- "Distance" between rankings
- two rankings on $\{1, \dots, n\}$
- assume one ranking is $1, \dots, n$ (up to relabling)

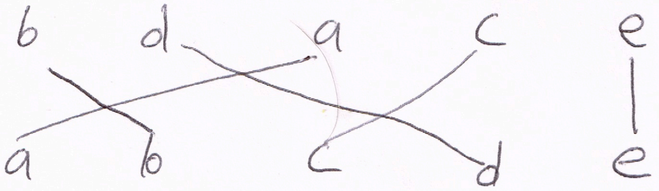
---

<u>Input</u>: $\underbrace{a_1, \dots, a_n}_{\text{permutation of } \{1, \dots, n\}}$

<u>Output</u>: # of inversions in $a_1, \dots, a_n$

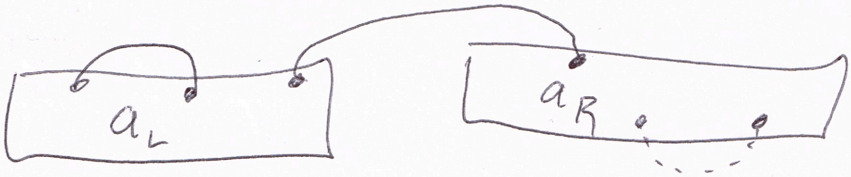
$(i, j)$ is an inversion if (i) $i < j$
(ii) $a_i > a_j$

<u>ex</u> $n=5$



each crossing is an inversion

worst case: reverse order $O(n^2)$ inversions



Count number of crossing inversions recursively

Say: $a_L$ and $a_R$ are sorted

patchup problem (count # of crossings)
in the same problem

---

Run merge from merge sort.

1 - - - - n/2
$a_L$
$b_j$
$a_R$
i
j

```
If $a_i \le b_j$   // $(i,j)$ is not an inversion
    i++;
else   // $a_i > b_j$
    // $(i,j)$ is an inversion
    // $(i+1, j)$      $a_{i+1} > a_i (> b_j)$
                       also an inversion
       ⋮
    // $(i' \geq i, j)$   all inversions
          ↳ $a_{i'} > a_i (> b_j)$
    j++;
    count ~~#~~ += $\frac{n}{2} - i$;
```

inside merge loop

$\frac{n}{2} - i$ inversions

$O(n)$ time