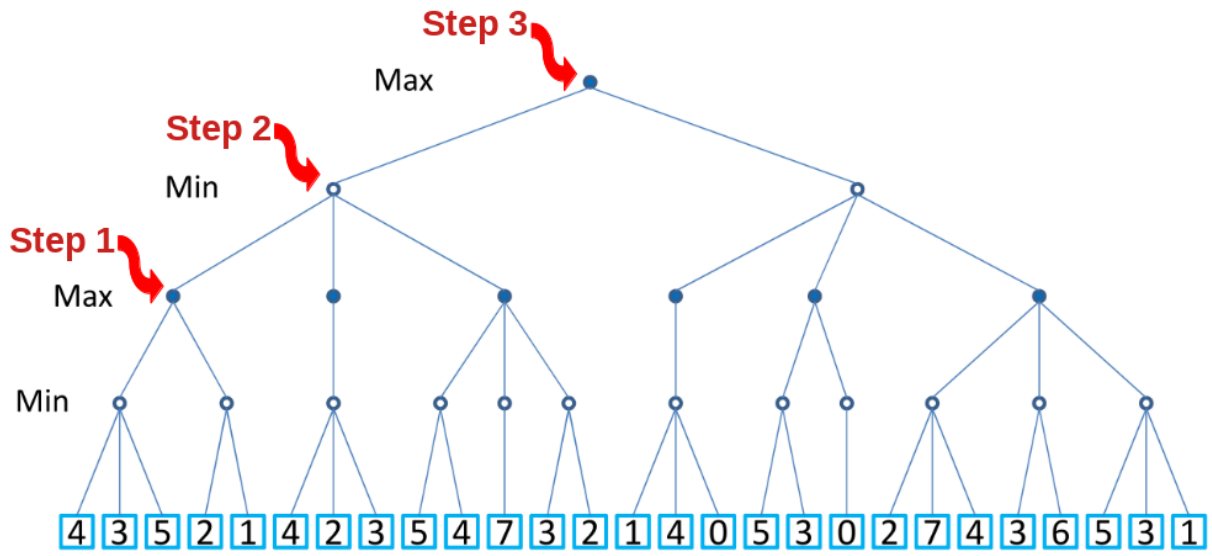
- Beta pruning:
  - eg. if a MIN node's β = 6, then the search can prune branches from a MAX descendant that has an α >= 6.
  - if ancestor β <= child α → prune



incompatible...
so stop searching the right branch;
the value cannot come from there!



on the following search tree:



Step 1: Perform the Alpha-Beta procedure (left-to-right) until you reached the node marked with "Step 1". Circle each node that you explored and show which subtrees are cut off by the algorithm (if any).

Step 2: Now continue with the algorithm until you reached the node marked "Step 2", marking explored nodes and cut subtrees as before.

Step 3: Complete the algorithm until you calculated the value for the root node in the same fashion.

How many nodes did the algorithm explore (out of 27 possible): ..... ?