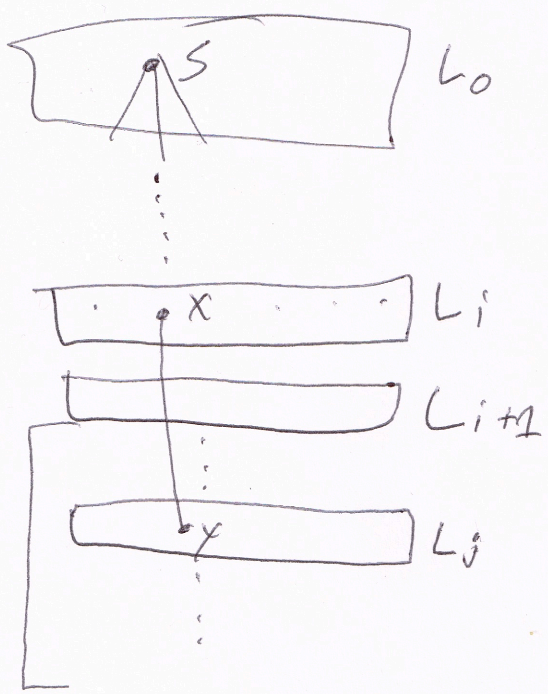
Theorem: If T is a BFS tree for $G=(V,E)$
and $x \in L_i$, $y \in L_j$, $(x,y) \in E$

then i and j differ by ≤ 1
$(|i-j| \leq 1)$

Proof: w.l.o.g. (without loss of generality) $i \leq j$
i.e. if $j < i$, switch i and j so $i \leq j$

Proof by contradiction

assume $i < j-1$ which implies $i \nleq j$
$j > i+1$



By the BFS algorithm,
$y \in L_{i+1}$
Edge (x,y) can't be skipped by the algorithm, so
y must be in the next Level.

Contradiction of $j > i+1$

Generic Search Algorithm: Add a connected component if possible

Output The connected component of the graph starting at node s. $CC(s)$

Explore(s)

1. $R \leftarrow \{s\}$
2. while $\exists (u,w) \in E$ s.t. $u \in R \wedge w \notin R$ {

   $R \leftarrow R \cup \{w\}$

   }

3. Return $R^* \leftarrow R$

<u>Theorem</u>: R^* is the connected component of s

(implies BFS(s) and DFS(s) return $CC(s)$ as well)

<u>Proof</u>: If we show $R^* \subseteq CC(s)$ and $CC(s) \subseteq R^*$ then $R^* = CC(s)$

~~$R^* \subseteq CC(s)$~~

<u>$R^* \subseteq CC(s)$</u>: $w \in R^* \Rightarrow w \in CC(s)$

w is added to R^* only if there $\exists$ a path from s to w which implies they are connected.

$CC(s) \subseteq R^*$: $\quad w \in CC(s) \Rightarrow w \in R^*$

Proof by contradiction

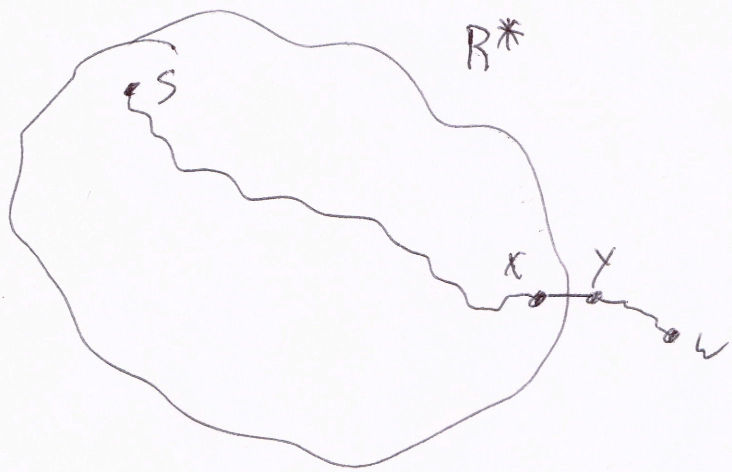
Assume the statement is false.

$w \in CC(s)$ $\quad$ T / ~~F~~ $\quad$ $w \in CC(s)$

$w \in R^*$ $\quad$ ~~T~~ / F $\quad$ $w \notin R^*$

show a contradiction.

Since $w \notin R^*$ then



$(x,y) \in E$ for some $x \in R^*$, $y \notin R^*$ since there must be a path from s to w.

By the algorithm, the edge (x,y) must be traversed and y should be in R^*. X this contradicts the algorithm.

∎