<u>Thm:</u> For a graph G s.t. $|V| = 2k$ for some $k \in \mathbb{N}$ even # of vtcs

s.t. $\forall v \in V: \deg(v) = d$ for some $d \in \mathbb{N}$ all degs same

and s.t. $\chi'(G) = d$: can be clrd with $\Delta(G) = d$ colrs

G' s.t. $G' = (G \setminus \{u,v\}) + \{u,x\} + \{x,v\}$ for some $\{u,v\} \in E$, $x \notin V$

$\Rightarrow \chi'(G') = d+1$

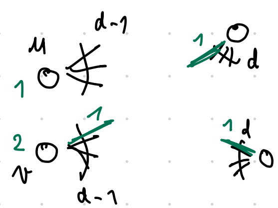
We have the following construction:

We need to show that in any coloring of $\bar{G} = G \setminus \{u,v\}$, $f_c(u) = f_c(v)$
where $f_c(u)$ ... free clr at u

Suppose for contra that $f_c(u) = 1$ & $f_c(v) = 2$ for some clring c

$\therefore \forall x \neq u,v : f_c(x) = \emptyset$ (Since deg $x = d$ which is # of clrs available)

$\Rightarrow \forall x \neq u,v : \exists$ edge w/ clr (1)



- also there is no edge w clr. 1 incid on u ; also $\exists$ edge from v w/ clr 1

$\therefore$ we have odd # of vtcs s.t. each has $\exists$ one incid edge of clr 1

$\Rightarrow$ impossible bcs so colored edges should constitute a matching ↯

$\Rightarrow$ each coloring of $\bar{G}$ using d clrs has $f(v) = f(u)$ $\Rightarrow$ $\{x,u\}$ & $\{x,v\}$ must have same clr if only d clrs is to be used ↯