① $\dfrac{x^2+x}{x^2+4x-12} = 1 + \dfrac{-3x+12}{x^2+4x-12}$

- dělení: $(x^2+x) : (x^2+4x-12) = 1$

$$-(x^2+4x-12)$$

$$0-3x+12$$

- rozklad jmenovatele:

$x^2+4x-12 = (x+6)(x-2)$ ... diskriminant ... nebo odhadem

- obecný tvar rozkladu + výpočet konstant

$$\frac{-3x+12}{(x+6)(x-2)} = \frac{A}{x+6} + \frac{B}{x-2}$$

$$-3x+12 = A(x-2) + B(x+6)$$

$x=2 \Rightarrow -3\cdot 2+12 = A\cdot 0 + B(2+6)$

$6 = 8B \Rightarrow B = \frac{6}{8} = \frac{3}{4}$

$x=-6 \Rightarrow -3\cdot(-6)+12 = A(-6-2)+B\cdot 0$

$30 = -8A \Rightarrow A = -\frac{30}{8} = -\frac{15}{4}$

- výsledek

$$\frac{x^2+x}{x^2+4x-12} = 1 + \frac{-\frac{15}{4}}{x+6} + \frac{\frac{3}{4}}{x-2} =$$

$$= 1 - \frac{15}{4(x+6)} + \frac{3}{4(x-2)}$$

(2) $\dfrac{18-5x-7x^2}{5x^3-26x^2+44x-24}$

výsledek

$\dfrac{-2}{x-2} - \dfrac{5}{(x-2)^2} + \dfrac{3}{5x-6}$

- dělit nemusíme
- rozklad jmenovatele

$5x^3-26x^2+44x-24 = 5(x-2)(x-2)\left(x-\frac{6}{5}\right) =$

$= (x-2)^2 \cdot (5x-6)$

	5	-26	44	-24
2	5	-16	12	0

$5x^2-16x+12 \Rightarrow D = 16^2-4\cdot 5\cdot 12 = 16$

$x_{1,2} = \dfrac{16 \pm 4}{2\cdot 5} = \begin{cases} \frac{20}{10} = 2 \\ \frac{12}{10} = \frac{6}{5} \end{cases}$

- obecný tvar rozkladu

$$\frac{18-5x-7x^2}{5x^3-26x^2+44x-24} = \frac{18-5x-7x^2}{(x-2)^2\cdot(5x-6)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{5x-6}$$

- výpočet konstant

$$\frac{18-5x-7x^2}{(x-2)^2(5x-6)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{5x-6}$$

$$\frac{18-5x-7x^2}{(x-2)^2(5x-6)} = \frac{A(x-2)(5x-6) + B(5x-6) + C(x-2)^2}{(x-2)^2\cdot(5x-6)}$$

$5x^2-10x-6x+12$; x^2-4x+4

$18-5x-7x^2 = 5Ax^2-16Ax+12A+5Bx-6B+Cx^2-4Cx+4C$

x^2: $-7 = 5A+C \Rightarrow C = -7-5A$

x^1: $-5 = -16A+5B-4C$

x^0: $18 = 12A-6B+4C$

$\left.\begin{matrix} 1. \\ 2. \end{matrix}\right\} + \quad 13 = -4A-B \Rightarrow B = -4A-13$

do 2. rovnice

$-5 = -16A+5(-4A-13)-4(-7-5A)$ (*)

$-5 = -16A-20A-65+28+20A$

$32 = -16A \Rightarrow \boxed{A = -2}$

$\boxed{B} = -4\cdot(-2)-13 = \boxed{-5}$

$\boxed{C} = -7-5\cdot(-2) = \boxed{3}$

③ $\frac{2x+2}{(x-1)(x^2+1)^2}$

- dělení i rozklad jmenovatele už máme
- obecný tvar rozkladu:

$$\frac{2x+2}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$\frac{2x+2}{(x-1)(x^2+1)^2} = \frac{A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1)}{(x-1)(x^2+1)^2}$$

$$2x+2 = A(x^4+2x^2+1) + (Bx+C)(x^3-x^2+x-1) + Dx^2 - Dx + Ex - E$$

$$2x+2 = Ax^4+2Ax^2+A+Bx^4-Bx^3+Bx^2-Bx+Cx^3-Cx^2+Cx-C+Dx^2-Dx+Ex-E$$

x^4: $0 = A+B \Rightarrow A=-B \Rightarrow B=-1$

x^3: $0 = -B+C \Rightarrow C=B=-A$ (*) $\Rightarrow C=-1$

x^2: $0 = 2A+B-C+D$

x^1: $2 = -B+C-D+E$ } + $2 = 2A+E \Rightarrow E=2-2A$ (**)

x^0: $2 = A-C-E$

(*) a (**) do poslední rovnice

$2 = A-(-A)-(2-2A)$

$2 = A+A-2+2A$

$4 = 4A \Rightarrow A=1$ $(\Rightarrow B=-1,\ C=-1)$

$E = 2-2A = 2-2 = 0$

dosazení do 4. rovnice (u x^1):

$2 = -(-1)+(-1)-D+0$

$2 = 1-1-D \Rightarrow D=-2$

výsledek

$$\frac{2x+2}{(x-1)(x^2+1)^2} = \frac{1}{x-1} + \frac{-x-1}{x^2+1} + \frac{-2x}{(x^2+1)^2}$$

(4) $\frac{x^4+6x^2+x-2}{x^4+2x^3}$ (*) $= 1 + \frac{-2x^3+6x^2+x-2}{x^4+2x^3}$

- **dělení:** $(x^4+6x^2+x-2):(x^4+2x^3)=1$

$$-(x^4+2x^3)$$

$$-2x^3+6x^2+x-2$$

- **rozklad jmenovatele:**

$$x^4+2x^3 = x^3(x+2)$$

- **obecný tvar rozkladu**

$$\frac{-2x^3+6x^2+x-2}{x^3(x+2)} = \frac{A}{x^3}+\frac{B}{x^2}+\frac{C}{x}+\frac{D}{x+2}$$

$$\frac{-2x^3+6x^2+x-2}{x^3(x+2)} = \frac{A(x+2)+Bx(x+2)+Cx^2(x+2)+Dx^3}{x^3(x+2)}$$

$$-2x^3+6x^2+x-2 = Ax+2A+Bx^2+2Bx+Cx^3+2Cx^2+Dx^3$$

x^3: $-2 = C+D \Rightarrow -2 = \frac{5}{2}+D \Rightarrow D = -2-\frac{5}{2} = -\frac{9}{2}$

x^2: $6 = B+2C \Rightarrow 2C = 6-B \Rightarrow 2C = 6-1 \Rightarrow C = \frac{5}{2}$

x^1: $1 = A+2B \Rightarrow 2B = 1-A = 1-(-1) = 2$

x^0: $-2 = 2A \Rightarrow A = -1$ $\Downarrow$ $B = 1$

výsledek

(*) $f(x) = 1 - \frac{1}{x^3} + \frac{1}{x^2} + \frac{5}{2x} - \frac{9}{2(x+2)}$