Maclaurinův polynom 3. stupně
pro $f(x) = \arcsin x$

$$T_3(x) = f(0) + \frac{f'(0)\cdot x}{1!} + \frac{f''(0)\cdot x^2}{2!} + \frac{f'''(0)\cdot x^3}{3!}$$

$x_0 = 0$

$f(0) = \arcsin 0 = 0$

$f'(x) = \frac{1}{\sqrt{1-x^2}} \quad f'(0) = 1$

$$f''(x) = \left(\frac{1}{\sqrt{1-x^2}}\right)' = \left((1-x^2)^{-\frac{1}{2}}\right)' =$$

$$= -\frac{1}{2}(1-x^2)^{-\frac{3}{2}}\cdot(-2x) = \frac{x}{\sqrt{(1-x^2)^3}} \quad f''(0) = 0$$

$$f'''(x) = \frac{(x)'\cdot\sqrt{(1-x^2)^3} - x\cdot\left(\sqrt{(1-x^2)^3}\right)'}{\left(\sqrt{(1-x^2)^3}\right)^2} =$$

$(1-x^2)^{\frac{3}{2}}$

$$= \frac{1\cdot\sqrt{(1-x^2)^3} - x\cdot\frac{3}{2}(1-x^2)^{\frac{1}{2}}\cdot(-2x)}{(1-x^2)^3}$$

$f'''(0) = 1$

$$\boxed{T_3(x)} = 0 + 1\cdot x + 0\cdot\frac{x^2}{2!} + \frac{1\cdot x^3}{3!} = \boxed{x + \frac{x^3}{6}}$$

Taylorův polynom 3. stupně pro $f(x) = \operatorname{arctg}\frac{1}{x}$ v $x_0 = 1$

$$T_3(x) = f(x) + \frac{f'(x)(x-1)}{1!} + \frac{f''(x)(x-1)^2}{2!} + \frac{f'''(x)(x-1)^3}{3!}$$

$$f(1) = \operatorname{arctg} 1 = \frac{\pi}{4}$$

$$f'(x) = \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \left(\frac{1}{x}\right)' = \frac{1}{1+\frac{1}{x^2}} \cdot (-1) \cdot x^{-2} = \frac{1}{\frac{x^2+1}{x^2}} \cdot \frac{-1}{x^2} =$$

$$= \frac{-1}{x^2+1} \qquad f'(1) = -\frac{1}{2}$$

$$f''(x) = \left(-\frac{1}{x^2+1}\right)' = \left(-1 \cdot (x^2+1)^{-1}\right)' = -1 \cdot (-1) \cdot (x^2+1)^{-2} \cdot 2x =$$

$$= \frac{2x}{(x^2+1)^2} \qquad f''(1) = \frac{2}{2^2} = \frac{1}{2}$$

$$f'''(x) = \frac{(2x)' \cdot (x^2+1)^2 - 2x \cdot \left((x^2+1)^2\right)'}{(x^2+1)^4} =$$

$$= \frac{2(x^2+1)^2 - 2x \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$$

$$f'''(1) = \frac{2 \cdot 2^2 - 2 \cdot 2 \cdot 2 \cdot 2}{2^4} = \frac{8-16}{16} = \frac{-8}{16} = -\frac{1}{2}$$

$$\boxed{T_3(x) = \frac{\pi}{4} + \left(-\frac{1}{2}\right)(x-1) + \frac{1}{4}(x-1)^2 - \frac{1}{12}(x-1)^3}$$

$$\frac{\frac{1}{2}}{2!} = \frac{1}{4} \qquad \frac{-\frac{1}{2}}{3!} = \frac{-\frac{1}{2}}{6} = -\frac{1}{12}$$

Určete limity:

a) $$\lim_{x\to\frac{\pi}{2}} \frac{\sin 2x - 2\cos x}{\cos^2 x} = \left[\frac{\sin \overbrace{2\cdot\frac{\pi}{2}}^{\pi} - 2\cos\frac{\pi}{2}}{\cos^2\frac{\pi}{2}} = \frac{0-0}{0} = \frac{0}{0}\right] =$$

$$\stackrel{L.P.}{=} \lim_{x\to\frac{\pi}{2}} \frac{\cos 2x\cdot 2 - 2(-\sin x)}{2\cos x\cdot(-\sin x)} = \left[\frac{(\cos \overbrace{2\cdot\frac{\pi}{2}}^{\pi})\cdot 2 + 2\sin\frac{\pi}{2}}{2\cos\frac{\pi}{2}\cdot(-\sin\frac{\pi}{2})} = \right.$$

$$\left. = \frac{2\cos\pi + 2\sin\frac{\pi}{2}}{-2\cos\frac{\pi}{2}\sin\frac{\pi}{2}} = \frac{-2+2}{-2\cdot 0\cdot 0}\right] = \left[\frac{0}{0}\right] \stackrel{L.P.}{=}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{2(-\sin 2x)\cdot 2 + 2\cos x}{-2(\underbrace{-\sin x\cdot\sin x + \cos x\cdot\cos x}_{(\cos x\sin x)'})} =$$

$$= \frac{-2(\sin \overbrace{2\cdot\frac{\pi}{2}}^{\pi})\cdot 2 + 2\cos\frac{\pi}{2}}{-2(-\sin\frac{\pi}{2}\cdot\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\cdot\cos\frac{\pi}{2})} = \frac{-2\cdot\overbrace{0}^{0}\cdot 2 + 2\cdot\overbrace{0}^{0}}{-2\cdot(-1\cdot 1 + 0\cdot 0)} = \frac{0}{2} = 0$$

b) $$\lim_{x\to 0} \frac{\ln\frac{\sin x}{x}}{1-\cos x} = \frac{\ln 1}{1-\cos 0} = \frac{0}{1-1} = \left[\frac{0}{0}\right] \;\; \text{L.P.}$$

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = \left[\frac{0}{0}\right] = \lim_{x\to 0}\frac{\cos x}{1} = 1}$$

$$\stackrel{L.P.}{=} \lim_{x\to 0} \frac{\frac{1}{\frac{\sin x}{x}}\cdot\left(\frac{\sin x}{x}\right)'}{0-(-\sin x)} = \lim_{x\to 0} \frac{\frac{1}{\frac{\sin x}{x}}\cdot\frac{\cos x\cdot x - \sin x\cdot 1}{x^2}}{\sin x} =$$

$$= \lim_{x\to 0} \frac{\cos x\cdot x - \sin x}{x\cdot\sin^2 x} = \left[\frac{0}{0}\right] \stackrel{L.P.}{=} \lim_{x\to 0} \frac{(-\sin x\cdot x + \cos x) - \cos x}{1\cdot\sin^2 x + x\cdot 2\sin x\cos x} =$$

$$= \lim_{x\to 0} \frac{-\sin x\cdot x}{\sin^2 x + x\cdot 2\sin x\cos x} = \lim_{x\to 0} \frac{-x}{\sin x + 2x\cos x} =$$

$$= \left[\frac{0}{0}\right] \stackrel{L.P.}{=} \lim_{x\to 0} \frac{-1}{\underbrace{\cos x}_{1} + \underbrace{2\cos x}_{2\cdot 1} + \underbrace{2x\cdot(-\sin x)}_{0}} = -\frac{1}{3}$$

asymptoty:

1 vodorovná pro $f(x) = e^{\frac{1}{x}}$

$\lim\limits_{x\to\infty} e^{\frac{1}{x}} = e^{\frac{1}{\infty} \to 0} = e^0 = 1$

$x \to -\infty \quad = e^{\frac{1}{-\infty} \to 0} = e^0 = 1$

$\boxed{y = 1 \text{ pro } x \to \pm\infty}$

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2 svislá pro $f(x) = \frac{1}{\ln x}$ $\to \ln x \dots x \in (0, \infty)$

$\to \ln x \neq 0$

$x \neq 1$

$D(f) = (0,1) \cup (1, \infty)$

$\lim\limits_{x\to 1} \frac{1}{\ln x} = \begin{cases} \lim\limits_{x\to 1^+} \frac{1}{\ln x} = \frac{1}{0+} = \infty \\ \lim\limits_{x\to 1^-} \frac{1}{\ln x} = \frac{1}{0^-} = -\infty \end{cases} \quad \boxed{x = 1}$

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3 šikmá pro $f(x) = \frac{x^2+1}{x}$

$a = \lim\limits_{x\to\infty} \frac{f(x)}{x} = \lim\limits_{x\to\infty} \frac{\frac{x^2+1}{x}}{x} = \lim\limits_{x\to\infty} \frac{x^2+1}{x^2} = \left[\frac{\infty}{\infty}\right] \overset{L.P.}{=}$

$= \lim\limits_{x\to\infty} \frac{\cancel{2x}}{\cancel{2x}} = \underline{\underline{1}}$

$b = \lim\limits_{x\to\infty} (f(x) - ax) = \lim\limits_{x\to\infty} \left(\frac{x^2+1}{x} - x\right) =$

$= \lim\limits_{x\to\infty} \frac{x^2+1-x^2}{x} = \lim\limits_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = \underline{\underline{0}}$

$y = 1 \cdot x + 0 \Rightarrow \boxed{y = x}$ $\boxed{\begin{array}{l}\text{pro } x \to \infty \\ \text{pro } x \to -\infty\end{array}}$

pozn. $\lim\limits_{x\to-\infty}$ stejný výpočet