# DERIVACE FUNKCE

① $y = e^{\sqrt{x^2+x+1}}$

$$y' = e^{\sqrt{x^2+x+1}} \cdot \left(\sqrt{x^2+x+1}\right)' = e^{\sqrt{x^2+x+1}} \cdot \left(\tfrac{1}{2}(x^2+x+1)\right)' =$$

$$= e^{\sqrt{x^2+x+1}} \cdot \frac{1}{2}(x^2+x+1)^{\frac{1}{2}-1} \cdot (x^2+x+1)' =$$

$$= e^{\sqrt{x^2+x+1}} \cdot \frac{1}{2}(x^2+x+1)^{-\frac{1}{2}} \cdot (2x+1) =$$

$$= \frac{(2x+1) \cdot e^{\sqrt{x^2+x+1}}}{2\sqrt{x^2+x+1}}$$

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② $y = \sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}} \quad \left( = \left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)^{\frac{1}{2}} \right)$

$$y' = \frac{1}{2}\left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)^{\frac{1}{2}-1} \cdot \left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)' =$$

$$= \frac{1}{2}\left(\operatorname{arctg} \ln \frac{x+1}{x+2}\right)^{-\frac{1}{2}} \cdot \frac{1}{1+\left(\ln \frac{x+1}{x+2}\right)^2} \cdot \left(\ln \frac{x+1}{x+2}\right)' =$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{1}{\frac{x+1}{x+2}} \cdot \left(\frac{x+1}{x+2}\right)' =$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{x+2}{x+1} \cdot \frac{(x+1)' \cdot (x+2) - (x+1) \cdot (x+2)'}{(x+2)^2}$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{\cancel{x+2}}{x+1} \cdot \frac{1 \cdot (x+2) - (x+1) \cdot 1}{(x+2)^{\cancel{2}}} =$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}}} \cdot \frac{1}{1+\ln^2 \frac{x+1}{x+2}} \cdot \frac{1}{x+1} \cdot \frac{\overbrace{x+2-x-1}^{1}}{x+2} =$$

$$= \frac{1}{2\sqrt{\operatorname{arctg} \ln \frac{x+1}{x+2}} \cdot \left(1+\ln^2 \frac{x+1}{x+2}\right) \cdot (x+1)(x+2)}$$

③ $y = \ln \frac{1}{x+\sqrt{x^2-1}}$

$$y' = \cancel{\frac{1}{x+\sqrt{x^2-1}}} = \frac{1}{\frac{1}{x+\sqrt{x^2-1}}} \cdot \Big( \overbrace{\frac{1}{x+\sqrt{x^2-1}}}^{(x+\sqrt{x^2-1})^{-1}} \Big)' =$$

$$= \left(x+\sqrt{x^2-1}\right) \cdot (-1)\left(x+\sqrt{x^2-1}\right)^{-1-1} \cdot \Big(x + \overbrace{\sqrt{x^2-1}}^{(x^2-1)^{\frac{1}{2}}}\Big)' =$$

$$= \left(x+\sqrt{x^2-1}\right) \cdot (-1)\left(x+\sqrt{x^2-1}\right)^{-2} \cdot \left(1 + \tfrac{1}{2}(x^2-1)^{\frac{1}{2}-1} \cdot (x^2-1)'\right) =$$

$$= \frac{\cancel{x+\sqrt{x^2-1}}}{-1 \cdot (x+\sqrt{x^2-1})^{\cancel{2}}} \cdot \left(1 + \tfrac{1}{\cancel{2}}(x^2-1)^{-\frac{1}{2}} \cdot \cancel{2}x\right) =$$

$$= \frac{-1}{x+\sqrt{x^2-1}} \cdot \left(1 + \frac{x}{\sqrt{x^2-1}}\right) = \frac{-1}{\cancel{x+\sqrt{x^2-1}}} \cdot \frac{\cancel{\sqrt{x^2-1}+x}}{\sqrt{x^2-1}} =$$

$$= \underline{\underline{\frac{-1}{\sqrt{x^2-1}}}}$$

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④ $y = \ln \operatorname{arctg} \frac{\sin x}{1+\cos x}$

$$y' = \frac{1}{\operatorname{arctg}\frac{\sin x}{1+\cos x}} \cdot \left(\operatorname{arctg}\frac{\sin x}{1+\cos x}\right)' =$$

$$= \frac{1}{\operatorname{arctg}\frac{\sin x}{1+\cos x}} \cdot \frac{1}{1+\left(\frac{\sin x}{1+\cos x}\right)^2} \cdot \left(\frac{\sin x}{1+\cos x}\right)' =$$

$$= \frac{1}{\operatorname{arctg}\frac{\sin x}{1+\cos x}} \cdot \frac{1}{1+\frac{\sin^2 x}{(1+\cos x)^2}} \cdot \frac{(\sin x)'(1+\cos x) - \sin x(1+\cos x)'}{(1+\cos x)^2} =$$

$$= \frac{1}{\operatorname{arctg}\frac{\sin x}{1+\cos x}} \cdot \frac{1}{\frac{(1+\cos x)^2+\sin^2 x}{\cancel{(1+\cos x)^2}}} \cdot \frac{\cos x(1+\cos x) - \sin x(-\sin x)}{\cancel{(1+\cos x)^2}} =$$

$$= \frac{1}{\operatorname{arctg}\frac{\sin x}{1+\cos x}} \cdot \frac{\cos x + \overbrace{\cos^2 x + \sin^2 x}^{1}}{1+2\cos x + \underbrace{\cos^2 x + \sin^2 x}_{1}} = \longrightarrow$$

= pokračování (4)

$$= \frac{1}{\operatorname{arctg}\frac{\sin x}{1+\cos x}} \cdot \frac{\cos x+1}{\underbrace{2+2\cos x}_{2(1+\cos x)}} = \frac{1}{2\operatorname{arctg}\frac{\sin x}{1+\cos x}}$$

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(5) $$y = \frac{1}{\log(3x^2+x+1)} = \left(\log(3x^2+x+1)\right)^{-1}$$

$$y' = (-1)\cdot\left(\log(3x^2+x+1)\right)^{-2}\cdot\left(\log(3x^2+x+1)\right)' =$$

$$= \frac{-1}{\log^2(3x^2+x+1)} \cdot \frac{1}{(3x^2+x+1)\cdot\ln 10}\cdot(3x^2+x+1)' =$$

$$= \frac{-(6x+1)}{(3x^2+x+1)\cdot\ln 10\cdot\log^2(3x^2+x+1)}$$

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(6) $$y = 5^{x^2-2x+1}$$

$$y' = 5^{x^2-2x+1}\cdot\ln 5\cdot(x^2-2x+1)' = (2x-2)\ln 5\cdot 5^{x^2-2x+1}$$

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(7) $$y = \frac{\cos x}{2\sin^2 x}$$

$$y' = \frac{(\cos x)'\,2\sin^2 x - \cos x\,(2\overbrace{\sin^2 x}^{(\sin x)^2})'}{(2\sin^2 x)^2} =$$

$$= \frac{-\sin x\cdot 2\sin^2 x - \cos x\cdot 2\cdot 2(\sin x)^{2-1}\cdot(\sin x)'}{4\sin^4 x} =$$

$$= \frac{-2\sin^3 x - 4\cos x\cdot\sin x\cdot(\cos x)}{4\sin^4 x} =$$

$1 = \cos^2 x + \cos^2 x$

$$= \frac{-2\sin^3 x - 4\cos^2 x\sin x}{4\sin^4 x} = \frac{-2\sin x(\sin^2 x + 2\cos^2 x)}{4\sin^4 x} = -\frac{1+\cos^2 x}{2\sin^3 x}$$

(8) $y = \frac{2x^3}{\sqrt{x}} + \frac{x}{2\sqrt[3]{x}} = 2\frac{x^3}{x^{\frac{1}{2}}} + \frac{1}{2}\frac{x}{x^{\frac{1}{3}}} = 2x^{\frac{5}{2}} + \frac{1}{2}x^{\frac{2}{3}}$

jen úpravy mocnin, pak až derivovat

$y' = 2\cdot\frac{5}{2}x^{\frac{5}{2}-1} + \frac{1}{2}\cdot\frac{2}{3}x^{\frac{2}{3}-1} =$

$= 5x^{\frac{3}{2}} + \frac{1}{3}x^{-\frac{1}{3}} = \underline{\underline{5\sqrt{x^3} + \frac{1}{3\sqrt[3]{x}}}}$

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(9) $y = \frac{x}{\sqrt{1-x^2}}\cdot\arcsin x$

$y' = \left(\frac{x}{\sqrt{1-x^2}}\right)'\cdot\arcsin x + \frac{x}{\sqrt{1-x^2}}\cdot(\arcsin x)' =$

$= \frac{(x)'\sqrt{1-x^2} - x\cdot\left(\overset{(1-x^2)^{\frac{1}{2}}}{\sqrt{1-x^2}}\right)'}{\left(\sqrt{1-x^2}\right)^2}\cdot\arcsin x + \frac{x}{\sqrt{1-x^2}}\cdot\frac{1}{\sqrt{1-x^2}} =$

$= \frac{1\cdot\sqrt{1-x^2} - x\cdot\frac{1}{2}(1-x^2)^{\frac{1}{2}-1 = -\frac{1}{2}}\cdot(1-x^2)'}{1-x^2}\cdot\arcsin x + \frac{x}{1-x^2} =$

$= \frac{\sqrt{1-x^2} - \frac{x}{2\sqrt{1-x^2}}\cdot(-2x)}{1-x^2}\cdot\arcsin x + \frac{x}{1-x^2} =$

$= \frac{\sqrt{1-x^2} + \frac{2x^2}{2\sqrt{1-x^2}}}{1-x^2}\cdot\arcsin x + \frac{x}{1-x^2} =$

$= \frac{\frac{1-x^2+x^2}{\sqrt{1-x^2}}}{1-x^2}\cdot\arcsin x + \frac{x}{1-x^2} = \underline{\underline{\frac{1}{\sqrt{1-x^2}\cdot(1-x^2)}\cdot\arcsin x + \frac{x}{1-x^2}}}$

lze ještě upravit takto: $\frac{\arcsin x + x\cdot\sqrt{1-x^2}}{\sqrt{1-x^2}\cdot(1-x^2)} = \underline{\underline{\frac{\arcsin x + x\cdot\sqrt{1-x^2}}{\sqrt{(1-x^2)^3}}}}$

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(10) $y = \operatorname{tg}\sqrt{x} + \operatorname{tg}\frac{\pi}{4}$ konstanta

$y' = \frac{1}{\cos^2\sqrt{x}}\cdot(\underset{x^{\frac{1}{2}}}{\sqrt{x}})' + 0 = \frac{1}{\cos^2\sqrt{x}}\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}} = \underline{\underline{\frac{1}{2\sqrt{x}\cos^2\sqrt{x}}}}$