COM 215: ELECTRIC CIRCUITS 3units

PURPOSE
To enable the learner to analyze circuits using different analysis methods

LEARNING OUTCOMES
The learners should be able to:-
1. Define basic circuit elements and apply Kirchoff's laws in problem solving
2. Determine electric field pattern for different arrangements of charges and describe the relationship between capacitance of a capacitor, charge on it and potential
3. Describe the circuit concepts
4. Describe the circuit laws
5. Derive and apply the analysis methods to circuits

COURSE CONTENT
Introduction
Electrical quantities, Force, work and power, Electric charge and current, Electric potential, Energy and electrical power, Constant and variable functions.
Circuit concepts
Passive and active elements, Sign conventions, Voltage current relations, Resistance, Inductance, Capacitance, Circuit diagrams, Non-linear resistors, voltage sources.
Circuit laws
Introduction, Kirchoff's voltage law, Kirchoff's current law, Circuit elements in series, Circuit elements in parallel, Voltage division, Current division
Analysis methods
The branch current method, The mesh current method, Matrices and determinants, The node voltage method, Input and output resistance, Transfer resistance, Network reductions, Superposition, Thevenin's and Norton's theorem

MODE OF DELIVERY
Discussion, Lecture, Demonstration

MATERIAL AND EQUIPMENT
Inductors, capacitors, resistors, Cathode ray oscilloscope, diodes, transformers, ammeters, voltmeters,

COURSE ASSESSMENT

Continuous assessment Tests	30%
• Class Tests	20%
• Assignments & Practicals	10%
End of semester examination	70%
Total	**100%**

PEER REVIEW
Monitoring by the head of department and fellow lectures, evaluation forms completed by students.

CORE REFERENCES

1. Duffin W. J. (1973*), Electricity and Magnetism (2nd* edition), Maidenhead, McGraw-Hill Book company
2. Bleany B. I and Bleany B (1992), *Electricity and Magnetism* (3rd edition), Oxford. Oxford University press
3. Lorrain, P., and Corson, D.R (1970), *Electromagnetic fields,* New York. John Wiley and sons

1.0 INTRODUCTION

1.1 ELECTRICAL QUANTITIES AND SI UNITS

- The International System of Units (SI) will be used throughout this course. Four basic quantities and their SI units are listed in Table 1. The other three basic quantities and corresponding SI units, not shown in the table, are temperature in degrees kelvin (K), amount of substance in moles (mol) and luminous intensity in candelas (cd).
- All other units may be derived from the seven basic units.
- The electrical quantities and their symbols commonly used in electrical circuit analysis are listed in Table 1 below.

TABLE 1

Quantity	Symbol	SI Unit	Abbreviation
length	L, l	meter	m
mass	M, m	kilogram	kg
time	T, t	second	s
current	I, i	ampere	A

- Two supplementary quantities are plane angle (also called phase angle in electric circuit analysis) and solid angle. Their corresponding SI units are the radian (rad) and steradian (sr).
- Degrees are almost universally used for the phase angles in sinusoidal functions, for instance, sin(ωt + $30^{0)}$. Since it is in radians, this is a case of mixed units.
- The decimal multiples or submultiples of SI units should be used whenever possible. The symbols given in Table 3 are prefixed to the unit symbols of Tables 1 and 2. For example, mV is used for millivolt, 10^{-3} V, and MW for megawatt, 10^{6} W.

1.2 FORCE, WORK, AND POWER

- The derived units follow the mathematical expressions which relate the quantities. From ‘‘force equals mass times acceleration,’’ the newton (N) is defined as the unbalanced force that imparts an acceleration of 1 meter per second squared to a 1-kilogram mass. Thus, 1N = 1 kg.m/s^2
- Work results when a force acts over a distance. A joule of work is equivalent to a newton-meter: 1 J = 1N. m. Work and energy have the same units.
- Power is the rate at which work is done or the rate at which energy is changed from one form to another. The unit of power, the watt (W), is one joule per second (J/s).

TABLE 2

Quantity	Symbol	SI Unit	Abbreviation
electric charge	Q, q	coulomb	C
electric potential	V, v	volt	V
resistance	R	ohm	Ω
conductance	G	siemens	S
inductance	L	henry	H
capacitance	C	farad	F
frequency	f	hertz	Hz
force	F, f	newton	N
energy, work	W, w	joule	J
power	P, p	watt	W
magnetic flux	ϕ	weber	Wb
magnetic flux density	$\mathbf{B}$	tesla	T

Example 1.1.

In simple rectilinear motion a 10-kg mass is given a constant acceleration of 2.0 m/s^2.

(a) Find the acting force F. (b) If the body was at rest at t = 0, x = 0, find the position, kinetic energy, and power for t = 4 s.

Solution

(a) $$F = ma = (10\,\text{kg})(2.0\,\text{m/s}^2) = 20.0\,\text{kg}\cdot\text{m/s}^2 = 20.0\,\text{N}$$

(b) At $t = 4\,\text{s}$,
$$x = \tfrac{1}{2}at^2 = \tfrac{1}{2}(2.0\,\text{m/s}^2)(4\,\text{s})^2 = 16.0\,\text{m}$$
$$\text{KE} = Fx = (20.0\,\text{N})(16.0\,\text{m}) = 3200\,\text{N}\cdot\text{m} = 3.2\,\text{kJ}$$
$$P = \text{KE}/t = 3.2\,\text{kJ}/4\,\text{s} = 0.8\,\text{kJ/s} = 0.8\,\text{kW}$$

1.3 ELECTRIC CHARGE AND CURRENT

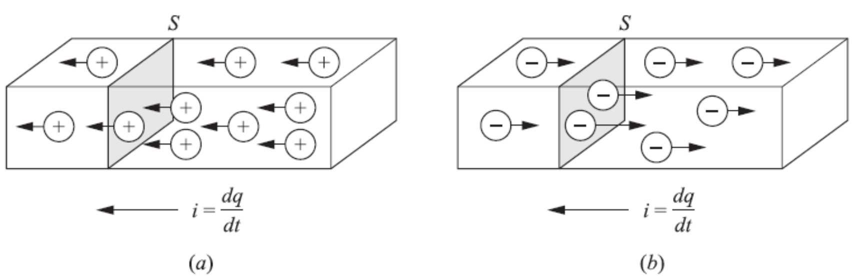
- The unit of current, the ampere (A), is defined as the constant current in two parallel conductors of infinite length and negligible cross section, 1 meter apart in vacuum, which produces a force between the conductors of 2 x 10^{-7} newtons per meter length.
- A more useful concept, however, is that current results from charges in motion, and 1 ampere is equivalent to 1 coulomb of charge moving across a fixed surface in 1 second. Thus, in time-variable functions, $i(A) = dq/dt(C/s)$

  The derived unit of charge, the coulomb (C), is equivalent to an ampere-second.
- The moving charges may be positive or negative. Positive ions, moving to the left in a liquid or plasma suggested in Fig. 1(a), produce a current *i*, also directed to the left. If these ions cross the plane surface S at the rate of one coulomb per second, then the resulting current is 1 ampere. Negative ions moving to the right as shown in Fig. 1(b) also produce a current directed to the left.

TABLE 3

Prefix	Factor	Symbol
pico	10^{-12}	p
nano	10^{-9}	n
micro	10^{-6}	μ
milli	10^{-3}	m
centi	10^{-2}	c
deci	10^{-1}	d
kilo	10^{3}	k
mega	10^{6}	M
giga	10^{9}	G
tera	10^{12}	T

Fig 1



- Of more importance in electric circuit analysis is the current in metallic conductors which takes place through the motion of electrons that occupy the outermost shell of the atomic structure. In copper, for example, one electron in the outermost shell is only loosely bound to the central nucleus and moves freely from one atom to the next in the crystal structure. At normal temperatures there is constant, random motion of these electrons. A reasonably accurate picture of conduction in a copper conductor is that approximately 8.5×10^{28} conduction electrons per cubic meter are free to move. The electron charge is $-e = -1.602 \times 10^{-19}$ C, so that for a current of one ampere approximately 6.24×10^{18} electrons per second would have to pass a fixed cross section of the conductor.

EXAMPLE 1.2. A conductor has a constant current of five amperes. How many electrons pass a fixed point on the conductor in one minute?

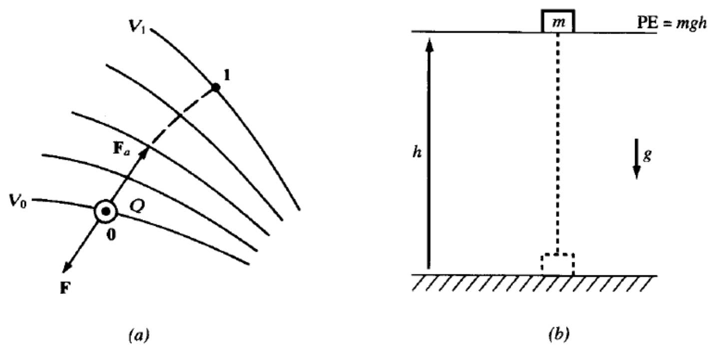
$$5\,\mathrm{A} = (5\,\mathrm{C/s})(60\,\mathrm{s/min}) = 300\,\mathrm{C/min}$$

$$\frac{300\,\mathrm{C/min}}{1.602 \times 10^{-19}\,\mathrm{C/electron}} = 1.87 \times 10^{21}\,\mathrm{electrons/min}$$

1.4 ELECTRIC POTENTIAL

- An electric charge experiences a force in an electric field which, if unopposed, will accelerate the particle containing the charge. Of interest here is the work done to move the charge against the field as suggested in Fig. 2(a).
- Thus, if 1 joule of work is required to move the charge Q, 1 coulomb from position 0 to position 1, then position 1 is at a potential of 1 volt with respect to position 0; 1V = 1 J /C. W = QV
- This electric potential is capable of doing work just as the mass in Fig. 2(b), which was raised against the gravitational force g to a height h above the ground plane. The potential energy *mgh* represents an ability to do work when the mass m is released. As the mass falls, it accelerates and this potential energy is converted to kinetic energy.

Fig 2



Example 1.3. In an electric circuit an energy of 9.25 µJ is required to transport 0.5 µC from point a to point b. What electric potential difference exists between the two points?

$$1 \text{ volt} = 1 \text{ joule per coulomb} \qquad V = \frac{9.25 \times 10^{-6}\,\mathrm{J}}{0.5 \times 10^{-6}\,\mathrm{C}} = 18.5\,\mathrm{V}$$

1.5 ENERGY AND ELECTRICAL POWER

- The rate, in joules per second, at which energy is transferred, is electric power in watts. Furthermore, the product of voltage and current yields the electric power, p = vi; 1W = 1V. 1 A. Also, V.A = (J/C).(C/s) = J/s = W
- In a more fundamental sense power is the time derivative p = dw/ dt, so that instantaneous power p is generally a function of time.

EXAMPLE 1.4. A resistor has a potential difference of 50.0V across its terminals and 120.0C of charge per minute passes a fixed point. Under these conditions at what rate is electric energy converted to heat?

$$(120.0\,\mathrm{C/min})/(60\,\mathrm{s/min}) = 2.0\,\mathrm{A} \qquad P = (2.0\,\mathrm{A})(50.0\,\mathrm{V}) = 100.0\,\mathrm{W}$$

Since $1\,\mathrm{W} = 1\,\mathrm{J/s}$, the rate of energy conversion is one hundred joules per second.

1.6 CONSTANT AND VARIABLE FUNCTIONS

To distinguish between constant and time-varying quantities, capital letters are employed for the constant quantity and lowercase for the variable quantity. For example, a constant current of 10 amperes is written I = 10.0 A, while a 10-ampere time-variable current is written i = 10.0 f (t) A. Examples of common functions in circuit analysis are the sinusoidal function i = 10.0 sin ωt(A) and the exponential function v = 15.0 e^{-at} (V).

Solved Problems

1. The force applied to an object moving in the x direction varies according to F = 12/x^2 (N).
   a) Find the work done in the interval 1m ≤ x ≤ 3m.
   b) What constant force acting over the same interval would result in the same work?

(*a*) $$dW = F\,dx \qquad \text{so} \qquad W = \int_1^3 \frac{12}{x^2}\,dx = 12\left[\frac{-1}{x}\right]_1^3 = 8\,\mathrm{J}$$

(*b*) $$8\,\mathrm{J} = F_c(2\,\mathrm{m}) \qquad \text{or} \qquad F_c = 4\,\mathrm{N}$$

2. Electrical energy is converted to heat at the rate of 7.56kJ/min in a resistor which has 270 C/min passing through. What is the voltage difference across the resistor terminals?

From $P = VI$,

$$V = \frac{P}{I} = \frac{7.56 \times 10^3\,\mathrm{J/min}}{270\,\mathrm{C/min}} = 28\,\mathrm{J/C} = 28\,\mathrm{V}$$

3. A certain circuit element has a current i = 2.5 sin ωt (mA), where ω is the angular frequency in rad/s, and a voltage difference v = 45 sin ωt (V) between terminals. Find the average power P_{avg} and the energy W_T transferred in one period of the sine function.

Energy is the time-integral of instantaneous power:

$$W_T = \int_0^{2\pi/\omega} vi\,dt = 112.5 \int_0^{2\pi/\omega} \sin^2 \omega t\,dt = \frac{112.5\pi}{\omega}\ (\mathrm{mJ})$$

The average power is then

$$P_{avg} = \frac{W_T}{2\pi/\omega} = 56.25\,\mathrm{mW}$$

Note that P_{avg} is independent of ω.

4. The unit of energy commonly used by electric utility companies is the kilowatt-hour (kWh).
   a) How many joules are in 1kWh?
   b) A color television set rated at 75W is operated from 7:00 p.m. to 11:30 p.m. What total energy does this represent in kilowatt-hours and in megajoules?

(*a*) $1\,\text{kWh} = (1000\,\text{J/s})(3600\,\text{s/h}) = 3.6\,\text{MJ}$

(*b*) $(75.0\,\text{W})(4.5\,\text{h}) = 337.5\,\text{Wh} = 0.3375\ \text{kWh}$
$(0.3375\,\text{kWh})(3.6\,\text{MJ/kWh}) = 1.215\,\text{MJ}$

5. An AWG #12 copper wire, a size in common use in residential wiring, contains approximately 2.77×10^{23} free electrons per meter length, assuming one free conduction electron per atom. What percentage of these electrons will pass a fixed cross section if the conductor carries a constant current of 25.0 A?

$$\frac{25.0\,\text{C/s}}{1.602 \times 10^{-19}\ \text{C/electron}} = 1.56 \times 10^{20}\ \text{electron/s}$$

$$(1.56 \times 10^{20}\ \text{electrons/s})(60\,\text{s/min}) = 9.36 \times 10^{21}\ \text{electrons/min}$$

$$\frac{9.36 \times 10^{21}}{2.77 \times 10^{23}}(100) = 3.38\%$$

6. How many electrons pass a fixed point in a 100-watt light bulb in 1 hour if the applied constant voltage is 120 V?

$$100\,\text{W} = (120\,\text{V}) \times I(\text{A}) \qquad I = 5/6\,\text{A}$$

$$\frac{(5/6\,\text{C/s})(3600\,\text{s/h})}{1.602 \times 10^{-19}\ \text{C/electron}} = 1.87 \times 10^{22}\ \text{electrons per hour}$$

7. A typical 12 V auto battery is rated according to ampere-hours. A 70-A. h battery, for example, at a discharge rate of 3.5 A has a life of 20 h. (a) Assuming the voltage remains constant, obtain the energy and power delivered in a complete discharge of the preceding battery. (b) Repeat for a discharge rate of 7.0 A.

(*a*) $(3.5\,\text{A})(12\,\text{V}) = 42.0\,\text{W (or J/s)}$
$(42.0\,\text{J/s})(3600\,\text{s/h})(20\,\text{h}) = 3.02\,\text{MJ}$

(*b*) $(7.0\,\text{A})(12\,\text{V}) = 84.0\,\text{W}$
$(84.0\,\text{J/s})(3600\,\text{s/h})(10\,\text{h}) = 3.02\,\text{MJ}$

The ampere-hour rating is a measure of the energy the battery stores; consequently, the energy transferred for total discharge is the same whether it is transferred in 10 hours or 20 hours. Since power is the rate of energy transfer, the power for a 10-hour discharge is twice that in a 20-hour discharge.

Exercises

1. Obtain the work and power associated with a force of 7.5×10^{-4} N acting over a distance of 2 meters in an elapsed time of 14 seconds. Ans. 1.5 mJ, 0.107mW
2. Obtain the work and power required to move a 5.0-kg mass up a frictionless plane inclined at an angle of 30^0 with the horizontal for a distance of 2.0m along the plane in a time of 3.5 s. Ans. 49.0 J, 14.0W
3. Work equal to 136.0 joules is expended in moving 8.5×10^{18} electrons between two points in an electric circuit. What potential difference does this establish between the two points? Ans. 100V
4. A pulse of electricity measures 305 V, 0.15 A, and lasts 500 ms. What power and energy does this represent? Ans. 45.75 W, 22.9 mJ
5. A unit of power used for electric motors is the horsepower (hp), equal to 746 watts. How much energy does a 5-hp motor deliver in 2 hours? Express the answer in MJ. Ans. 26.9MJ
6. The capacitance of a circuit element is defined as Q / V, where Q is the magnitude of charge stored in the element and V is the magnitude of the voltage difference across the element. The SI derived unit of capacitance is the farad (F). Express the farad in terms of the basic units.
Ans. $1\text{F} = 1\text{A}^2.\text{s}^4/\text{kg} \,.\, \text{m}^2$

2. CIRCUIT CONCEPTS

2.1 PASSIVE AND ACTIVE ELEMENTS

- An electrical device is represented by a circuit diagram or network constructed from series and parallel arrangements of two-terminal elements.
- The analysis of the circuit diagram predicts the performance of the actual device. A two-terminal element in general form is shown in Fig. 2-1, with a single device represented by the rectangular symbol and two perfectly conducting leads ending at connecting points A and B.

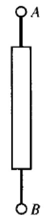


Fig 2.1

- Active elements are voltage or current sources which are able to supply energy to the network. Resistors, inductors, and capacitors are passive elements which take energy from the sources and either convert it to another form or store it in an electric or magnetic field.
- Figure 2.2 illustrates seven basic circuit elements. Elements (a) and (b) are voltage sources and (c) and (d) are current sources.
- A voltage source that is not affected by changes in the connected circuit is an independent source, illustrated by the circle in Fig. 2.2(a).
- A dependent voltage source which changes in some described manner with the conditions on the connected circuit is shown by the diamond-shaped symbol in Fig. 2.2(b).
- Current sources may also be either independent or dependent and the corresponding symbols are shown in (c) and (d). The three passive circuit elements are shown in Fig. 2.2(e), (f), and (g).

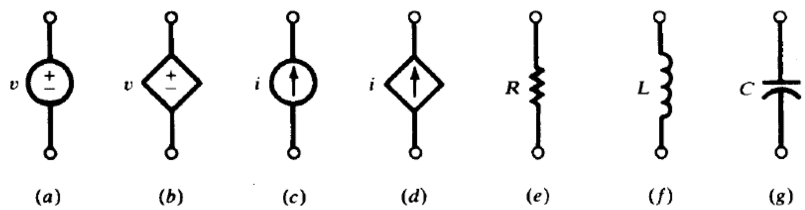


Fig 2.2

- The circuit diagrams presented here are termed lumped-parameter circuits, since a single element in one location is used to represent a distributed resistance, inductance, or capacitance. For example, a coil consisting of a large number of turns of insulated wire has resistance throughout the entire length of the wire. Nevertheless, a single resistance lumped at one place as in Fig. 2.3(b) or (c) represents the distributed resistance. The inductance is likewise lumped at one place, either in series with the resistance as in (b) or in parallel as in (c).

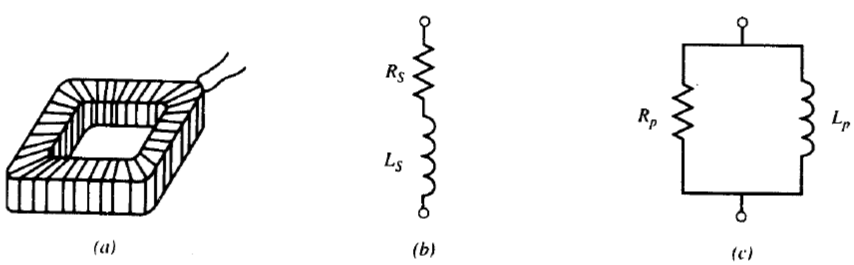


Fig 2.3

- In general, a coil can be represented by either a series or a parallel arrangement of circuit elements.
- The frequency of the applied voltage may require that one or the other be used to represent the device.

2.2 SIGN CONVENTIONS

- A voltage function and a polarity must be specified to completely describe a voltage source. The polarity marks, + and - , are placed near the conductors of the symbol that identifies the voltage source.
- If, for example, $v = 10.0 \sin \omega t$ in Fig. 2.4(a), terminal A is positive with respect to B for $0 > \omega t > \pi$ and B is positive with respect to A for $\pi > \omega t > 2\pi$ for the first cycle of the sine function.

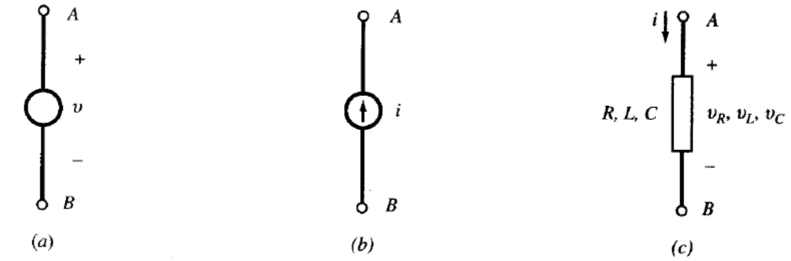


Fig. 2.4

- Similarly, a current source requires that a direction be indicated, as well as the function, as shown in Fig. 2.4(b). For passive circuit elements R, L, and C, shown in Fig. 2.4(c), the terminal where the current enters is generally treated as positive with respect to the terminal where the current leaves.
- The sign on power is illustrated by the dc circuit of Fig. 2.5(a) with constant voltage sources $V_A = 20.0V$ and $V_B = 5.0V$ and a single 5 Ω resistor. The resulting current of 3.0A is in the clockwise direction.
- Considering now Fig. 2.5(b), power is absorbed by an element when the current enters the element at the positive terminal. Power, computed by VI or I^2R, is therefore absorbed by both the resistor and the V_B source, 45.0W and 15W respectively. Since the current enters V_A at the negative terminal, this element is the power source for the circuit. P = VI = 60.0W confirms that the power absorbed by the resistor and the source V_B is provided by the source V_A.

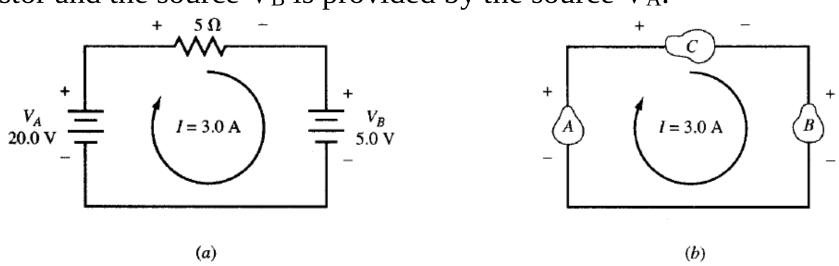


Fig. 2.5

2.3 VOLTAGE-CURRENT RELATIONS

- The passive circuit elements resistance R, inductance L, and capacitance C are defined by the manner in which the voltage and current are related for the individual element. For example, if the voltage v and current i for a single element are related by a constant, then the element is a resistance, R is the constant of proportionality, and v = Ri.
- Similarly, if the voltage is the time derivative of the current, then the element is an inductance, L is the constant of proportionality, and v = L di/dt.
- Finally, if the current in the element is the time derivative of the voltage, then the element is a capacitance, C is the constant of proportionality, and i = C dv/dt. Table 2.1 summarizes these

relationships for the three passive circuit elements. Note the current directions and the corresponding polarity of the voltages.

TABLE 2.1

Circuit element	Units	Voltage	Current	Power
Resistance (i ↓, $+$ v $-$)	ohms (Ω)	$v = Ri$ (Ohms's law)	$i = \frac{v}{R}$	$p = vi = i^2R$
Inductance (i ↓, $+$ v $-$)	henries (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + k_1$	$p = vi = Li\frac{di}{dt}$
Capacitance (i ↓, $+$ v $-$)	farads (F)	$v = \frac{1}{C}\int i\,dt + k_2$	$i = C\frac{dv}{dt}$	$p = vi = Cv\frac{dv}{dt}$

2.4 RESISTANCE

- All electrical devices that consume energy must have a resistor (also called a resistance) in their circuit model. Inductors and capacitors may store energy but over time return that energy to the source or to another circuit element.
- Power in the resistor, given by p = vi = i^2R = v^2/R, is always positive as illustrated in Example 2.1 below. Energy is then determined as the integral of the instantaneous power

$$w = \int_{t_1}^{t_2} p\,dt = R\int_{t_1}^{t_2} i^2\,dt = \frac{1}{R}\int_{t_1}^{t_2} v^2\,dt$$

Example 2.1.

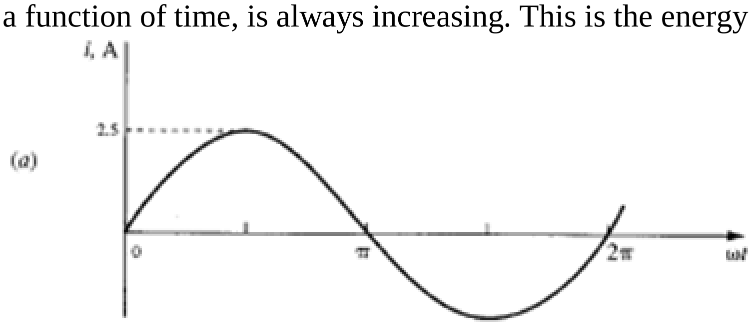
A 4.0Ω resistor has a current *i* = 2.5 sin ωt (A). Find the voltage, power, and energy over one Cycle, ω = 500 rad/s.

$$v = Ri = 10.0\sin\omega t \text{ (V)}$$

$$p = vi = i^2R = 25.0\sin^2\omega t \text{ (W)}$$

$$w = \int_0^t p\,dt = 25.0\left[\frac{t}{2} - \frac{\sin 2\omega t}{4\omega}\right] \text{ (J)}$$

The plots of *i*, *p*, and *w* shown in Fig. 2.6 illustrate that p is always positive and that the energy w, although a function of time, is always increasing. This is the energy absorbed by the resistor.



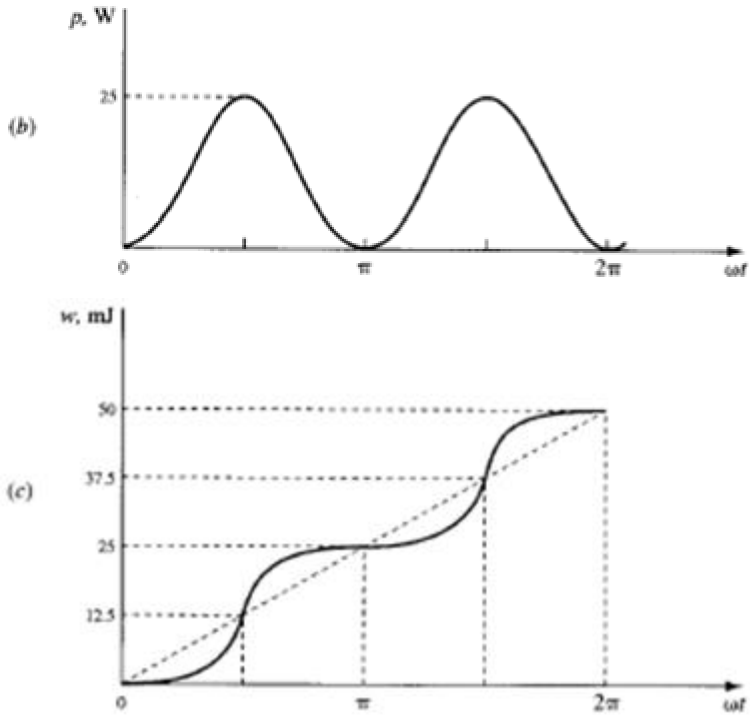


Fig 2.6

2.5 INDUCTANCE

- The circuit element that stores energy in a magnetic field is an inductor (also called an inductance). With time-variable current, the energy is generally stored during some parts of the cycle and then returned to the source during others. When the inductance is removed from the source, the magnetic field will collapse; in other words, no energy is stored without a connected source. Coils found in electric motors, transformers, and similar devices can be expected to have inductances in their circuit models.
- Even a set of parallel conductors exhibits inductance that must be considered at most frequencies. The power and energy relationships are as follows.

$$p = vi = L\frac{di}{dt}i = \frac{d}{dt}\left[\frac{1}{2}Li^2\right]$$

$$w_L = \int_{t_1}^{t_2} p\,dt = \int_{t_1}^{t_2} Li\,dt = \frac{1}{2}L[i_2^2 - i_1^2]$$

Energy stored in the magnetic field of an inductance is

$w_L = \frac{1}{2}Li^2.$

Example 2.2.

In the interval 0 > t > (π/50) s a 30-mH inductance has a current *i = 10.0 sin 50t (A).* Obtain the voltage, power, and energy for the inductance.

$$v = L\frac{di}{dt} = 15.0\cos 50t\ (\mathrm{V}) \qquad p = vi = 75.0\sin 100t\ (\mathrm{W}) \qquad w_L = \int_0^t p\,dt = 0.75(1-\cos 100t)\ (\mathrm{J})$$

As shown in Fig. 2.7, the energy is zero at t = 0 and t = (π/50)s. Thus, while energy transfer did occur over the interval, this energy was first stored and later returned to the source.

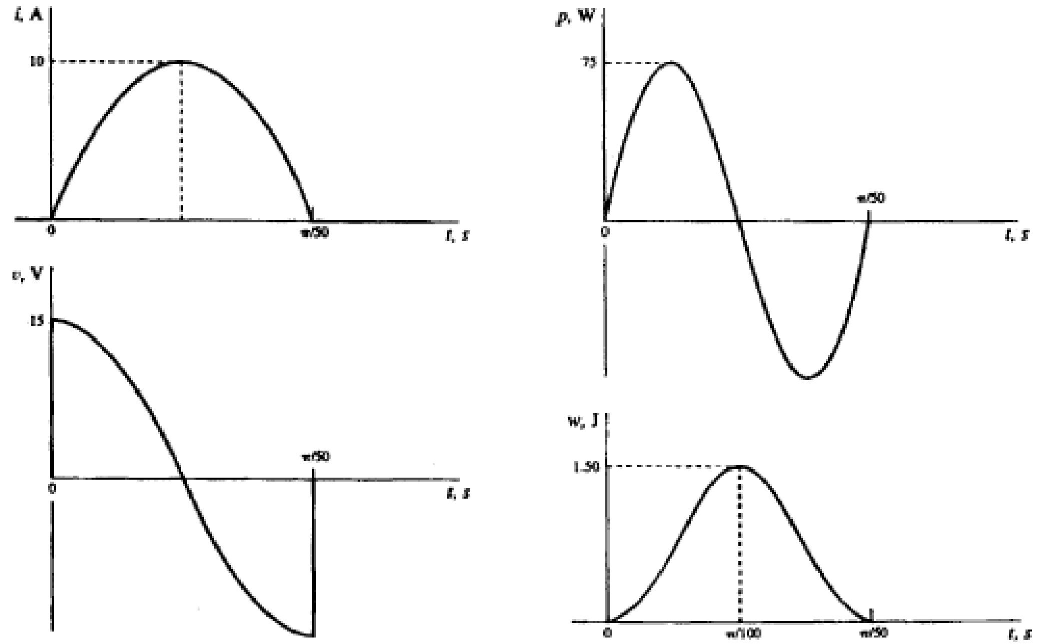


Fig. 2.7

2.6 CAPACITANCE

- The circuit element that stores energy in an electric field is a capacitor (also called capacitance). When the voltage is variable over a cycle, energy will be stored during one part of the cycle and returned in the next. While an inductance cannot retain energy after removal of the source because the magnetic field collapses, the capacitor retains the charge and the electric field can remain after the source is removed. This charged condition can remain until a discharge path is provided, at which time the energy is released. The charge, q = C*v*, on a capacitor results in an electric field in the dielectric which is the mechanism of the energy storage. In the simple parallel-plate capacitor there is an excess of charge on one plate and a deficiency on the other. It is the equalization of these charges that takes place when the capacitor is discharged. The power and energy relationships for the capacitance are as follows.

$$p = vi = Cv\frac{dv}{dt} = \frac{d}{dt}\left[\frac{1}{2}Cv^2\right]$$

$$w_C = \int_{t_1}^{t_2} p\,dt = \int_{t_1}^{t_2} Cv\,dv = \frac{1}{2}C[v_2^2 - v_1^2]$$

The energy stored in the electric field of capacitance is $w_c = 1/2\ Cv^2$.

Example 2.3. In the interval 0 > t > 5π ms, a 20-μF capacitance has a voltage v = 50.0 sin 200t (V). Obtain the charge, power, and energy. Plot w_C assuming w = 0 at t = 0.

$$q = Cv = 1000 \sin 200t\ (\mu C)$$

$$i = C\frac{dv}{dt} = 0.20 \cos 200t\ (A)$$

$$p = vi = 5.0 \sin 400t\ (W)$$

$$w_C = \int_{t_1}^{t_2} p\,dt = 12.5[1 - \cos 400t]\ (mJ)$$

In the interval 0 > t > 2.5 π ms the voltage and charge increase from zero to 50.0V and 1000 μC, respectively.

Figure 2.8 shows that the stored energy increases to a value of 25 mJ, after which it returns to zero as the energy is returned to the source.

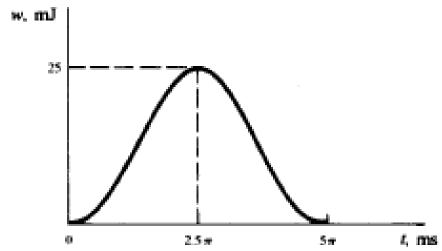


Figure 2.8

2.7 CIRCUIT DIAGRAMS

- Every circuit diagram can be constructed in a variety of ways which may look different but are in fact identical. The diagram presented in a problem may not suggest the best of several methods of solution. Consequently, a diagram should be examined before a solution is started and redrawn if necessary to show more clearly how the elements are interconnected. An extreme example is illustrated in Fig. 2.9, where the three circuits are actually identical.
- In Fig. 2.9(a) the three ‘‘junctions’’ labeled A are shown as two ‘‘junctions’’ in (b). However, resistor R_4 is bypassed by a short circuit and may be removed for purposes of analysis. Then, in Fig. 2.9(c) the single junction A is shown with its three meeting branches.

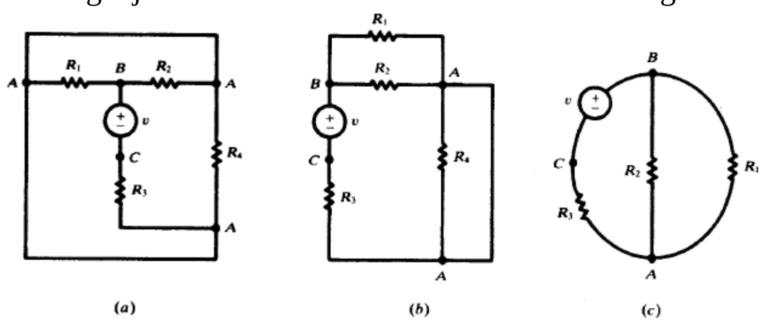
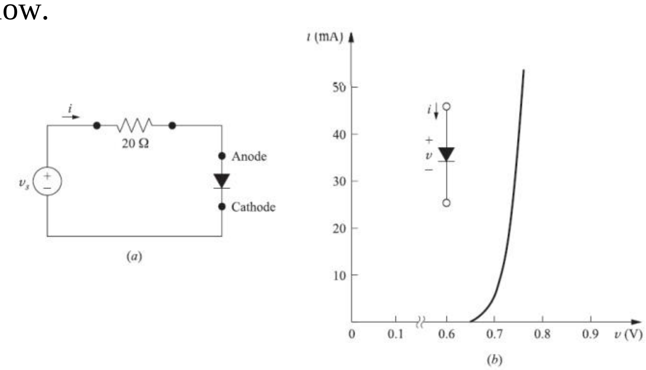


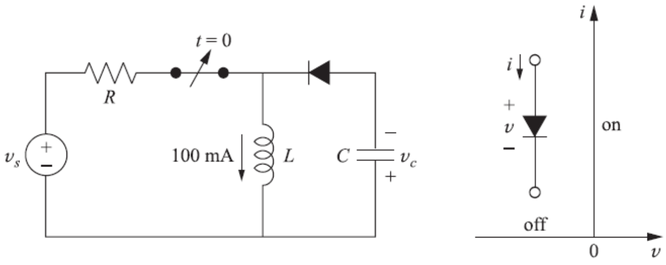
Fig. 2.9

2.8 NONLINEAR RESISTORS

- The current-voltage relationship in an element may be instantaneous but not necessarily linear. The element is then modeled as a nonlinear resistor. An example is a filament lamp which at higher voltages draws proportionally less current. Another important electrical device modeled as a nonlinear resistor is a diode. A diode is a two-terminal device that, roughly speaking, conducts electric current in one direction (from anode to cathode, called forward-biased) much better than the opposite direction (reverse-biased).
- The circuit symbol for the diode and an example of its current-voltage characteristic are shown in Fig. below.



- The arrow is from the anode to the cathode and indicates the forward direction ($i > 0$). A small positive voltage at the diode's terminal biases the diode in the forward direction and can produce a large current. A negative voltage biases the diode in the reverse direction and produces little current even at large voltage values. An ideal diode is a circuit model which works like a perfect switch.
- From Fig. below



(a) (b)

- Its (i, v) characteristic is

$$\begin{cases} v = 0 & \text{when } i \geq 0 \\ i = 0 & \text{when } v \leq 0 \end{cases}$$

- The static resistance of a nonlinear resistor operating at (I, V) is R = V/I .
- Its dynamic resistance is r = ΔV/ΔI which is the inverse of the slope of the current plotted versus voltage. Static and dynamic resistances both depend on the operating point.

Example 2.4. The current and voltage characteristic of a semiconductor diode in the forward direction is measured and recorded in the following table:

v (V)	0.5	0.6	0.65	0.66	0.67	0.68	0.69	0.70	0.71	0.72	0.73	0.74	0.75
i (mA)	2×10^{-4}	0.11	0.78	1.2	1.7	2.6	3.9	5.8	8.6	12.9	19.2	28.7	42.7

In the reverse direction (i.e., when v < 0), $i = 4 \times 10^{-15}$ A. Using the values given in the table, calculate the static and dynamic resistances (R and r) of the diode when it operates at 30 mA, and find its power consumption p.

$$R = \frac{V}{I} \approx \frac{0.74}{28.7 \times 10^{-3}} = 25.78\,\Omega$$
$$r = \frac{\Delta V}{\Delta I} \approx \frac{0.75 - 0.73}{(42.7 - 19.2) \times 10^{-3}} = 0.85\,\Omega$$
$$p = VI \approx 0.74 \times 28.7 \times 10^{-3}\text{W} = 21.238\,\text{mW}$$

Example 2.5. The current and voltage characteristic of a tungsten filament light bulb is measured and recorded in the following table. Voltages are DC steady-state values, applied for a long enough time for the lamp to reach thermal equilibrium.

v (V)	0.5	1	1.5	2	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8
i (mA)	4	6	8	9	11	12	13	14	15	16	17	18	18	19	20

Find the static and dynamic resistances of the filament and also the power consumption at the operating points

(a) i = 10 mA;

(b) (b) i = 15 mA.

$$R = \frac{V}{I}, \qquad r = \frac{\Delta V}{\Delta I}, \qquad p = VI$$

(*a*) $R \approx \frac{2.5}{10 \times 10^{-3}} = 250\,\Omega,\ r \approx \frac{3-2}{(11-9) \times 10^{-3}} = 500\,\Omega,\ p \approx 2.5 \times 10 \times 10^{-3}\,\text{W} = 25\,\text{mW}$

(*b*) $R \approx \frac{5}{15 \times 10^{-3}} = 333\,\Omega,\ r \approx \frac{5.5-4.5}{(16-14) \times 10^{-3}} = 500\,\Omega,\ p \approx 5 \times 15 \times 10^{-3}\,\text{W} = 75\,\text{mW}$

2.9 VOLTAGE SOURCES

- Any device that produces voltage output continuously is known as a *voltage source*. There are two types of voltage sources, namely; direct voltage source and alternating voltage source.

- **Direct voltage source.** A device which produces direct voltage output continuously is called *a direct voltage source*. Common examples are cells and d.c. generators. An important characteristic of a direct voltage source is that it maintains the same polarity of the output voltage *i.e.* positive and negative terminals remain the same.
- When load resistance R_L is connected across such a source, current flows from positive terminal to negative terminal *via* the load [See Fig. 2.20 (*i*)]. This is called *direct current* because it has just one direction. The current has one direction as the source maintains the same polarity of output voltage.
- The opposition to load current inside the d.c. source is known as *internal resistance* R_i. The equivalent circuit of a d.c. source is the generated *e.m.f.* E_g in series with internal resistance R_i of the source as shown in Fig. 2.20 (*ii*). Referring to Fig. 2.20 (*i*), it is clear that:

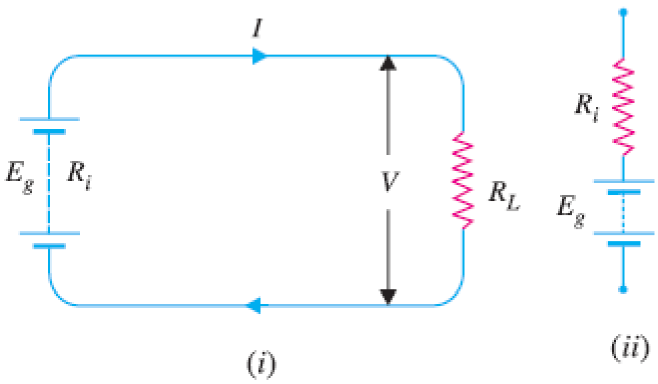


Fig 2.20

$$\text{Load current, } I = \frac{E_g}{R_L + R_i}$$

$$\text{Terminal voltage, } V = (E_g - I R_i) \quad \text{or} \quad I R_L$$

(*ii*) Alternating voltage source. A device which produces alternating voltage output continuously is known as *alternating voltage source e.g.* a.c. generator. An important characteristic of alternating voltage source is that it periodically reverses the polarity of the output voltage. When load impedance Z_L is connected across such a source, current flows through the circuit that periodically reverses in direction. This is called *alternating current.*

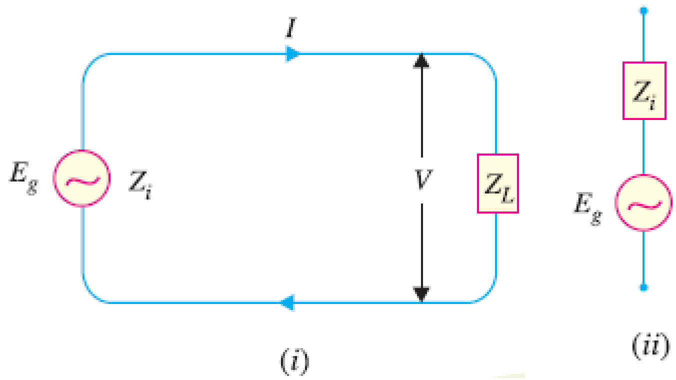


Fig. 2.21

The opposition to load current inside the a.c. source is called its *internal impedance* Z_i. The equivalent circuit of an a.c. source is the generated *e.m.f.* E_g (*r.m.s.*) in series with internal impedance Z_i of the source as shown in Fig. 2.21(*ii*). Referring to Fig. 2.21 (*i*), it is clear that:

$$\text{Load current, } I\,(r.m.s.) = \frac{E_g}{Z_L + Z_i}$$

$$\text{Terminal voltage, } V = (E_g - I Z_i)^{**} \quad \text{or} \quad I Z_L$$

1.9 Constant Voltage Source

A voltage source which has very low internal impedance as compared with external load impedance is known as a **constant voltage source.**

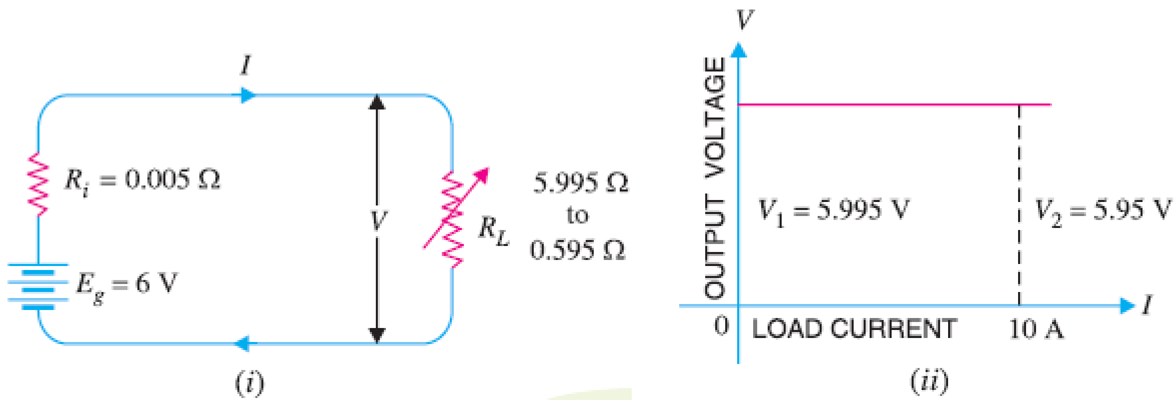


Fig 2.22

- In such a case, the output voltage nearly remains the same when load current changes.
- Fig. 2.22 (*i*) illustrates a constant voltage source. It is a d.c. source of 6 V with internal resistance R_i = 0.005 Ω. If the load current varies over a wide range of 1 to 10 A, for any of these values, the internal drop across R_i (= 0.005 Ω) is less than 0.05 volt. Therefore, the voltage output of the source is between 5.995 to 5.95 volts. This can be considered constant voltage compared with the wide variations in load current.
- Fig. 2.22 (*ii*) shows the graph for a constant voltage source. It may be seen that the output voltage remains constant in spite of the changes in load current. Thus as the load current changes from 0 to 10 A, the output voltage essentially remains the same (*i.e.* $V_1 = V_2$). A constant voltage source is represented as shown in Fig. 2.23.

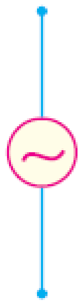


Fig. 2.23

Example 2.6 A lead acid battery fitted in a truck develops 24V and has an internal resistance of 0.01 Ω. It is used to supply current to head lights etc. If the total load is equal to 100 watts, find:

(i) Voltage drop in internal resistance

(ii) Terminal voltage

Solution.

Generated voltage, $E_g = 24$ V

Internal resistance, $R_i = 0.01\ \Omega$

Power supplied, $P = 100$ watts

(*i*) Let I be the load current.

Now $P = E_g \times I$ ($\because$ For an ideal source, $V \simeq E_g$)

$$\therefore \quad I = \frac{P}{E_g} = \frac{100}{24} = 4.17\text{ A}$$

$$\therefore \quad \text{Voltage drop in } R_i = I R_i = 4.17 \times 0.01 = \mathbf{0.0417\ V}$$

(*ii*) Terminal Voltage, $V = E_g - I R_i$

$= 24 - 0.0417 = \mathbf{23.96\ V}$

Comments: It is clear from the above example that when internal resistance of the source is quite small, the voltage drop in internal resistance is very low. Therefore, the terminal voltage substantially remains constant and the source behaves as a constant voltage source irrespective of load current variations.

1.10 Constant Current Source

A voltage source that has a very high internal impedance as compared with external load impedance is considered as a **constant current source.**

In such a case, the load current nearly remains the same when the output voltage changes.

Fig. 2.24 (*i*) illustrates a constant current source. It is a d.c. source of 1000 V with internal resistance R_i = 900 k Ω. Here, load R_L varies over 3 : 1 range from 50 k Ω to 150 k Ω. Over this variation of load R_L, the circuit current *I* is essentially constant at 1.05 to 0.95 mA or approximately 1 mA. It may be noted that output voltage *V* varies approximately in the same 3 : 1 range as R_L, although load current essentially remains constant at 1mA.

The beautiful example of a constant current source is found in vacuum tube circuits where the tube acts as a generator having internal resistance as high as 1 M Ω.

Fig. 2.24 (*ii*) shows the graph of a constant current source. It is clear that current remains constant even when the output voltage changes substantially. The following points may be noted regarding the constant current source:

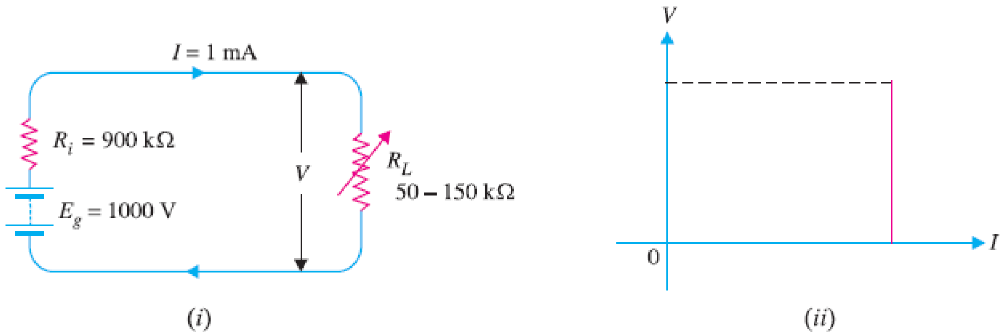


Fig. 2.24

(i) Due to high internal resistance of the source, the load current remains essentially constant as the load R_L is varied.

(ii) The output voltage varies approximately in the same range as R_L, although current remains constant.

(iii) The output voltage V is much less than the generated voltage E_g because of high $I R_i$ drop.

Fig. 2.25 shows the symbol of a constant current source.

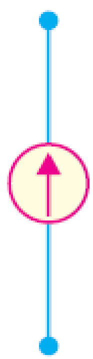


Fig. 2.25

Note:

* Resistance in case of a d.c. source

** Now $I = \dfrac{E_g}{R_L + R_i}$. Since $R_i >> R_L$, $I = \dfrac{E_g}{R_i}$

As both E_g and R_i are constants, I is constant.

Example 2.7. *A d.c. source generating 500 V has an internal resistance of 1000 Ω. Find the load current if load resistance is (i) 10 Ω (ii) 50 Ω and (iii) 100 Ω.*

Solution.

Generated voltage, $E_g = 500$ V

Internal resistance, $R_i = 1000\ \Omega$

(i) When $R_L = 10\ \Omega$

Load current, $I = \dfrac{E_g}{R_L + R_i} = \dfrac{500}{10 + 1000} = \mathbf{0.495\ A}$

(ii) When $R_L = 50\ \Omega$

Load current, $I = \dfrac{500}{50 + 1000} = \mathbf{0.476\ A}$

(iii) When $R_L = 100\ \Omega$

Load current, $I = \dfrac{500}{100 + 1000} = \mathbf{0.454\ A}$

It is clear from the above example that load current is essentially constant since $R_i >> R_L$.

1.11 Conversion of Voltage Source into Current Source

Fig. 2.26 shows a constant voltage source with voltage V and internal resistance R_i. *Fig. 2.27* shows its equivalent current source. It can be easily shown that the two circuits behave electrically the same way under all conditions.

(*i*) If in *Fig. 2.26*, the load is open-circuited (*i.e.* $R_L \to \infty$), then voltage across terminals A and B is V. If in *Fig. 2.27*, the load is open-circuited (*i.e.* $R_L \to \infty$), then all current I (= V/R_i) flows through R_i, yielding voltage across terminals $AB = I\,R_i = V$. Note that open-circuited voltage across AB is V for both the circuits and hence they are electrically equivalent.

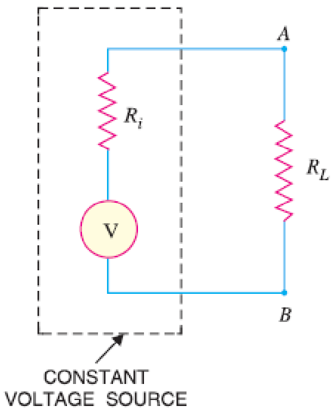


Fig. 2.26

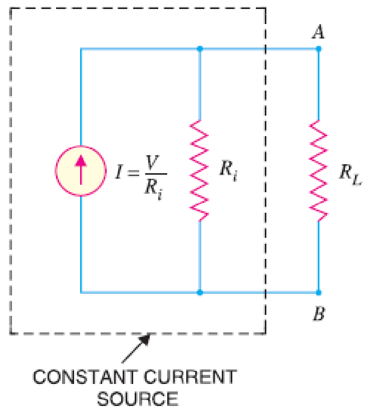


Fig. 2.27

(*ii*) If in Fig. 2.26, the load is short-circuited (*i.e.* $R_L = 0$), the short circuit current is given by:

$$I_{short} = \frac{V}{R_i}$$

If in Fig. 2.27, the load is short-circuited (*i.e.* $R_L = 0$), the current I (= V/R_i) bypasses R_i in favour of shortcircuit. It is clear that current (= V/R_i) is the same for the two circuits and hence they are electrically equivalent.

Thus to convert a constant voltage source into a constant current source, the following procedure may be adopted:

(a) Place a short - circuit across the two terminals in question (terminals AB in the present case) and find the short-circuit current. Let it be I. Then I is the current supplied by the equivalent current source.

(b) Measure the resistance at the terminals with load removed and sources of *e.m.f.s* replaced by their internal resistances if any. Let this resistance be R.

(c) Then equivalent current source can be represented by a single current source of magnitude I in parallel with resistance R.

Note. To convert a current source of magnitude I in parallel with resistance R into voltage source, Voltage of voltage source, $V = I\,R$

Resistance of voltage source, $R = R$

Thus voltage source will be represented as voltage V in series with resistance R.

Example 2.8. *Convert the constant voltage source shown in Fig.* 2.28 *into constant current source.*

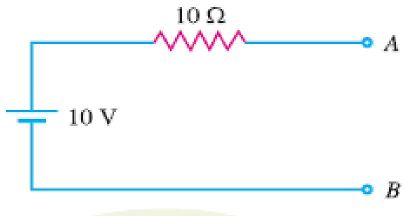


Fig. 2.28

Solution. The solution involves the following steps:

(i) Place a short across *AB* in Fig. 2.29 and find the short-circuit current *I*.
Clearly, $I = 10/10 = 1$ A
Therefore, the equivalent current source has a magnitude of 1 A.

(ii) Measure the resistance at terminals *AB* with load removed and 10 V source replaced by its internal resistance. The 10 V source has negligible resistance so that resistance at terminals *AB* is $R = 10\ \Omega$.

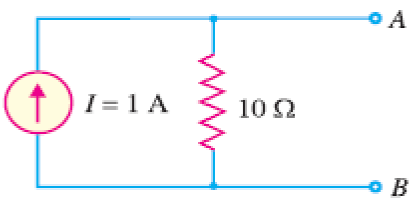


Fig. 2.29

(iii) The equivalent current source is a source of 1 A in parallel with a resistance of 10 Ω as shown in Fig. 2.29

Example 2.9. *Convert the constant current source in Fig. 2.30 into equivalent voltage source.*

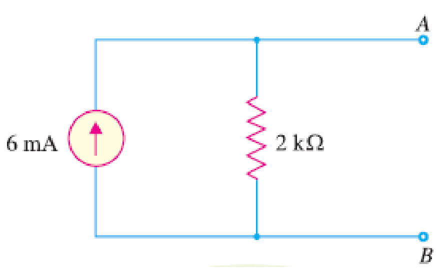


Fig 2.30

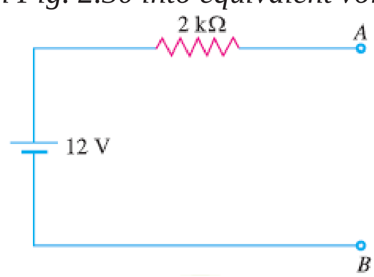


Fig. 2.31

Solution. The solution involves the following steps:

Note: Fortunately, no load is connected across AB. Had there been load across AB, it would have been removed.

(i) To get the voltage of the voltage source, multiply the current of the current source by the internal resistance *i.e.*
Voltage of voltage source = $I\,R = 6$ mA x 2 kΩ = 12V

(ii) The internal resistance of voltage source is 2 kΩ.
The equivalent voltage source is a source of 12 V in series with a resistance of 2 kΩ as shown in Fig. 2.31.

Note. The voltage source should be placed with +ve terminal in the direction of current flow.

Solved Problems

1. A 25.0 Ω resistance has a voltage v = 150.0 sin 377t (V). Find the corresponding current *i* and Power *p*.

$$i = \frac{v}{R} = 6.0 \sin 377t \text{ (A)} \qquad p = vi = 900.0 \sin^2 377t \text{ (W)}$$

2. The current in a 5 resistor increases linearly from zero to 10A in 2 ms. At t = 2^+ ms the current is again zero, and it increases linearly to 10A at t = 4 ms. This pattern repeats each 2 ms. Sketch the corresponding v.

Since $v = Ri$, the maximum voltage must be (5)(10) = 50 V. In Fig. 2-10 the plots of i and v are shown. The identical nature of the functions is evident.

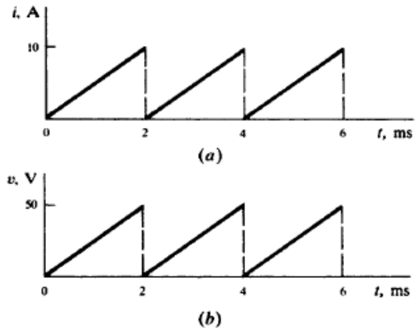


Fig 2.10

3. An inductance of 2.0mH has a current $i = 5.0(1 - e^{-5000t})$ (A). Find the corresponding voltage and the maximum stored energy.

$$v = L\frac{di}{dt} = 50.0e^{-5000t} \text{ (V)}$$

In Fig. 2-11 the plots of i and v are given. Since the maximum current is 5.0 A, the maximum stored energy is

$$W_{\max} = \frac{1}{2} L I_{\max}^2 = 25.0 \text{ mJ}$$

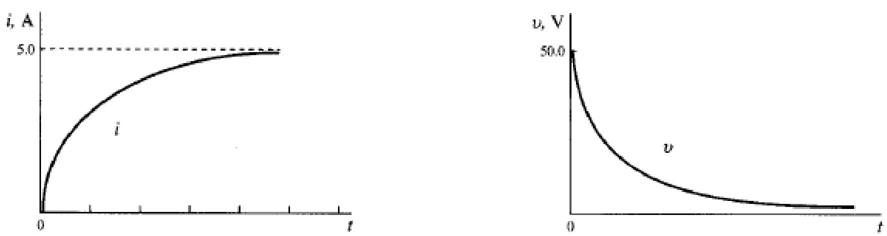


Fig 2.11

4. An inductance of 3.0mH has a voltage that is described as follows: for 0 > t > 2 ms, V = 15.0V and, for 2 > t > 4 ms, V = -30.0 V. Obtain the corresponding current and sketch v_L and i for the given intervals.

For $0 > t > 2\,\text{ms}$,

$$i = \frac{1}{L}\int_0^t v\,dt = \frac{1}{3\times10^{-3}}\int_0^t 15.0\,dt = 5\times10^3 t\ (\text{A})$$

For $t = 2\,\text{ms}$,

$$i = 10.0\ \text{A}$$

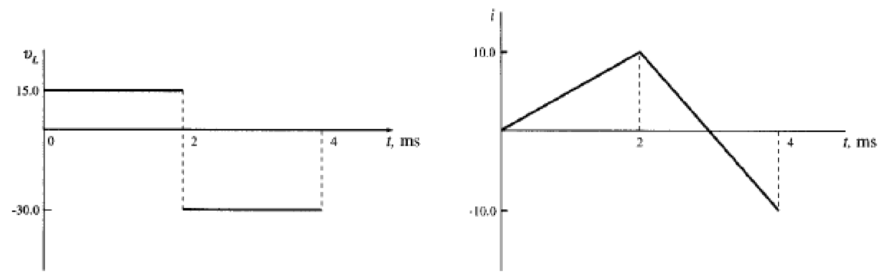
For $2 > t > 4\,\text{ms}$,

$$\begin{aligned} i &= \frac{1}{L}\int_{2\times10^{-3}}^t v\,dt + 10.0 + \frac{1}{3\times10^{-3}}\int_{2\times10^{-3}}^t -30.0\,dt \\ &= 10.0 + \frac{1}{3\times10^{-3}}[-30.0t + (60.0\times10^{-3})]\ (\text{A}) \\ &= 30.0 - (10\times10^3 t)\ (\text{A}) \end{aligned}$$

See Fig. 2-12.

5. A capacitance of 60.0 mF has a voltage described as follows: $0 > t > 2$ ms, $v = 25.0 \times 10^3 t$ (V). Sketch i, p, and w for the given interval and find W_{max}.

For $0 > t > 2\,\text{ms}$,



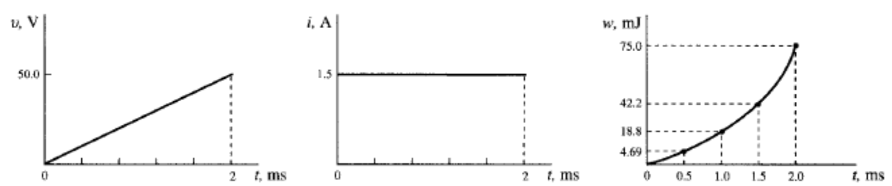
$$\begin{aligned} i &= C\frac{dv}{dt} = 60\times10^{-6}\frac{d}{dt}(25.0\times10^3 t) = 1.5\,\text{A} \\ p &= vi = 37.5\times10^3 t\ (\text{W}) \\ w_C &= \int_0^t p\,dt = 1.875\times10^4 t^2\ (\text{mJ}) \end{aligned}$$

See Fig. 2-13.

$$W_{max} = (1.875\times10^4)(2\times10^{-3})^2 = 75.0\,\text{mJ}$$

or

$$W_{max} = \frac{1}{2}CV_{max}^2 = \frac{1}{2}(60.0\times10^{-6})(50.0)^2 = 75.0\,\text{mJ}$$



6. A 20.0-µF capacitance is linearly charged from 0 to 400 µC in 5.0 ms. Find the voltage function and W_{max}.

$$q = \left(\frac{400\times10^{-6}}{5.0\times10^{-3}}\right)t = 8.0\times10^{-2}t\ (\text{C})$$

$$v = q/C = 4.0\times10^3 t\ (\text{V})$$

$$V_{max} = (4.0\times10^3)(5.0\times10^{-3}) = 20.0\,\text{V} \qquad W_{max} = \frac{1}{2}CV_{max}^2 = 4.0\,\text{mJ}$$

7. A series circuit with R = 2Ω, L = 2 mH, and C ¼ 500 µF has a current which increases linearly from zero to 10A in the interval $0 \le t \le 1$ ms, remains at 10A for $1\text{ ms} \le t \le 2$ ms, and decreases linearly from 10A at t = 2 ms to zero at t = 3 ms. Sketch v_R, v_L, and v_C.

v_R must be a time function identical to i, with $V_{max} = 2(10) = 20$ V.
For $0 < t < 1$ ms,

$$\frac{di}{dt} = 10 \times 10^3 \text{ A/s} \qquad \text{and} \qquad v_L = L\frac{di}{dt} = 20 \text{ V}$$

When $di/dt = 0$, for $1 \text{ ms} < t < 2 \text{ ms}$, $v_L = 0$.

Assuming zero initial charge on the capacitor,

$$v_C = \frac{1}{C}\int i\,dt$$

For $0 \le t \le 1$ ms,

$$v_C = \frac{1}{5 \times 10^{-4}}\int_0^t 10^4\, t\,dt = 10^7 t^2 \text{ (V)}$$

This voltage reaches a value of 10 V at 1 ms. For $1 \text{ ms} < t < 2 \text{ ms}$,

$$v_C = (20 \times 10^3)(t - 10^{-3}) + 10 \text{ (V)}$$

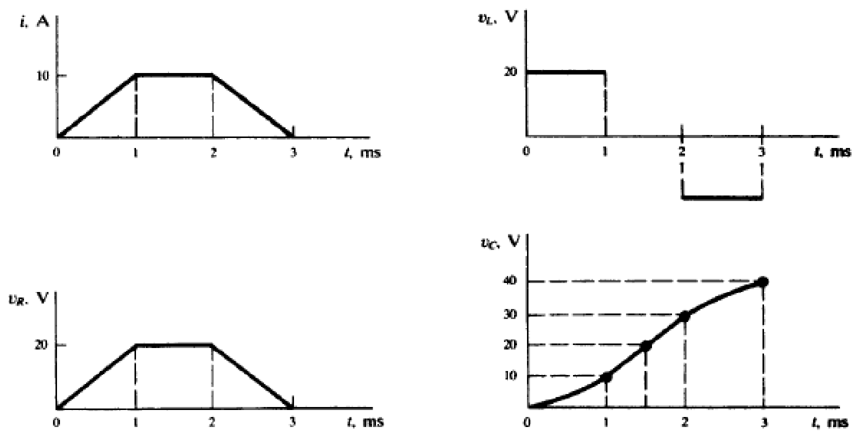
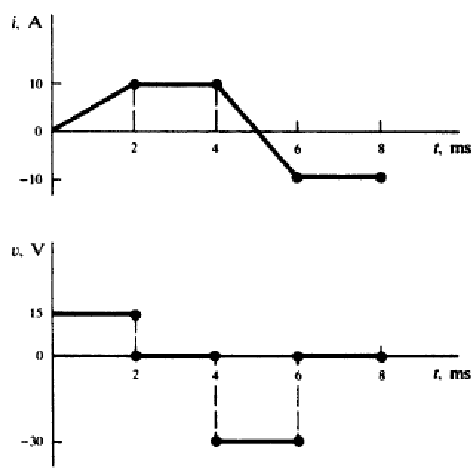


Fig 2.14

8. A single circuit element has the current and voltage functions graphed in Fig. 2-15. Determine the element.



The element cannot be a resistor since v and i are not proportional. v is an integral of i. For 2ms < t < 4 ms, i ≠ 0 but v is constant (zero); hence the element cannot be a capacitor. For 0 < t < 2 ms,

$$\frac{di}{dt} = 5 \times 10^3 \text{ A/s} \qquad \text{and} \qquad v = 15 \text{ V}$$

Consequently,

$$L = v\bigg/\frac{di}{dt} = 3 \text{ mH}$$

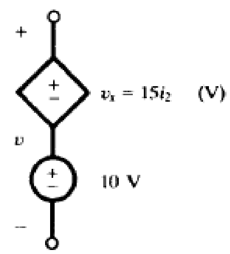
(Examine the interval 4 ms < t < 6 ms; L must be the same.)

9. Obtain the voltage v in the branch shown in Fig. 2.16 for (a) i_2 = 1A, (b) i_2 = - 2A, (c) i_2 = 0A. Voltage v is the sum of the current-independent 10-V source and the current-dependent voltage source v_x. Note that the factor 15 multiplying the control current carries the units _.

(a) $v = 10 + v_x = 10 + 15(1) = 25\text{ V}$

(b) $v = 10 + v_x = 10 + 15(-2) = -20\text{ V}$

(c) $v = 10 + 15(0) = 10\text{ V}$



10. Find the power absorbed by the generalized circuit element in Fig. 2-17, for (a) v = 50 V, (b) v = -50 V.

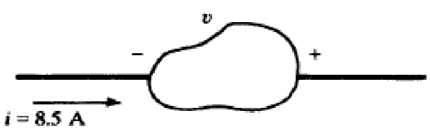


Fig. 2.17

Since the current enters the element at the negative terminal,

(a) $p = -vi = -(50)(8.5) = -425\text{ W}$

(b) $p = -vi = -(-50)(8.5) = 425\text{ W}$

11. Find the power delivered by the sources in the circuit of Fig. 2.18.

$$i = \frac{20 - 50}{3} = -10\text{ A}$$

The powers absorbed by the sources are:

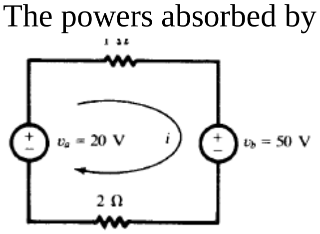


Fig 2.18

$p_a = -v_a i = -(20)(-10) = 200\text{ W}$

$p_b = v_b i = (50)(-10) = -500\text{ W}$

Since power delivered is the negative of power absorbed, source v_b delivers 500W and source v_a absorbs 200 W. The power in the two resistors is 300 W.

12. A 25.0-Ω resistance has a voltage v = 150.0 sin 377t (V). Find the power p and the average power p_{avg} over one cycle.

$i = v/R = 6.0 \sin 377t\text{ (A)}$

$p = vi = 900.0 \sin^2 377t\text{ (W)}$

The end of one period of the voltage and current functions occurs at $377t = 2\pi$. For P_{avg} the integration is taken over one-half cycle, $377t = \pi$. Thus,

$$P_{avg} = \frac{1}{\pi}\int_0^{\pi} 900.0\sin^2(377t)d(377t) = 450.0 \text{ (W)}$$

13. Find the voltage across the 10.0 Ω resistor in Fig. 2.19 if the control current i_x in the dependent source is (a) 2A and (b) -1A.

$i = 4i_x - 4.0; \quad v_R = iR = 40.0i_x - 40.0 \text{ (V)}$

$i_x = 2; \quad v_R = 40.0 \text{ V}$

$i_x = -1; \quad v_R = -80.0 \text{ V}$

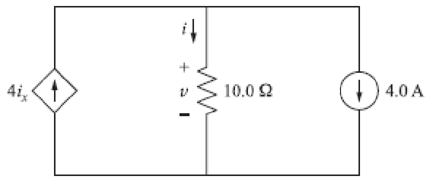


Fig. 2.19

Exercises

1. A resistor has a voltage of V = 1.5 mV. Obtain the current if the power absorbed is (a) 27.75nW and (b) 1.20 mW. Ans. 18.5 mA, 0.8mA
2. A resistance of 5.0Ω has a current i = 5.0 x 10^3t (A) in the interval $0 \geq t \geq 2$ ms. Obtain the instantaneous and average power. Ans. 125.0t^2 (W), 167.0 (W)
3. Current i enters a generalized circuit element at the positive terminal and the voltage across the element is 3.91 V. If the power absorbed is -25.0mW, obtain the current. Ans. - 6.4mA
4. Determine the single circuit element for which the current and voltage in the interval $0 \geq 10^3t \geq \pi$ are given by i = 2.0 sin 10^3t (mA) and v = 5.0 cos 10^3t (mV). Ans. An inductance of 2.5mH

3. CIRCUIT LAWS

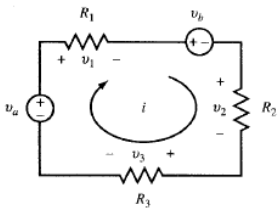
3.1 INTRODUCTION

- An electric circuit or network consists of a number of interconnected single circuit elements of the type described in Chapter 2. The circuit will generally contain at least one voltage or current source.
- The arrangement of elements results in a new set of constraints between the currents and voltages. These new constraints and their corresponding equations, added to the current-voltage relationships of the individual elements, provide the solution of the network.
- The underlying purpose of defining the individual elements, connecting them in a network, and solving the equations is to analyze the performance of such electrical devices as motors, generators, transformers, electrical transducers, and a host of electronic devices. The solution generally answers necessary questions about the operation of the device under conditions applied by a source of energy.

3.2 KIRCHHOFF'S VOLTAGE LAW

- For any closed path in a network, *Kirchhoff's voltage law (KVL)* states that the algebraic sum of the voltages is zero. Some of the voltages will be sources, while others will result from current in passive elements creating a voltage, which is sometimes referred to as a *voltage drop*. The law applies equally well to circuits driven by constant sources, DC, time variable sources, *v(t)* and *i(t)*, and to circuits driven by sources.
- The mesh current method of circuit analysis introduced in Chapter 4 is based on Kirchhoff's voltage law.

Example 3.1 Write the KVL equation for the circuit shown in Fig. 3.1



Starting at the lower left corner of the circuit, for the current direction as shown, we have

$$-v_a + v_1 + v_b + v_2 + v_3 = 0$$

$$-v_a + iR_1 + v_b + iR_2 + iR_3 = 0$$

$$v_a - v_b = i(R_1 + R_2 + R_3)$$

Example Consider Figure below with the following Parameters: $V_1 = 15V$, $V_2 = 7V$, $R_1 = 20\Omega$, $R_2 = 5\Omega$, $R_3 = 10\Omega$. Find current through R_3 using Kirchhoff's Voltage Law.

Solution:

We can see that there are two closed paths (loops) where we can apply KVL in, Loop 1 and 2 as shown in figure

From Loop 1 we get:

$$V_1 - V_{R3} - V_{R1} = 0$$

From Loop 2 we get:

$$V_2 - V_{R3} - V_{R2} = 0$$

The above results can further be simplified as follows: $V_1 - (I_1 - I_2) * R_3 - I_1 * R_1 = 0$

..... (1)

and

$V_2 + (I_1 - I_2) * R_3 - I_2 * R_2 = 0$

..... (2)

By equating above (1) and (2) we can eliminate I_2 and hence get the following:

..... (3)

We end up with the above three equations and now substitute the Values given in the above equations and solve the variables.

It is clear that: from (3)

.

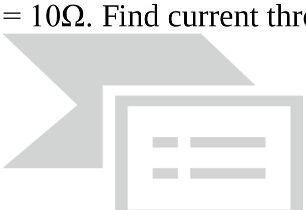
Substitute the Above Result into (2)

.

The Positive sign for I_2 only tells us that Current I_2 flows in the same direction to our initial assumed direction. Thus now we can calculate Current through R_3 as follows:

.

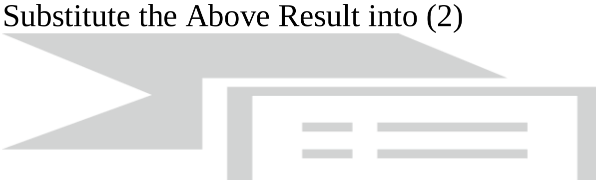
Example. Consider Figure below with the following Parameters: $V_1 = 15V$, $V_2 = 7V$, $R_1 = 20\Omega$, $R_2 = 5\Omega$, $R_3 = 10\Omega$. Find current through R_3 using Kirchhoff's Voltage Law.



Solution

From KVL we get the following:

Substitute the Above Result into (2)



Thus now we can calculate Current through R_3 as follows:

Thus Current through R_3 is effectively flowing in the same direction as I_1. See why you need to understand Passive sign Convention?

3.3 KIRCHHOFF'S CURRENT LAW

- The connection of two or more circuit elements creates a junction called a *node*.
- The junction between two elements is called a *simple node* and no division of current results. The junction of three or more elements is called a *principal node,* and here current division does take place. *Kirchhoff's current law* (KCL) states that the *algebraic sum* of the currents at a node is *zero*.
- It may be stated alternatively that the sum of the currents entering a node is equal to the sum of the currents leaving that node. The node voltage method of circuit analysis introduced in chapter 4 is based on equations written at the principal nodes of a network by applying Kirchhoff's current law. The basis for the law is the conservation of electric charge.

Example 3.2. Write the KCL equation for the principal node shown in Fig. 3.2.

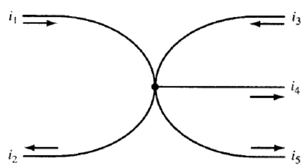
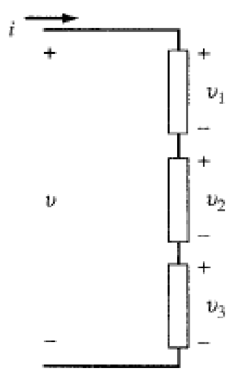


Fig 3.2

$$i_1 - i_2 + i_3 - i_4 - i_5 = 0$$
$$i_1 + i_3 = i_2 + i_4 + i_5$$

3.4 CIRCUIT ELEMENTS IN SERIES

- Three passive circuit elements in series connection as shown in Fig. 3.3 have the same current i. The voltages across the elements are v_1, v_2, and v_3. The total voltage v is the sum of the individual voltages; $v = v_1 + v_2 + v_3$.



If the elements are resistors,

$$\begin{aligned} v &= iR_1 + iR_2 + iR_3 \\ &= i(R_1 + R_2 + R_3) \\ &= iR_{eq} \end{aligned}$$

where a single equivalent resistance R_{eq} replaces the three series resistors. The same relationship between *i* and *v* will pertain.
For any number of resistors in series, we have $R_{eq} = R_1 + R_2 + \_\_\_$.
If the three passive elements are inductances,

$$\begin{aligned} v &= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} \\ &= (L_1 + L_2 + L_3) \frac{di}{dt} \\ &= L_{eq} \frac{di}{dt} \end{aligned}$$

Extending this to any number of inductances in series, we have $L_{eq} = L_1 + L_2 + \_\_\_$.
If the three circuit elements are capacitances, assuming zero initial charges so that the constants of integration are zero,

$$\begin{aligned} v &= \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt + \frac{1}{C_3}\int i\,dt \\ &= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)\int i\,dt \\ &= \frac{1}{C_{eq}}\int i\,dt \end{aligned}$$

The equivalent capacitance of several capacitances in series is
$1/C_{eq} = 1/C_1 + 1/C_2 + \cdots$.

Example 3.3. The equivalent resistance of three resistors in series is 750.0 Ω. Two of the resistors are 40.0 and 410.0Ω. What must be the ohmic resistance of the third resistor?

$$R_{eq} = R_1 + R_2 + R_3$$
$$750.0 = 40.0 + 410.0 + R_3 \qquad \text{and} \qquad R_3 = 300.0\,\Omega$$

Example 3.4. Two capacitors, $C_1 = 2.0\mu$ F and $C_2 = 10.0\ \mu F$, are connected in series. Find the equivalent capacitance. Repeat if C_2 is 10.0 pF.

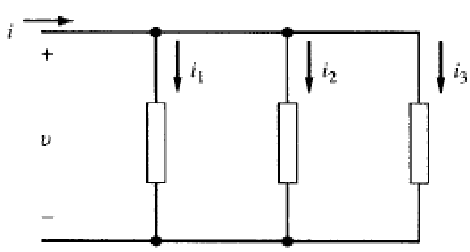
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2.0 \times 10^{-6})(10.0 \times 10^{-6})}{2.0 \times 10^{-6} + 10.0 \times 10^{-6}} = 1.67\,\mu F$$

If $C_2 = 10.0$ pF,

$$C_{eq} = \frac{(2.0 \times 10^{-6})(10.0 \times 10^{-12})}{2.0 \times 10^{-6} + 10.0 \times 10^{-12}} = \frac{20.0 \times 10^{-18}}{2.0 \times 10^{-6}} = 10.0\,pF$$

where the contribution of 10.0 x 10^{-12} to the sum $C_1 + C_2$ in the denominator is negligible and therefore it can be omitted.
Note: When two capacitors in series differ by a large amount, the equivalent capacitance is essentially equal to the value of the smaller of the two.

3.5 CIRCUIT ELEMENTS IN PARALLEL
For three circuit elements connected in parallel as shown in Fig. 3.4, KCL states that the current *I* entering the principal node is the sum of the three currents leaving the node through the branches.



$$i = i_1 + i_2 + i_3$$

If the three passive circuit elements are resistances,

$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)v = \frac{1}{R_{eq}}v$$

For several resistors in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

The case of two resistors in parallel occurs frequently and deserves special mention. The equivalent resistance of two resistors in parallel is given by the *product over the sum*.

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Example 3.5. Obtain the equivalent resistance of (a) two 60.0 Ω resistors in parallel and (b) three 60.0 Ω resistors in parallel.

(*a*) $$R_{eq} = \frac{(60.0)^2}{120.0} = 30.0\,\Omega$$

(*b*) $$\frac{1}{R_{eq}} = \frac{1}{60.0} + \frac{1}{60.0} + \frac{1}{60.0} \qquad R_{eq} = 20.0\,\Omega$$

Note: For n identical resistors in parallel the equivalent resistance is given by R = n.
Combinations of inductances in parallel have similar expressions to those of resistors in parallel:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \qquad \text{and, for two inductances,} \qquad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Example 3.6. Two inductances L_1 = 3.0mH and L_2 = 6.0mH are connected in parallel. Find L_{eq}.

$$\frac{1}{L_{eq}} = \frac{1}{3.0\,\text{mH}} + \frac{1}{6.0\,\text{mH}} \qquad \text{and} \qquad L_{eq} = 2.0\,\text{mH}$$

With three capacitances in parallel,

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} = (C_1 + C_2 + C_3)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

For several parallel capacitors, $C_{eq} = C_1 + C_2 +$_ _ _, which is of the same form as resistors in series.

3.6 VOLTAGE DIVISION

- A set of series-connected resistors as shown in Fig. 3.5 is referred to as a voltage divider. The concept extends beyond the set of resistors illustrated here

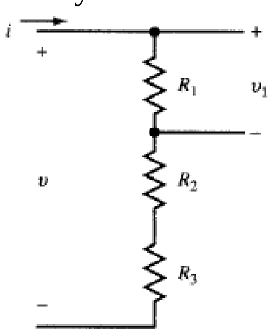


FIG 3.5

Since $v_1 = iR_1$ and $v = i(R_1 + R_2 + R_3)$,

$$v_1 = v\left(\frac{R_1}{R_1 + R_2 + R_3}\right)$$

Example 3.7. A voltage divider circuit of two resistors is designed with a total resistance of the two resistors equal to 50.0Ω. If the output voltage is 10 percent of the input voltage, obtain the values of the two resistors in the circuit.

$$\frac{v_1}{v} = 0.10 \qquad 0.10 = \frac{R_1}{50.0 \times 10^3}$$

from which $R_1 = 5.0\ \Omega$ and $R_2 = 45.0\ \Omega$.

3.7 CURRENT DIVISION

A parallel arrangement of resistors as shown in Fig. 3.6 results in a current divider. The ratio of the branch current i_1 to the total current i illustrates the operation of the divider.

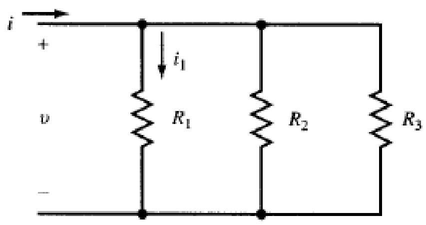


Fig. 3.6

$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} \qquad \text{and} \qquad i_1 = \frac{v}{R_1}$$

Then
$$\frac{i_1}{i} = \frac{1/R_1}{1/R_1 + 1/R_2 + 1/R_3} = \frac{R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

For a two-branch current divider we have

$$\frac{i_1}{i} = \frac{R_2}{R_1 + R_2}$$

This may be expressed as follows: The ratio of the current in one branch of a two-branch parallel circuit to the total current is equal to the ratio of the resistance of the other branch resistance to the sum of the two resistances

Example 3.8. A current of 30.0mA is to be divided into two branch currents of 20.0mA and 10.0mA by a network with an equivalent resistance equal to or greater than 10.0Ω. Obtain the branch resistances.

$$\frac{20\,\text{mA}}{30\,\text{mA}} = \frac{R_2}{R_1 + R_2} \qquad \frac{10\,\text{mA}}{30\,\text{mA}} = \frac{R_1}{R_1 + R_2} \qquad \frac{R_1R_2}{R_1 + R_2} \geq 10.0\,\Omega$$

Solving these equations yields $R_1 \geq 15.0\,\Omega$ and $R_2 \geq 30.0\,\Omega$.

Solved Problems

1. Find V_3 and its polarity if the current I in the circuit of Fig. 3.7 is 0.40 A.

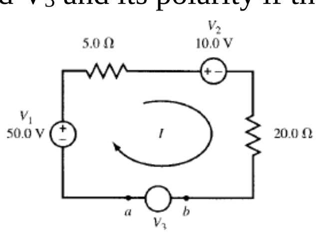


Fig. 3.7

Assume that V_3 has the same polarity as V_1. Applying KVL and starting from the lower left corner,

$$V_1 - I(5.0) - V_2 - I(20.0) + V_3 = 0$$
$$50.0 - 2.0 - 10.0 - 8.0 + V_3 = 0$$
$$V_3 = -30.0\,\text{V}$$

Terminal b is positive with respect to terminal a.

2. Obtain the currents I_1 and I_2 for the network shown in Fig. 3.8.

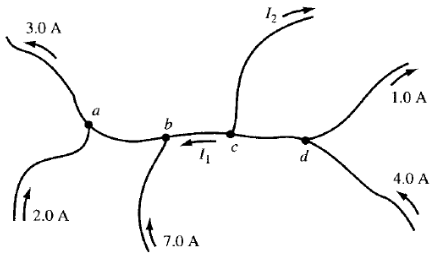


FIG 3.8

a and *b* comprise one node. Applying KCL,

$$2.0 + 7.0 + I_1 = 3.0 \qquad \text{or} \qquad I_1 = -6.0\,\text{A}$$

Also, *c* and *d* comprise a single node. Thus,

$$4.0 + 6.0 = I_2 + 1.0 \qquad \text{or} \qquad I_2 = 9.0\,\text{A}$$

3. Find the current *I* for the circuit shown in Fig. 3.9.

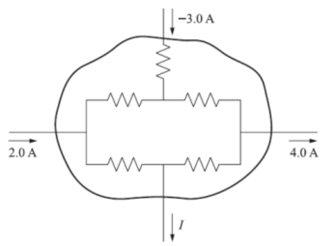
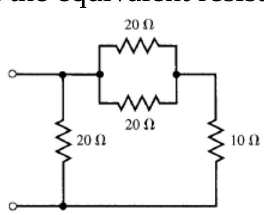


Fig 3.9

The branch currents within the enclosed area cannot be calculated since no values of the resistors are given. However, KCL applies to the network taken as a single node. Thus,

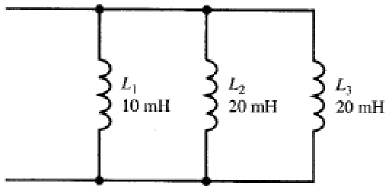
$$2.0 - 3.0 - 4.0 - I = 0 \qquad \text{or} \qquad I = -5.0\,\text{A}$$

4. Find the equivalent resistance for the circuit shown in Fig. 3.10.



The two 20-Ω resistors in parallel have an equivalent resistance $R_{eq} = [(20)(20)/(20+20)] = 10\,\Omega$. This is in series with the 10-Ω resistor so that their sum is 20 Ω. This in turn is in parallel with the other 20-Ω resistor so that the overall equivalent resistance is 10 Ω.

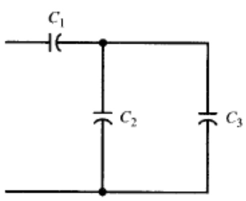
5. Determine the equivalent inductance of the three parallel inductances shown in Fig. 3.11.



The two 20-mH inductances have an equivalent inductance of 10 mH. Since this is in parallel with the 10-mH inductance, the overall equivalent inductance is 5 mH. Alternatively,

$$\frac{1}{L_{eq}}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}=\frac{1}{10\,\mathrm{mH}}+\frac{1}{20\,\mathrm{mH}}+\frac{1}{20\,\mathrm{mH}}=\frac{4}{20\,\mathrm{mH}} \qquad \text{or} \qquad L_{eq}=5\,\mathrm{mH}$$

6. Express the total capacitance of the three capacitors in Fig. 3.12.

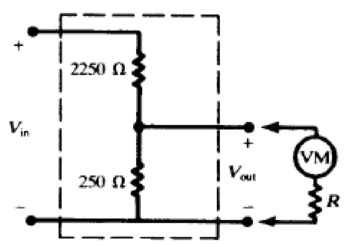


For C_2 and C_3 in parallel, $C_{eq}=C_2+C_3$. Then for C_1 and C_{eq} in series,

$$C_T=\frac{C_1C_{eq}}{C_1+C_{eq}}=\frac{C_1(C_2+C_3)}{C_1+C_2+C_3}$$

7. The circuit shown in Fig. 3.13 is a voltage divider, also called an *attenuator*. When it is a single resistor with an adjustable tap, it is called a *potentiometer*, or *pot*. To discover the effect of loading, which is caused by the resistance R of the voltmeter VM, calculate the ratio V_{out}/V_{in} for (a) $R=\infty$ (b) 1MΩ, (c) 10kΩ, (d) 1kΩ.

(*a*) $$V_{out}/V_{in}=\frac{250}{2250+250}=0.100$$



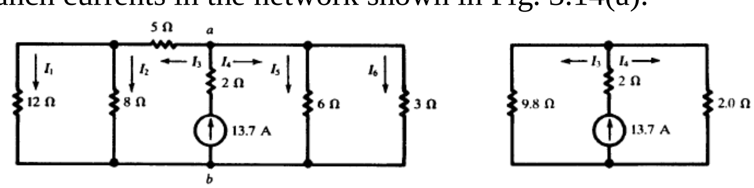
(*b*) The resistance R in parallel with the 250-Ω resistor has an equivalent resistance

$$R_{eq}=\frac{250(10^6)}{250+10^6}=249.9\,\Omega \qquad \text{and} \qquad V_{out}/V_{in}=\frac{249.9}{2250+249.9}=0.100$$

(*c*) $$R_{eq}=\frac{(250)(10\,000)}{250+10\,000}=243.9\,\Omega \qquad \text{and} \qquad V_{out}/V_{in}=0.098$$

(*d*) $$R_{eq}=\frac{(250)(1000)}{250+1000}=200.0\,\Omega \qquad \text{and} \qquad V_{out}/V_{in}=0.082$$

8. Find all branch currents in the network shown in Fig. 3.14(a).



(*a*) (*b*)

Fig. 3.14

The equivalent resistances to the left and right of nodes a and b are

$$R_{eq(left)} = 5 + \frac{(12)(8)}{20} = 9.8\ \Omega$$
$$R_{eq(right)} = \frac{(6)(3)}{9} = 2.0\ \Omega$$

Now referring to the reduced network of Fig. 3-14(b),

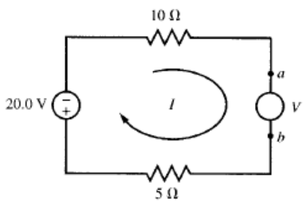
$$I_3 = \frac{2.0}{11.8}(13.7) = 2.32\ \text{A}$$
$$I_4 = \frac{9.8}{11.8}(13.7) = 11.38\ \text{A}$$

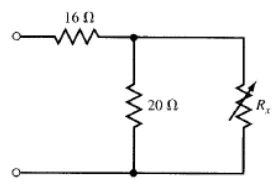
Then referring to the original network,

$$I_1 = \frac{8}{20}(2.32) = 0.93\ \text{A} \qquad I_2 = 2.32 - 0.93 = 1.39\ \text{A}$$
$$I_5 = \frac{3}{9}(11.38) = 3.79\ \text{A} \qquad I_6 = 11.38 - 3.79 = 7.59\ \text{A}$$

Exercises

1. Find the source voltage V and its polarity in the circuit shown in Fig. 3.15 if (a) I = 2.0A and (b) I = - 2.0A. Ans. (a) 50V, *b* positive; (b) 10V, *a* positive.
2. Find R_{eq} for the circuit of Fig. 3.16 for (a) $R_x = \infty$, (b) $R_x = 0$, (c) $R_x = 5\Omega$. *Ans*. (a) 36Ω; (b) 16Ω; (c) 20Ω





3. An inductance of 8.0mH is in series with two inductances in parallel, one of 3.0mH and the other 6.0 mH. Find L_{eq}. *Ans*. 10.0mH
4. Show that for the three capacitances of equal value shown in Fig. 3.17 C_{eq} = 1.5C:

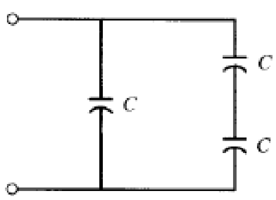


Fig 3.17

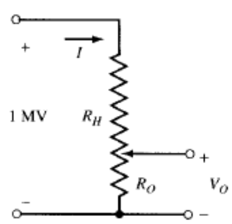


fig 3.18

5. Find R_H and R_O for the voltage divider in Fig. 3.18 so that the current I is limited to 0.5A when V_O = 100 V. *Ans*: R_H = 2MΩ; R_O = 200Ω.
6. Using voltage division, calculate V_1 and V_2 in the network shown in Fig. 3.19. Ans. 11.4 V, 73.1V

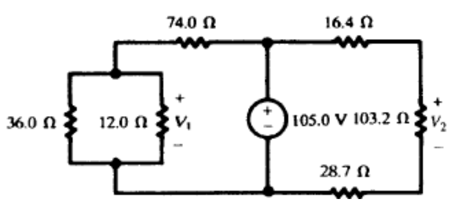


Fig. 3.19

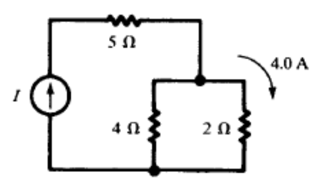


Fig. 3.20

7. Obtain the source current *I* and the total power delivered to the circuit in Fig. 3.20. Ans. 6.0 A, 228W
8. Show that for four resistors in parallel the current in one branch, for example the branch of R_4, is related to the total current by

$$I_4 = I_T\left(\frac{R'}{R_4 + R'}\right) \qquad \text{where } R' = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

Note: This is similar to the case of current division in a two-branch parallel circuit where the other resistor has been replaced by R'.

9. A power transmission line carries current from a 6000-V generator to three loads, A, B, and C. The loads are located at 4, 7, and 10km from the generator and draw 50, 20, and 100 A, respectively. The resistance of the line is 0.1Ω/km; see Fig. 3.21. (a) Find the voltage at loads A, B, C. (b) Find the maximum percentage voltage drop from the generator to a load.

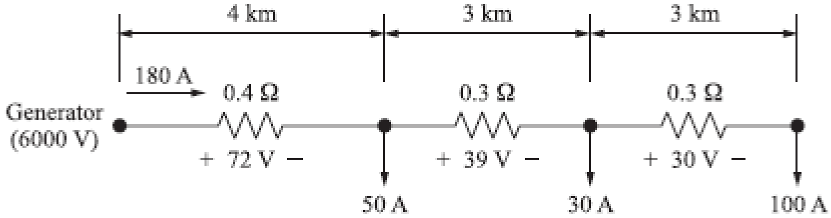


Fig. 3.21

Ans. (*a*) $v_A = 5928$ V, $v_B = 5889$ V, $v_C = 5859$ V; (*b*) 2.35 percent

<u>4.0 ANALYSIS METHODS</u>

4.1 THE BRANCH CURRENT METHOD

- In the branch current method a current is assigned to each branch in an active network. Then Kirchhoff's current law is applied at the principal nodes and the voltages between the nodes employed to relate the currents. This produces a set of simultaneous equations which can be solved to obtain the currents.

Example 4.1

Obtain the current in each branch of the network shown in Fig. 4.1 using the branch current method.

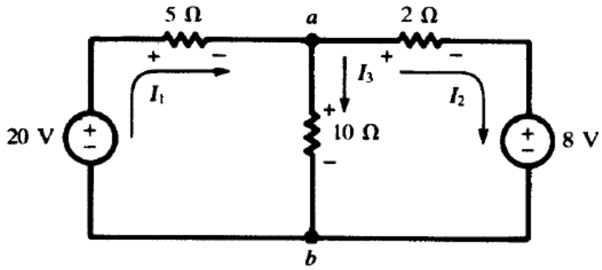


Fig. 4.1

Currents I_1, I_2, and I_3 are assigned to the branches as shown. Applying KCL at node a,

$$I_1 = I_2 + I_3 \qquad (1)$$

The voltage V_{ab} can be written in terms of the elements in each of the branches; $V_{ab} = 20 - I_1(5)$, $V_{ab} = I_3(10)$ and $V_{ab} = I_2(2) + 8$. Then the following equations can be written

$$20 - I_1(5) = I_3(10) \qquad (2)$$

$$20 - I_1(5) = I_2(2) + 8 \qquad (3)$$

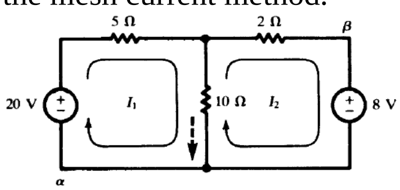
Solving the three equations (1), (2), and (3) simultaneously gives $I_1 = 2\,\text{A}$, $I_2 = 1\,\text{A}$, and $I_3 = 1\,\text{A}$.

Other directions may be chosen for the branch currents and the answers will simply include the appropriate sign. In a more complex network, the branch current method is difficult to apply because it does not suggest either a starting point or a logical progression through the network to produce the necessary equations. It also results in more independent equations than either the mesh current or node voltage method requires.

4.2 THE MESH CURRENT METHOD

In the mesh current method a current is assigned to each window of the network such that the currents complete a closed loop. They are sometimes referred to as loop currents. Each element and branch therefore will have an independent current. When a branch has two of the mesh currents, the actual current is given by their algebraic sum. The assigned mesh currents may have either clockwise or counterclockwise directions, although at the outset it is wise to assign to all of the mesh currents a clockwise direction. Once the currents are assigned, Kirchhoff's voltage law is written for each loop to obtain the necessary simultaneous equations.

Example 4.2 Obtain the current in each branch of the network shown in Fig. 4.2 (same as Fig. 4.1) using the mesh current method.



The currents I_1 and I_2 are chosen as shown on the circuit diagram. Applying KVL around the left loop, starting at point α,

$$-20 + 5I_1 + 10(I_1 - I_2) = 0$$

and around the right loop, starting at point β,

$$8 + 10(I_2 - I_1) + 2I_2 = 0$$

Rearranging terms,

$$15I_1 - 10I_2 = 20 \quad (4)$$
$$-10I_1 + 12I_2 = -8 \quad (5)$$

Solving (4) and (5) simultaneously results in $I_1 = 2$ A and $I_2 = 1$ A. The current in the center branch, shown dotted, is $I_1 - I_2 = 1$ A. In Example 4.1 this was branch current I_3.

The currents do not have to be restricted to the windows in order to result in a valid set of simultaneous equations, although that is the usual case with the mesh current method. For example, see Problem 4.6, where each of the currents passes through the source. In that problem they are called loop currents. The applicable rule is that each element in the network must have a current or a combination of currents and no two elements in different branches can be assigned the same current or the same combination of currents.

4.4 THE NODE VOLTAGE METHOD

The network shown in Fig. 4.4(a) contains five nodes, where 4 and 5 are simple nodes and 1, 2, and 3 are principal nodes. In the node voltage method, one of the principal nodes is selected as the reference and equations based on KCL are written at the other principal nodes. At each of these other principal nodes, a voltage is assigned, where it is understood that this is a voltage with respect to the reference node. These voltages are the unknowns and, when determined by a suitable method, result in the network solution.

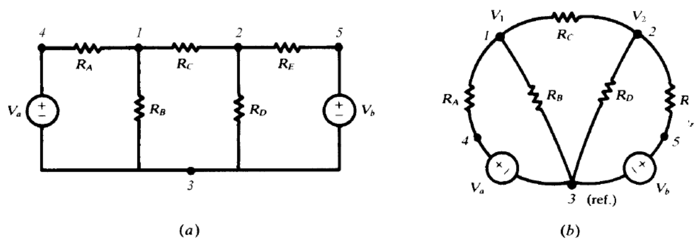


Fig. 4.4

The network is redrawn in Fig. 4.4(b) and node 3 selected as the reference for voltages V_1 and V_2. KCL requires that the total current out of node 1 be zero:

$$\frac{V_1 - V_a}{R_A} + \frac{V_1}{R_B} + \frac{V_1 - V_2}{R_C} = 0$$

Similarly, the total current out of node 2 must be zero:

$$\frac{V_2 - V_1}{R_C} + \frac{V_2}{R_D} + \frac{V_2 - V_b}{R_E} = 0$$

(Applying KCL in this form does not imply that the actual branch currents all are directed out of either node. Indeed, the current in branch 1–2 is necessarily directed *out of* one node and into the other.) Putting the two equations for V_1 and V_2 in matrix form,

$$\begin{bmatrix} \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C} + \frac{1}{R_D} + \frac{1}{R_E} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_a/R_A \\ V_b/R_E \end{bmatrix}$$

Note the symmetry of the coefficient matrix. The 1,1- element contains the sum of the reciprocals of all resistances connected to note *1*; the 2,2-element contains the sum of the reciprocals of all resistances connected to node *2*. The 1,2- and 2,1-elements are each equal to the negative of the sum of the reciprocals of the resistances of all branches joining nodes *1* and *2*. (There is just one such branch in the present circuit.)

On the right-hand side, the current matrix contains V_a / R_A and V_b / R_E, the driving currents. Both these terms are taken positive because they both drive a current *into* a node.

EXAMPLE 4.5 Solve the circuit of Example 4.2 using the node voltage method.

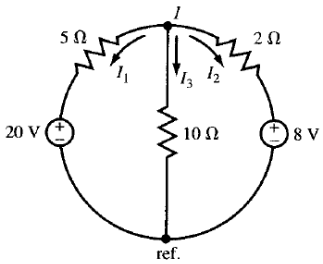


Fig. 4.5

The circuit is redrawn in Fig. 4-5. With two principal nodes, only one equation is required. Assuming the currents are all directed out of the upper node and the bottom node is the reference,

$$\frac{V_1 - 20}{5} + \frac{V_1}{10} + \frac{V_1 - 8}{2} = 0$$

from which $V_1 = 10\,\text{V}$. Then, $I_1 = (10 - 20)/5 = -2\,\text{A}$ (the negative sign indicates that current I_1 flows into node *1*); $I_2 = (10 - 8)/2 = 1\,\text{A}$; $I_3 = 10/10 = 1\,\text{A}$. Current I_3 in Example 4.2 is shown dotted.

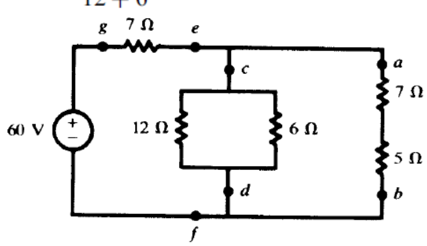
4.7 NETWORK REDUCTION

The mesh current and node voltage methods are the principal techniques of circuit analysis. However, the equivalent resistance of series and parallel branches (Sections 3.4 and 3.5), combined with the voltage and current division rules, provide another method of analyzing a network. This method is tedious and usually requires the drawing of several additional circuits. Even so, the process of reducing the network provides a very clear picture of the overall functioning of the network in terms of voltages, currents, and power. The reduction begins with a scan of the network to pick out series and parallel combinations of resistors.

EXAMPLE 4.6 Obtain the total power supplied by the 60-V source and the power absorbed in each resistor in the network of Fig. 4.8.

$$R_{ab} = 7 + 5 = 12\,\Omega$$

$$R_{cd} = \frac{(12)(6)}{12 + 6} = 4\,\Omega$$

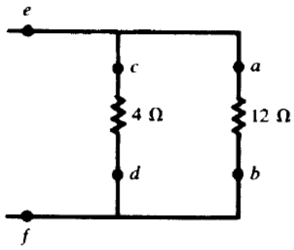


These two equivalents are in parallel (Fig. 4-9), giving

$$R_{ef} = \frac{(4)(12)}{4 + 12} = 3\,\Omega$$

Then this 3-Ω equivalent is in series with the 7-Ω resistor (Fig. 4-10), so that for the entire circuit,

$$R_{eq} = 7 + 3 = 10\,\Omega$$



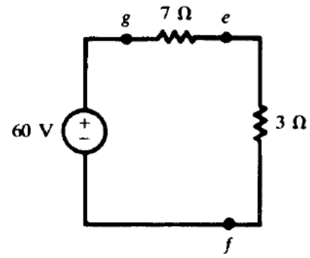


Fig. 4-9 Fig. 4-10

The total power absorbed, which equals the total power supplied by the source, can now be calculated as

$$P_T = \frac{V^2}{R_{eq}} = \frac{(60)^2}{10} = 360\ \text{W}$$

This power is divided between R_{ge} and R_{ef} as follows:

$$P_{ge} = P_{7\Omega} = \frac{7}{7+3}(360) = 252\ \text{W} \qquad P_{ef} = \frac{3}{7+3}(360) = 108\ \text{W}$$

Power P_{ef} is further divided between R_{cd} and R_{ab} as follows:

$$P_{cd} = \frac{12}{4+12}(108) = 81\ \text{W} \qquad P_{ab} = \frac{4}{4+12}(108) = 27\ \text{W}$$

Finally, these powers are divided between the individual resistances as follows:

$$P_{12\Omega} = \frac{6}{12+6}(81) = 27\ \text{W} \qquad P_{7\Omega} = \frac{7}{7+5}(27) = 15.75\ \text{W}$$

$$P_{6\Omega} = \frac{12}{12+6}(81) = 54\ \text{W} \qquad P_{5\Omega} = \frac{5}{7+5}(27) = 11.25\ \text{W}$$

4.8 SUPERPOSITION

A linear network which contains two or more independent sources can be analyzed to obtain the various voltages and branch currents by allowing the sources to act one at a time, then superposing the results. This principle applies because of the linear relationship between current and voltage. With dependent sources, superposition can be used only when the control functions are external to the network containing the sources, so that the controls are unchanged as the sources act one at a time. Voltage sources to be suppressed while a single source acts are replaced by short circuits; current sources are replaced by open circuits. Superposition cannot be directly applied to the computation of power, because power in an element is proportional to the square of the current or the square of the voltage, which is nonlinear.

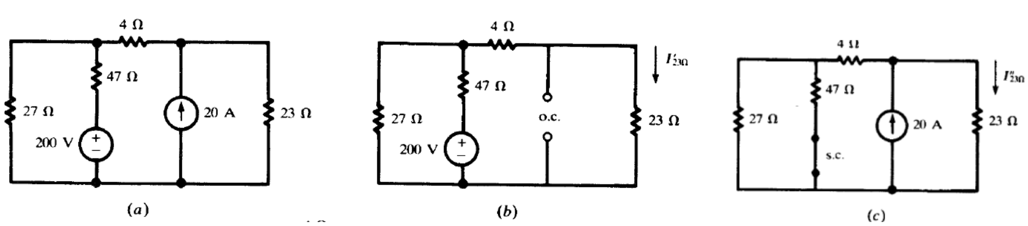
As a further illustration of superposition consider equation (7) of Example 4.4:

$$I_1 = V_1\left(\frac{\Delta_{11}}{\Delta_R}\right) + V_2\left(\frac{\Delta_{21}}{\Delta_R}\right) + V_3\left(\frac{\Delta_{31}}{\Delta_R}\right)$$

which contains the superposition principle implicitly. Note that the three terms on the right are added to result in current I_1. If there are sources in each of the three meshes, then each term contributes to the current I_1. Additionally, if only mesh 3 contains a source, V_1 and V_2 will be zero and I_1 is fully determined by the third term.

EXAMPLE 4.7 Compute the current in the 23-Ω resistor of Fig. 4-11(a) by applying the superposition principle.

With the 200-V source acting alone, the 20-A current source is replaced by an open circuit, Fig. 4-11(b).



$$R_{eq} = 47 + \frac{(27)(4+23)}{54} = 60.5\ \Omega$$

$$I_T = \frac{200}{60.5} = 3.31\ \text{A}$$

$$I'_{23\Omega} = \left(\frac{27}{54}\right)(3.31) = 1.65\ \text{A}$$

When the 20-A source acts alone, the 200-V source is replaced by a short circuit, Fig. 4-11(c). The equivalent resistance to the left of the source is

$$R_{eq} = 4 + \frac{(27)(47)}{74} = 21.15\ \Omega$$

Then

$$I''_{23\Omega} = \left(\frac{21.15}{21.15+23}\right)(20) = 9.58\ \text{A}$$

The total current in the 23-Ω resistor is

$$I_{23\Omega} = I'_{23\Omega} + I''_{23\Omega} = 11.23\ \text{A}$$

4.9 THE´ VENIN'S AND NORTON'S THEOREMS

THEVENIN'S THEOREM

Sometimes it is desirable to find a particular branch current in a circuit as the resistance of that branch is varied while all other resistances and voltage sources remain constant. For instance, in the circuit shown in Fig. 4-12, it may be desired to find the current through R_L for five values of R_L, assuming that R_1, R_2, R_3 and E remain constant. In such situations, the solution can be obtained readily by applying *Thevenin's theorem* stated below:

Any two-terminal network containing a number of e.m.f. sources and resistances can be replaced by an equivalent series circuit having a voltage source E_0 in series with a resistance R_0 where,

E_0 = open circuited voltage between the two terminals.

R_0 = the resistance between two terminals of the circuit obtained by looking "in" at the terminals with load removed and voltage sources replaced by their internal resistances, if any.

To understand the use of this theorem, consider the two-terminal circuit shown in Fig 4-12. The circuit enclosed in the dotted box can be replaced by one voltage E_0 in series with resistance R_0 as shown in Fig. 4-13. The behaviour at the terminals *AB* and *A'B'* is the same for the two circuits, independent of the values of R_L connected across the terminals.

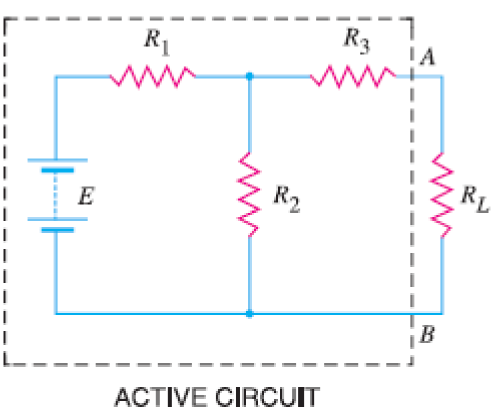


Fig 4-12

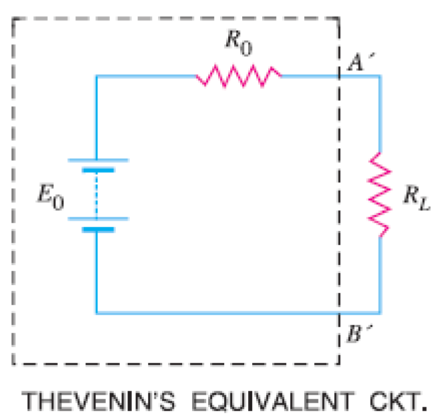


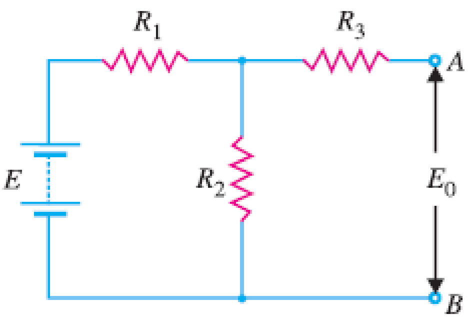
fig 4-13

(*i*) Finding E_0. This is the voltage between terminals *A* and *B* of the circuit when load R_L is removed. Fig. 1.25 shows the circuit with load removed. The voltage drop across R_2 is the desired voltage E_0.

$$\text{Current through } R_2 = \frac{E}{R_1 + R_2}$$

$$\therefore \quad \text{Voltage across } R_2,\ E_0 = \left(\frac{E}{R_1 + R_2}\right) R_2$$

Thus, voltage E_0 is determined



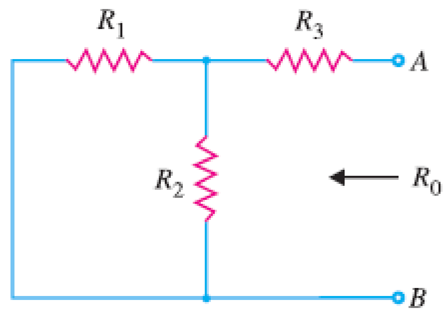


Fig 4-14 fig 4-15

Note: Solution can also be obtained by applying Kirchhoff's laws but it requires a lot of labour.

(*ii*) Finding R_0. This is the resistance between terminals *A* and *B* with load removed and e.m.f. reduced to zero (See Fig. 4-15).

$\therefore$ Resistance between terminals *A* and *B* is

$$R_0 = \text{parallel combination of } R_1 \text{ and } R_2 \text{ in series with } R_3$$

$$= \frac{R_1 R_2}{R_1 + R_2} + R_3$$

Thus, the value of R_0 is determined. Once the values of E_0 and R_0 are determined, then the current through the load resistance R_L can be found out easily (Refer to Fig. 4-12).

Procedure for Finding Thevenin Equivalent Circuit

i. Open the two terminals (*i.e.* remove any load) between which you want to find Thevenin equivalent circuit.
ii. Find the open-circuit voltage between the two open terminals. It is called Thevenin voltage E_0.
iii. Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Thevenin resistance R_0.
iv. Connect E_0 and R_0 in series to produce Thevenin equivalent circuit between the two terminals under consideration.
v. Place the load resistor removed in step (*i*) across the terminals of the Thevenin equivalent circuit. The load current can now be calculated using only Ohm's law and it has the same value as the load current in the original circuit.

Example 4.8. Using Thévenin's theorem, find the current through 100 Ω resistance connected across terminals A and B in the circuit of Fig. 4-16

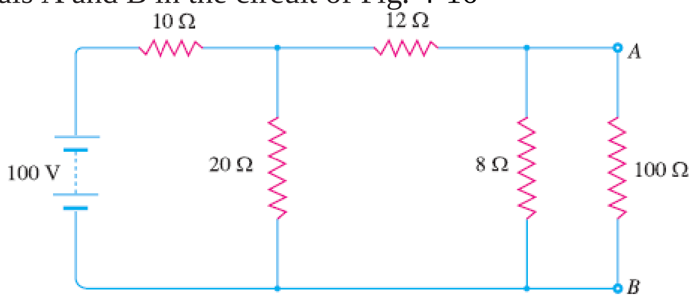


Fig 4-16

Solution.

(*i*) Finding E_0. It is the voltage across terminals *A* and *B* with 100 Ω resistance removed as shown in Fig. 4-17.

E_0 = (Current through 8 Ω) · 8 Ω = 2.5* · 8 = 20 V

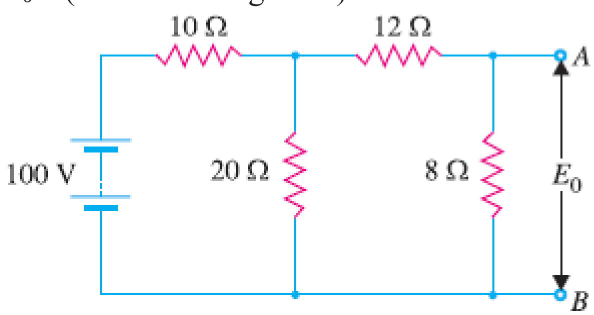


Fig 4-17

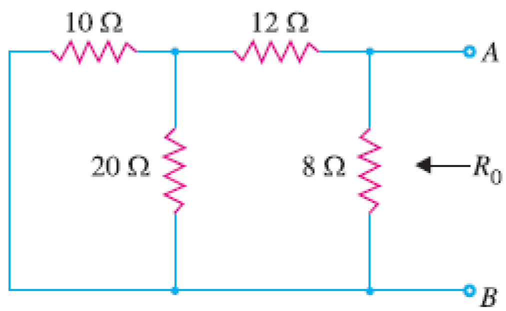


Fig 4-18

(*ii*) Finding R_0. It is the resistance between terminals *A* and *B* with 100 Ω removed and voltage source short circuited as shown in Fig. 4-18.

R_0 = Resistance looking in at terminals *A* and *B* in Fig. 4-18

$$= \frac{\left[\dfrac{10 \times 20}{10 + 20} + 12\right] 8}{\left[\dfrac{10 \times 20}{10 + 20} + 12\right] + 8}$$

$$= 5.6\ \Omega$$

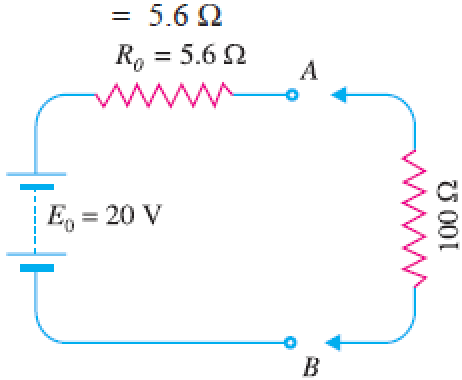


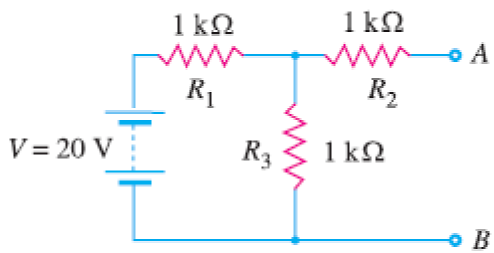
Fig 4-19

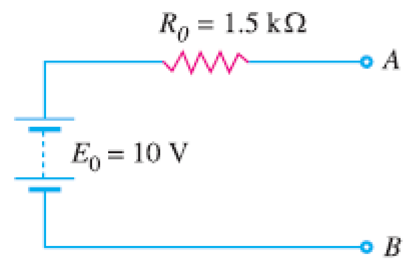
Therefore, Thévenin's equivalent circuit will be as shown in Fig. 4-19. Now, current through 100 Ω resistance connected across terminals *A* and *B* can be found by applying Ohm's law.

$$\text{Current through } 100\ \Omega \text{ resistor} = \frac{E_0}{R_0 + R_L} = \frac{20}{5.6 + 100} = \mathbf{0.19\ A}$$

Example 4.9. Find the Thévenin's equivalent circuit for Fig. 4

Solution. The Thévenin's voltage E_0 is the voltage across terminals *A* and *B*. This voltage is equal to the voltage across R_3. It is because terminals *A* and *B* are open circuited and there is no current flowing through R_2 and hence no voltage drop across it.





$$\therefore \qquad E_0 = \text{Voltage across } R_3$$

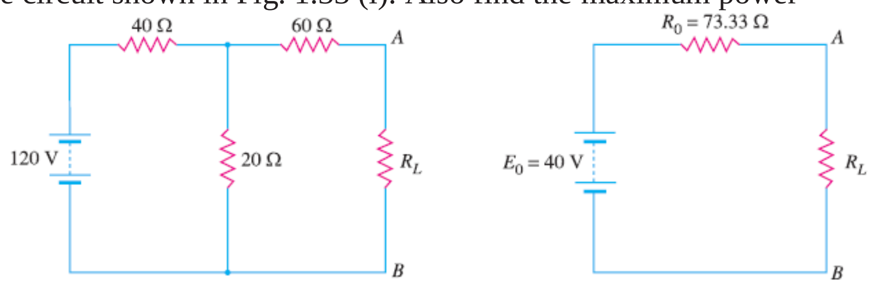
$$= \frac{R_3}{R_1 + R_3} \times V = \frac{1}{1+1} \times 20 = 10\ \text{V}$$

The Thévenin's resistance R_0 is the resistance measured between terminals *A* and *B* with no load (*i.e.* open at terminals *A* and *B*) and voltage source replaced by a short circuit.

$$\therefore \qquad R_0 = R_2 + \frac{R_1 R_3}{R_1 + R_3} = 1 + \frac{1 \times 1}{1+1} = 1.5\ \text{k}\Omega$$

Therefore, Thévenin's equivalent circuit will be as shown in Fig. 1.32.

Example 1.10. Calculate the value of load resistance R_L to which maximum power may be transferred from the circuit shown in Fig. 1.33 (i). Also find the maximum power



(*i*) (*ii*)

Solution. We shall first find Thévenin's equivalent circuit to the left of terminals *AB* in Fig. 1.33 (*i*).

$$E_0 = \text{Voltage across terminals } AB \text{ with } R_L \text{ removed}$$

$$= \frac{120}{40 + 20} \times 20 = 40\ \text{V}$$

$$R_0 = \text{Resistance between terminals } A \text{ and } B \text{ with } R_L \text{ removed and 120 V source replaced by a short}$$

$$= 60 + (40\ \Omega \,||\, 20\ \Omega) = 60 + (40 \times 20)/60 = 73.33\ \Omega$$

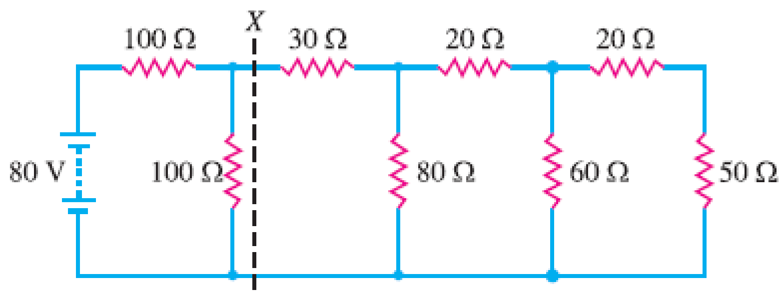
The Thévenin's equivalent circuit to the left of terminals *AB* in Fig. 1.33 (*i*) is E_0 (= 40 V) in series with R_0 (= 73.33 Ω). When R_L is connected between terminals *A* and *B*, the circuit becomes as shown in Fig. 1.33 (*ii*). It is clear that maximum power will be transferred when

$$R_L = R_0 = 73.33\ \Omega$$

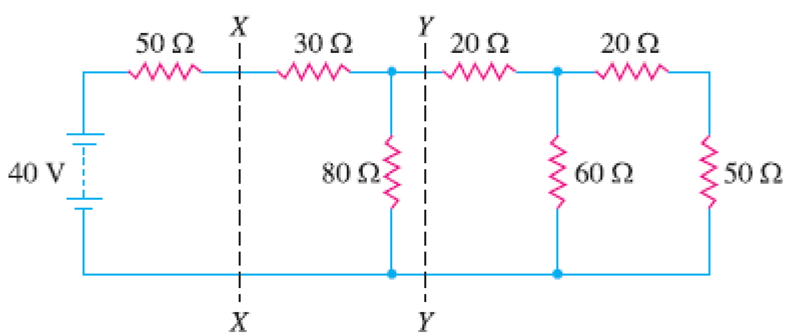
$$\text{Maximum power to load} = \frac{E_0^2}{4\,R_L} = \frac{(40)^2}{4 \times 73.33} = 5.45\ \text{W}$$

Comments. This shows another advantage of Thévenin's equivalent circuit of a network. Once Thévenin's equivalent resistance R_0 is calculated, it shows at a glance the condition for maximum power transfer. Yet Thevenin's equivalent circuit conveys another information. Thus referring to Fig. 1.33 (*ii*), the maximum voltage that can appear across terminals *A* and *B* is 40 V. This is not so obvious from the original circuit shown in Fig. 1.33 (*i*).

Example 1.11. Calculate the current in the 50 Ω resistor in the network shown in Fig. 1.34.



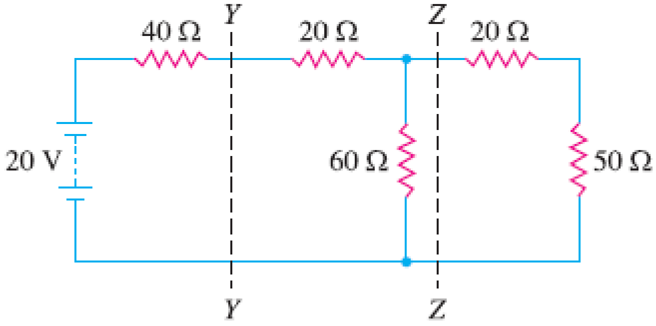
Solution. We shall simplify the circuit shown in Fig. 1.34 by the repeated use of Thévenin's theorem. We first find Thévenin's equivalent circuit to the left of *XX*.



$$E_0 = \frac{80}{100 + 100} \times 100 = 40\text{V}$$

$$R_0 = 100 \,||\, 100 = \frac{100 \times 100}{100 + 100} = 50\ \Omega$$

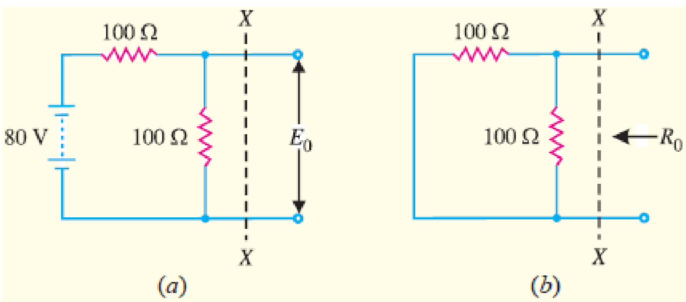
Therefore, we can replace the circuit to the left of *XX* in Fig. 1.34 by its Thévenin's equivalent circuit *viz.* E_0 (= 40V) in series with *R*0 (= 50 Ω). The original circuit of Fig. 1.34 then reduces to the one shown in Fig. 1.35. We shall now find Thévenin's equivalent circuit to left of *YY* in Fig. 1.35.



$$E_0' = \frac{40}{50+30+80} \times 80 = 20 \text{ V}$$

$$R_0' = (50+30) \parallel 80 = \frac{80 \times 80}{80+80} = 40\ \Omega$$

We can again replace the circuit to the left of *YY* in Fig. 1.35 by its Thévenin's equivalent circuit. Therefore, the original circuit reduces to that shown in Fig. 1.36.



$$E_0 = \text{Current in } 100\ \Omega \times 100\ \Omega = \frac{80}{100+100} \times 100 = 40\text{V}$$ See fig (a)

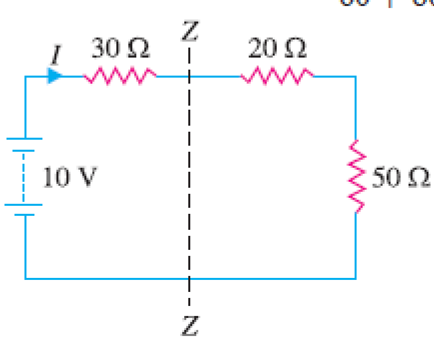
R_0 = Resistance looking in the open terminals in Fig. (*b*)

$$= 100 \parallel 100 = \frac{100 \times 100}{100+100} = 50\ \Omega$$

Using the same procedure to the left of *ZZ*, we have,

$$E_0'' = \frac{20}{40+20+60} \times 60 = 10\text{V}$$

$$R_0'' = (40+20) \parallel 60 = \frac{60 \times 60}{60+60} = 30\ \Omega$$

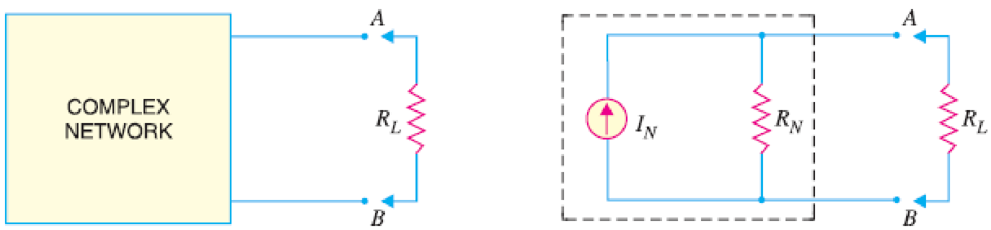


The original circuit then reduces to that shown in Fig. 1.37. By Ohm's law, current I in 50 Ω resistor is

$$I = \frac{10}{30+20+50} = 0.1 \text{ A}$$

1.15 Norton's Theorem

Fig. 1.38 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. The network in the box may contain any number of resistors and e.m.f. sources connected in any manner. But according to Norton, the entire circuit behind terminals *A* and *B* can be replaced by a current source of output I_N in parallel with a single resistance R_N as shown in Fig. 1.38 (*ii*). The value of I_N is determined as mentioned in Norton's theorem. The resistance R_N is the same as Thévenin's resistance R_0. Once Norton's equivalent circuit is determined [See Fig. 1.38 (*ii*)], then current through any load R_L connected across terminals *AB* can be readily obtained.



(*i*) (*ii*)

Hence Norton's theorem as applied to d.c. circuits may be stated as under: *Any network having two terminals A and B can be replaced by a current source of output I_N in parallel with a resistance R_N.*

(i) *The output I_N of the current source is equal to the current that would flow through AB when terminals A and B are short circuited.*

(ii) *The resistance R_N is the resistance of the network measured between terminals A and B with load (R_L) removed and sources of e.m.f. replaced by their internal resistances, if any.*

Norton's theorem is *converse* of Thévenin's theorem in that Norton equivalent circuit uses a current generator instead of voltage generator and resistance R_N (which is the same as R_0) in parallel with the generator instead of being in series with it.

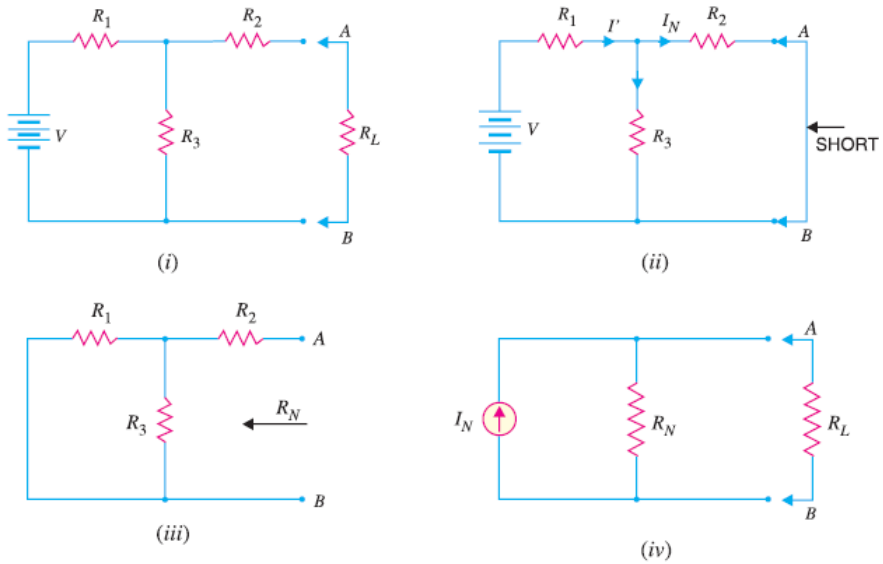
Illustration. Fig. 1.39 illustrates the application of Norton's theorem. As far as circuit behind terminals *AB* is concerned [See Fig. 1.39 (*i*)], it can be replaced by a current source of output I_N in parallel with a resistance R_N as shown in Fig. 1.39 (*iv*). The output I_N of the current generator is equal to the current that would flow through *AB* when terminals *A* and *B* are short-circuited as shown in Fig. 1.39 (*ii*). The load R_2 on the source when terminals *AB* are short-circuited is given by:

$$R' = R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

$$\text{Source current, } I' = \frac{V}{R'} = \frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$\text{Short-circuit current, } I_N = \text{Current in } R_2 \text{ in Fig. 1.39 } (ii)$$

$$= I' \times \frac{R_3}{R_2 + R_3} = \frac{V R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$



To find R_N, remove the load R_L and replace the voltage source by a short circuit because its resistance is assumed zero [See Fig. 1.39 (*iii*)].

$\therefore$ R_N = Resistance at terminals *AB* in Fig. 1.39 (*iii*).

$$= R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Thus the values of I_N and R_N are known. The Norton equivalent circuit will be as shown in Fig. 1.39 (*iv*).

1.16 Procedure for Finding Norton Equivalent Circuit

(i) Open the two terminals (*i.e.* remove any load) between which we want to find Norton equivalent circuit.

(ii) Put a short-circuit across the terminals under consideration. Find the short-circuit current flowing in the short circuit. It is called Norton current I_N.

(iii) Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Norton's resistance R_N. It is easy to see that $R_N = R_0$.

(iv) Connect I_N and R_N in parallel to produce Norton equivalent circuit between the two terminals under consideration.

(v) Place the load resistor removed in step (*i*) across the terminals of the Norton equivalent circuit. The load current can now be calculated by using current-divider rule. This load current will be the same as the load current in the original circuit.

Example 1.12. Using Norton's theorem, find the current in 8 Ω resistor in the network shown in Fig. 1.40 (i).

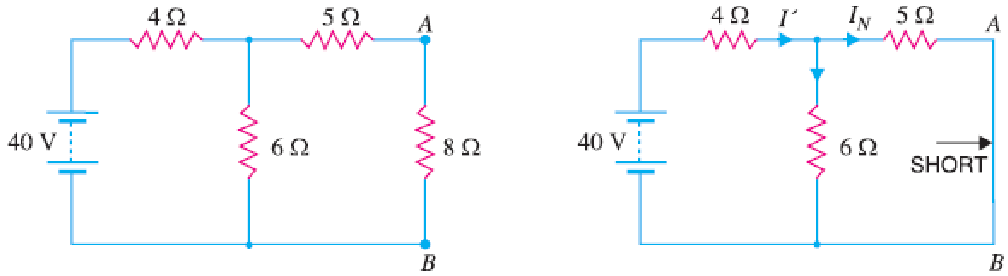
Solution. We shall reduce the network to the left of *AB* in Fig. 1.40 (*i*) to Norton's equivalent circuit. For this purpose, we are required to find I_N and R_N.

(*i*) With load (*i.e.*, 8 Ω) removed and terminals *AB* short circuited [See Fig. 1.40 (*ii*)], the current that flows through *AB* is equal to I_N. Referring to Fig. 1.40 (*ii*),

$$\text{Load on the source} = 4\,\Omega + 5\,\Omega \parallel 6\,\Omega$$

$$= 4 + \frac{5 \times 6}{5 + 6} = 6.727\,\Omega$$

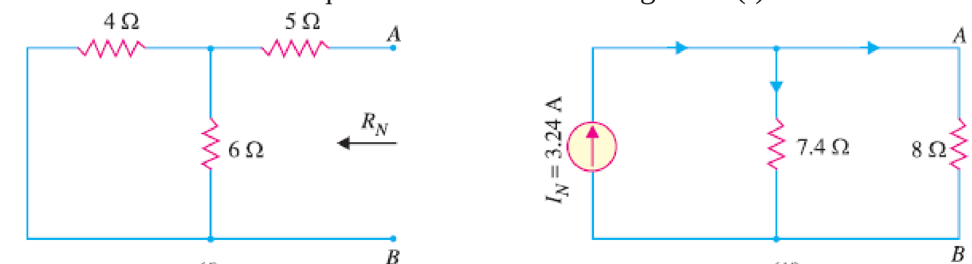
Source current, $I' = 40/6.727 = 5.94$ A



(*i*) (*ii*)

$$\therefore \quad \text{Short-circuit current in } AB,\ I_N = I' \times \frac{6}{6+5} = 5.94 \times 6/11 = 3.24 \text{ A}$$

(*ii*) With load (*i.e.*, 8 Ω) removed and battery replaced by a short (since its internal resistance is assumed zero), the resistance at terminals *AB* is equal to R_N as shown in Fig. 1.41 (*i*).



(*i*) (*ii*)

$$R_N = 5\,\Omega + 4\,\Omega \parallel 6\,\Omega = 5 + \frac{4 \times 6}{4+6} = 7.4\,\Omega$$

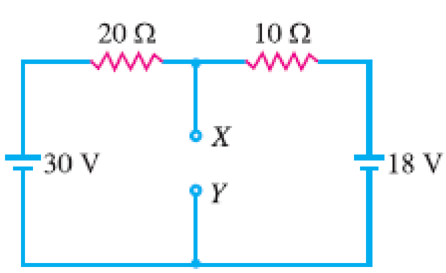
The Norton's equivalent circuit behind terminals *AB* is I_N (= 3.24 A) in parallel with R_N (= 7.4 Ω). When load (*i.e.*, 8 Ω) is connected across terminals *AB*, the circuit becomes as shown in Fig. 1.41 (*ii*). The current source is supplying current to two resistors 7.4 Ω and 8 Ω in parallel.

$$\therefore \quad \text{Current in 8 } \Omega \text{ resistor} = 3.24 \times \frac{7.4}{8+7.4} = \mathbf{1.55\ A}$$

Example 1.13. Find the Norton equivalent circuit at terminals X – Y in Fig. 1.42.

Solution. We shall first find the Thevenin equivalent circuit and then convert it to an equivalent current source. This will then be Norton equivalent circuit.

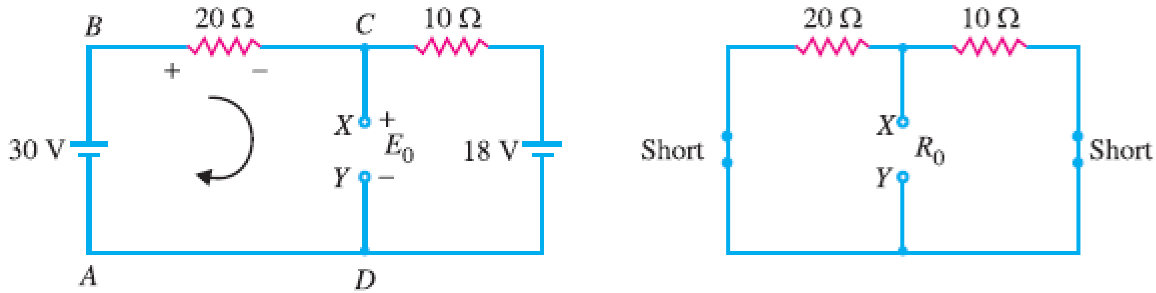
Finding Thevenin Equivalent circuit. To find E_0, refer to Fig. 1.43 (*i*). Since 30 V and 18 V sources are in opposition, the circuit current *I* is given by:



$$I = \frac{30 - 18}{20 + 10} = \frac{12}{30} = 0.4 \text{ A}$$

Applying Kirchhoff's voltage law to loop *ABCDA*, we have,

$30 - 20 \times 0.4 - E_0 = 0 \qquad \therefore \; E_0 = 30 - 8 = 22 \text{ V}$



(i) (ii)

To find R_0, we short both voltage sources as shown in Fig. 1.43 (*ii*). Notice that 10 Ω and 20 Ω resistors are then in parallel.

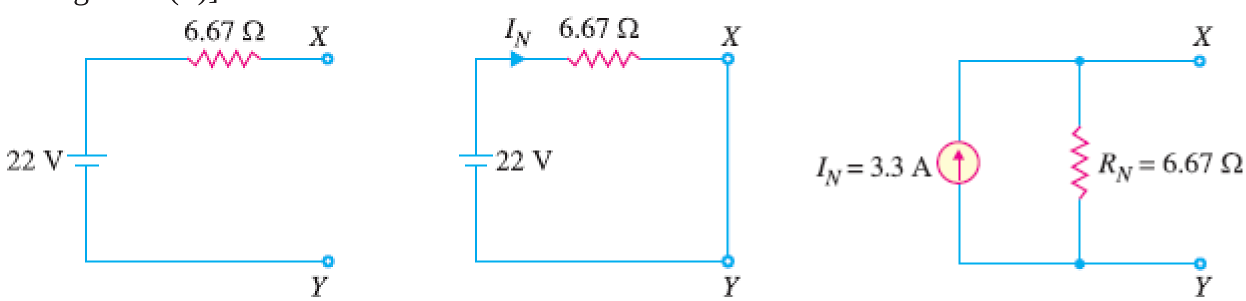
$$\therefore \qquad R_0 = 10\ \Omega \,||\, 20\ \Omega = \frac{10 \times 20}{10 + 20} = 6.67\Omega$$

Therefore, Thevenin equivalent circuit will be as shown in Fig. 1.44 (*i*). Now it is quite easy to convert it into equivalent current source.

$$I_N = \frac{E_0}{R_0} = \frac{22}{6.67} = 3.3\text{A}$$

$$R_N = R_0 = 6.67\ \Omega$$

[See Fig. 1.44 (*ii*)]



(i) (ii) (iii)

Fig. 1.44 (*iii*) shows Norton equivalent circuit. Observe that the Norton equivalent resistance has the same value as the Thevenin equivalent resistance. Therefore, R_N is found exactly the same way.

Example 1.14. *Show that when Thevenin's equivalent circuit of a network is converted into Norton's equivalent circuit,* $I_N = E_0 / R_0$ *and* $R_N = R_0$*. Here* E_0 *and* R_0 *are Thevenin voltage and Thevenin resistance respectively*.

Solution. Fig. 1.45 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. Thévenin's equivalent circuit of this network will be as shown in Fig. 1.45 (*ii*). To find Norton's equivalent circuit, we are to find I_N and R_N. Referring to Fig. 1.45 (*ii*),

I_N = Current flowing through short-circuited *AB* in Fig. 1.45 (*ii*)

$= E_0/R_0$

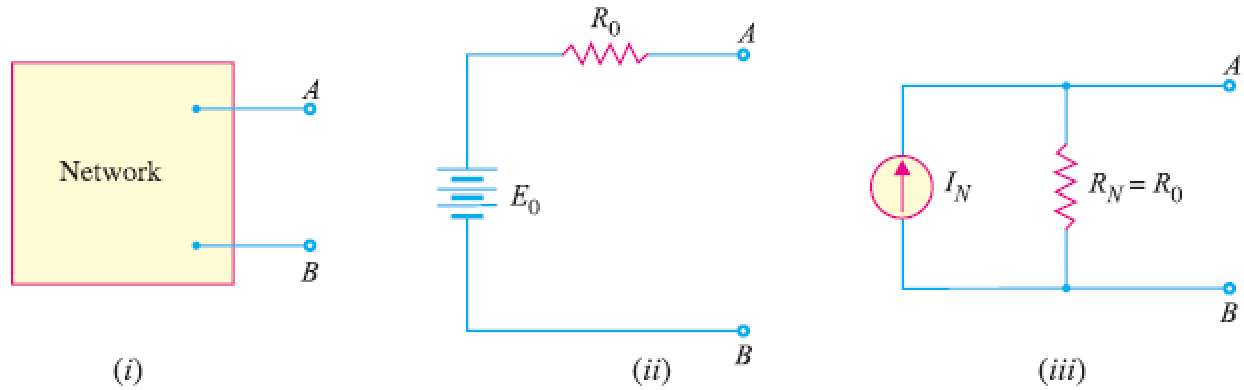
R_N = Resistance at terminals *AB* in Fig. 1.45 (*ii*)

$= R_0$

Fig. 1.45 (*iii*) shows Norton's equivalent circuit. Hence we arrive at the following two important conclusions:

(i) To convert Thévenin's equivalent circuit into Norton's equivalent circuit,

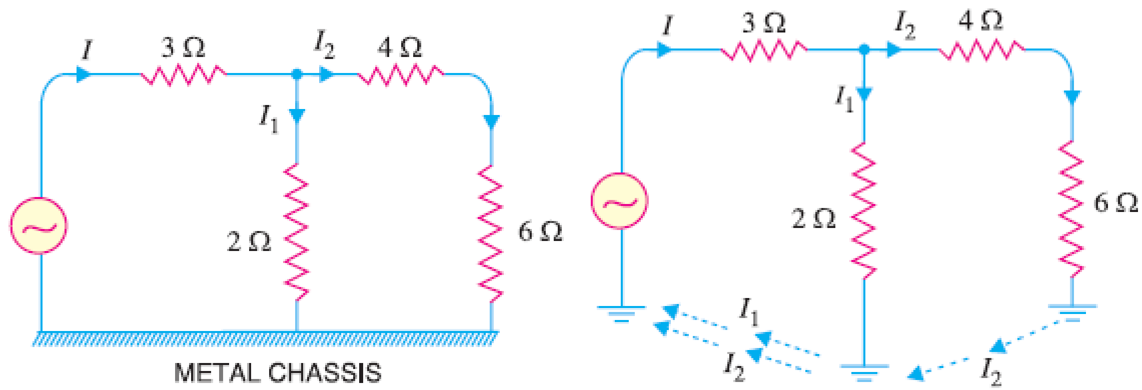
$I_N = E_0/R_0$; $R_N = R_0$



(ii) To convert Norton's equivalent circuit into Thevenin's equivalent circuit, $E_0 = I_N R_N$; $R_0 = R_N$

1.17 Chassis and Ground

It is the usual practice to mount the electronic components on a metal base called *chassis*. For example, in Fig. 1.46, the voltage source and resistors are connected to the chassis. As the resistance of chassis is very low, therefore, it provides a conducting path and may be considered as a piece of wire.



It is customary to refer to the chassis as *ground*. Fig. 1.47 shows the symbol for chassis. It may be seen that all points connected to chassis are shown as grounded and represent the same potential. The adoption of this scheme (*i.e.* showing points of same potential as grounded) often simplifies the electronic circuits. In our further discussion, we shall frequently use this scheme.

EXAMPLE 4.8 Obtain the The´venin and Norton equivalent circuits for the active network in Fig. 4-13(a). With terminals *ab* open, the two sources drive a clockwise current through the 3-Ω and 6-Ω resistors [Fig. 4-13(b)].

$$I = \frac{20+10}{3+6} = \frac{30}{9}\ \text{A}$$

Since no current passes through the upper right 3-Ω resistor, the The′venin′s voltage can be taken from either active branch:

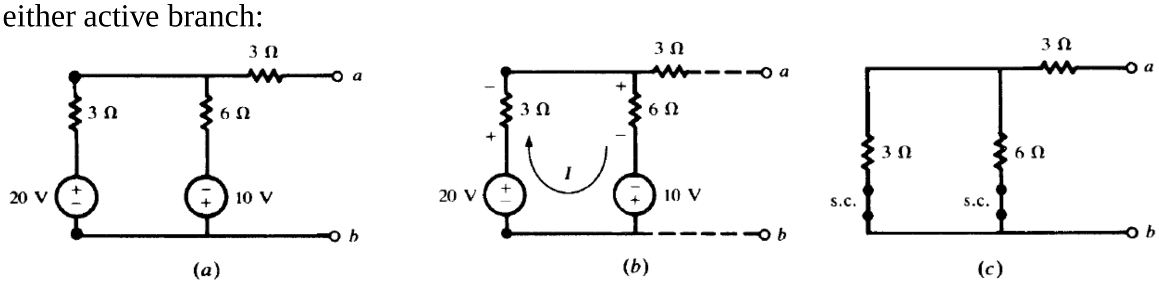


Fig. 4-21

$$V_{ab} = V' = 20 - \left(\frac{30}{9}\right)(3) = 10\,\text{V}$$

$$V_{ab} = V' = \left(\frac{30}{9}\right)6 - 10 = 10\,\text{V}$$

The resistance R' can be obtained by shorting out the voltage sources [Fig. 4.13(c)] and finding the equivalent resistance of this network at terminals ab:

$$R' = 3 + \frac{(3)(6)}{9} = 5\,\Omega$$

When a short circuit is applied to the terminals, current $I_{s.c}$ results from the two sources. Assuming that it runs through the short from a to b, we have, by superposition,

$$I_{s.c.} = I' = \left(\frac{6}{6+3}\right)\left[\frac{20}{3+\frac{(3)(6)}{9}}\right] - \left(\frac{3}{3+3}\right)\left[\frac{10}{6+\frac{(3)(3)}{6}}\right] = 2\,\text{A}$$

Figure 4-14 shows the two equivalent circuits. In the present case, V', R', and I' were obtained independently. Since they are related by Ohm's law, any two may be used to obtain the third.

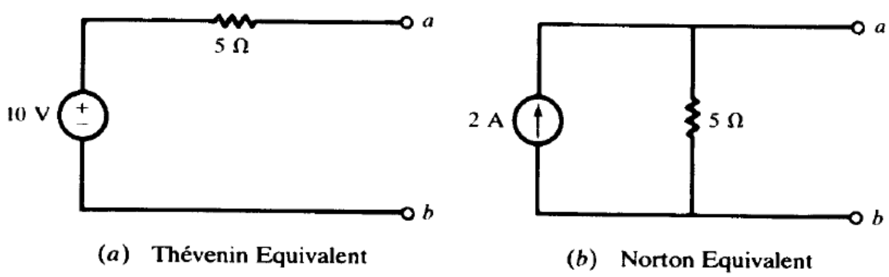


Fig 4-22

The usefulness of The´venin and Norton equivalent circuits is clear when an active network is to be examined under a number of load conditions, each represented by a resistor. This is suggested in Fig. 4-15, where it is evident that the resistors $R_1; R_2; \ldots, R_n$ can be connected one at a time, and the resulting current and power readily obtained. If this were attempted in the original circuit using, for example, network reduction, the task would be very tedious and time-consuming.

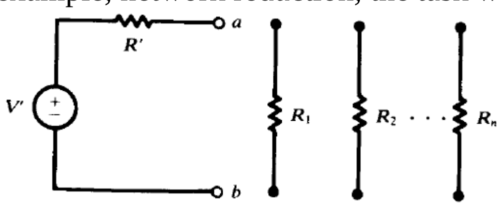


Fig. 4-23

4.10 MAXIMUM POWER TRANSFER THEOREM

When load is connected across a voltage source, power is transferred from the source to the load. The amount of power transferred will depend upon the load resistance. If load resistance R_L is made equal to the internal resistance R_i of the source, then maximum power is transferred to the load R_L. This is known as *maximum power transfer theorem* and can be stated as follows:

Maximum power is transferred from a source to a load when the load resistance is made equal to the internal resistance of the source.

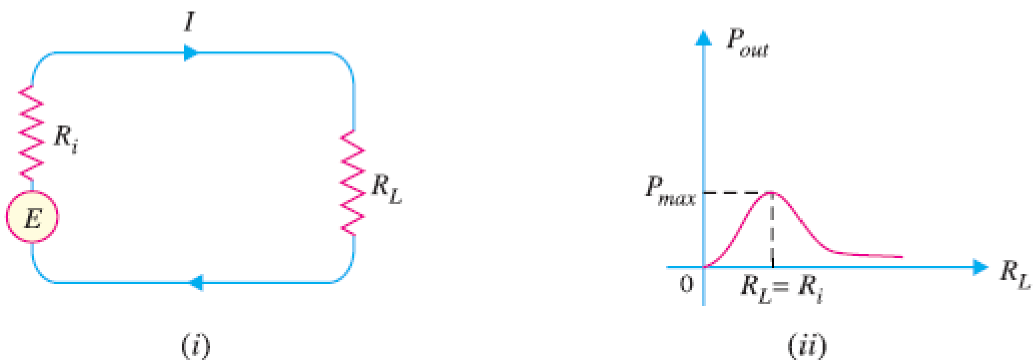
This applies to d.c. as well as a.c. power.

To prove this theorem mathematically, consider a voltage source of generated voltage E and internal resistance R_i and delivering power to a load resistance R_L [See Fig. 1.20 (*i*)]. The current I flowing through the circuit is given by:

$$I = \frac{E}{R_L + R_i}$$

Power delivered to the load,

$$P = I^2 R_L = \left(\frac{E}{R_L + R_i}\right)^2 R_L$$



(*i*) (*ii*)

Fig 4-24

For a given source, generated voltage E and internal resistance R_i are constant. Therefore, power delivered to the load depends upon R_L. In order to find the value of R_L for which the value of P is maximum, it is necessary to differentiate eq. (*i*) w.r.t. R_L and set the result equal to zero.

Thus, $$\frac{dP}{dR_L} = E^2\left[\frac{(R_L + R_i)^2 - 2R_L(R_L + R_i)}{(R_L + R_i)^4}\right] = 0$$

or $$(R_L + R_i)^2 - 2R_L(R_L + R_i) = 0$$

or $$(R_L + R_i)(R_L + R_i - 2R_L) = 0$$

or $$(R_L + R_i)(R_i - R_L) = 0$$

Note: As power is concerned with resistance only, therefore, this is true for both a.c. and d.c. power.

Since $(R_L + R_i)$ cannot be zero,

$\therefore$ $$R_i - R_L = 0$$

or $$R_L = R_i$$

i.e. *Load resistance* = *Internal resistance*

Thus, for maximum power transfer, load resistance R_L must be equal to the internal resistance R_i of the source.

Under such conditions, the load is said to be *matched* to the source. Fig. 1.20 (*ii*) shows a graph of power delivered to R_L as a function of R_L. It may be mentioned that efficiency of maximum power transfer is *50% as one-half of the total generated power is dissipated in the internal resistance R_i of the source.

Applications. Electric power systems never operate for maximum power transfer because of low efficiency and high voltage drops between generated voltage and load. However, in the electronic circuits, maximum power transfer is usually desirable. For instance, in a public address system, it is desirable to have load (*i.e.* speaker) "matched" to the amplifier so that there is maximum transference of power from the amplifier to the speaker. In such situations, efficiency is sacrificed at the cost of high power transfer.

Example 1.5. *A generator develops 200 V and has an internal resistance of 100 Ω. Find the power delivered to a load of (i) 100 Ω (ii) 300 Ω. Comment on the result.*

Solution.

Generated voltage, $E = 200\ \text{V}$

Internal resistance, $R_i = 100\ \Omega$

(*i*) When load $R_L = 100\ \Omega$

$$\text{Load current, } I = \frac{E}{R_L + R_i} = \frac{200}{100 + 100} = 1\ \text{A}$$

$$\therefore \quad \text{Power delivered to load} = I^2 R_L = (1)^2 \times 100 = 100 \text{ watts}$$

$$\text{Total power generated} = I^2 (R_L + R_i) = 1^2 (100 + 100) = 200 \text{ watts}$$

Thus, out of 200 W power developed by the generator, only 100W has reached the load *i.e.* efficiency is 50% only.

(*ii*) When load $R_L = 300\ \Omega$

$$\text{Load current, } I = \frac{E}{R_L + R_i} = \frac{200}{300 + 100} = 0.5\ \text{A}$$

$$\text{Power delivered to load} = I^2 R_L = (0.5)^2 \times 300 = 75 \text{ watts}$$

$$\text{Total power generated} = I^2 (R_L + R_i) = (0.5)^2 (300 + 100) = 100 \text{ watts}$$

Thus, out of 100 watts of power produced by the generator, 75 watts is transferred to the load *i.e.* efficiency is 75%.

Comments. Although in case of $R_L = R_i$, a large power (100 W) is transferred to the load, but there is a big wastage of power in the generator. On the other hand, when R_L is ***not*** equal to R_i, the power transfer is less (75 W) but smaller part is wasted in the generator *i.e.* efficiency is high. Thus, it depends upon a particular situation as to what the load should be. If we want to transfer maximum power (*e.g.* in amplifiers) irrespective of efficiency, we should make $R_L = R_i$. However, if efficiency is more important (*e.g.* in power systems), then internal resistance of the source should be considerably smaller than the load resistance.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{I^2 R_L}{I^2 (R_L + R_i)}$$

$$= R_L / 2\,R_L = 1/2 = 50\% \qquad (\because R_L = R_i)$$

Electronic devices develop small power. Therefore, if too much efficiency is sought, a large number of such devices will have to be connected in series to get the desired output. This will distort the output as well as increase the cost and size of equipment.

Example 1.6. *An audio amplifier produces an alternating output of 12 V before the connection to a load. The amplifier has an equivalent resistance of 15 Ω at the output. What resistance the load need to have to produce maximum power ? Also calculate the power output under this condition.*

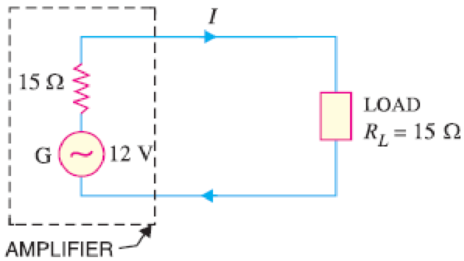


Fig 4-25

Solution. In order to produce maximum power, the load (*e.g.* a speaker) should have a resistance of 15 Ω to match the amplifier. The equivalent circuit is shown in Fig. 1.21.

∴ Load required, $R_L = \mathbf{15\ \Omega}$

$$\text{Circuit current, } I = \frac{V}{R_T} = \frac{12}{15+15} = 0.4\text{ A}$$

Power delivered to load, $P = I^2 R_L = (0.4)^2 \times 15 = \mathbf{2.4\ W}$

Example 1.7. *For the a.c. generator shown in Fig. 1.22 (i), find (i) the value of load so that maximum power is transferred to the load (ii) the value of maximum power.*

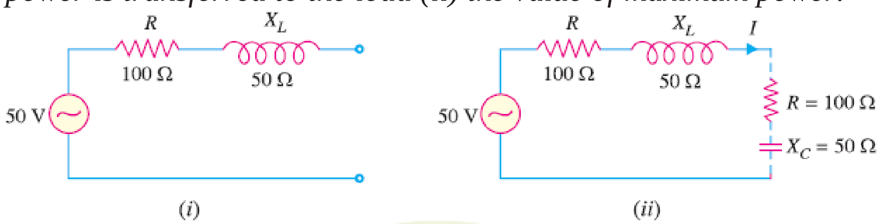


Fig 4-26

Solution.

(*i*) In a.c. system, maximum power is delivered to the load impedance (Z_L) when load impedance is conjugate of the internal impedance (Z_i) of the source. Now in the problem, $Z_i = (100 + j50)\Omega$. For maximum power transfer, the load impedance should be conjugate of internal impedance *i.e.* Z_L should be $(100 - j50)\ \Omega$. This is shown in dotted line in Fig. 1.22 (*ii*).

∴ $Z_L = \mathbf{(100 - j50)\ \Omega}$

(*ii*) Total impedance, $Z_T = Z_i + Z_L = (100 + j50) + (100 - j50) = 200\ \Omega$*

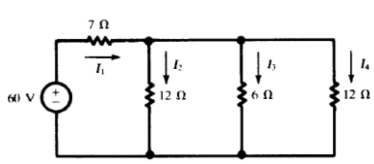
$$\text{Circuit current, } I = \frac{V}{Z_T} = \frac{50}{200} = 0.25\text{ A}$$

Maximum power transferred to the load $= I^2 R_L = (0.25)^2 \times 100 = \mathbf{6.25\ W}$

Note: Note that by making internal impedance and load impedance conjugate, the reactive terms cancel. The circuit then consists of internal and external resistances only. This is quite logical because power is only consumed in resistances as reactances (X_L or X_C) consume no power.

Solved Problems

1. 4.1 Use branch currents in the network shown in Fig. below to find the current supplied by the 60-V source.



KVL and KCL give:

$$I_2(12) = I_3(6) \qquad (10)$$
$$I_2(12) = I_4(12) \qquad (11)$$
$$60 = I_1(7) + I_2(12) \qquad (12)$$
$$I_1 = I_2 + I_3 + I_4 \qquad (13)$$

Substituting (*10*) and (*11*) in (*13*),

$$I_1 = I_2 + 2I_2 + I_2 = 4I_2 \qquad (14)$$

Now (*14*) is substituted in (*12*):

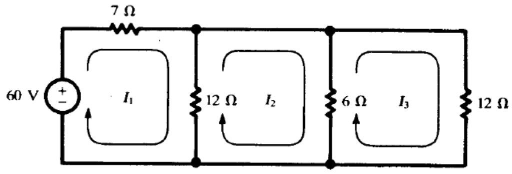
$$60 = I_1(7) + \tfrac{1}{4}I_1(12) = 10I_1 \quad \text{or} \quad I_1 = 6\text{ A}$$

2. 4.4 In Problem 2, obtain Rinput;1 and use it to calculate I1.

$$R_{\text{input},1} = \frac{\Delta_R}{\Delta_{11}} = \frac{2880}{\begin{vmatrix} 18 & -6 \\ -6 & 18 \end{vmatrix}} = \frac{2880}{288} = 10\,\Omega$$

Then $$I_1 = \frac{60}{R_{\text{input},1}} = \frac{60}{10} = 6\text{ A}$$

3. 4.2 Solve Problem 1 by the mesh current method.



Applying KVL to each mesh (see Fig. 4-18) results in

$$60 = 7I_1 + 12(I_1 - I_2)$$
$$0 = 12(I_2 - I_1) + 6(I_2 - I_3)$$
$$0 = 6(I_3 - I_2) + 12I_3$$

Rearranging terms and putting the equations in matrix form,

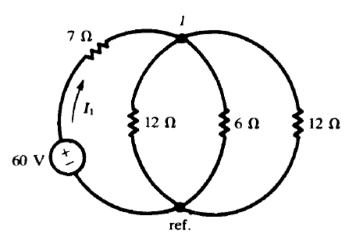
$$\begin{aligned} 19I_1 - 12I_2 \qquad &= 60 \\ -12I_1 + 18I_2 - 6I_3 &= 0 \\ -6I_2 + 18I_3 &= 0 \end{aligned} \qquad \text{or} \qquad \begin{bmatrix} 19 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

Using Cramer's rule to find I_1,

$$I_1 = \begin{vmatrix} 60 & -12 & 0 \\ 0 & 18 & -6 \\ 0 & -6 & 18 \end{vmatrix} \div \begin{vmatrix} 19 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 18 \end{vmatrix} = 17\,280 \div 2880 = 6\text{ A}$$

4. 4.3 Solve the network of Problems 1 and 2 by the node voltage method. See Fig. BELOW
   With two principal nodes, only one equation is necessary.

$$\frac{V_1 - 60}{7} + \frac{V_1}{12} + \frac{V_1}{6} + \frac{V_1}{12} = 0$$



from which $V_1 = 18$ V. Then,

$$I_1 = \frac{60 - V_1}{7} = 6\text{ A}$$

5. 4.5 Obtain $R_{transfer;12}$ and $R_{transfer;13}$ for the network of Problem 2 and use them to calculate I_2 and I_3.

The cofactor of the 1,2-element in Δ_R must include a negative sign:

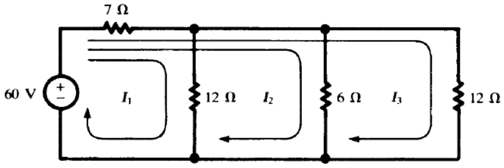
$$\Delta_{12} = (-1)^{1+2}\begin{vmatrix} -12 & -6 \\ 0 & 18 \end{vmatrix} = 216 \qquad R_{\text{transfer},12} = \frac{\Delta_R}{\Delta_{12}} = \frac{2880}{216} = 13.33\,\Omega$$

Then, $I_2 = 60/13.33 = 4.50\,\text{A}$.

$$\Delta_{13} = (-1)^{1+3}\begin{vmatrix} -12 & 18 \\ 0 & -6 \end{vmatrix} = 72 \qquad R_{\text{transfer},13} = \frac{\Delta_R}{\Delta_{13}} = \frac{2880}{72} = 40\,\Omega$$

Then, $I_3 = 60/40 = 1.50\,\text{A}$.

6. Solve Problem 1 by use of the loop currents indicated in Fig. BELOW. The elements in the matrix form of the equations are obtained by inspection, following the rules of Section 4.2.



$$\begin{bmatrix} 19 & 7 & 7 \\ 7 & 13 & 7 \\ 7 & 7 & 19 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 60 \\ 60 \end{bmatrix}$$

Thus,

$$\Delta_R = \begin{bmatrix} 19 & 7 & 7 \\ 7 & 13 & 7 \\ 7 & 7 & 19 \end{bmatrix} = 2880$$

Notice that in Problem 4.2, too, $\Delta_R = 2880$, although the elements in the determinant were different. *All valid sets of meshes or loops yield the same numerical value for* Δ_R. The three numerator determinants are

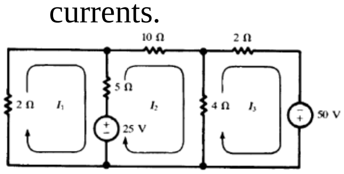
$$N_1 = \begin{vmatrix} 60 & 7 & 7 \\ 60 & 13 & 7 \\ 60 & 7 & 19 \end{vmatrix} = 4320 \qquad N_2 = 8642 \qquad N_3 = 4320$$

Consequently,

$$I_1 = \frac{N_1}{\Delta_R} = \frac{4320}{2880} = 1.5\,\text{A} \qquad I_2 = \frac{N_2}{\Delta_R} = 3\,\text{A} \qquad I_3 = \frac{N_3}{\Delta_R} = 1.5\,\text{A}$$

The current supplied by the 60-V source is the sum of the three loop currents, $I_1 + I_2 + I_3 = 6\,\text{A}$.

7. Write the mesh current matrix equation for the network of Fig. BELOW by inspection, and solve for the currents.



$$\begin{bmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -25 \\ 25 \\ 50 \end{bmatrix}$$

Solving,

$$I_1 = \begin{vmatrix} -25 & -5 & 0 \\ 25 & 19 & -4 \\ 50 & -4 & 6 \end{vmatrix} \div \begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix} = (-700) \div 536 = -1.31\,\text{A}$$

Similarly,

$$I_2 = \frac{N_2}{\Delta_R} = \frac{1700}{536} = 3.17\,\text{A} \qquad I_3 = \frac{N_3}{\Delta_R} = \frac{5600}{536} = 10.45\,\text{A}$$

8. Solve Problem 7 by the node voltage method.

The circuit has been redrawn in Fig. 4-22, with two principal nodes numbered *1* and *2* and the third chosen as the reference node. By KCL, the net current out of node *1* must equal zero.

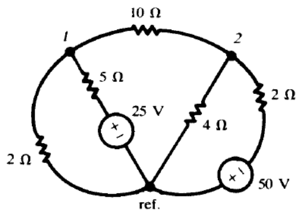


Fig. 4-22

$$\frac{V_1}{2}+\frac{V_1-25}{5}+\frac{V_1-V_2}{10}=0$$

Similarly, at node *2*,

$$\frac{V_2-V_1}{10}+\frac{V_2}{4}+\frac{V_2+50}{2}=0$$

Putting the two equations in matrix form,

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{5}+\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{4}+\frac{1}{2} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{vmatrix} 5 \\ -25 \end{vmatrix}$$

The determinant of coefficients and the numerator determinants are

$$\Delta=\begin{vmatrix} 0.80 & -0.10 \\ -0.10 & 0.85 \end{vmatrix}=0.670$$

$$N_1=\begin{vmatrix} 5 & -0.10 \\ -25 & 0.85 \end{vmatrix}=1.75 \qquad N_2=\begin{vmatrix} 0.80 & 5 \\ -0.10 & -25 \end{vmatrix}=-19.5$$

From these,

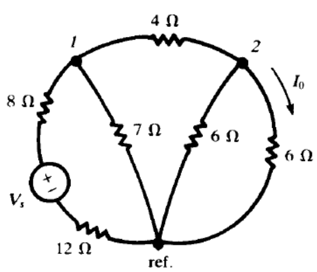
$$V_1=\frac{1.75}{0.670}=2.61\text{ V} \qquad V_2=\frac{-19.5}{0.670}=-29.1\text{ V}$$

In terms of these voltages, the currents in Fig. 4-21 are determined as follows:

$$I_1=\frac{-V_1}{2}=-1.31\text{ A} \qquad I_2=\frac{V_1-V_2}{10}=3.17\text{ A} \qquad I_3=\frac{V_2+50}{2}=10.45\text{ A}$$

9. For the network shown in Fig. below, find Vs which makes I_0 = 7.5 mA.
   The node voltage method will be used and the matrix form of the equations written by inspection.

$$\begin{bmatrix} \frac{1}{20}+\frac{1}{7}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{6}+\frac{1}{6} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} V_s/20 \\ 0 \end{bmatrix}$$



Solving for V_2,

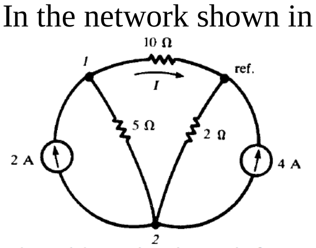
$$V_2=\frac{\begin{vmatrix} 0.443 & V_s/20 \\ -0.250 & 0 \end{vmatrix}}{\begin{vmatrix} 0.443 & -0.250 \\ -0.250 & 0.583 \end{vmatrix}}=0.0638V_s$$

Then

$$7.5\times10^{-3}=I_0=\frac{V_2}{6}=\frac{0.0638V_s}{6}$$

from which $V_s=0.705\text{ V}$.

10. In the network shown in Fig. below, find the current in the 10-Ω resistor.



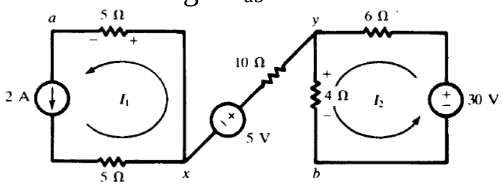
The nodal equations in matrix form are written by inspection.

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{10} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{2} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$V_1=\frac{\begin{vmatrix} 2 & -0.20 \\ -6 & 0.70 \end{vmatrix}}{\begin{vmatrix} 0.30 & -0.20 \\ -0.20 & 0.70 \end{vmatrix}}=1.18\text{ V}$$

Then, $I=V_1/10=0.118\text{ A}$.

11. Find the voltage V_{ab} in the network shown in Fig below

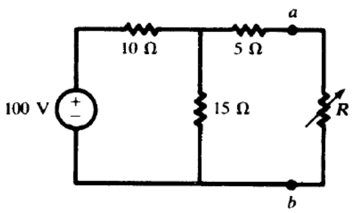


The two closed loops are independent, and no current can pass through the connecting branch.

$$I_1 = 2\,\text{A} \qquad I_2 = \frac{30}{10} = 3\,\text{A}$$

$$V_{ab} = V_{ax} + V_{xy} + V_{yb} = -I_1(5) - 5 + I_2(4) = -3\,\text{V}$$

12. Find the value of the adjustable resistance R which results in maximum power transfer across the terminals *ab* of the circuit shown in Fig. below.



First a Thévenin equivalent is obtained, with $V' = 60\,\text{V}$ and $R' = 11\,\Omega$. By Section 4.10, maximum power transfer occurs for $R = R' = 11\,\Omega$, with

$$P_{max} = \frac{V'^2}{4R'} = 81.82\,\text{W}$$