Topological Ordering

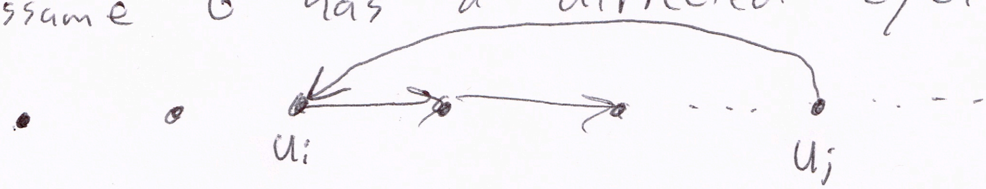
Input: Directed graph $G=(V,E)$
(Assum: G is a DAG)

Output: An ordering $u_1, u_2, \dots, u_n$
s.t. ~~##~~ $\boxed{(u_i, u_j) \in E \Rightarrow i < j}$

Proposition: If G has a topological ordering, then G is a DAG

Proof: ... by contradiction

Assume G has a dirrected cycle (C)



Let i be the smallest index
s.t. $u_i \in C$ $\cancel{(u_i, u_j) \in E}$ $(u_j, u_i) \in E$

but $j > i$

$\therefore$ (u_j, u_i) is backwords.

$\therefore$ G is a DAG

Greedy Algorithms

- easy algorithms
- more difficult proofs (than the algos)

two primary proof ideas
- greedy stays ahead - today
- Exchange argument - next week

Interval Scheduling

inputs: Set of n intervals $[s(i), f(i))$ — maximize # of intervals scheduled.

- sort by $f(i)$
- loop:
  - schedule the earliest $f(i)$ (insert i to A)
  - remove all conflicts with interval i
- output Solution set A

Correctness proof: Greedy stays ahead of any optimal solution.

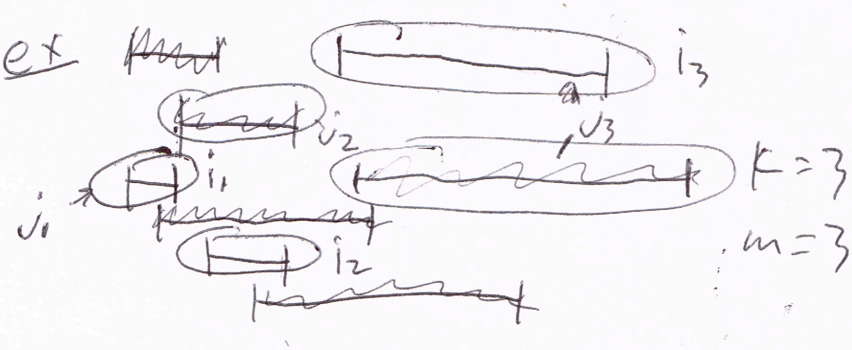
Proof: Assume an optimal solution $\mathcal{O}$

$A = i_1, i_2, \ldots, i_k \qquad |A| = k$

$\mathcal{O} = j_1, j_2, \ldots, j_m \qquad |\mathcal{O}| = m$

assume A and $\mathcal{O}$ are sorted by time (start or finish are same since there are no conflicts)

ex ~~Aud~~



Greedy stays ahead is ~~usually~~ usually a proof by induction

We want to prove that $f(i_r) \le f(j_r)$, by induction

base case: $r=1$ $\quad f(i_1) \le f(j_1)$

by definition of our Algo

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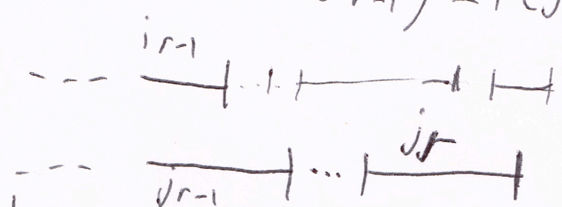
induction: $r>1$ $\quad$ IH: assume true for $r-1$ i.e. $f(i_{r-1}) \le f(j_{r-1})$

show true for r



Greedy algorithm always has the choice of j_r and maybe an earlier interval.

$\therefore f(i_r) \le f(j_r)$.

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finish the proof!!!... by contradiction $\quad |A| = k$

$|O| = m$

assume $m > k$ show contradiction.

at $r = k$ we know $f(i_k) \le f(j_k)$

Greedy can pick j_{k+1}. By the definition of the algorithm, we will pick j_{k+1} and $|A| \geq k+1 = k$ ⚡

$\therefore m \le k$

Since O is optimal, $m \not< k$

$\therefore m = k$

∎