CSE 331 - Summer 2015

HOMEWORK 3

Due Tuesday, July 28, 2015 @ 2:10pm

IMPORTANT: The stated deadline on assignments will be strictly enforced. I will go over solutions at the deadline and will not accept submissions after the solutions have been presented.

Homework can be submitted at any time before the deadline as either a hard copy or electronically. If submitting electronically, use a recognizable filename (ex. "`homework3.pdf`") and submit with `submit_cse331` or as an email attachment sent to me.

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1. (40 points) You are given an array A with n entries, with each entry holding a distinct number. You are told that the sequence of values $A[1], A[2], \ldots, A[n]$ has only one positive number at index p (you are not given p). Furthermore, you know the array is *unimodal*: The values in the array entries increase from index 1 to index p and decrease from index p to index n.

   Give an algorithm to find p by reading at most $O(\log n)$ entries of A.

   Correct Algorithm: 30 points

   Runtime Analysis: 10 points

2. (45 points) Chapter 5, Exercise 1:

   You are interested in analyzing some hard-to-obtain data from two separate databases. Each database contains n numerical values (so there are $2n$ values total) and you may assume that no two values are the same. You'd like to determine the median of this set of $2n$ values, which we will define here to be the n^{th} smallest values.

   However, the only way you can access these values is through *queries* to the databases. In a single query, you can specify a value k to one of the two databases, and the chosen database will return the k^{th} smallest value that it contains. Since queries are expensive, you would like to compute the median using as few queries as possible.

   Give an algorithm that finds the median value using at most $O(\log n)$ queries.

   Correct algorithm: 25 points

   Proof of correctness: 10 points

   Runtime analysis: 10 points

3. (15 points) Given a directed graph $G = (V, E)$, a vertex $s \in V$ is called a *sink* if there are incoming edges from every other vertex to s but no outgoing edge from s, i.e. $|\{(u, s) \in E\}| = |V| - 1$ and $|\{(s, u) \in E\}| = 0$.

   Present an $O(n)$ time algorithm to find out if G has a sink and if so, output it. (Recall that $n = |V|$). Your algorithm is given G in its adjacency matrix format.

   (*Note:* The solution in mind is something like a divide and conquer algorithm, though you don't have to use divide and conquer.)

   Correct algorithm with $O(n)$ runtime: 15 points