Def: Graph $G = (V, E)$

set of vertices/nodes (V); set of edges (E)

$$E \subseteq V \times V$$

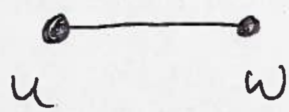
Types

(i) Directed graph

$(u,w) \in E \not\Rightarrow (w,u) \in E$ (not necessarily imply)

(ii) Undirected graph

$$(u,w) \in E \Rightarrow (w,u) \in E$$



Ex: (i) TV host example: undirected
(ii) Internet: undirected
(iii) Facebook: Undirected
(iv) Wikipedia: Directed.
(v) Airline routes: Undirected

$\equiv$

Every undirected graph is directed

Assumptions: Unless stated o/w

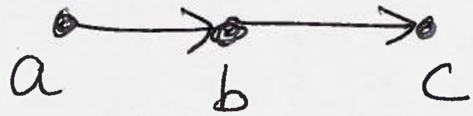
(i) $(u,u) \notin E$ (ii) a graph will be undirected

$G = (V, E)$

Def: Path sequence

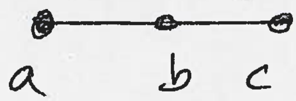
$u_1, \ldots, u_k$ $\quad u_i \in V$

$(u_i, u_{i+1}) \in E \quad 1 \le i \le k-1$

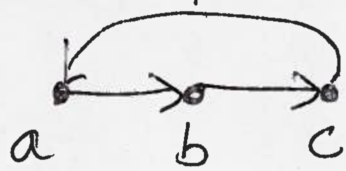


(path fr a to c but not c to a)

a is conneted to c but c ~~a~~ is not connected to a.



(both paths exist)



a, b, a, b, c X

Simple path: no vertices are repeated.

a, b, c is ✓.

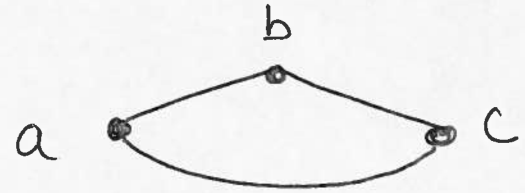
Assumption: A path is a simple path.

Def: u, w is connected iff $\exists$ a path from u to w. (undirected graph)

u, w strongly connected iff $\exists$ $u \rightsquigarrow w$ & $w \rightsquigarrow u$

Def. G is (strongly) connected iff every pair of vertices u & w are (strongly connected)

<u>Def</u>: Path length of $u_1, \ldots, u_k$ is $k-1$.



distance between u & w is the shortest length of any $u \sim w$ path

<u>Def</u>: Cycle $(k \geq 3)$ ⇝ undirected (directed $k \geq 2$)

$u_1, \ldots, u_k$ $\quad$ $u_1, \ldots, u_{k-1}$ are distinct

$u_k = u_1$

$(u_i, u_{i+1}) \in E$

THM. Any 2 out of 3 conditions below $\Rightarrow$ 3rd cond.

(G is undirected)

(1) G is ~~undirected~~ & connected

(2) G has no cycles

(3) G has $n-1$ edges

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(1) + (2) $\Rightarrow$ (3)

Lemma: A tree T on n vertices has exactly $n-1$ edges

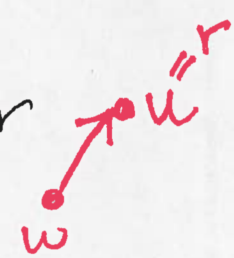
Pf. idea: Pick $T = (V, E)$, $r \in V$ as root & root T at r

- For each edge (u, w) orient the edge towards the vertex closer to r.

Claim 1: Every non-root vertex u, has exactly 1 outgoing edge

Claim 2: r has 0 outgoing edge. ✓

Claim 3: Every edge is outgoing edge for exactly one non-root vertex. ✓



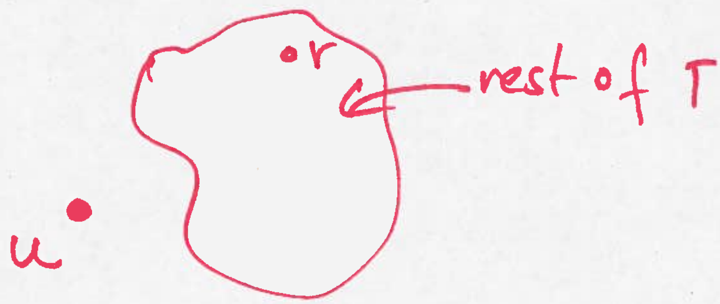
$\Rightarrow |E| = \#$ non-root vertices $= n-1$

Pf of Claim 1: Consider $u \in V \setminus \{r\}$

Case 1: u has exactly 1 outgoing edge ✓

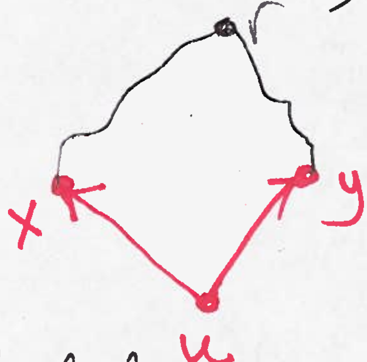
Case 2: u has exactly 0 outgoing edges

$\Rightarrow$



Contradiction, as this $\Rightarrow$ T is not connected.

Case 3: u has ≥ 2 outgoing edges.



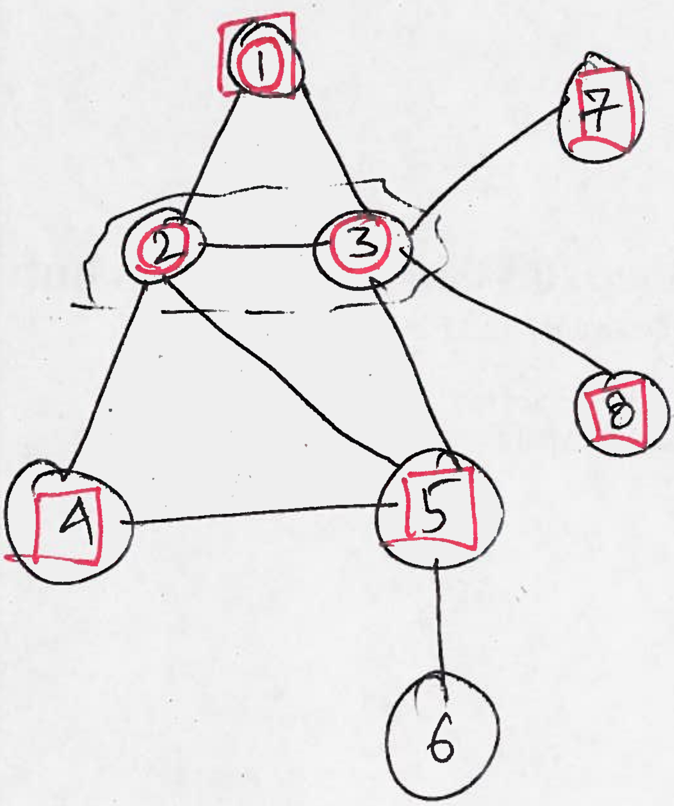
(1) as T is connected $\Rightarrow \exists\ x \sim r$ & $y \sim r$

$\Rightarrow$ Contradiction $\Rightarrow$ T has a cycle. $\blacksquare$

I/P: $G = (V, E)$, $s \in V$

O/P: All $w \in V$ s.t. s & w are connected.

$\Rightarrow$ can solve the s,t-connectivity problem.



$s = 1$.

Answer: $\{1, \ldots, 8\}$

→ Output s first

→ Output all u s.t. $(s, u) \in E$

→ Output all friends of friend of s.

Assumption: given $u \in V$, easy to compute all w s.t. $(u, w) \in E$