Chapter 4 Group Worksheet

Problem 1. From the homework:

[4.4 #60] Sketch the graph of a single function f satisfying all the given conditions.

(a) $f'(x) < 0$ for $x < 0$

(b) $f'(x) > 0$ for $x > 0$

(c) $f''(x) < 0$ for $|x| > 2$

(d) $f''(x) > 0$ for $|x| < 2$

Problem 2. From an old exam:

In 1919, physicist Alfref Betz showed that fraction of wind power extracted from a wind turbine is given by

$$F(r) = \frac{1}{2}(1 - r^2)(1 + r)$$

where $r = v_2/v_1$. Here the wind enters a turbine with speed v_1 and exits with speed v_2. If $0 \leq r \leq 1$, find the value of r that maximizes F. Be sure to justify your answer.

Problem 3. For in-class discussion:

a) Find the linear approximation to $f(x) = (1 + x)^\alpha$ about $x = 0$, where α is a constant.

b) Einstein's equation is $E = mc^2$ which relates mass to energy. c is the speed of light and the mass of particle moving with speed v is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Use (a) to approximate m for $x = v^2/c^2$ small.

c) Insert your approximation into Einstein's equation to show that once you replace x by $\frac{v^2}{c^2}$,

$$E \approx m_0(1 + \frac{v^2}{2c^2})c^2 = m_0c^2 + \frac{1}{2}m_0v^2$$

d) Explain why Newtonian physics, in this case, is just a linearization of Einstein's theory of relativity.