Fadil Army Renatha
2055301039 - 2 IT A

Sun, January 9
2022

- Matematika Diskrit -

1) Buktikan melalui induksi Matematika bahwa ...

(a) $1(2) + 2(3) + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

untuk semua $n \geq 1$

answer.

misal $p = 1$

- $n(n+1) = \frac{n(n+1)(n+2)}{3}$

$1(1+1) = \frac{1(1+1)(1+2)}{3}$

$2 = \frac{(2)(3)}{3}$

$\boxed{2 = 2}$

→ Basis Induksi : $p(1)$ benar.

- memperlihatkan bahwa $p(n+1)$ benar.

$= 1(2) + 2(3) + \dots + n(n+1) + n+1(n+1+1)$

$= \frac{n(n+1)(n+2)}{3} + (n+1)(n+2)$

$= \frac{(n+1)(n+2)(n+3)}{3}$

TERBUKTI !.

(b) $\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

untuk semua $n \geq 1$

Answer.

- misal $n = 1$

$\frac{1}{n(n+1)} = \frac{n}{n+1}$

$\frac{1}{1(1+1)} = \frac{1}{1+1}$

$\boxed{\frac{1}{2} = \frac{1}{2}}$ → Basis Induksi $n=1$ Benar.

- memperlihatkan bahwa $p(n+1)$ benar.

$+ \dots + \frac{1}{n(n+1)} + \frac{1}{n+1(n+2)} = \frac{n+1}{n+2}$

$= \frac{n}{n+1} + \frac{1}{n+1(n+2)} = \frac{n+1}{n+2}$

$\frac{n(n+2)}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)} = \frac{n+1}{n+2}$

$\frac{n^2+2n+1}{(n+1)(n+2)} = \frac{n+1}{n+2}$

$\frac{(n+1)(\cancel{n+1})}{(\cancel{n+1})(n+2)} = \frac{n+1}{n+2}$

$\frac{n+1}{n+2} = \frac{n+1}{n+2}$

TERBUKTI !

(c) $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2$

$= \frac{n(2n-1)(2n+1)}{3}$

untuk semua $n \geq 1$

answer.

- misal $n = 1$

$(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

$(2(1)-1)^2 = \frac{1(2(1)-1)(2(1)+1)}{3}$

$1 = \frac{1(1)(3)}{3}$

$\boxed{1 = 1}$

↳ Basis indeks $n = 1$ Benar

- Perlihatkan bahwa $P(n+1)$ adalah benar!

$\cdots + (2n-1)^2 + (2n+1-1)^2$

$= \frac{n+1(2n+1-1)(2n+1+1)}{3}$

$= \frac{n(2n-1)(2n+1)}{3} + (2n)^2$

$= \frac{n+1(2n+(2n+2)}{3}$

Terbukti!

(d) $1 + a + a^2 + \cdots + a^n = \frac{1-a^{n+1}}{1-a}$

untuk semua $n \geq 0$ dan $a \neq 1$

answer.

- misal $n = 0$,

$a^0 = \frac{1-a^{0+1}}{1-a}$

$1 = \frac{1-a}{1-a}$

$\boxed{1 = 1}$

↳ Basis indeks $n = 0$ Benar.

- Perlihatkan bahwa $P(n+1)$ adalah benar.

$\cdots + a^n + a^{n+1} = \frac{1-a^{n+1+1}}{1-a}$

$= \frac{1-a^{n+1}}{1-a} + a^{n+1}$

$= \frac{1-a^{n+2}}{1-a}$

↳ using Least common denominator.

TERBUKTI!

(e) $3 + 3.5 + 3.5^2 + \ldots + 3.5^n$

$= \frac{3(5^{n+1} - 1)}{4}$

untuk semua $n \geq 0$

Answer.

- misal $n = 0$

$3.5^n = \frac{3(5^{n+1} - 1)}{4}$

$3.5^0 = \frac{3(5^{0+1} - 1)}{4}$

$3 = \frac{3(4)}{4}$

$3 = 3$

↳ Basis Indeks $n = 0$ terbukti Benar.

- memperlihatkan bahwa $P(n+1)$ adalah benar.

$\ldots + 3.5^n + 3.5^{n+1} = \frac{3(5^{n+1+1} - 1)}{4}$

$= \frac{3(5^{n+1} - 1)}{4} + 3(5^{n+1})$

$= \frac{3(5^{n+2} - 1)}{4}$

TERBUKTI !

2) Buktikan melalui Induksi matematika bahwa $n^4 - 4n^2$ habis dibagi 3 untuk Semua bilangan bulat $n \geq 2$

answer.

- misal $n = 1$

$2^4 - 4(2)^2 = 16 - 16$

$= 0$

↳ Benar karna habis dibagi 3

- Pemburktian bahwa $P(n+1)$ habis dibagi 3.

$= (n+1)^4 - 4(n+1)^2$

$= n^4 + 4n^3 + 2n^2 - 4n - 3$

misal $n = 2$

$= 2^4 + 4(2^3) + 2(2^2) - 4(2) - 3$

$= 45$

↳ Habis dibagi 3

TERBUKTI !

Page 222

1) Apakah 19 habis membagi bilangan bulat berikut :

(a) 89

answer.

$\frac{89}{19} = 4,7 \rightarrow$ tidak habis

(b) 561

answer.

$\frac{561}{19} = 29,5 \rightarrow$ tidak habis

(c) 209

answer.

$\frac{209}{19} = 11 \rightarrow$ habis

(d) 773

answer.

$\frac{773}{19} = 40,7 \rightarrow$ tidak habis

(e) 8721

answer.

$\frac{8721}{19} = 456 \rightarrow$ habis

2) Carilah bilangan bulat $q \& r$ sehingga

$m = nq + r$

(a) $m = 45$, $n = 6$

answer.

Teorema Euclidean

$0 \leq r \leq n$

$45 = 6q + r$

$45 : 6 = 7 \text{ R } 3$

maka, $q = 7 \& r = 3$

(b) $m = 66$, $n = 11$

answer.

$66 = 11q + r$

$66 : 11 = 6 \text{ R } 0$

maka $q = 6 \& r = 0$

(c) $m = 106$, $n = 12$

answer.

$106 = 12q + r$

$106 : 12 = 8 \text{ R } 10$

maka $q = 8 \& r = 10$

(d) $m = 0$, $n = 47$

$0 = 47q + r$

$0 : 47 = 0 \text{ R } 0$

maka, $q = 0 \& r = 0$

(e) $m = -221$, $n = 12$

answer.

$-221 = 12q + r$

$-221 : 12 = -18$ R -5

maka $q = -18$ & $r = -5$

↳ salah karna syarat

$0 \leq r \leq n$

(f) $m = -246$, $n = 49$

answer.

$-246 = 49q + r$

$-246 : 49 = -5$ R -1

maka $q = -5$ & $r = -1$

↳ salah karna syarat

$0 \leq r \leq n$

12) Tentukan PBB dari pasangan bilangan bulat a & b berikut:

(a) 220, 1400

answer.

$m = 1400$, $n = 220$

$m = nq + r$

$1400 = 220 \times 6 + 80$

↳ 6 R 80 PBB (1400, 220)

$220 = 80 \times 2 + 60$

↳ 2 R 60 PBB (220, 80)

$80 = 60 \times 1 + 20$

↳ 1 R 20 PBB (80, 60)

$60 = 20 \times 3 + 0$

↳ 3 R 0 PBB (60, 20)

Jadi, PBB (1400, 220) = PBB (220, 80)
= PBB (80, 60) = PBB (60, 20)

(b) 315, 825

answer.

$m = 825$, $n = 315$

$825 = 315 \times 2 + 195$

↳ 2 R 195 PBB (825, 315)

$315 = 195 \times 1 + 120$

↳ 1 R 120 PBB (315, 195)

$195 = 120 \times 1 + 75$

↳ 1 R 75 PBB (195, 75)

$120 = 75 \times 1 + 45$

↳ 1 R 45 PBB (120, 75)

$75 = 45 \times 1 + 30$

↳ 1 R 30 PBB (75, 45)

$45 = 30 \times 1 + 15$

↳ 1 R 15 PBB (45, 30)

$30 = 15 \times 2 + 0$

↳ 2 R 0 PBB (30, 15)

Jadi, PBB (825, 315) = PBB (315, 195)
= PBB (195, 75) = PBB (120, 75)
= PBB (75, 45) = PBB (45, 30)

© 110 , 273

answer .

m = 273 , n = 110

273 = 110 x 2 + 53
↳ 2 R 53 PBB (273, 110)

110 = 53 x 2 + 4
↳ 2 R 4 PBB (110 , 53)

53 = 4 x 13 + 1
↳ 13 R 1 PBB (53, 4)

4 = 1 x 4 + 0
↳ 4 R 0 PBB (4 , 1)

Jadi, PBB (273, 110) = PBB (110, 53) = PBB (53, 4) = PBB (4, 1) =

(d) 2475 , 32670

answer .

m = 32670 , n = 2475

32670 = 2475 x 13 + 495,
↳ 13 R 495 PBB (32670, 2475)

2475 = 495 x 5 + 0
↳ 5 R 0 PBB (2475 , 495)

Jadi, PBB (32670, 2475) = PBB (2475, 495) =

(e) -456, 688

answer .

688 = -456 x -1 + 223
↳ -1 R 223 PBB(688 , -456)

-456 = 223 x -2 + (-10)
↳ -2 R -10 PBB (-456, 223)

223 = -10 x -22 + 3
↳ -22 R 3 PBB (223 , -10)

-10 = 3 x -3 + (-1)
↳ -3 R -1 PBB (-10, 3)

3 = -1 x -3 + 0
↳ -3 R 0 PBB (3 , -1)

Jadi, PBB (688, -456) = PBB (-456, 223) = PBB (223, -10) = PBB (-10, 3) = PBB (3 , -1) =

(18) Pecahkan kekongruenan lanjar berikut !

(a) $4x \equiv 5 \pmod 8$

answer .

x = (5 + 8k) / 4 → bukan bil bulat

Tidak ada solusi .

(b) $2x \equiv 7 \pmod{17}$

answer .

x = ((7 + 17k) / 2 → k harus ganjil

- k = 1 ; x = 12
- k = 3 ; x = 29
- k = 5 ; x = 46
- k = -1 ; x = -5
- k = -3 ; x = -22
- k = -5 ; x = -39

maka yang memenuhi adalah
{12, 29, 46 ... dan -5, -22, -39..}

Rumus umum x :

x = 12 + 17 k , k ∈ bil bulat

ⓒ $5x \equiv 10 \pmod{12}$

answer .

$x = (10 + 12k) / 5$

↳ 12k harus 0 / kelipatan 5.

- $k = 0 ; x = 2$
- $k = 5 ; x = 14$
- $k = 10 ; x = 26$
- $k = -5 ; x = -10$
- $k = -10 ; x = -22$
- $k = -15 ; x = -34$

maka nilai yang memenuhi

$\{ 2, 14, 26 \ldots$ dan $-10, -22, -34 \ldots \}$

Rumus umum nilai x :

$x = 2 + 12k$, k bil bulat.

(9) Tentukan invers dari a modulo m berikut :

ⓐ $a = 34, \quad m = 5$

answer .

$34 = 2 . 15 + 4$

$15 = 4 . 3 + 3$

$3 = 3 . 1 + 0$

$1 = 4 - 1 (15 - 3 . 4)$

$1 = -1 . 15 + 4 (34 - 2 . 15)$

$1 = 4 . 34 - 9 . 15$

invers = 4

ⓑ $a = 178 , \quad m = 62 .$

answer .

$178 = 62 . 2 + 54$

$62 = 54 . 1 + 8$

$54 = 8 . 6 + 6$

$8 = 6 . 1 + 2$

$6 = 2 . 3 + 0$

$2 = 8 - 1 (54 - 6 . 8)$

$2 = -1 . 54 + 7 (62 - 1 . 54)$

$2 = 7 . 62 - 8 (178 - 2 . 62)$

$2 = -8 . 178 + 23 . 62$

invers = -8 .

ⓒ $a = -341 , \quad m = 17$

answer .

$-341 = 17 . -21 + 16$

$17 = 1 . 16 + 1$

$16 = 16 . 1 + 0$

$1 = 17 - 1 (-341 + 21 . 17)$

$1 = -1 - 341 + 21 . 17$

invers = -1 .

24) ISBN sebuah buku mengenai algoritma adalah 0-201-57p859-1 yang dalam hal ini, p adalah angka. Berapa nilai p?

Answer .

Karna terdapat 11 digit maka menggunakan modulo 12.

$k = 1$ maka .

$$1 = \left(\sum_{i=1}^{10} i\, x_i\right) \bmod 12$$

$= 1.0 + 2.2 + 3.0 + 4.1 + 5.5 + 6.7$

$+ 7.P + 8.8 + 9.5 + 10.9 \bmod 12$

$1 = 274 + 7p \bmod 12$

$1 = 10 - 5p \bmod 12$

$1 = 12q + (10 - 5p)$

$p = \dfrac{12q + 9}{5}$

karna p bil asli maka

$\dfrac{12q + 9}{5}$ harus kelipatan 5

maka $12q + 9 = 2(q + 2) \bmod 5$

didapat $q = \ldots -7, -2, 3, 8, 13 \ldots$

karna p bil asli dan satu digit

maka $q = 3$. Dengan demikian

$p = 9$ Maka ISBN adalah

0 - 201 - 579859 - 1 ═ .

25) Tunjukkan bagaimana sekumpulan data dengan kunci - kunci berikut : 714, 631, 26, 373, 775, 906, 909, 2025, 42, 4, 136, 1028 ditempatkan di dalam memori dengan fungsi hash $h(k) = \bmod 17$!