I Aggregate Method

insert	old capacity	new capacity	insert cost	copy cost	total cost
1	0	1	1	-	1
2	1	2	1	1	2
3	2	4	1	2	3
4	4	4	1	-	1
5	4	8	1	4	5
6	8	8	1	-	1
7	8	8	1	-	1
8	8	8	1	-	1
9	8	16	1	8	9
⋮	⋮	⋮	⋮	⋮	⋮

Insert cost = n

Copy cost = $1 + 2 + 4 + \dots + 2^{\log_2(n-1)}$

$a = 1 \quad h = 2 \qquad S = \dfrac{(a(h^k - 1))}{(h-1)}$

$k = \log_2(n-1) + 1$

$S = \dfrac{(1 \cdot 2^k - 1)}{(2-1)} = 2^{\log_2(n-1)+1} - 1$

dar $\quad 2^{\log_2(n-1)+1} = 2 \cdot 2^{\log_2(n-1)} \leq 2 \cdot (n-1) < 2n \implies S < 2n$

Total cost = $n + S < n + 2n = 3n$

delete	size before	capacity before	delete cost	copy cost	total cost
1	n	n	1	-	1
2	n-1	n	1	-	1
3	n-2	n	1	⋮	1
⋮	⋮	⋮	⋮	⋮	⋮
n/2+1	n/2	n	1	-	1
n/2 +2	n/2-1	n	1	⋮	1
⋮	⋮	⋮	1	n/4	n/4+1
3n/4+1	n/4	n/2	1	-	1
3n/4+2	n/4 -1	⋮	⋮	⋮	⋮
⋮					

Delete cost $= n$

Copy cost $= n/4 + n/8 + n/16 + \ldots + 1$

$a = \dfrac{n}{4} \quad h = \dfrac{1}{2} \qquad S = \dfrac{a}{(1-h)} \cdot (1 - h^K)$

$S = \dfrac{n}{4} \cdot \dfrac{1}{\left(1 - \frac{1}{2}\right)} \cdot \left(1 - \left(\frac{1}{2}\right)^K\right) = \dfrac{n}{4} \cdot 2 \cdot \left(1 - \left(\frac{1}{2}\right)^K\right) = \dfrac{n}{2} \cdot \left(1 - \left(\frac{1}{2}\right)^K\right)$

$K \nearrow \Rightarrow \left(\dfrac{1}{2}\right)^K \to 0 \Rightarrow S \to \dfrac{n}{2}$

Total cost $= n + \dfrac{n}{2} = \dfrac{3n}{2} < 2n$

Mix of Insert and delete

m operations
m insert op. $\qquad$ Total cost: $3m + (2n - m) = 3m + 2n - 2m = 2n + m \leq 3n$
$n - m$ delete op.

$\Rightarrow$ For n operatations $\Rightarrow O(n)$

Amortized Cost per Operation

$\dfrac{\text{Total cost}}{\text{Nr. of op.}} = \dfrac{O(n)}{n} = O(1)$

III Potential Method

$$\phi(\Delta) = \begin{cases} 2 \cdot size(\Delta) - capacity(\Delta), & dacă\ \alpha(\Delta) \geq 1/2 \\ capacity(\Delta)/2 - size(\Delta), & dacă\ \alpha(\Delta) < 1/2 \end{cases}$$

$$\alpha(\Delta) = size(\Delta) / capacity(\Delta)$$

1) Insert (prima inserare)

Cost actual : $c_1 = 1$

$\phi(\Delta_1) - \phi(\Delta_0) = 1$

Cost amortizat : $\hat{c}_1 = 1 + 1 = 2$

2) Insert fără redimensionare
$(\alpha \geq 1/2)$

Cost actual $c_i = 1$

$\phi(\Delta_i) - \phi(\Delta_{i-1}) = 1$

Cost amortizat $\hat{c}_i = 1 + 2 = 3$

3) Insert cu redimensionare (tabel plin)

$c_i = 1 + size(\Delta_{i-1})$

$\phi(\Delta_i) - \phi(\Delta_{i-1}) = 2 - capacity(\Delta_{i-1})$

$\hat{c}_i = (1 + capacity(\Delta_{i-1})) + (2 - capacity(\Delta_{i-1})) = 3$

4) Delete fără redimensionare $(\alpha \geq 1/2)$

$c_i = 1$

$\phi(\Delta_i) - \phi(\Delta_{i-1}) = -2$

$\hat{c}_i = 1 + (-2) = -1$

5) Delete fără redimensionare
$(1/4 < \alpha < 1/2)$

$c_i = 1$

$\phi(\Delta_i) - \phi(\Delta_{i-1}) = ~~\cancel{1}~~$

$\hat{c}_i = ~~\cancel{\text{...}}~~$

$1 + 1 = 2$

6) Delete cu redimensionare $(\alpha = 1/4)$

$c_i = size(\Delta_{i-1})$

$\phi(\Delta_i) - \phi(\Delta_{i-1}) = -size(\Delta_{i-1})$

$\hat{c}_i = size(\Delta_{i-1}) + (-size(\Delta_{i-1})) = 0$

Operație	Stare după	α	ϕ	cost actual	potențial	cost amortizat
Inițial	[ ]	0	0	–	–	–
Insert A	[A]	1	1	1	+1	2
Insert B	[A, B]	1	2	2	+1	3
Insert C	[A, B, C]	3/4	2	3	0	3
Delete C	[A, B]	1/2	0	1	-2	-1
Delete B	[A]	1/4	1	1	+1	2
Delete A	[ ]	0	1	1	0	1

Pentru n operații (m insert și n - m delete)

Cost amortizat total = $3m + 2(n-m) = 2m + n \le 3n$

Total amortizat = Total actual + $\phi(\Delta_n) - \phi(\Im_0) \Rightarrow$
                                        ≥ 0 $= 0$

$3n = $ Total amortizat $\ge$ Total actual
  O(n)

$C = 3$

Cost amortizat
pe operație O(1)

II Accounting Method

Insert : taxăm 3 credite
  - 1 credit pt. operație
  - 2 credite pt. stocare

Delete : taxăm 2 credite - 1 pt. op.
  1 credit alocat în slotul gol dacă acesta se află
  în jumătatea stângă a tabelului

1. Insert fără redimensionare
   cost actual : 1
   cost amortizare : 3 (1 inserare + 2 stocare)
   Diferența de 2 credite ⇒ invariant 1

2. Insert cu redimensionare (când tabelul este plin):
   Cost actual : 1+ size ()
   Cost copiere size
   Se dublează capacitatea $K \to 2K$
   K/2 sunt în jumătatea dreaptă
   fiecare element are 2 credite ⇒ K credite total
   K credite plătesc copierea celor K elem.
   După redimensionare, toate K+1 elem. sunt în stânga
   Cost amortizate : 3

1. Delete fără redim.
   Cost actual : 1.
   Cost amortizat : 2
   Diferența ⇒ invariant 2

2. Delete cu redim. (când tabelul
   devine 1/4 plin)
   Cost actual : 1 + size/4
   Cost copiere : size/4
   Cost amortizare : 2

n Operații , m insert, n - m Delete
Cost amortizat total 3m (Insert) , 2(n-m)(Delete)
Total = $3m + 2(n-m) = 2n + m \le 3n$
  Complexitate O(n)