

HOMEWORK 0

Due Thursday, June 4, 2015 @ 2:10pm (Optional)

This homework is to refresh your memory on stuff you've seen in CSE 191 and/or CSE 250. This homework is intended for you to start thinking about proofs again. **This HW will not count towards your grade.** I will grade any submitted HW— the idea is to give you feedback where the stakes are much lower. If you are not comfortable writing proofs then it is recommended that you submit this one. On the other hand, if you have written many proofs and are comfortable with them, then you can probably skip this one.

1. (**What is a proof?**) (40 points) Consider the following “proof”:

- Brad Pitt has a beard:¹



- Every goat has a beard:²



- Hence, Brad Pitt is a goat.

State precisely where the “proof” above fails logically.

Follow-up question: Can you prove that Brad Pitt is not a goat?

2. (**Proof by Contradiction**) (45 points) Assume that the following are true:

- Every blockbuster movie has a hero.
- Jake Sully and Neytiri are dating.
- The highest grossing movie ever is a blockbuster.

¹Image Source: <http://www.waleg.com/celebrities/archives/019283.html>

²Image Source: Myers, P., R. Espinosa, C. S. Parr, T. Jones, G. S. Hammond, and T. A. Dewey. 2006. The Animal Diversity Web (online). Accessed August 28, 2010 at <http://animaldiversity.org>.

- A hero in a movie never dies.
- The movie **Avatar** has made the most money ever.
- The heroine always dates the hero.
- Neytiri is the heroine of **Avatar**.

Prove by contradiction that **Jake Sully** is alive at the end of the movie **Avatar**. Please clearly state any assumptions that you needed to make in the proof.

3. (**Induction**) (15 points) Let $n \geq 1$ be an integer. Given n men and n women, a *perfect matching* is a way to assign every man to a unique woman and vice-versa. For example for $n = 2$, where **Brad Pitt** and **Billy Bob Thornton** are the two men and **Jennifer Aniston** and **Angelina Jolie** are the two women, one possible perfect matching consist of the two pairs (**Brad Pitt**, **Jennifer Aniston**) and (**Billy Bob Thornton**, **Angelina Jolie**).

Prove that the total number of perfect matchings when you have n men and n women is $n!$. (Recall $n! = n \times (n - 1) \times (n - 2) \times \cdots \times 2 \times 1$.)

(*Hint:* Use induction. Also note that the number of perfect matching just depends on n and is *independent* of the identities of the men and women.)