Implementação de funções booleanas

Quatro Passos

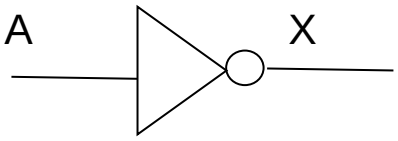
1. Identificar na tabela verdade as linhas que contém o valor 1 na coluna de resultado

2. Para tais linhas, fazer um AND considerando todas as variáveis. Usa-se o ponto para indicar AND entre as variáveis de entrada e, quando necessário, colocar os inversores (uma barra sobre a variável deve ser usada para indicar que seu valor é invertido, ou seja, igual a zero)

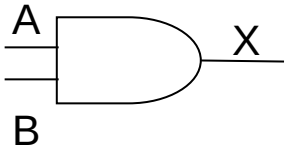
3. Fazer um OR de todos os termos do produto para obter a função X correspondente. Usa-se o sinal + para indicar OR.

4. Implementar o circuito equivalente à função obtida, usando os símbolos estudados. Para tanto, represente as variáveis de entrada do lado esquerdo e X do lado direito. Quando necessário, devem ser representados os inversores

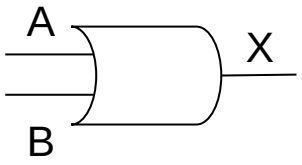
NOT



AND

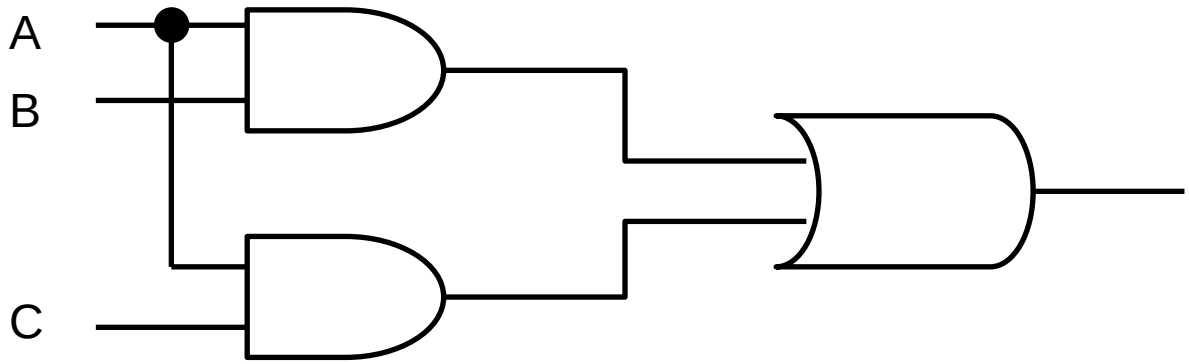


OR



Exercício

Dado o circuito a seguir, elabore a função equivalente e a tabela verdade.



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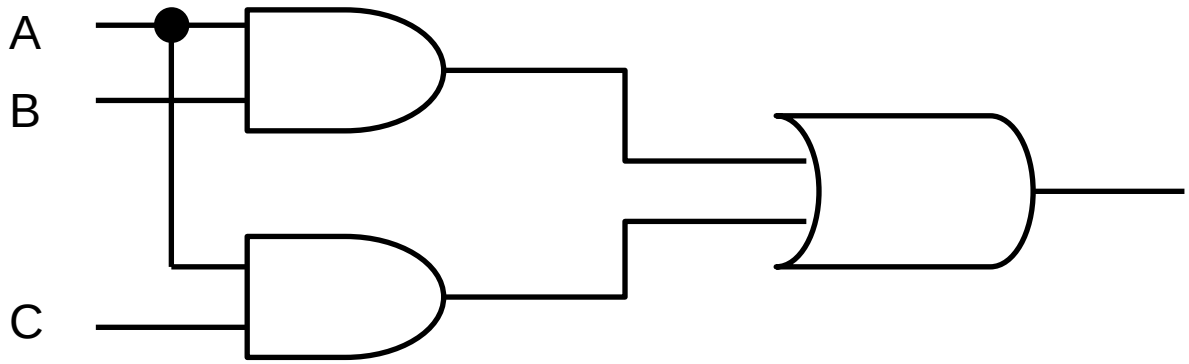
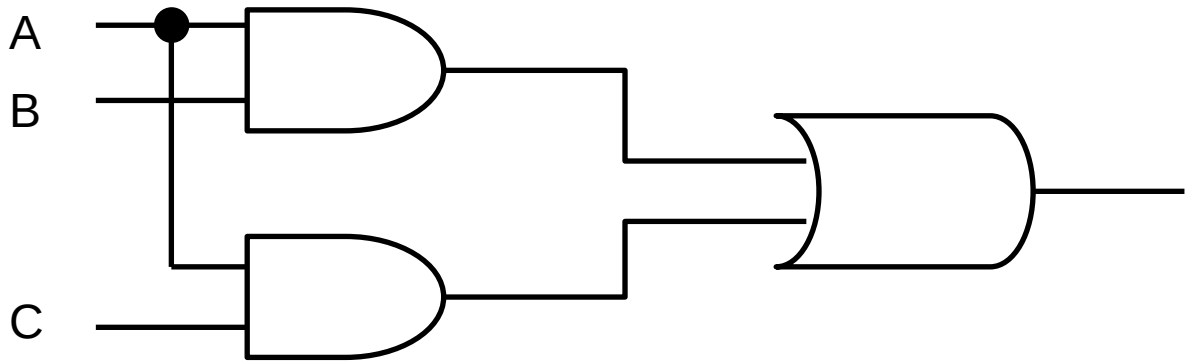


A.B

Exercício

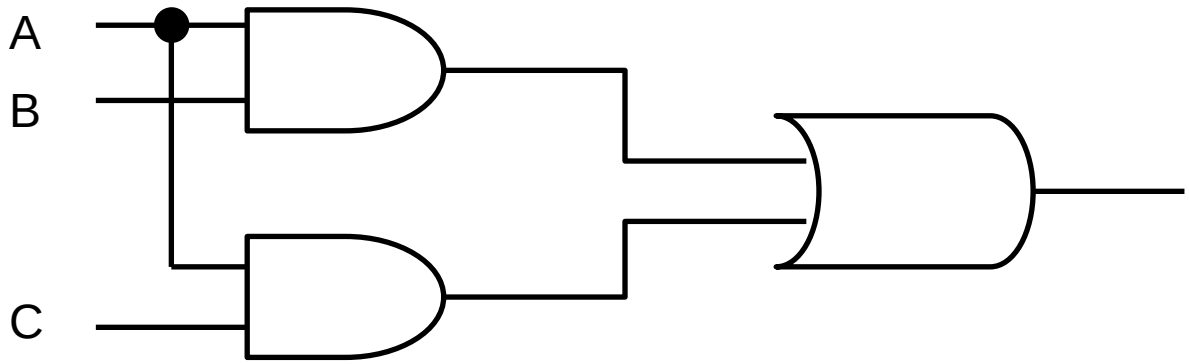
Dado o circuito a seguir, elabore a função equivalente e a tabela verdade.



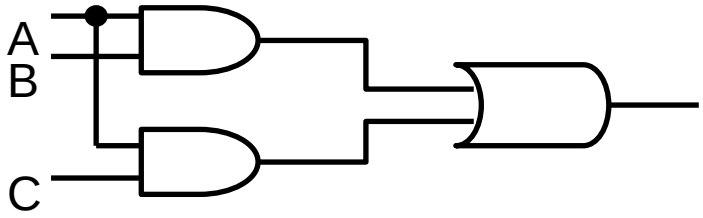
A.B A.C

Exercício

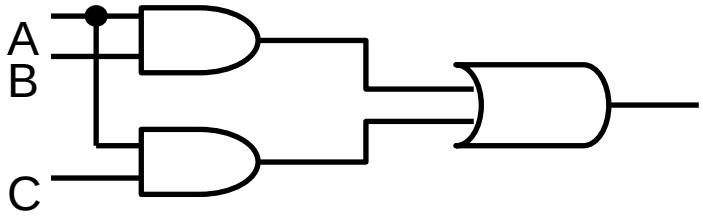
Dado o circuito a seguir, elabore a função equivalente e a tabela verdade.



A.B + A.C

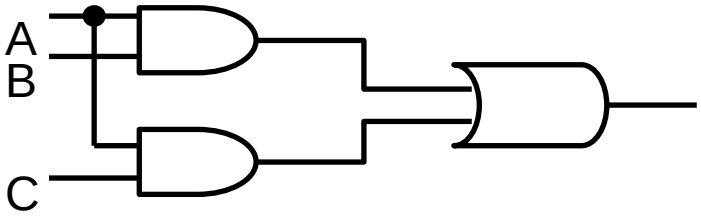


A.B + A.C



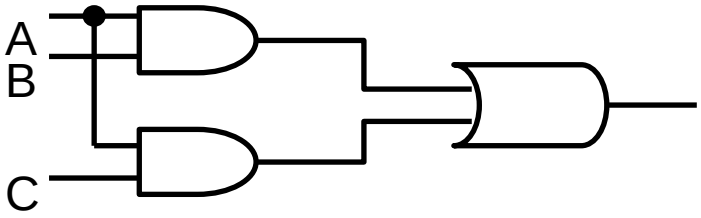
A.B + A.C

A	B	C	AB	AC	A.B + A.C



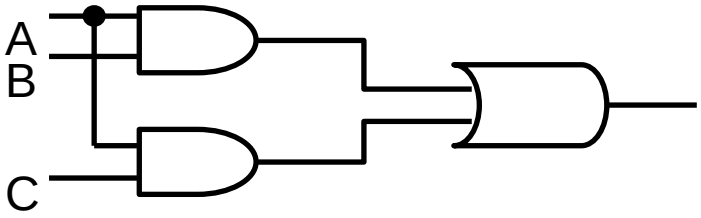
A.B + A.C

A	B	C	AB	AC	A.B + A.C
0					
0					
0					
0					
1					
1					
1					
1					



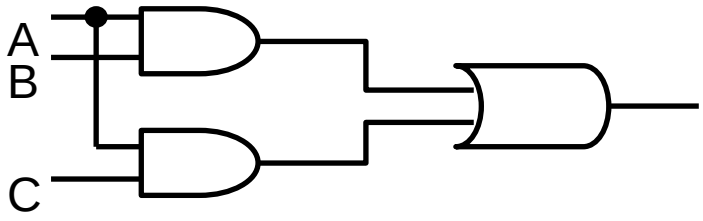
A.B + A.C

A	B	C	AB	AC	A.B + A.C
0	0				
0	0				
0	1				
0	1				
1	0				
1	0				
1	1				
1	1				



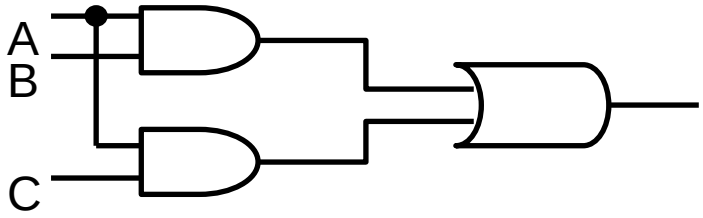
A.B + A.C

A	B	C	AB	AC	A.B + A.C
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



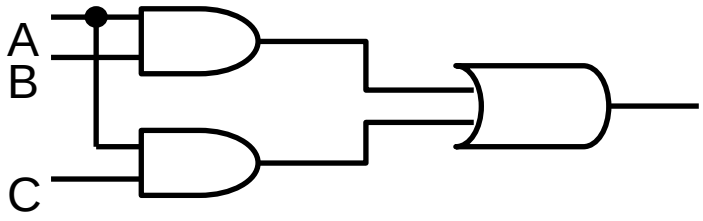
A.B + A.C

A	B	C	AB	AC	A.B + A.C
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



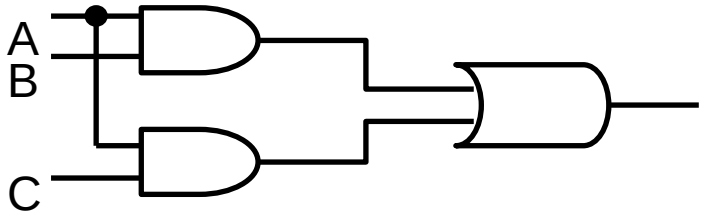
A.B + A.C

A	B	C	AB	AC	A.B + A.C
0	0	0	0		
0	0	1	0		
0	1	0	0		
0	1	1	0		
1	0	0	0		
1	0	1	0		
1	1	0	1		
1	1	1	1		



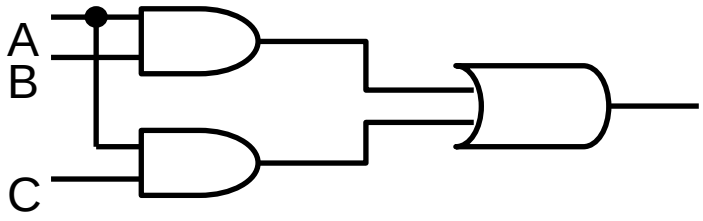
A.B + A.C

A	B	C	AB	AC	A.B + A.C
0	0	0	0		
0	0	1	0		
0	1	0	0		
0	1	1	0		
1	0	0	0		
1	0	1	0		
1	1	0	1		
1	1	1	1		



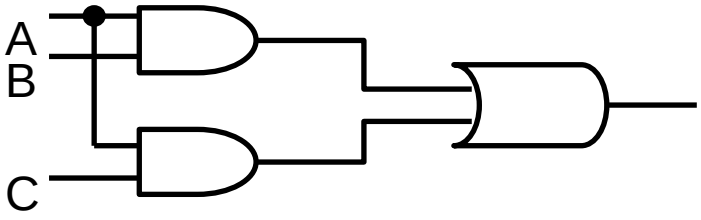
A.B + A.C

A	B	C	AB	AC	A.B + A.C
0	0	0	0	0	
0	0	1	0	0	
0	1	0	0	0	
0	1	1	0	0	
1	0	0	0	0	
1	0	1	0	1	
1	1	0	1	0	
1	1	1	1	1	



A.B + A.C

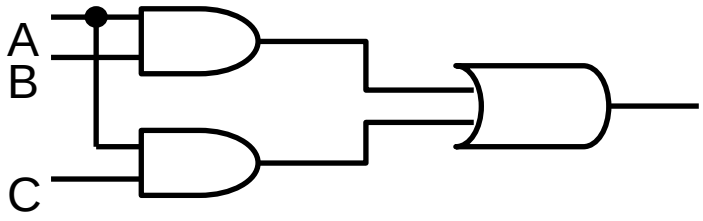
A	B	C	AB	AC	A.B + A.C
0	0	0	0	0	
0	0	1	0	0	
0	1	0	0	0	
0	1	1	0	0	
1	0	0	0	0	
1	0	1	0	1	
1	1	0	1	0	
1	1	1	1	1	



SOLUÇÃO FINAL

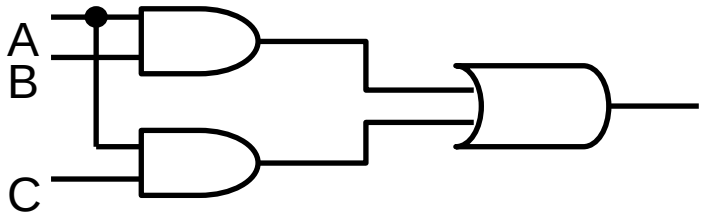
A.B + A.C

A	B	C	AB	AC	A.B + A.C
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1



A.B + A.C

A	B	C	AB	AC	A.B + A.C
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1



A.B + A.C

A	B	C	AB	AC	A.B + A.C
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

$$A\overline{B}.C + A.B\overline{C} + A.B.C$$

$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

Regras válidas da Álgebra:

Comutativa: $A \cdot B = B \cdot A$ $\qquad A + B = B + A$

Distributiva: $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

Existência de elementos neutros: $A \cdot 1 = A$ $\qquad A + 0 = A$

Propriedades do complemento: $A \cdot \overline{A} = 0$ $\qquad A + \overline{A} = 1$

$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

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$$A.\overline{B}.C + A.B.\overline{C} + A.B.C =$$

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$$A.\overline{B}.C + A.B.\overline{C} + A.B.C =$$

Distributiva

$$A.(\overline{B}.C + B.\overline{C} + B.C) =$$

$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

Regras válidas da Álgebra:

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$$A.(\overline{B}.C + B.\overline{C} + B.C) =$$

Distributiva

$$A.(\overline{B}.C + B(\overline{C} + C)) =$$

$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

Regras válidas da Álgebra:

Comutativa:	$A \cdot B = B \cdot A$	$A + B = B + A$
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$$A.\overline{B}.C + A.B.\overline{C} + A.B.C =$$

$$A.(\overline{B}.C + B.\overline{C} + B.C) =$$

$$A.(\overline{B}.C + B(\overline{C} + C)) =$$

$$A.(\overline{B}.C + B)$$

Complemento + Elemento neutro

$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

Regras válidas da Álgebra:

Comutativa:	$A \cdot B = B \cdot A$	$A + B = B + A$
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$$A.(\overline{B}.C + B.\overline{C} + B.C) =$$

$$A.(\overline{B}.C + B(\overline{C} + C)) =$$

$$A.(\overline{B}.C + B)$$

$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

Regras válidas da Álgebra:

Comutativa: $A \cdot B = B \cdot A$ $\qquad A + B = B + A$

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$$A.(\overline{B}.C + B.\overline{C} + B.C) =$$

$$A.(\overline{B}.C + B(\overline{C} + C)) =$$

$$A.(\overline{B}.C + B)$$

para B verdadeiro, resultado depende de B
para B falso, resultado depende de C

$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

Regras válidas da Álgebra:

Comutativa: $A \cdot B = B \cdot A$ $\qquad A + B = B + A$

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Propriedades do complemento: $A \cdot \overline{A} = 0$ $\qquad A + \overline{A} = 1$

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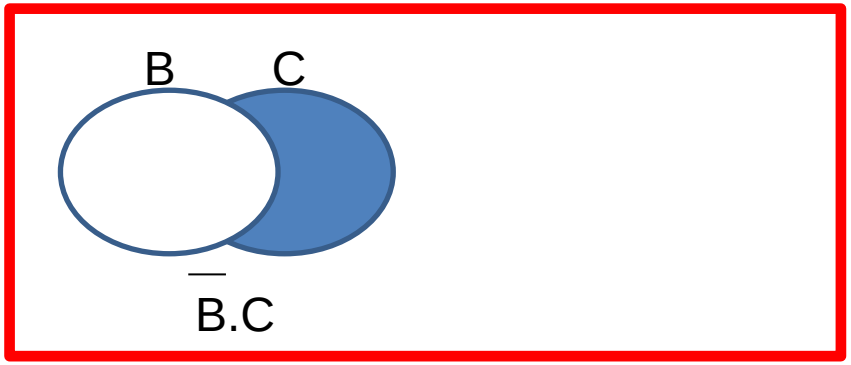
$$A.(\overline{B}.C + B.\overline{C} + B.C) =$$

$$A.(\overline{B}.C + B(\overline{C} + C)) =$$

$$A.(\overline{B}.C + B)$$

para B verdadeiro, resultado depende de B
para B falso, resultado depende de C



$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

Regras válidas da Álgebra:

Comutativa: $A \cdot B = B \cdot A$ $\qquad A + B = B + A$

Distributiva: $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

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Propriedades do complemento: $A \cdot \overline{A} = 0$ $\qquad A + \overline{A} = 1$

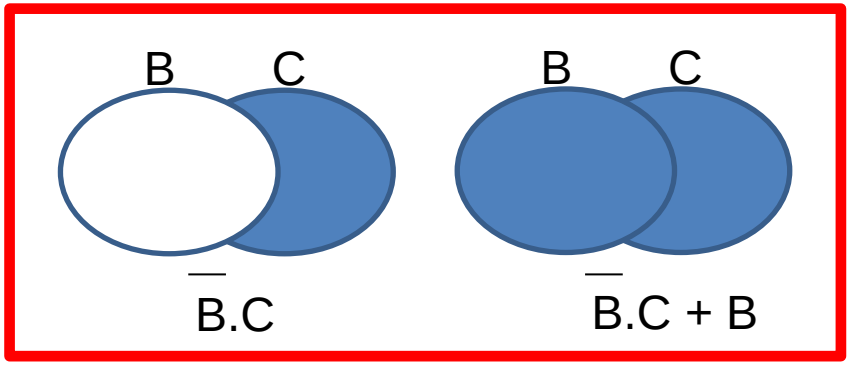
$$A.\overline{B}.C + A.B.\overline{C} + A.B.C =$$

$$A.(\overline{B}.C + B.\overline{C} + B.C) =$$

$$A.(\overline{B}.C + B(\overline{C} + C)) =$$

$$A.(\overline{B}.C + B)$$

para B verdadeiro, resultado depende de B
para B falso, resultado depende de C



$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

Regras válidas da Álgebra:

Comutativa: $A \cdot B = B \cdot A$ $\qquad A + B = B + A$

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$$A.\overline{B}.C + A.B.\overline{C} + A.B.C =$$

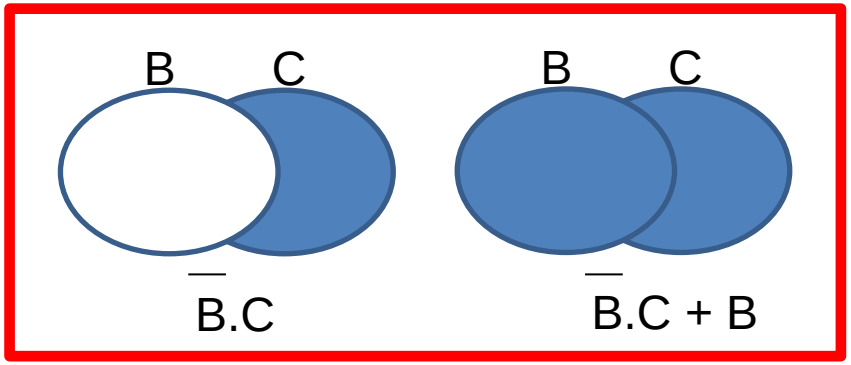
$$A.(\overline{B}.C + B.\overline{C} + B.C) =$$

$$A.(\overline{B}.C + B(\overline{C} + C)) =$$

$$A.(\overline{B}.C + B)$$

$$A.(B + C) =$$

para B verdadeiro, resultado depende de B
para B falso, resultado depende de C



$$A.B + A.C = A.\overline{B}.C + A.B.\overline{C} + A.B.C$$

Regras válidas da Álgebra:

Comutativa: $A \cdot B = B \cdot A$ $\qquad A + B = B + A$

Distributiva: $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

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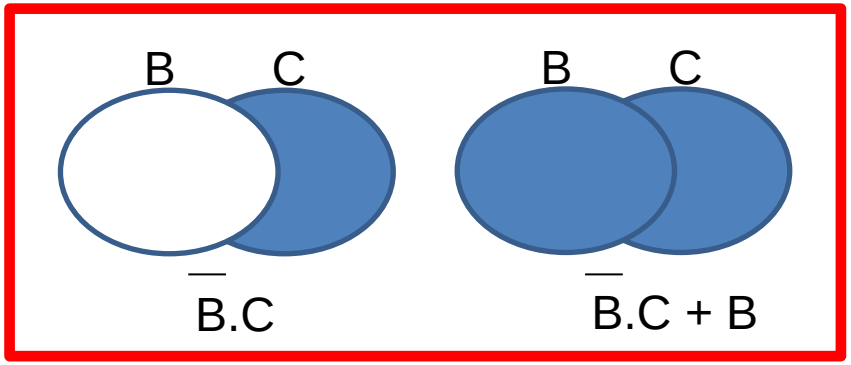
$A.\overline{B}.C + A.B.\overline{C} + A.B.C =$

$A.(\overline{B}.C + B.\overline{C} + B.C) =$

$A.(\overline{B}.C + B(\overline{C} + C)) =$



$A.(\overline{B}.C + B)$ { **para B verdadeiro, resultado depende de B**
para B falso, resultado depende de C

$A.(B + C) = A.B + A.C$ (Distributiva)

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.C.(\overline{B} + B) + \overline{A}.C.(\overline{B} + B) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.C.(\overline{B} + B) + \overline{A}.C.(\overline{B} + B) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.C.(\overline{B} + B) + \overline{A}.C.(\overline{B} + B) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.C + \overline{A}.C + \overline{A}.\overline{B}.\overline{C} =$$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.C.(\overline{B} + B) + \overline{A}.C.(\overline{B} + B) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.C + \overline{A}.C + \overline{A}.\overline{B}.\overline{C} =$$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.C.(\overline{B} + B) + \overline{A}.C.(\overline{B} + B) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.C + \overline{A}.C + \overline{A}.\overline{B}.\overline{C} =$$

$$C.(A + \overline{A}) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.C.(\overline{B} + B) + \overline{A}.C.(\overline{B} + B) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.C + \overline{A}.C + \overline{A}.\overline{B}.\overline{C} =$$

$$C.(A + \overline{A}) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.C.(\overline{B} + B) + \overline{A}.C.(\overline{B} + B) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.C + \overline{A}.C + \overline{A}.\overline{B}.\overline{C} =$$

$$C.(A + \overline{A}) + \overline{A}.\overline{B}.\overline{C} =$$

$$C + \overline{A}.\overline{B}.\overline{C}$$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.C.(\overline{B} + B) + \overline{A}.C.(\overline{B} + B) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.C + \overline{A}.C + \overline{A}.\overline{B}.\overline{C} =$$

$$C.(A + \overline{A}) + \overline{A}.\overline{B}.\overline{C} =$$

$$C + \overline{A}.\overline{B}.\overline{C}$$

- para C verdadeiro, resultado depende de C
- para C falso, resultado depende de $\overline{A}.\overline{B}$

$$A.B.C + A.\overline{B}.C + \overline{A}.B.C + \overline{A}.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} = C + \overline{A}.\overline{B}$$

$$A.C.(\overline{B} + B) + \overline{A}.C.(\overline{B} + B) + \overline{A}.\overline{B}.\overline{C} =$$

$$A.C + \overline{A}.C + \overline{A}.\overline{B}.\overline{C} =$$

$$C.(A + \overline{A}) + \overline{A}.\overline{B}.\overline{C} =$$

$C + \overline{A}.\overline{B}.\overline{C}$

- para C verdadeiro, resultado depende de C
- para C falso, resultado depende de $\overline{A}.\overline{B}$

$$C + \overline{A}.\overline{B}$$

Referências

Cap. 1, 3 e 7 (seção inicial) – Fundamentos Matemáticos para a Ciência da Computação