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MTH 325 - 01
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Learning Module: Representation and Properties of Relations

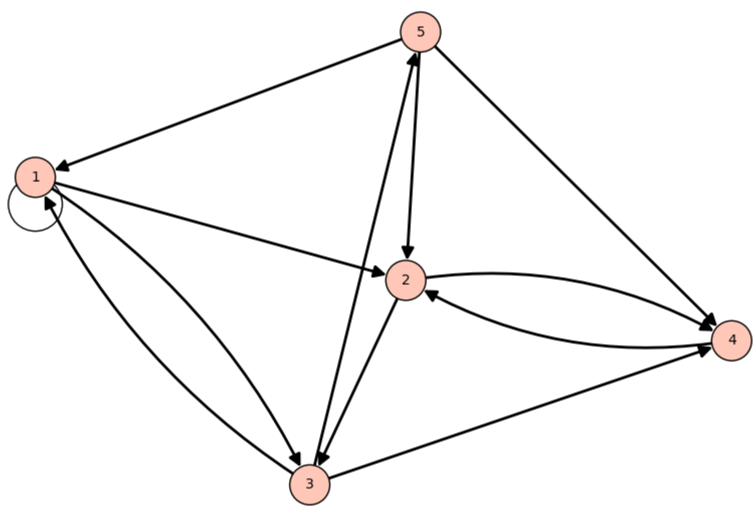
1.

a. dict = {1:[1,2,3], 2:[3,4], 3:[1,4,5], 4:[2], 5:[1, 2, 4]}

b.

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

c. digraph = DiGraph({1:[1,2,3], 2:[3,4], 3:[1,4,5], 4:[2], 5:[1, 2, 4]})
digraph.plot()



d. **Reflexive** - *No*. (2, 2) is not in the relation.
Symmetric - *No*. (5, 2) is in the relation but (2, 5) is not.
Antisymmetric - *No*. (2, 4) and (4, 2) are both in the relation.
Transitive - *No*. (1, 3) and (3, 4) are in the relation but (1, 4) is not.

e. R^2 = {(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (4,3), (4,4), (5,1), (5,2), (5,3), (5,4)}

R^3 = {(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,4), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5)}

I got this answer by composing R with itself once for R^2 and twice for R^3. To do these compositions, I found the binary product of R with itself.

2.

a. D = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (1,10), (1,11), (1,12), (2,2), (2,4), (2,6), (2,8), (2,10), (2,12), (3,3), (3,6), (3,9), (3,12), (4,4), (4,8), (4,12), (5,5), (5,10), (6,6), (6,12), (7,7), (8,8), (9,9), (10,10), (11,11), (12,12)}

   P = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (1,10), (1,11), (1,12), (2,1), (2,3), (2,5), (2,7), (2,9), (2,11), (3,1), (3,2), (3,4), (3,5), (3,7), (3,8), (3,10), (3,11), (4,1), (4,3), (4,5), (4,7), (4,9), (4,11), (5,1), (5,2), (5,3), (5,4), (5,6), (5,7), (5,8), (5,9), (5,11), (5,12), (6,1), (6,5), (6,7), (6,11), (7,1), (7,2), (7,3), (7,4), (7,5), (7,6), (7,8), (7,9), (7,10), (7,11), (7,12), (8,1), (8,3), (8,5), (8,7), (8,9), (8,11), (9,1), (9,2), (9,4), (9,5), (9,7), (9,8), (9,10), (9,11), (10,1), (10,3), (10,7), (10,9), (10,11), (11,1), (11,2), (11,3), (11,4), (11,5), (11,6), (11,7), (11,8), (11,9), (11,10), (11,12), (12,1), (12,5), (12,7), (12,11)}

b. D

$$\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

P

$$\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}$$

c. D^2

$$\begin{pmatrix} 1&1&1&1&1&1&1&1&1&1&1&1 \\ 0&1&0&1&0&1&0&1&0&1&0&1 \\ 0&0&1&0&0&1&0&0&1&0&0&1 \\ 0&0&0&1&0&0&0&1&0&0&0&1 \\ 0&0&0&0&1&0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

I got this result by taking the boolean product of D and itself. This matrix is identical to the matrix D. I think this is because D is transitive, so there are no paths of length 2 in D^2 that are not already in D.

d.

$$\begin{pmatrix} 1&1&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

I got this result by doing an and operation on each element of the matrix. For instance, the element at [1,1] in the result equals (D[1,1] and P[1,1]). Only ordered pairs that are present in both D and P are present in the result.

e. No. Matrix composition is not commutative (the order of the operands matters) since it is based on matrix multiplication. When multiplying two matrices, A*B does not equal B*A. This is also true for the binary product of a matrix, which is used to calculate the composite of two relations.

3.

a. 500,500
When a = 1, every b is greater than or equal to a, so we have 1000 1s in the first row. When a = 2, every b except when b = 1 is greater than or equal to a, so we have 999 1s in the second row. This pattern of one less 1 per row continues until we get to the end of the sequence. When a = 999, we only have two 1s in the row. When a = 1000, we only have one 1 in the row. If you add up every integer from 1 to 1000, you get 500,500.

b. 999
Each row can have at most one entry that is a 1 since only one b will satisfy the equation for each a. Each row will have that one entry that is a 1 except for the row where a = 1000. When a = 1000, a + b will never equal 1000 since the lowest number that b can be is 1.

c. 500,500
This one is very similar to part a. When a = 1, every b that you add to it will give a sum that is less than or equal to 1001, so the first row will have 1000 1s. When a = 2, every b except 1000 will give a sum less than or equal to 1001. This pattern will continue until the last row when a = 1000 and only when b = 1 will you get a sum that is less than or equal to 1001 so there is only one 1 in the last row. If you add up every integer from 1 to 1000, you get 500,500.

4. The whole first question was easy for me (except part e). I am very confident in my ability to translate ordered pairs to dictionaries and then use that dictionary in sage to create digraphs and zero-one matrices. I'm also confident in my understanding of the properties of relations. Part e of the first question threw me off a little bit because I didn't quite understand what R^2 was. I initially thought that R^2 only contained the paths of length 2 from R. I was confused why there were additional points in R^2 after I calculated it at first. I was able to resolve this confusion by looking back at the book. The second question wasn't too difficult for me either, but it was pretty time consuming. I was easily able to determine which ordered pairs belonged in D and P. It was also easy to convert those ordered pairs to zero-one matrices. After my struggle with part e of the first question, part c on the second question was much easier, though more time consuming given the size of the matrix. The meet operation in part d was very easy and I knew the answer to part e because of my experience with matrix multiplication in the past. Question 3 had me stumped for a little while, but the second two parts were much easier after I figured the first part out.

I needed to figure out how to render matrices in Latex to create the zero-one matrices. I was able to come up with a quick python program that outputs the Latex code needed to render the matrix. I then went over to a Latex editor, pasted the output from my program in, and took a screenshot of the matrix it generated to paste into my word document. I will definitely start the next learning module sooner than I did this one.