



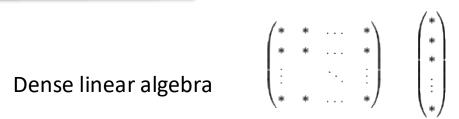


Sparse BLAS Working Group: On the path to defining a standard for sparse BLAS operations

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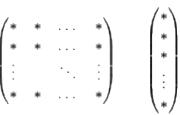
Sparse linear algebra

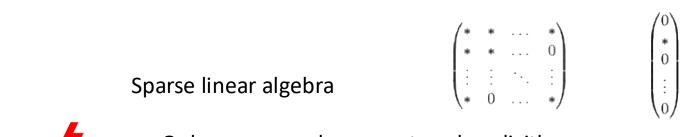
$$\begin{pmatrix} * & * & \dots & * \\ * & * & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & 0 & \dots & * \end{pmatrix} \qquad \begin{pmatrix} 0 \\ * \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ * \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Dense linear algebra $\begin{pmatrix} * & * & \cdots & * \\ * & * & \cdots & * \\ \vdots & & \ddots & \vdots \\ * & * & \cdots & * \end{pmatrix} \begin{pmatrix} * \\ * \\ \vdots \\ * \end{pmatrix}$

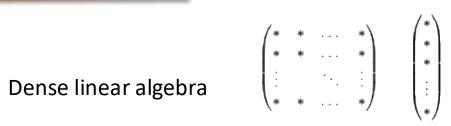
All entries are stored explicitly



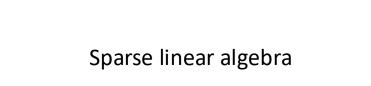




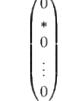
• Only nonzero values are stored explicitly (and sometimes also zeros for performance reasons or because the matrix changes over time and a zero becomes nonzero)



- All entries are stored explicitly
- One standard format (excpt. for row/col major)

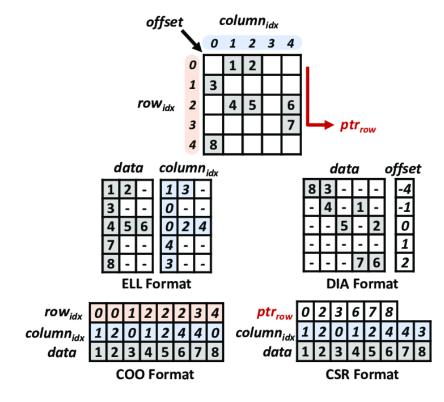




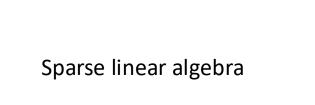




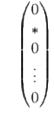
- Only nonzero values are stored explicitly Different formats: COO, CSR, ELL, DIA...



- Dense linear algebra $\begin{pmatrix} * & * & \cdots & * \\ * & * & \cdots & * \\ \vdots & * & \ddots & \vdots \\ * & * & \cdots & * \end{pmatrix}$
- All entries are stored explicitly
- One standard format (excpt. for row/col major)
- Elements are stored in sorted order





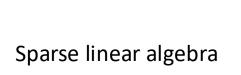


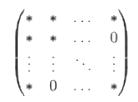


- Only nonzero values are stored explicitly
- Different formats: COO, CSR, ELL, DIA...
 Not always and stored explicitle
 - Not always sorted E.g. when elements are added in COO, or if diagonal elements are stored first, etc.

Dense linear algebra

- $\begin{pmatrix} * & * & \dots & * \\ * & * & \dots & * \\ \vdots & & \ddots & \vdots \\ * & * & \dots & * \end{pmatrix} \qquad \begin{pmatrix} * \\ * \\ \vdots \\ * \end{pmatrix}$
- All entries are stored explicitly
- One standard format (excpt. for row/col major)
- Elements are stored in sorted order
- Vectors and matrices are conceptually the same



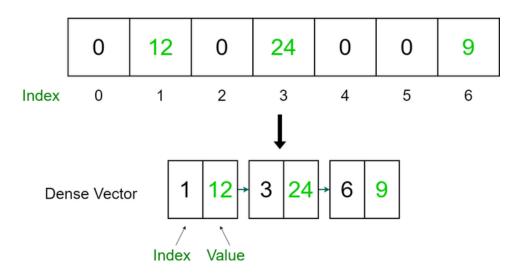






- Only nonzero values are stored explicitly
- Different formats: COO, CSR, ELL, DIA...
- Not always sorted
- Vectors use a different storage format

Sparse Vector



Dense linear algebra

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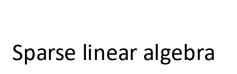
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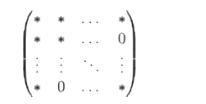
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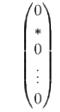
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- Vectors and matrices are conceptually the same
- Size of output is known upfront -> single kernel









- Only nonzero values are stored explicitly
- Different formats: COO, CSR, ELL, DIA...
- Not always sorted
- Vectors use a different storage format
- Generally, the memory requirement is not known before (e.g. sparse matrix multiply)
 - 1) symbolic phase
 - 2) memory allocation
 - 3) numeric phase

- Dense linear algebra $\begin{pmatrix} * & * & \cdots & * \\ * & * & \cdots & * \\ \vdots & & \ddots & \vdots \\ * & * & * \end{pmatrix} \begin{pmatrix} * \\ * \\ \vdots \\ \vdots \end{pmatrix}$
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- Fortran/C interface adopted by most vendors

Sparse linear algebra





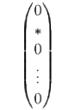


- Only nonzero values are stored explicitly
- Different formats: COO, CSR, ELL, DIA...
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- Generally, the memory requirement is not known before
- Vendors increasingly use a C++ interface, no agreement Plethora of options makes a C-interface "impossible"

- Dense linear algebra $\begin{pmatrix} * & * & \cdots & * \\ * & * & \cdots & * \\ \vdots & & \ddots & \vdots \\ * & * & & * \end{pmatrix}$
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- Elements are stored in sorted order
- Vectors and matrices are conceptually the same
- Size of output is known upfront -> single kernel
- Fortran/C interface adopted by most vendors
- **Exceptions are clearly handled**

Sparse linear algebra







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- Not always sorted
- Vectors use a different storage format
 - Generally, the memory requirement is not known before
- Vendors increasingly use a C++ interface, no agreement
 - No consistent handling of NaN/Inf: What if an implicit matrix zero is multiplied with a *NaN*? Checking for NaN in the vector introduces overhead

$$\begin{pmatrix} 0 & * & \dots & * \\ 0 & * & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & * \end{pmatrix} \quad \begin{pmatrix} NaN \\ * \\ \vdots \\ * \end{pmatrix} \quad =?$$

The working group

Cross-Institutional working group, loose collaboration, not everyone attends every meeting, workshops, SC 2024 BoF; (Vendor) library developers, Application specialists, standard enthusiasts; Weekly virtual meetings, Sparse BLAS design proposal is working document;



2nd Sparse BLAS workshop, 2024

Intel Corporation
NVIDIA
AMD
Arm
MathWorks
University of California, Berkeley
Intel Labs
ORNL,
Karlsruhe Institute of Technology
LLNL
Sandia National Laboratories

MIT

The working group

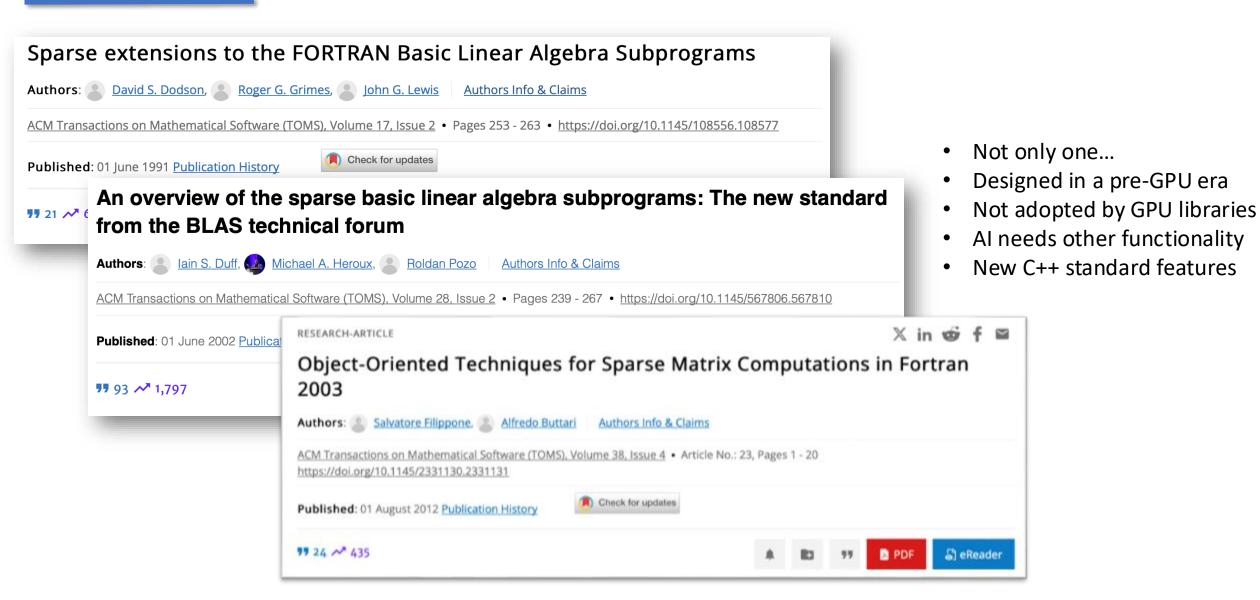
Cross-Institutional working group, loose collaboration, not everyone (Vendor) library developers, Application specialists, standard enthus Weekly virtual meetings, Sparse BLAS design proposal is working do



2nd Sparse BLAS work

Interface for Sparse Linear Algebra Operations September 19, 2024 Contents 1 Introduction and Motivation 2 Related Efforts 3 Functionality Scope of the Sparse BLAS 4 Supported Sparse Matrix Storage Formats keley 5 API Design blogy 6 Execution Model (Short Version) 19 7 Numerical considerations 7.3 Consistent exception handling and reproducibility 24 8 Future extensions A Appendix

But there already is a sparse BLAS standard?



Should libraries own the sparse objects or a non-owning interface?

- Owning allows for more optimization as the opaque object can use any storage format
- Owning makes it impossible for the user to easily access the data or modify the data
- Packing/Unpacking routines introduce overhead maybe error prone?

First step: view-based non-owning interface.

Owning or non-owning?

- Owning allows for more optimization as the opaque object can use any storage format
- Owning makes it impossible for the user to easily access the data or modify the data
- Packing/Unpacking routines introduce overhead maybe error prone?

Most use cases want to apply operation to application-owned data.

--> We decide to go for light-weight "views" of the data

Light-weight views with additional data?

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Most use cases want to apply operation to application-owned data.

- --> We decide to go for light-weight "views" of the data
 - Very limited optimization potential
 - Where can we store additional matrix information?
 - Create a copy?
 - --> We add a "bag" to the "view" for additional data

Light-weight views with additional data?

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 - Very limited optimization potential
 - Where can we store additional matrix information?
 - Create a copy?
 - --> We add a "bag" to the "view" for additional data
 - matrixHandle
 - How should the space for additional data be allocated?

Library-internal data allocation

- Some applications want to control all GPU memory with their allocator (Matlab, etc.)
- We need the possibility to pass this allocator to the library
- If no allocator is passed to the library, the standard allocator is used

```
using namespace sparseblas;
csr_view<float> A_view(values, rowptr, colind, shape, nnz);
// create a handle to hold some internal stuff more than view
matrix_handle A(A_view, allocator);
```

9/24/24

Which data format?

- We focus on the most universal and popular formats: CSR, CSC, COO
- We handle dense matrices/vectors as C++20 mdspan

Type	Name	Description	
index_type	nrows	Number of rows of the matrix	
index_type	ncols	Number of columns of the matrix	
offset_type	nnz	Structural number of non-zeros	
offset_type array	rowptr	Row pointer array (length: nrows+1	
index_type array	colindx	Column indices array (length: nnz)	
scalar_type array	values	Structural values (length: nnz)	
base_type	$index_base$	Base indexing	

Type	Name	Description
<pre>index_type index_type offset_type index_type array index_type array scalar_type array base_type</pre>	nrows ncols nnz rowindx colindx values index_base	Number of rows of the matrix Number of columns of the matrix Structural number of non-zeros Row index array (length: nnz) Column index array (length: nnz) Structural values (length: nnz) Base indexing

Table 2: CSR input parameters

Table 4: COO input parameters

Which operations?

Operation	Notation
Scaling	$A := \alpha A$
Transpose and Conjugate Transpose	$A := A^T, A := A^H$
Infinity Matrix Norm	$lpha := \ A\ _{\mathrm{inf}}$
Frobenius Matrix Norm	$\alpha := \ A\ _F$
Element-wise Multiplication	$C := A \cdot * B$
Sparse Matrix – Sparse Matrix Addition	C := A + B
Sparse Matrix – Sparse Matrix Multiplication	$C := A \cdot B + D$
Sparse Matrix – Dense Matrix Multiplication	$Y := A \cdot X + Y$
Sparse Matrix – Dense Vector Multiplication	$y = A \cdot x$
Triangular Solve	Solve $A \cdot x = y$ for x
Sparse Matrix Format Conversion	$B = \operatorname{sparse}(A)$
Predicate Selection	B = A(predicate)
Sampled dense dense matrix multiplication	$C\langle \text{mask} \rangle = A \cdot B$

Table 1: List of the Sparse BLAS functionalities. Uppercase letters represent matrices; lowercase represent vectors. Letters from the start of the alphabet are used for sparse matrices/vectors; letters from the end are used for dense matrices/vectors. α is a scalar value. Note that scaling may be applied to any input matrix/vector, and the transpose/conjugate transpose operation may be applied to any input matrix.

What if an operation needs temporary buffers?

- An operation takes a state object that can contain temporary buffers
- Some applications want to control all GPU memory with their allocator (Matlab, etc.)
- We need the possibility to pass this allocator to the library
- If no allocator is passed to the library, the standard allocator is used also in the state object

```
using namespace sparseblas; csr_view<float> A(values, rowptr, colind, shape, nnz); // scale() overwrites the values of A scale_state_t state(allocator); scale(policy, state, 2.3, A); Listing 1: Scaling, A := \alpha A
```

What is the execution policy?

- Handles the execution
 (like a ROCm/CUDA stream, sycl queue)
- Sequential / parallel
- Synchronous / asynchronous

Preserving explicit zeros?

Storing zeros explicitly can have advantages:

- Block-sparse matrix: sparse matrix with dense blocks (excluded for now?)
- Matrix entries change over the application time, zeros become nonzero
- Better performance through padding (cache line, coalesced access...)
- What if we convert from one format to another? Do we preserve the explicit zeros?
 - Current discussion: yes, conversion preserves zeros, except for conversion from dense.

How about NaN propagation?

$$\begin{pmatrix} 0 & * & \dots & * \\ 0 & * & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & * \end{pmatrix} \quad \begin{pmatrix} NaN \\ * \\ \vdots \\ * \end{pmatrix} \quad =?$$

- Checking right-hand-side for NaN/Inf by default is too expensive
- This leads to inconsistency:

a format storing an explicit zero will get a different result than a format storing an implicit zero

- Plan for different execution modes:
 - Permissive (as fast as possible)
 - Consistent (check for NaN/Inf)
 - Reproducible (bitwise reproducibility)

- 1. inspect phase (optional) prepare any potential optimizations for subsequent phases, or may do nothing
- 2. compute phase computing the size of the sparse output structure, which typically requires significant work
- 3. allocation phase allocating the memory for the sparse output data structure and placing it in the output matrix object to be filled
- 4. fill phase complete execution of the operation and filling the output structure with the result.

```
using namespace sparseblas; csr_view<float> A(values, rowptr, colind, shape, nnz); 

// scale() overwrites the values of A 
scale_state_t state(allocator); 
scale(policy, state, 2.3, A); 

Listing 1: Scaling, A := \alpha A
```

```
using namespace sparseblas;
csr_view<float> A(values, rowptr, colind, shape, nnz);
auto X = std::mdspan(raw_x.data(), k, n);
auto Y = std::mdspan(raw_y.data(), m, n);

float alpha = 1.1, beta = 1.2;

multiply_state_t state(allocator);
multiply_inspect(policy, state, scaled(alpha, A), X, beta, Y)); // optional
multiply_compute(policy, bind_info{scaled(alpha, A), info}, X, beta, Y);
multiply_fill(policy, bind_info{scaled(alpha, A), info}, X, beta, Y);
```

Listing 9: Sparse Matrix – Dense Matrix Multiplication, $Y = \alpha A \cdot X + \beta Y$

```
using namespace sparseblas;
csr_view<float> A_view(values, rowptr, colind, shape, nnz);
// create a handle to hold some internal stuff more than view
matrix_handle A(A_view, allocator);
// ... likewise for B, D
csr_view<float> C(m, n);
multiply_state_t state(allocator);
multiply_inspect(policy, state, transposed(A),
                          B, C, D); // optional
// it will store the transposed A in the matrix_handle
multiply_symbolic_compute(policy, state, transposed(A),
                          B, C, D);
index_t nnz = state.get_result_nnz();
// allocate C rowptr/colind arrays and put in C
multiply_symbolic_fill(policy, state, scaled(alpha, transposed(A))
                       B, C, scaled(beta, D));
// C structure is now able to be used
multiply_numeric_compute(policy, state, transposed(A),
                                                                        Can be combined
                         B, C, D);
index_t nnz = state.get_result_nnz();
// allocate C values arrays and put in C (Note: this can also be done previously
// since nnz is already known from symbolic stage)
multiply_numeric_fill(policy, state, scaled(alpha, transposed(A)),
                      B, C, scaled(beta, D));
// C structure and values are now able to be used
Listing 8: Sparse Matrix – Sparse Matrix Multiplication with matrix_handle,
```

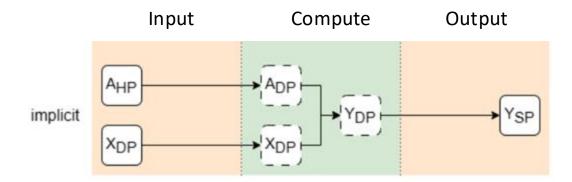
 $C = \alpha \cdot A^T \cdot B + \beta D$ with separate symbolic and numeric stages

How about out-of-place submatrices?

```
using namespace sparseblas;
csr_view<float> A_view(values, rowptr, colind, shape, nnz);
csr_view<float> B(values, rowptr, colind, shape, nnz);
auto x = std::mdspan(raw_x.data(), n);
auto b = std::mdspan(raw_b.data(), m);
auto pred = [](auto i, auto j, auto v) {
              return v > 0;
                                     Extract submatrix of non-negative values
            };
auto pred = [](auto i, auto j, auto v) {
              return (i < 10 ) && (j < 10);
            };
                                        Extract first left-top 10x10 submatrix
filter_state_t state(allocator);
// matrix_handle is opaque and may contain vendor optimization details
matrix_handle A(A_view, allocator);
filter_compute(policy, state, A, B, pred);
// allocate arrays for B
filter_fill(policy, state, A, B, pred);
```

Which precisions are relevant?

- Fp64, fp32, fp16, bf16 (+ complex variants)
- Mixed Precision (different inputs / outputs)
- Arithmetic precision is always the highest input/output precision (no performance loss)



Large number of routine variants -> C++ overloading

C++ may be doable, but what about C and Fortran?

- Combinatorial complexity make a Fortran/C interface infeasible.
- Maybe an interface for the most relevant combinations?
- Vendors prefer using internally a C++ interface for their sparse routine interfaces.

Aren't the vendors already doing it anyway?

• Yes... and they are all going in similar directions – it is only a question of moderating the process...

hipSPARSE ROCm/CUDA C API

Anyway, what is a standard?

- A standard is a standard because a significant portion of the community follows the same convention;
- We discuss how we do things in the different NLA libraries, and what the customers want;
- We try to find an interface that combines the good ideas;
- Ideally, we have vertical compatibility: efforts can adopt the interface we design w/o significant changes;
- We hope for horizontal compatibility (Fortran, C) but acknowledge this will be limited;





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Next step: present the interface design at a BoF at SC and request feedback from the broader community.

Working Toward an Interface for Sparse BLAS

Description: While sparse matrix computations are at the heart of many scientific and engineering applications, there exists no widely adopted interface standard. A reason for this may be the plethora of optimization options relevant to today's accelerator architectures. At the same time, many vendors already provide support for sparse matrix computations in proprietary libraries, but due to diverging architectural constraints, these libraries have different execution models, APIs, and formats supported. We started a cross-institutional effort involving academia and industry to define an API for sparse linear algebra operations. In the BoF, we present a blueprint and discuss considerations motivating design choices.

Event Type: Birds of a Feather

Add to Schedule

Time:

Thursday, 21 November 2024 12:15pm - 1:15pm EST

Location: B207

Registration Categories:

TP XO/EX

Links:

Website