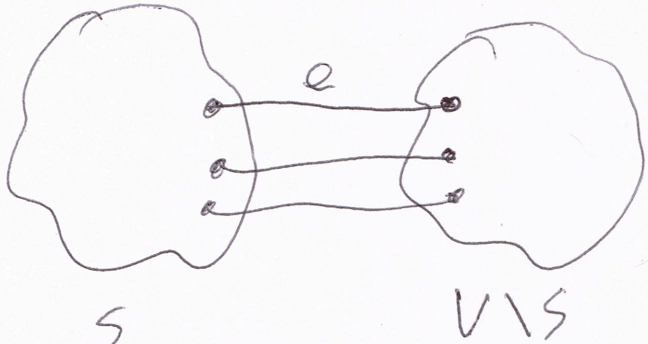
Cut property $\quad G=(V,E)$

assume all edge costs $c_e > 0$ are distinct
then for every $S \neq \emptyset$ and $S \subset V$



if e is the cheapest crossing edge, then
e is in all MST's for G.

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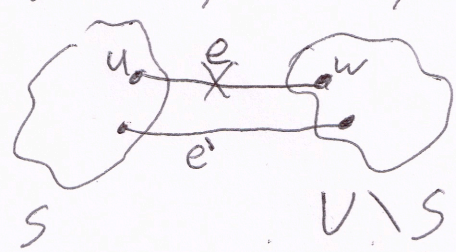
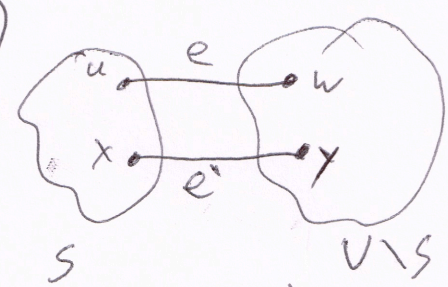
proof: by contradiction

for contradiction, assume $\exists$ an MST (V,T') and
a cut S, $V \setminus S$, $S \neq \emptyset$, $V \setminus S \neq \emptyset$

$\boxed{e \notin T'}$



By the "exchange argument", come up with
a tree T^* s.t. (V, T^*) is a spanning tree and
$cost(T^*) < cost(T')$



$$T^* = (T' \setminus \{e'\}) \cup \{e\}$$

$e' \in T' \qquad c_e < c_{e'}$

$$cost(T^*) = cost(T') - c_{e'} + c_e < cost(T')$$

For completeness: If there are multiple crossing edges in T',
add e, remove the e' that creates a cycle.