

AGM – Algoritmo de Prim



AGM – Algoritmo de Prim

- Nascimento: 1928 (101 anos).
- Terminou seu PhD na Universidade de Princeton em 1949.
- Trabalhou na General Electric (Segunda Guerra), Bell Laboratories (1958-1961); Sandia National Lab (acompanhou o Weapons Reliability Committee, e posteriormente virou vice-presidente).
- O algoritmo autointitulado “o algoritmo de Prim”, foi originalmente descoberto em 1930 pelo matemático Vojtěch Jarník e, posteriormente, de forma independente por Prim em 1957.



Árvore Geradora Mínima – Prim

- Diferente do algoritmo de Kruskal, o algoritmo de Prim alimenta uma **única árvore** durante a execução.
- **Ideia geral (Algoritmo de Prim):**
 - O algoritmo começa de um dado nó r e vai expandindo a árvore geradora até alcançar todos os nós de V .
 - Durante o processo, $X \subseteq E$ será sempre uma árvore (nunca uma floresta, como acontece em Kruskal).
- O algoritmo de Prim é guloso, uma vez que a árvore geradora é expandida a cada etapa a partir da inserção da aresta de menor peso possível.

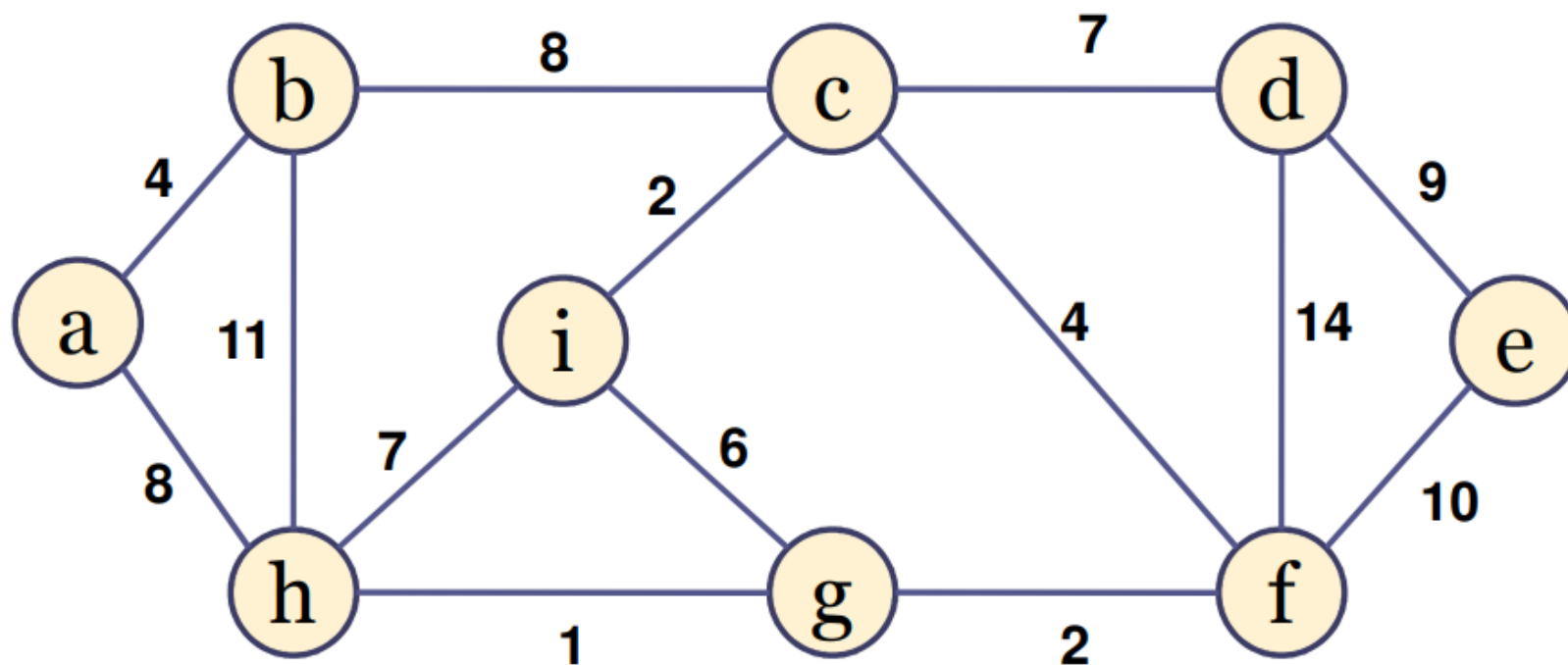
Árvore Geradora Mínima – Prim

Estruturas auxiliares do algoritmo:

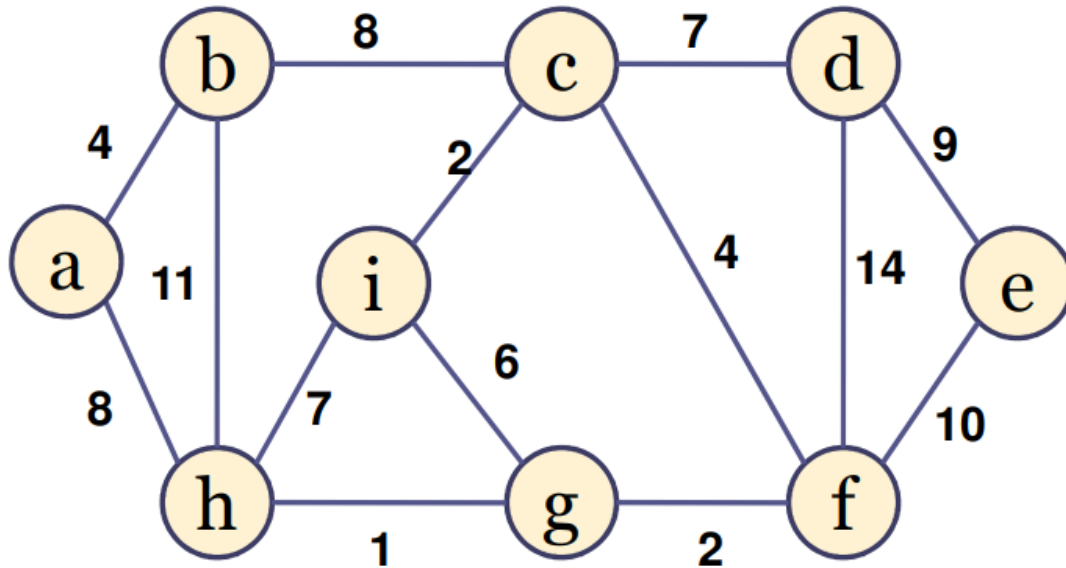
- Q : Nós que ainda não integram a AGM parcial, isto é, que ainda não fazem parte do conjunto X .
- $chave[u]$: peso da aresta mais leve do nó u que a conecta à AGM parcialmente construída.
- $\pi[u]$: nó pai do vértice u .

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Rodando um exemplo “na mão” para $r = a$



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$AGM_Prim(G(V, E, W), r)$

$X \leftarrow \{ \}$ //AGM

Para cada nó $u \in V$ faça

$chave(u) \leftarrow \infty$

$\pi(u) \leftarrow NULL$

Fim_para

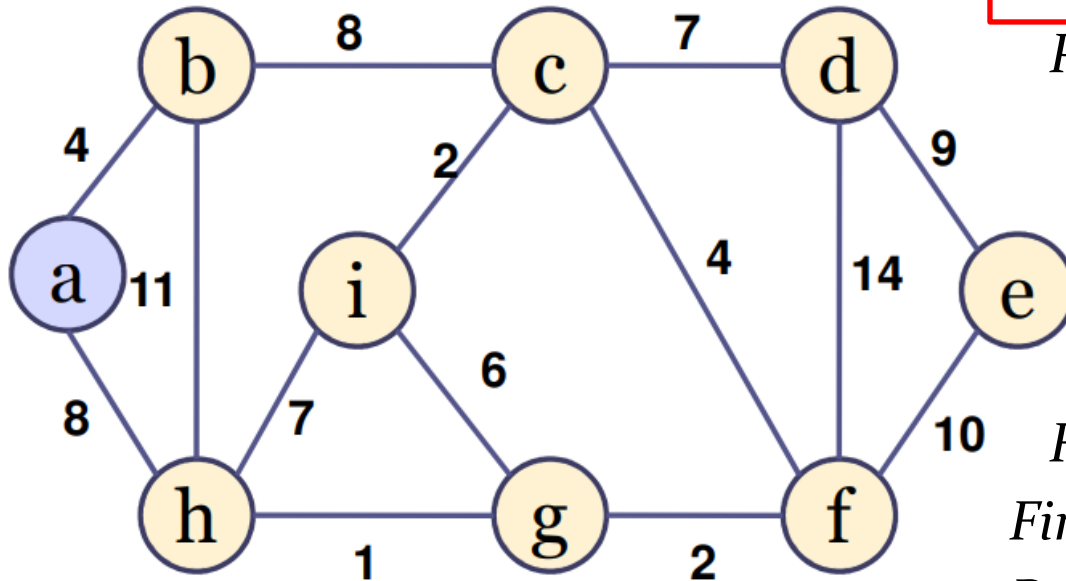
$chave[r] \leftarrow 0$

$Q \leftarrow V$

vértice	a	b	c	d	e	f	g	h	i
chave	0	∞	∞	∞	∞	∞	∞	∞	∞
π	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
Q	a	b	c	d	e	f	g	h	i

AGM – Algoritmo de Prim

➤ $u = a$



Enquanto $Q \neq \{ \}$

$u \leftarrow \text{Extrai_Min}(Q)$

$X \leftarrow X \cup \{(u, \pi[u])\}$ //se $u \neq r$

Para cada $v \in \text{Adj}[u]$ faça

Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

Fim_para

Fim_enquanto

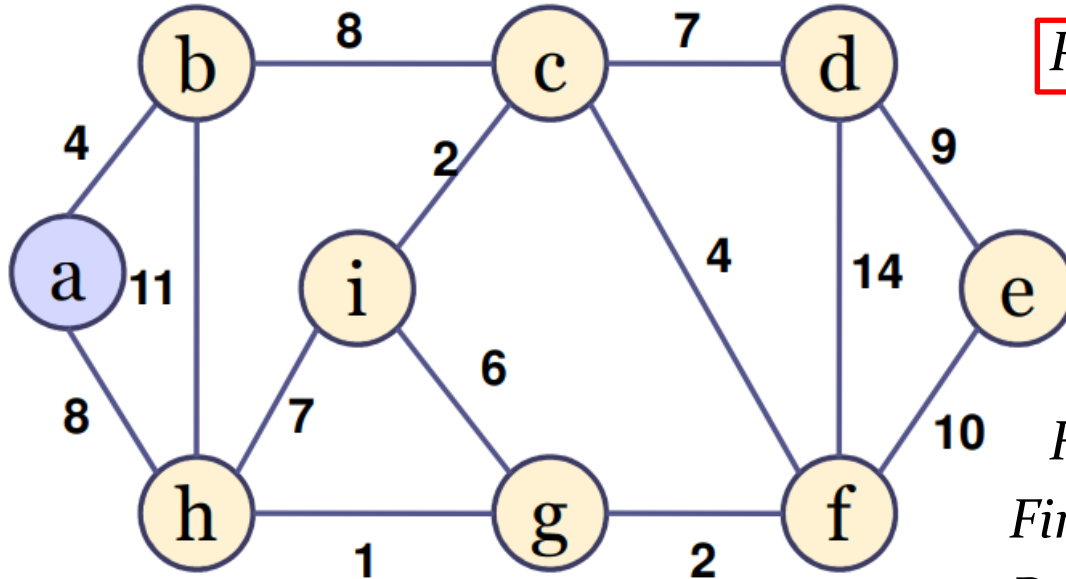
Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	∞	∞	∞	∞	∞	∞	∞	∞
π	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
Q	--	b	c	d	e	f	g	h	i

*Nota: $\text{Extrai_Min}(Q)$: Extrai o mínimo do conj. Q com relação à chave.

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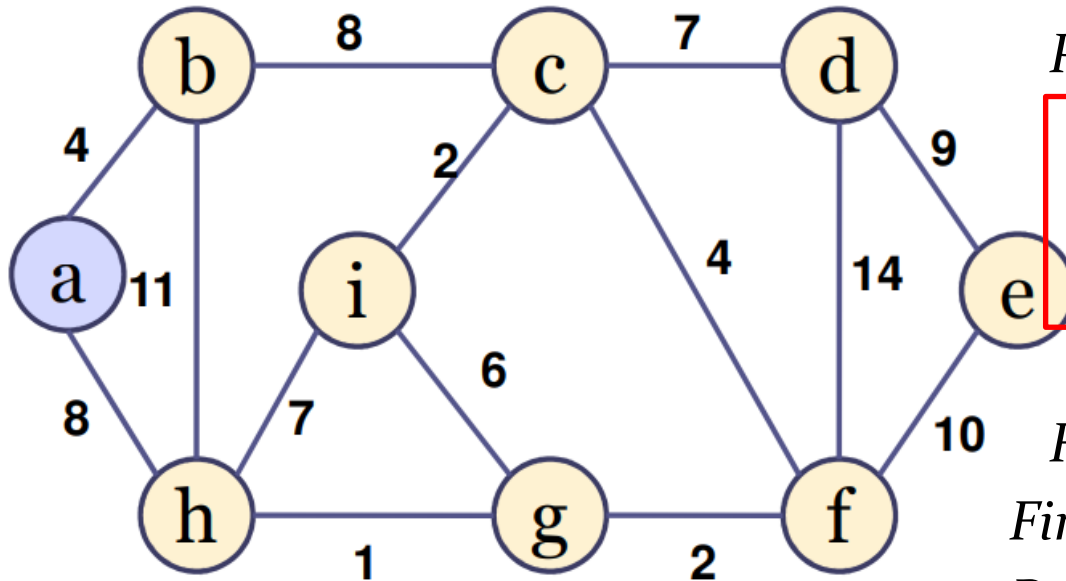
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	∞	∞	∞	∞	∞	∞	∞	∞
π	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL	NULL
Q	--	b	c	d	e	f	g	h	i

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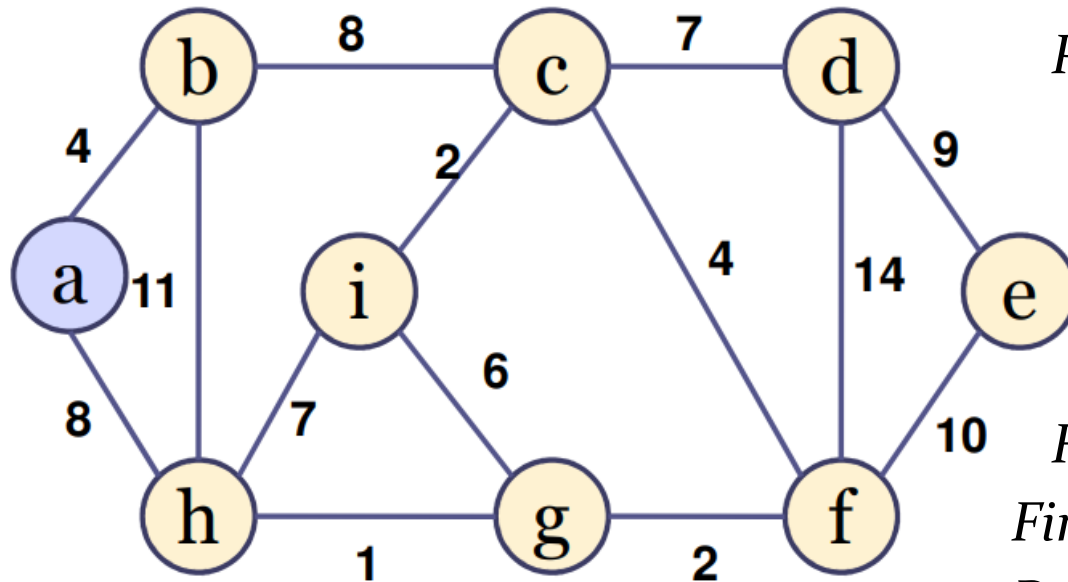
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	∞	∞	∞	∞	∞	8	∞
π	NULL	a	NULL	NULL	NULL	NULL	NULL	a	NULL
Q	--	b	c	d	e	f	g	h	i

AGM – Algoritmo de Prim

➤ $u = b$



Enquanto $Q \neq \{ \}$

$u \leftarrow \text{Extrai_Min}(Q)$

$X \leftarrow X \cup \{(u, \pi[u])\}$ //se $u \neq r$

Para cada $v \in \text{Adj}[u]$ faça

Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

Fim_para

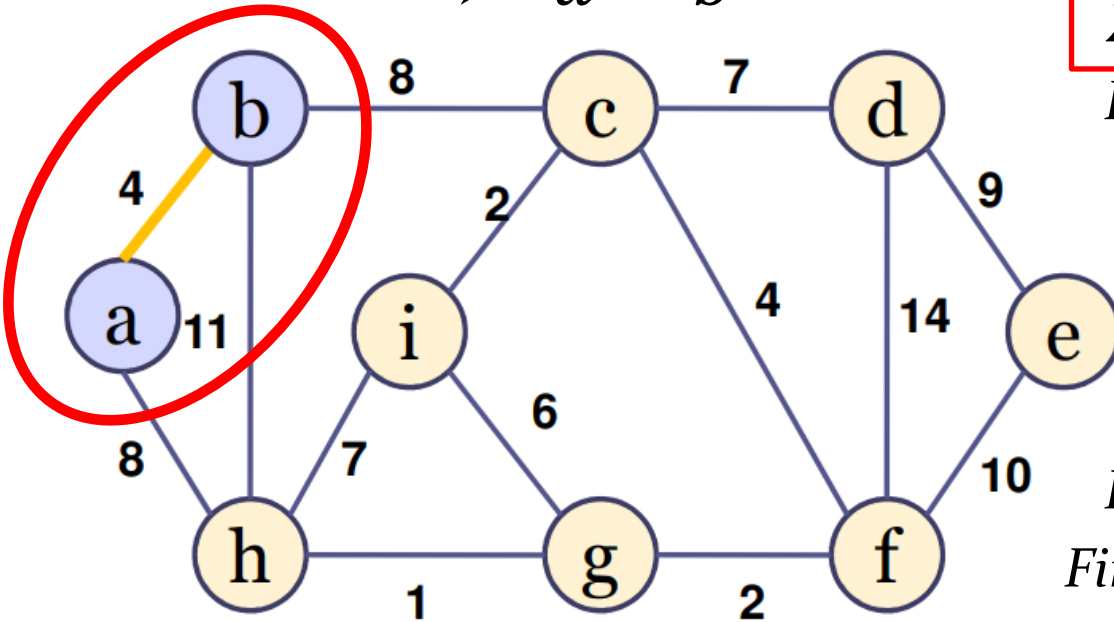
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	∞	∞	∞	∞	∞	8	∞
π	NULL	a	NULL	NULL	NULL	NULL	NULL	a	NULL
Q	--	--	c	d	e	f	g	h	i

AGM – Algoritmo de Prim

➤ $u = b$



Enquanto $Q \neq \{ \}$

$u \leftarrow \text{Extrai_Min}(Q)$

$X \leftarrow X \cup \{(u, \pi[u])\}$ //se $u \neq r$

Para cada $v \in \text{Adj}[u]$ faça

Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

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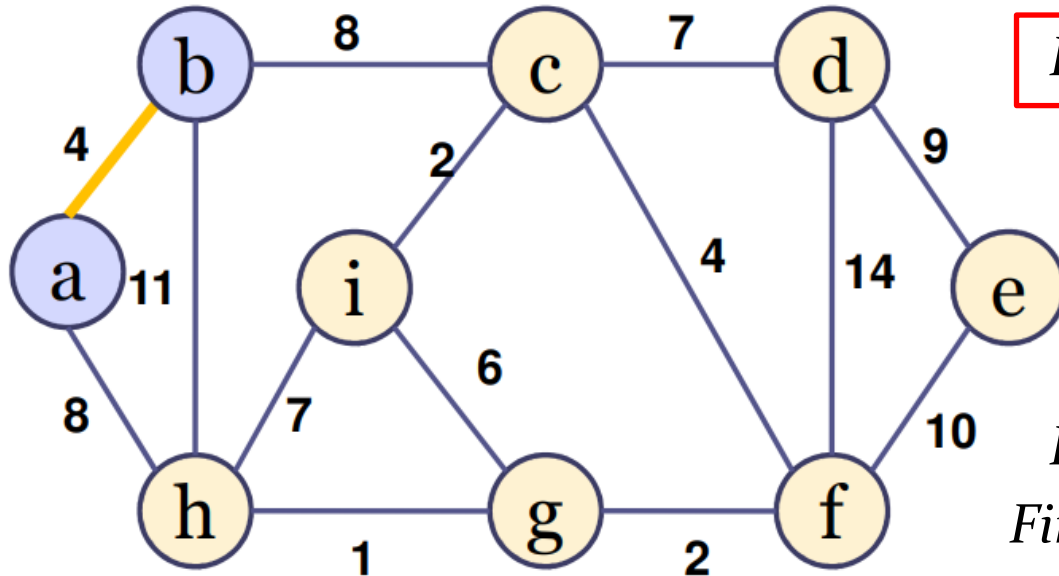
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Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	∞	∞	∞	∞	∞	8	∞
π	NULL	a	NULL	NULL	NULL	NULL	NULL	a	NULL
Q	--	--	c	d	e	f	g	h	i

AGM – Algoritmo de Prim

➤ $u = b$



Enquanto $Q \neq \{ \}$

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Para cada $v \in \text{Adj}[u]$ faça

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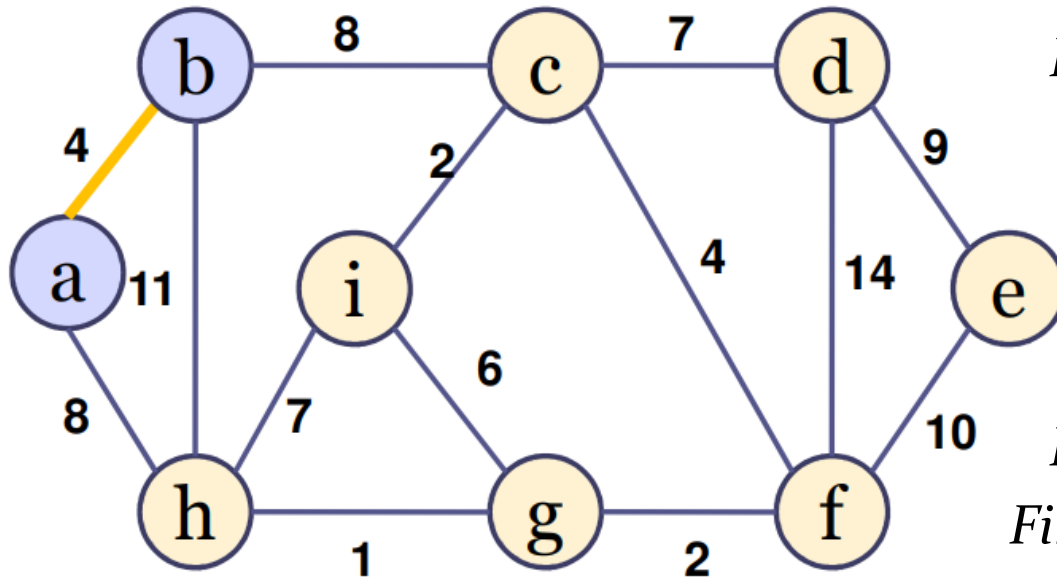
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	∞	∞	∞	∞	∞	8	∞
π	NULL	a	NULL	NULL	NULL	NULL	NULL	a	NULL
Q	--	--	c	d	e	f	g	h	i

AGM – Algoritmo de Prim

➤ $u = b$



Enquanto $Q \neq \{ \}$

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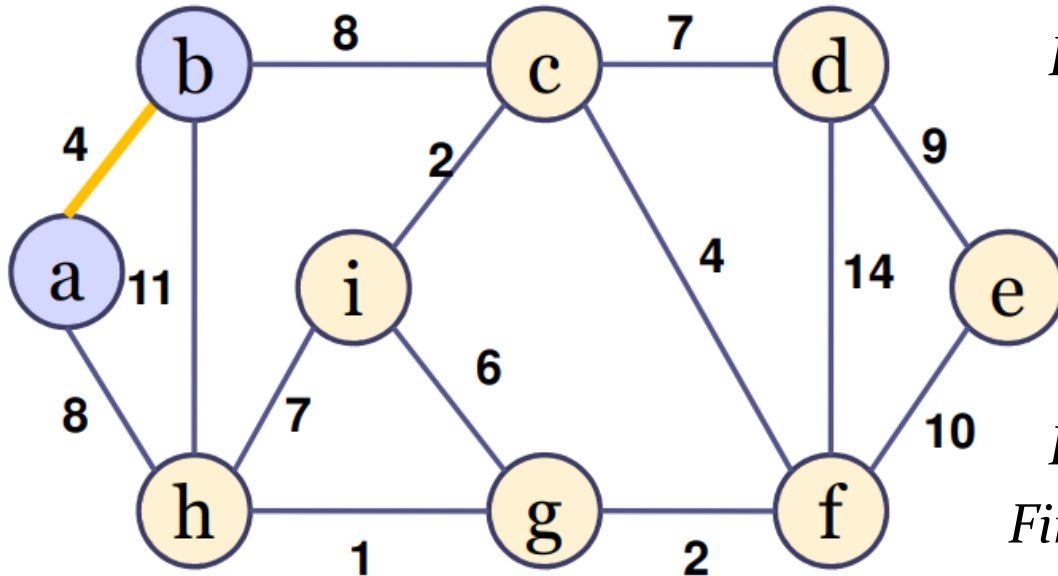
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	∞	∞	8	∞
π	NULL	a	b	NULL	NULL	NULL	NULL	a	NULL
Q	--	--	c	d	e	f	g	h	i

AGM – Algoritmo de Prim

➤ $u = ??$



Enquanto $Q \neq \{ \}$

$u \leftarrow \text{Extrai_Min}(Q)$

$X \leftarrow X \cup \{(u, \pi[u])\}$ //se $u \neq r$

Para cada $v \in \text{Adj}[u]$ faça

Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

Fim_para

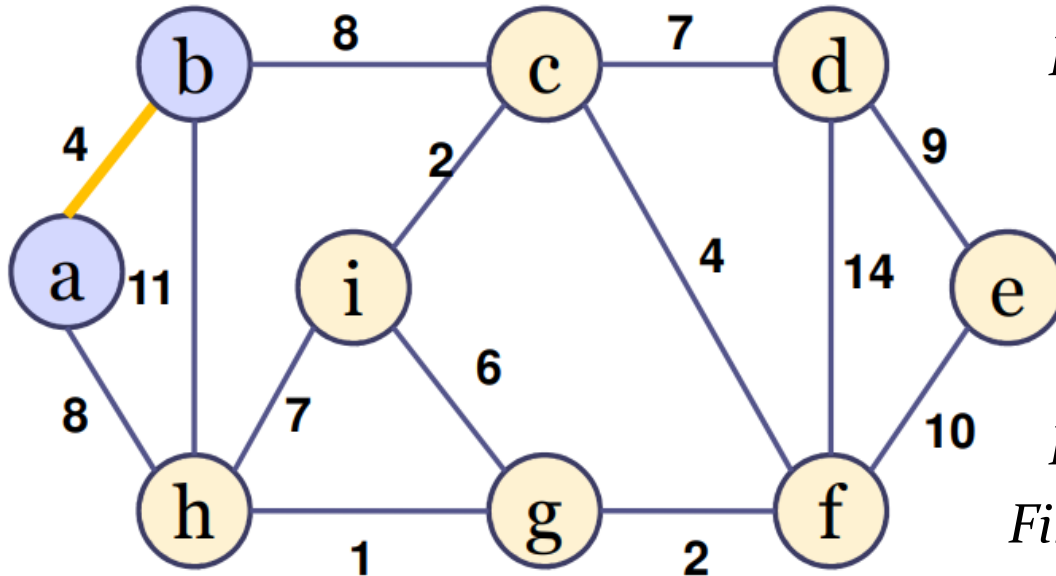
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	∞	∞	8	∞
π	NULL	a	b	NULL	NULL	NULL	NULL	a	NULL
Q	--	--	c	d	e	f	g	h	i

AGM – Algoritmo de Prim

➤ $u = h$



Enquanto $Q \neq \{ \}$

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Para cada $v \in \text{Adj}[u]$ faça

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Fim_se

Fim_para

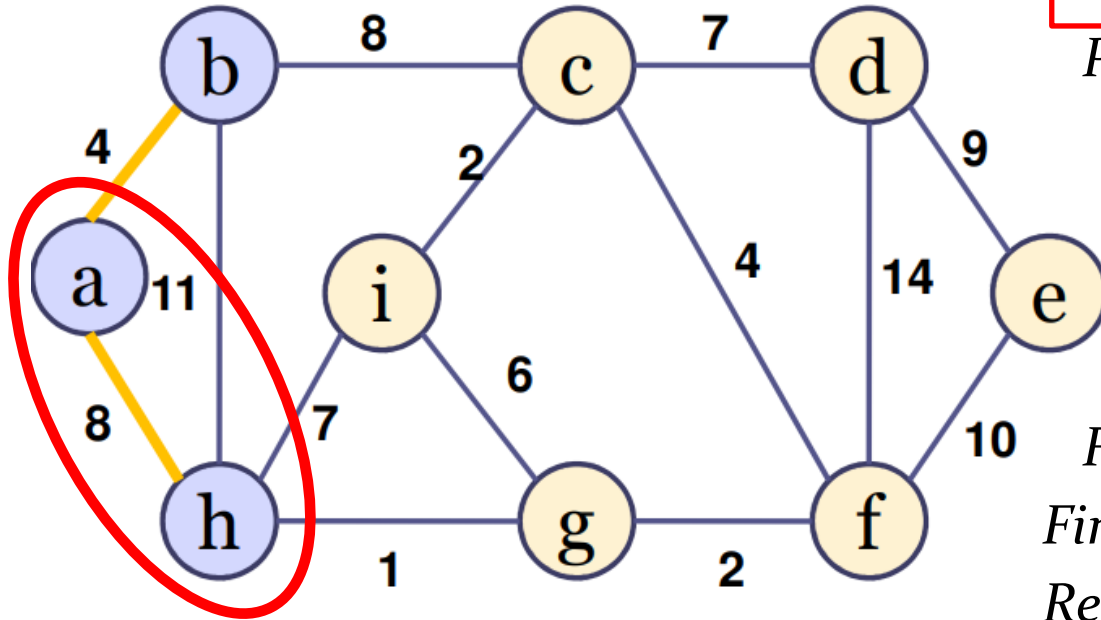
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	∞	∞	8	∞
π	NULL	a	b	NULL	NULL	NULL	NULL	a	NULL
Q	--	--	c	d	e	f	g	--	i

AGM – Algoritmo de Prim

➤ $u = h$



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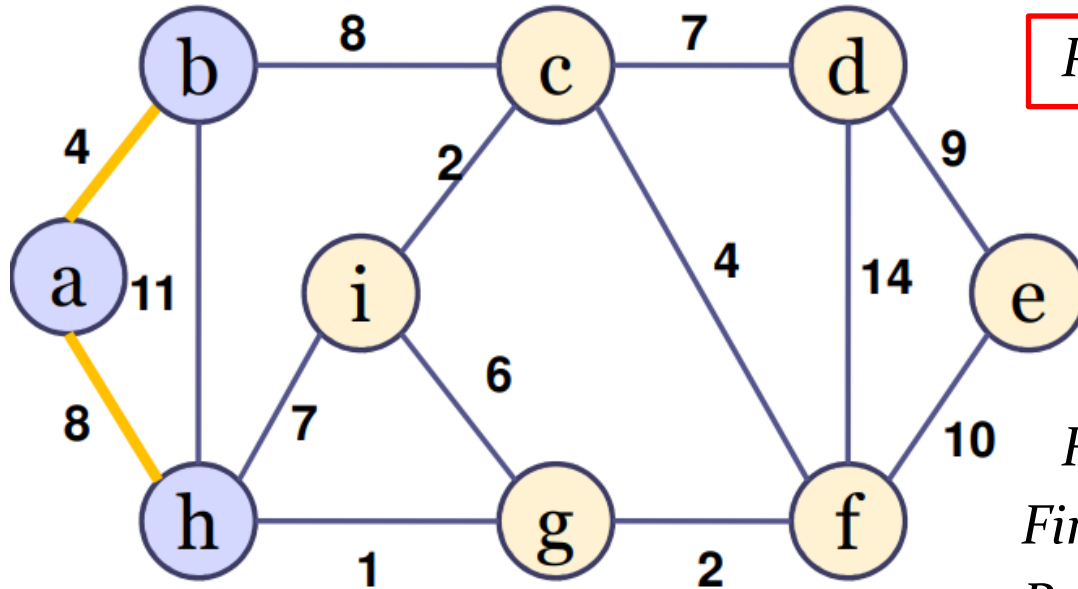
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Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	∞	∞	8	∞
π	NULL	a	b	NULL	NULL	NULL	NULL	a	NULL
Q	--	--	c	d	e	f	g	--	i

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➤ $u = h$



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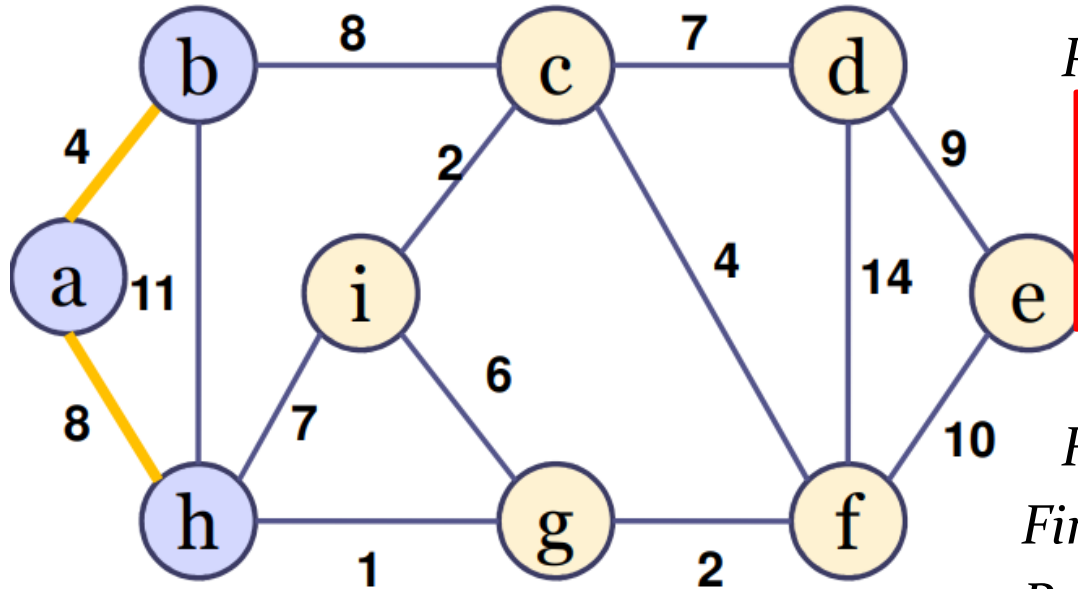
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	∞	∞	8	∞
π	NULL	a	b	NULL	NULL	NULL	NULL	a	NULL
Q	--	--	c	d	e	f	g	--	i

AGM – Algoritmo de Prim

➤ $u = h$



Enquanto $Q \neq \{ \}$

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Fim_para

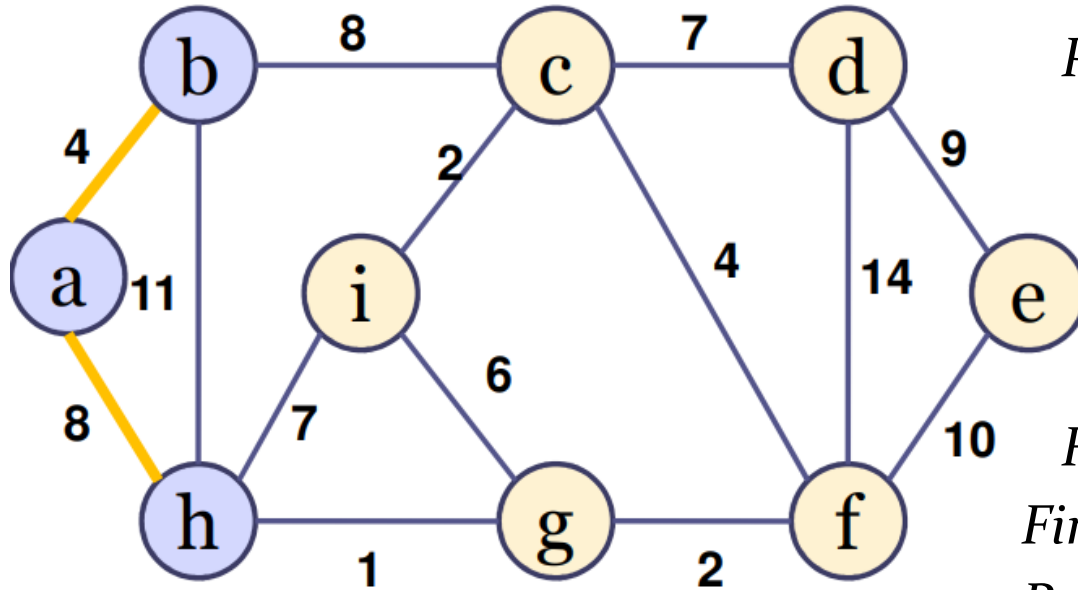
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	∞	1	8	7
π	NULL	a	b	NULL	NULL	NULL	h	a	h
Q	--	--	c	d	e	f	g	--	i

AGM – Algoritmo de Prim

➤ $u = g$



Enquanto $Q \neq \{ \}$

$u \leftarrow \text{Extrai_Min}(Q)$

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Para cada $v \in \text{Adj}[u]$ faça

Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

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Fim_se

Fim_para

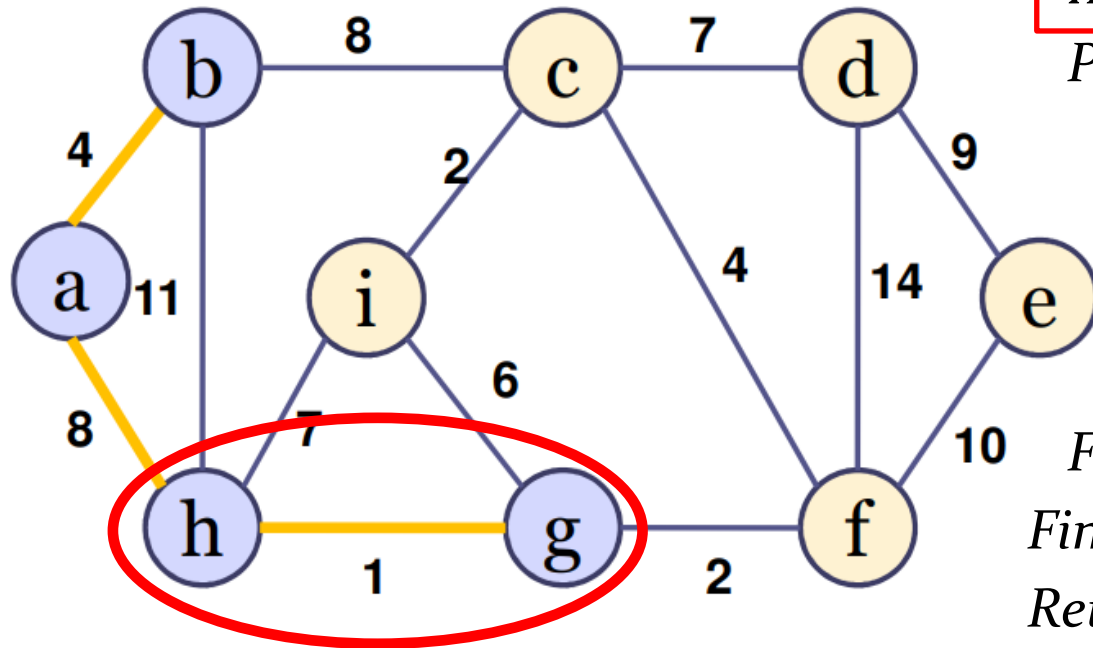
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	∞	1	8	7
π	NULL	a	b	NULL	NULL	NULL	h	a	h
Q	--	--	c	d	e	f	--	--	i

AGM – Algoritmo de Prim

➤ $u = g$



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Fim_se

Fim_para

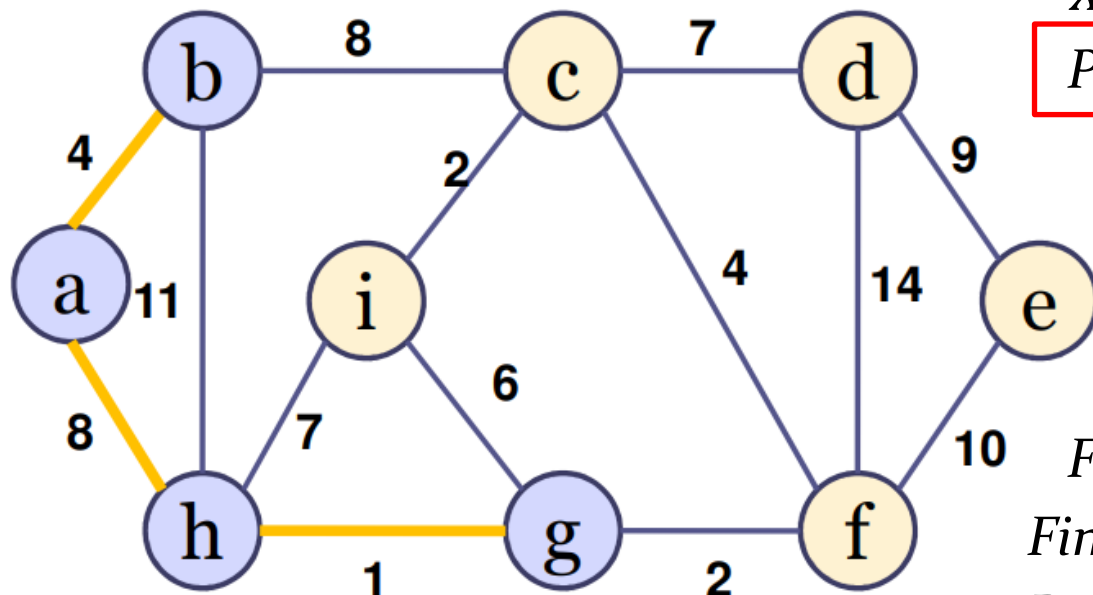
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	∞	1	8	7
π	NULL	a	b	NULL	NULL	NULL	h	a	h
Q	--	--	c	d	e	f	--	--	i

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➤ $u = g$



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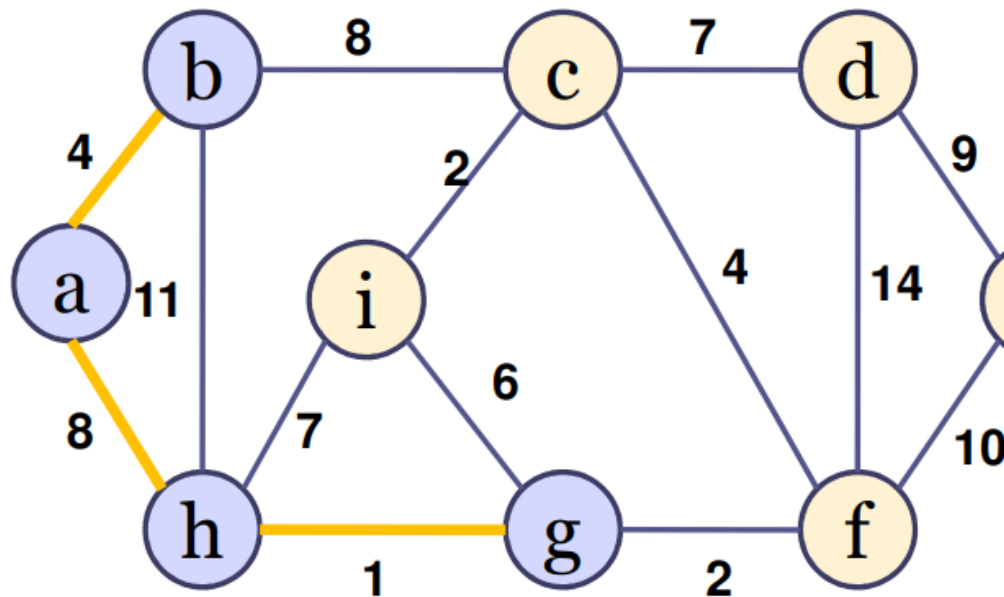
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	∞	1	8	7
π	NULL	a	b	NULL	NULL	NULL	h	a	h
Q	--	--	c	d	e	f	--	--	i

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➤ $u = g$



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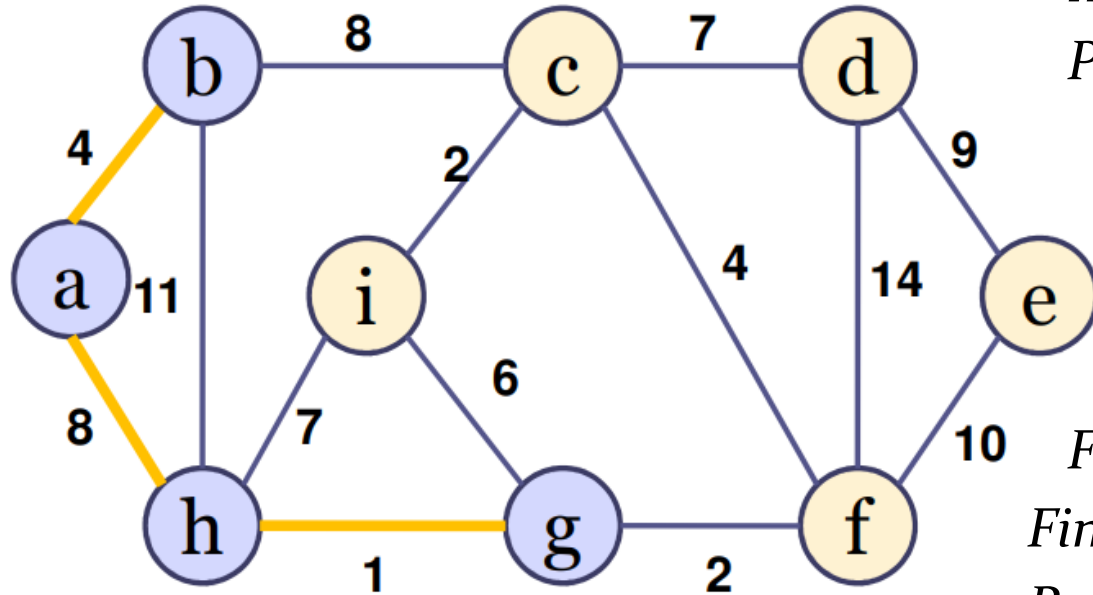
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	2	1	8	6
π	NULL	a	b	NULL	NULL	g	h	a	g
Q	--	--	c	d	e	f	--	--	i

AGM – Algoritmo de Prim

➤ $u = f$



Enquanto $Q \neq \{ \}$

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Para cada $v \in \text{Adj}[u]$ faça

Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

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Fim_se

Fim_para

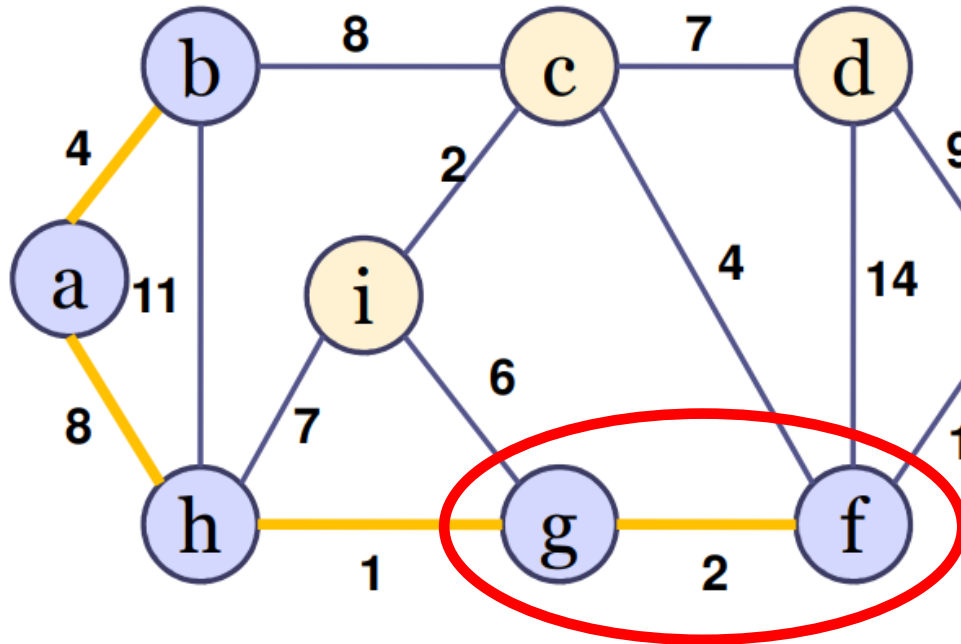
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	2	1	8	6
π	NULL	a	b	NULL	NULL	g	h	a	g
Q	--	--	c	d	e	--	--	--	i

AGM – Algoritmo de Prim

➤ $u = f$



Enquanto $Q \neq \{ \}$

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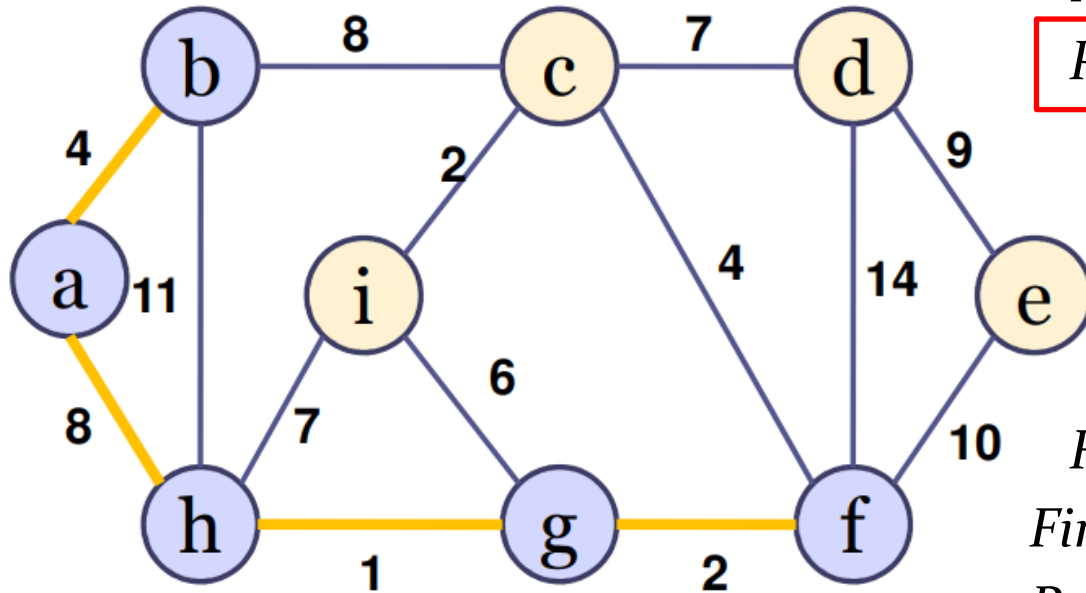
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	2	1	8	6
π	NULL	a	b	NULL	NULL	g	h	a	g
Q	--	--	c	d	e	--	--	--	i

AGM – Algoritmo de Prim

➤ $u = f$



Enquanto $Q \neq \{ \}$

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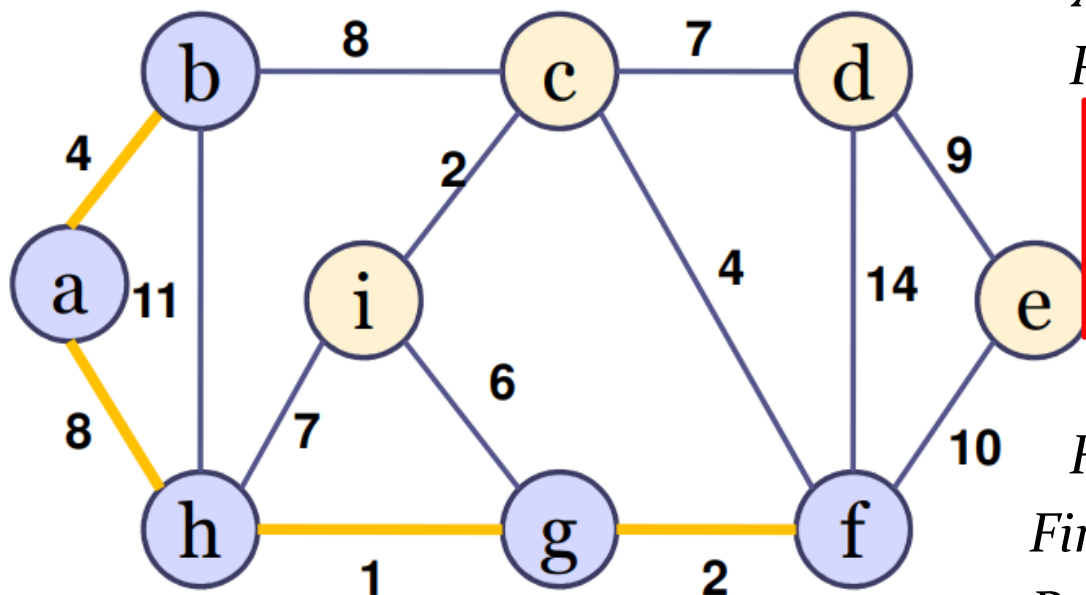
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	8	∞	∞	2	1	8	6
π	NULL	a	b	NULL	NULL	g	h	a	g
Q	--	--	c	d	e	--	--	--	i

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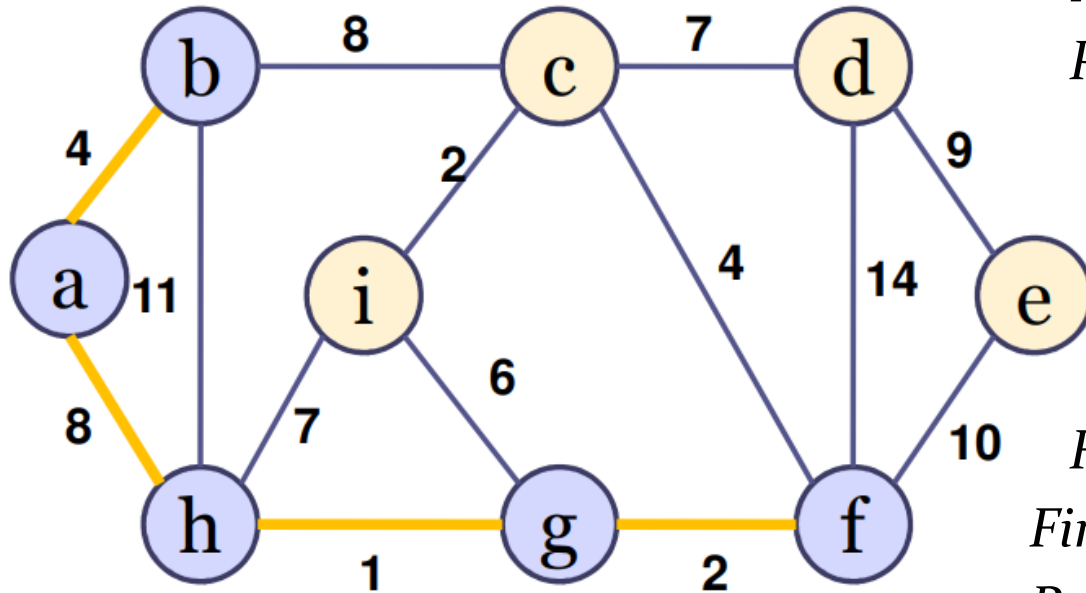
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	14	10	2	1	8	6
π	NULL	a	f	f	f	g	h	a	g
Q	--	--	c	d	e	--	--	--	i

AGM – Algoritmo de Prim

➤ $u = c$



Enquanto $Q \neq \{ \}$

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Para cada $v \in \text{Adj}[u]$ faça

Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

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$\pi[v] \leftarrow u$

Fim_se

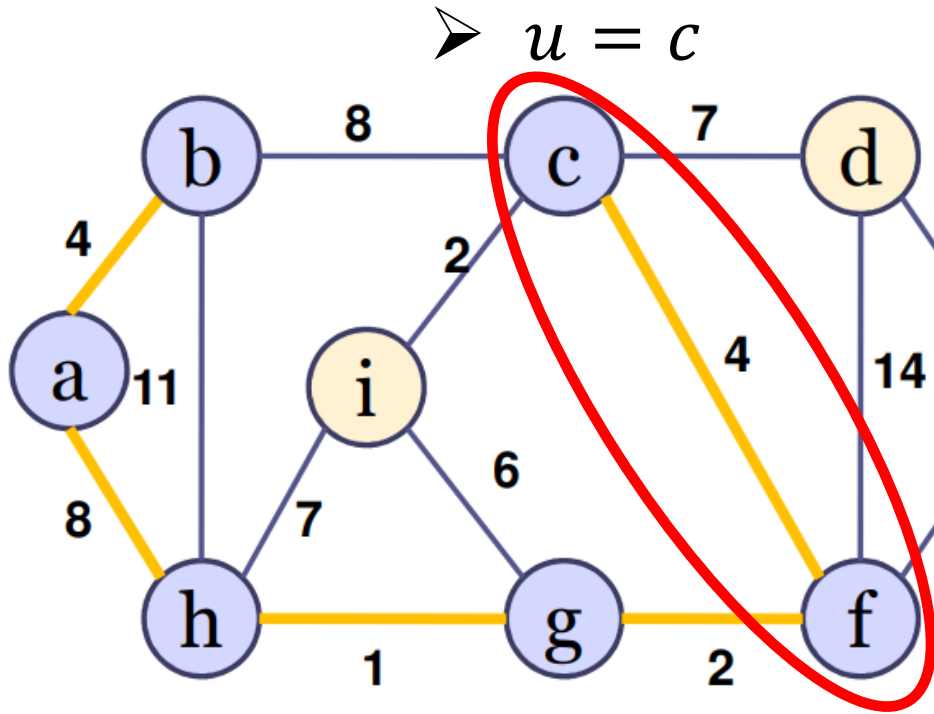
Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	14	10	2	1	8	6
π	NULL	a	f	f	f	g	h	a	g
Q	--	--	--	d	e	--	--	--	i

AGM – Algoritmo de Prim



Enquanto $Q \neq \{ \}$

$u \leftarrow \text{Extrai_Min}(Q)$

$X \leftarrow X \cup \{(u, \pi[u])\}$ //se $u \neq r$

Para cada $v \in \text{Adj}[u]$ faça

Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

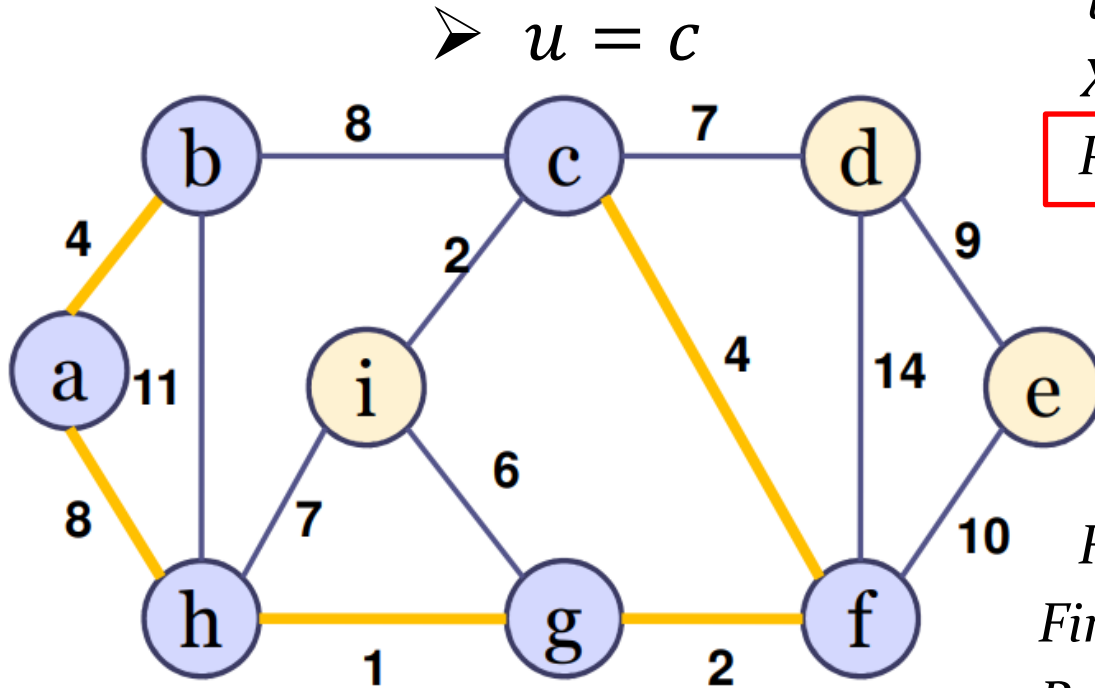
Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	14	10	2	1	8	6
π	NULL	a	f	f	f	g	h	a	g
Q	--	--	--	d	e	--	--	--	i

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Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

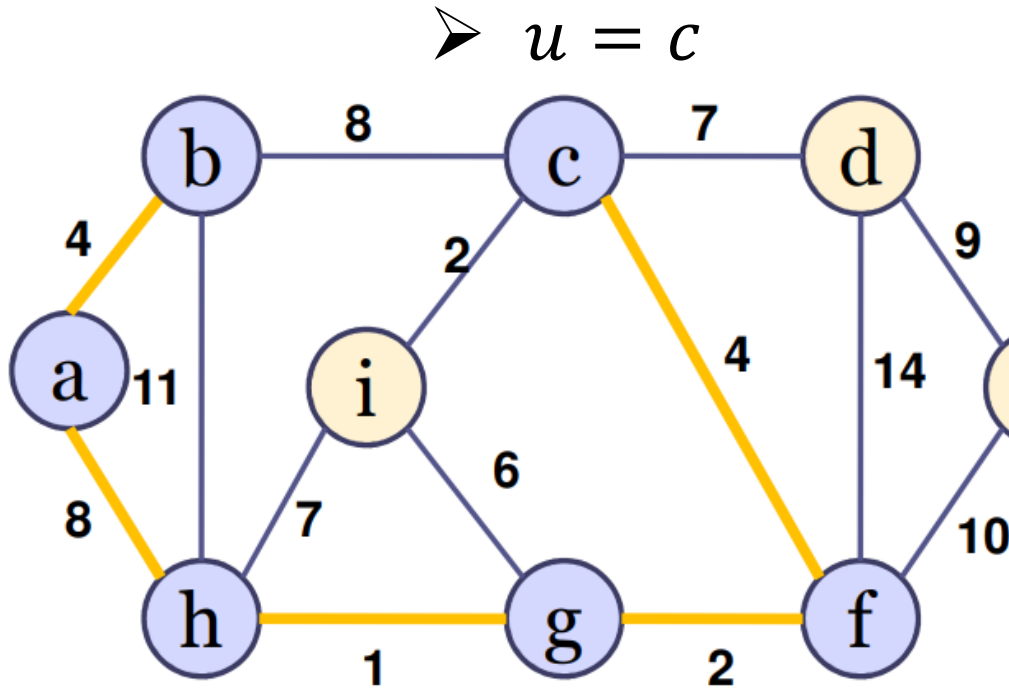
Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	14	10	2	1	8	6
π	NULL	a	f	f	f	g	h	a	g
Q	--	--	--	d	e	--	--	--	i

AGM – Algoritmo de Prim



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Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

Fim_para

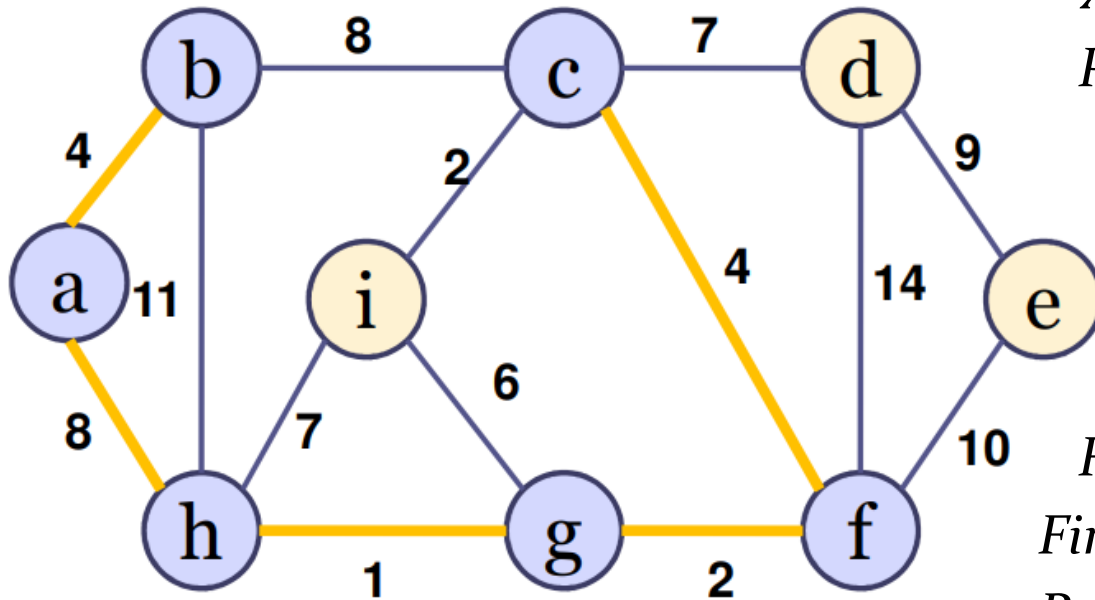
Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	7	10	2	1	8	2
π	NULL	a	f	c	f	g	h	a	c
Q	--	--	--	d	e	--	--	--	i

AGM – Algoritmo de Prim

➤ $u = i$



Enquanto $Q \neq \{ \}$

$u \leftarrow \text{Extrai_Min}(Q)$

$X \leftarrow X \cup \{(u, \pi[u])\}$ //se $u \neq r$

Para cada $v \in \text{Adj}[u]$ faça

Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

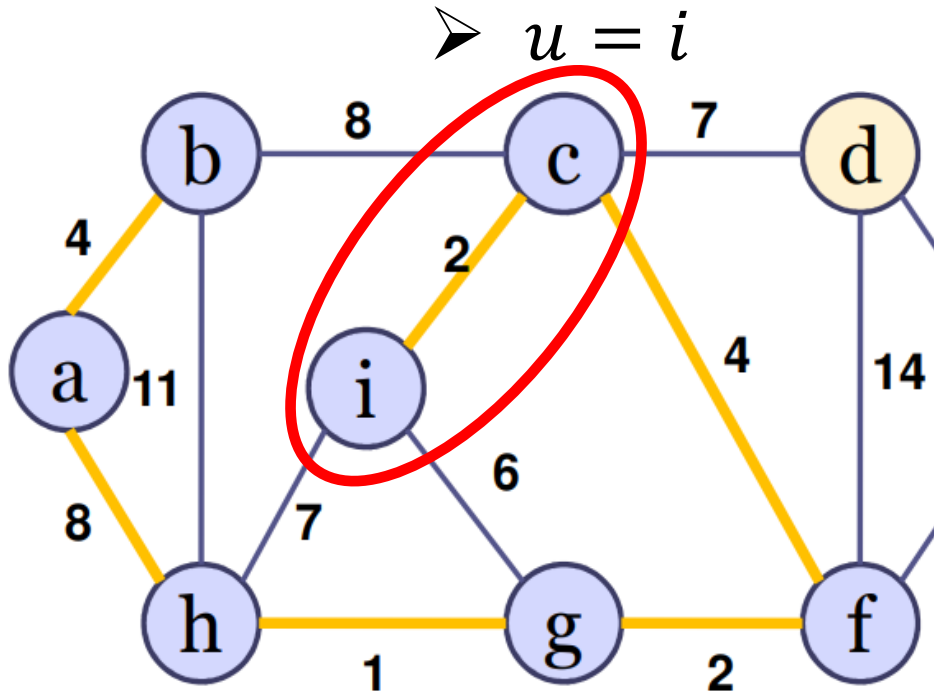
Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	7	10	2	1	8	2
π	NULL	a	f	c	f	g	h	a	c
Q	--	--	--	d	e	--	--	--	--

AGM – Algoritmo de Prim



Enquanto $Q \neq \{ \}$

$u \leftarrow \text{Extrai_Min}(Q)$

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$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

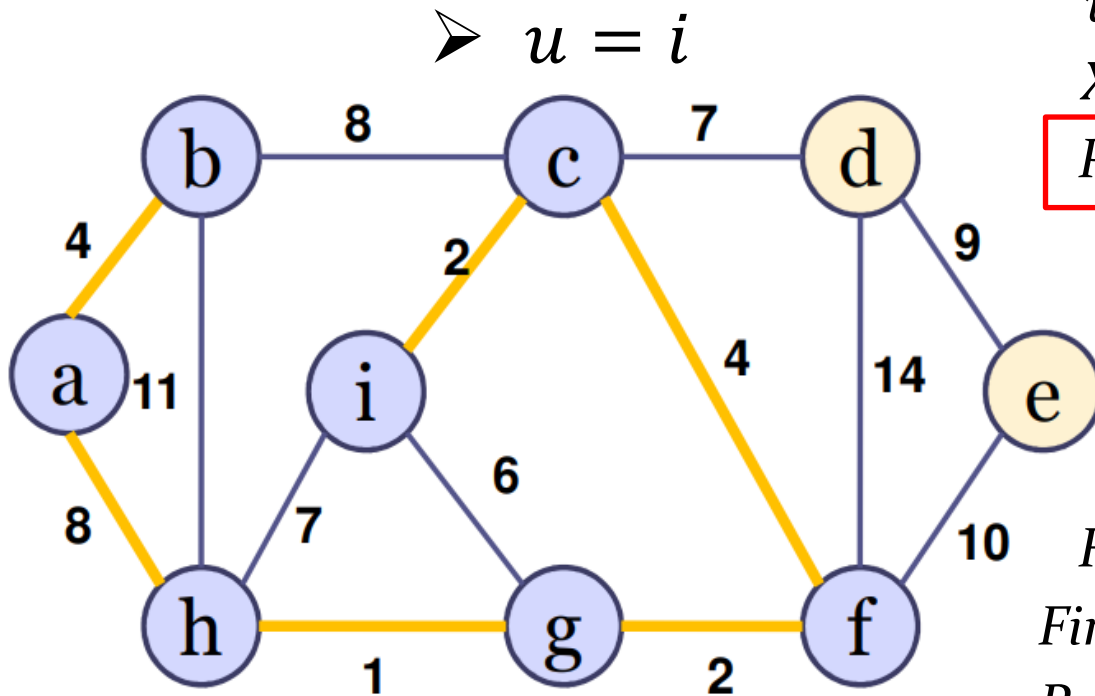
Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	7	10	2	1	8	2
π	NULL	a	f	c	f	g	h	a	c
Q	--	--	--	d	e	--	--	--	--

AGM – Algoritmo de Prim



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$\pi[v] \leftarrow u$

Fim_se

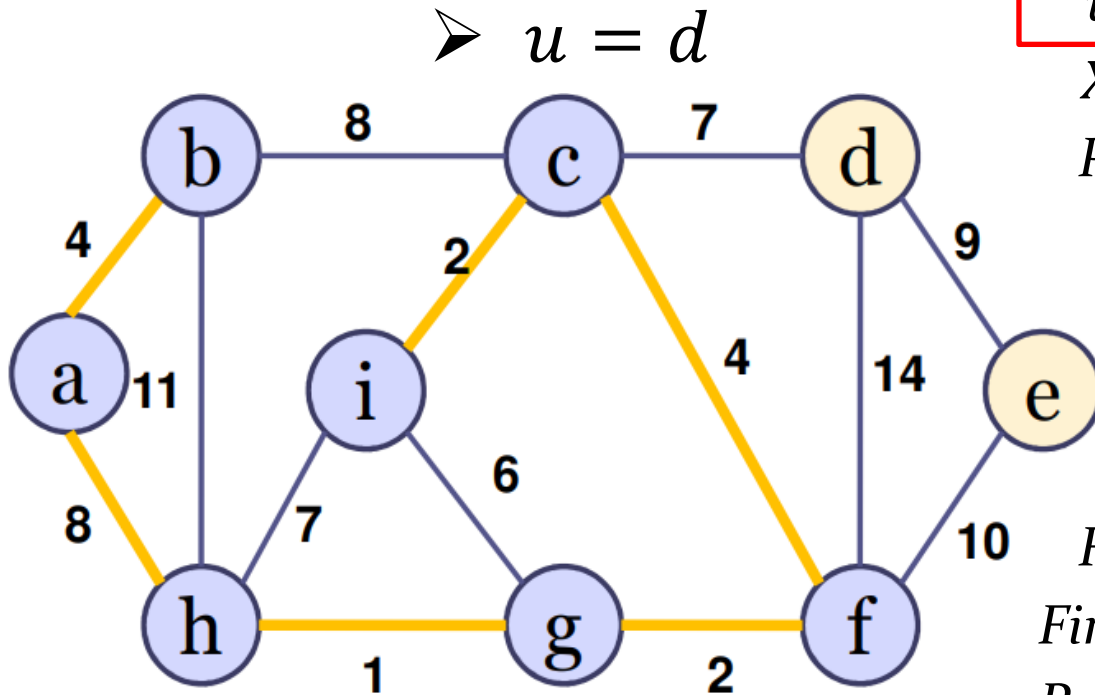
Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	7	10	2	1	8	2
π	NULL	a	f	c	f	g	h	a	c
Q	--	--	--	d	e	--	--	--	--

AGM – Algoritmo de Prim



Enquanto $Q \neq \{ \}$

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$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

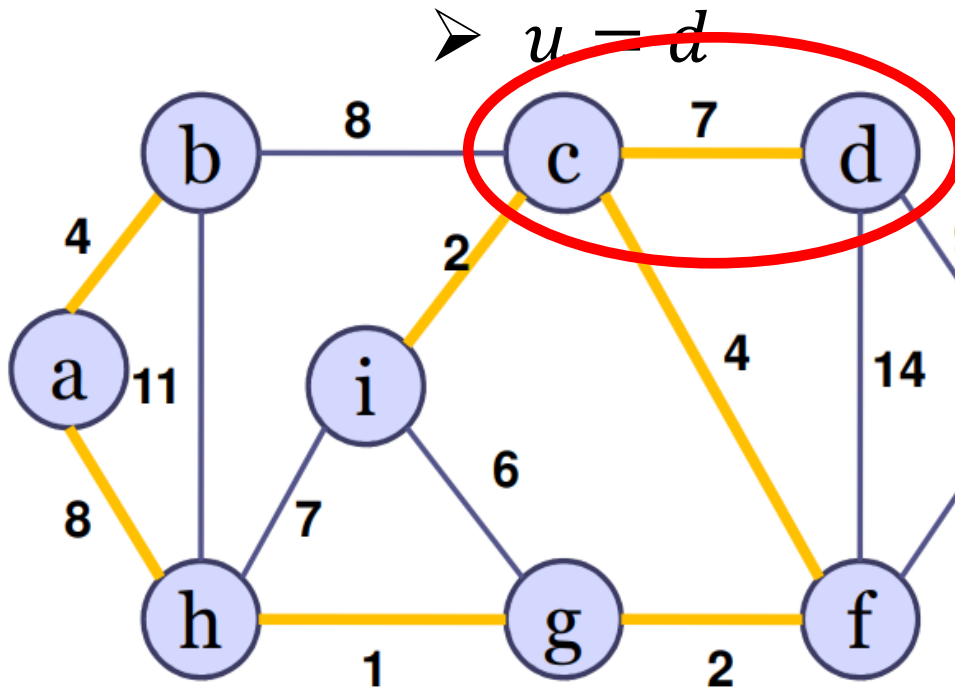
Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	7	10	2	1	8	2
π	NULL	a	f	c	f	g	h	a	c
Q	--	--	--	--	e	--	--	--	--

AGM – Algoritmo de Prim



Enquanto $Q \neq \{ \}$

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Se $(v \in Q)$ e $w(u, v) < \text{chave}[v]$

$\text{chave}[v] \leftarrow w(u, v)$

$\pi[v] \leftarrow u$

Fim_se

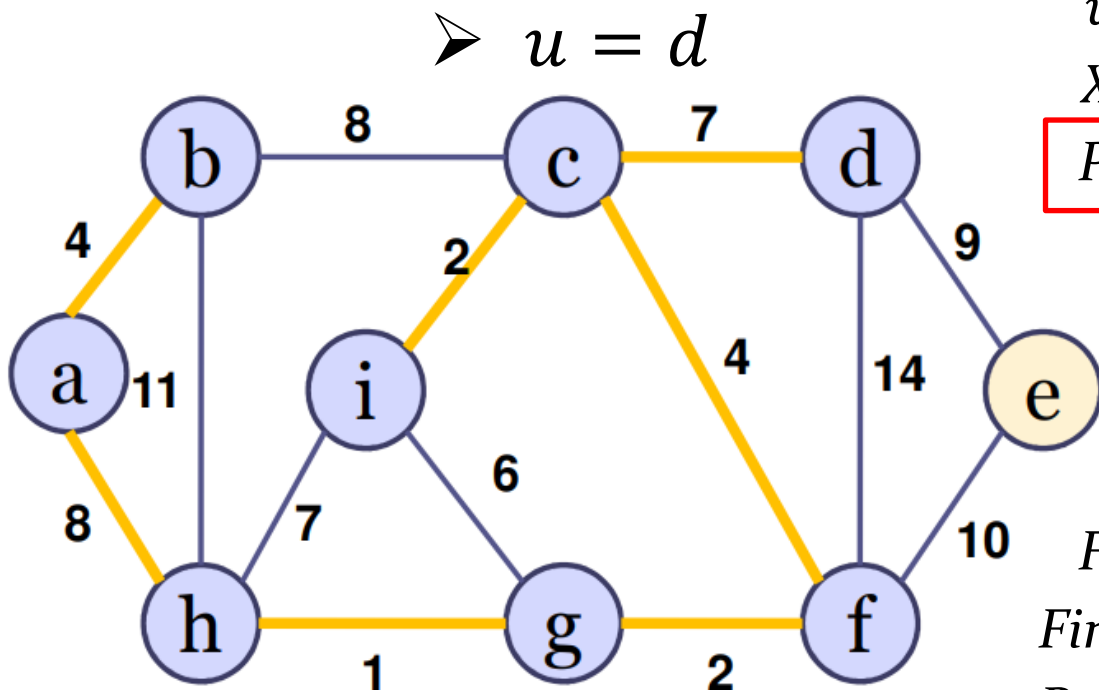
Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	7	10	2	1	8	2
π	NULL	a	f	c	f	g	h	a	c
Q	--	--	--	--	e	--	--	--	--

AGM – Algoritmo de Prim



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$\pi[v] \leftarrow u$

Fim_se

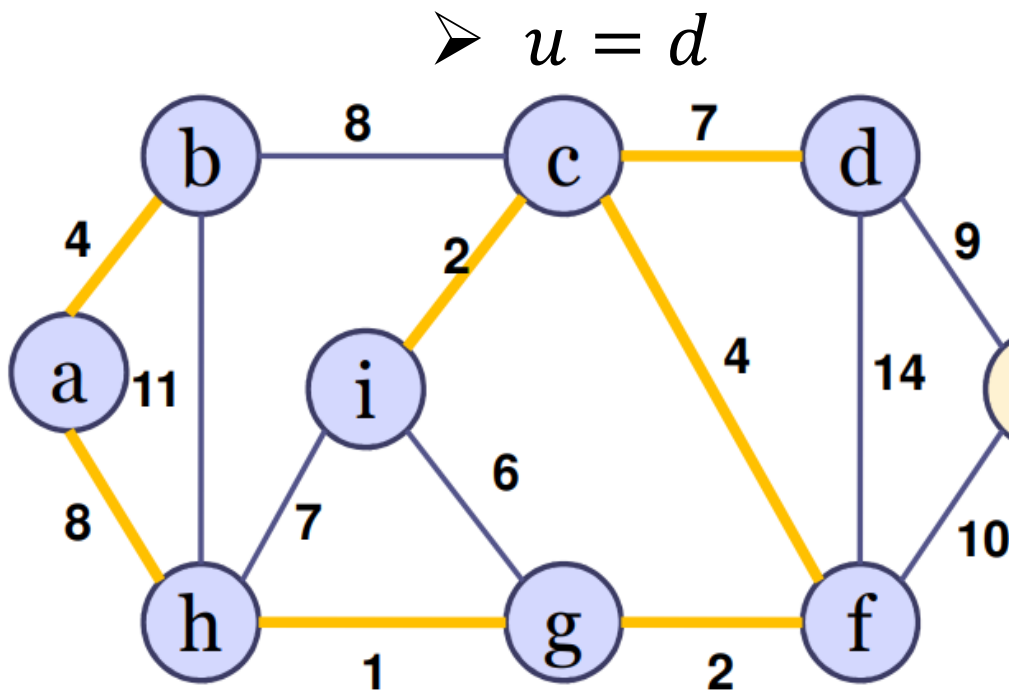
Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	7	10	2	1	8	2
π	NULL	a	f	c	f	g	h	a	c
Q	--	--	--	--	e	--	--	--	--

AGM – Algoritmo de Prim



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$\pi[v] \leftarrow u$

Fim_se

Fim_para

Fim_enquanto

Retorne X

vértice	a	b	c	d	e	f	g	h	i
chave	0	4	4	7	9	2	1	8	2
π	NULL	a	f	c	d	g	h	a	c
Q	--	--	--	--	e	--	--	--	--


$$u \leftarrow Extrai_Min(Q)$$

Para cada $v \in Adj[u]$ faça

$$chave[v] \leftarrow w(u, v)$$

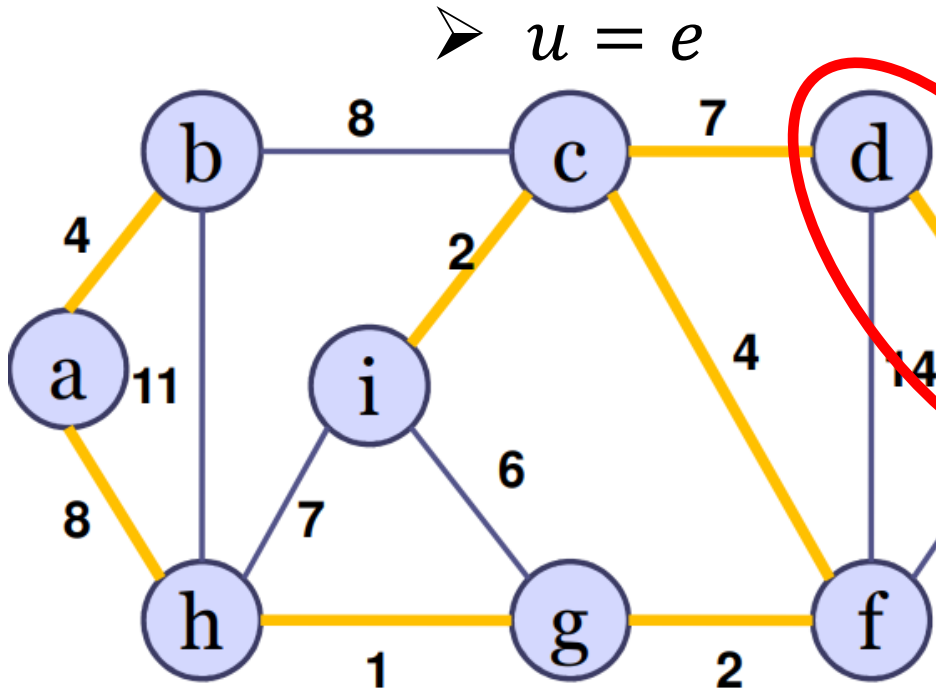
Fim_se

Fim_enquanto

Retorne X

e	9	d
--		

Country	Year	Value
China	2010	1.0
China	2011	1.0
China	2012	1.0
China	2013	1.0
China	2014	1.0
China	2015	1.0
China	2016	1.0
China	2017	1.0
China	2018	1.0
China	2019	1.0
China	2020	1.0
China	2021	1.0
China	2022	1.0
China	2023	1.0
China	2024	1.0
China	2025	1.0
China	2026	1.0
China	2027	1.0
China	2028	1.0
China	2029	1.0
China	2030	1.0
China	2031	1.0
China	2032	1.0
China	2033	1.0
China	2034	1.0
China	2035	1.0
China	2036	1.0
China	2037	1.0
China	2038	1.0
China	2039	1.0
China	2040	1.0
China	2041	1.0
China	2042	1.0
China	2043	1.0
China	2044	1.0
China	2045	1.0
China	2046	1.0
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China	2049	1.0
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China	2104	1.0
China	2105	1.0
China	2106	1.0
China	2107	1.0
China	2108	1.0
China	2109	1.0
China	2110	1.0
China	2111	1.0
China	2112	1.0
China	2113	1.0
China	2114	1.0
China	2115	1.0
China	2116	1.0
China	2117	1.0
China	2118	1.0
China	2119	1.0
China	2120	1.0
China	2121	1.0
China	2122	1.0
China	2123	1.0
China	2124	1.0
China	2125	1.0
China	2126	1.0
China	2127	1.0
China	2128	1.0
China	2129	1.0
China	2130	1.0
China	2131	1.0
China	2132	1.0</



Enquanto $Q \neq \{\}$

$$u \leftarrow Extrai_Min(Q)$$
$$X \leftarrow X \cup \{(u, \pi[u])\} // se\ u \neq r$$

Para cada $v \in Adj[u]$ faça

Se $(v \in Q)$ e $w(u, v) < chave[v]$

$$chave[v] \leftarrow w(u, v)$$
$$\pi[v] \leftarrow u$$

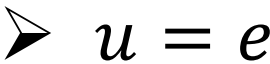
Fim_se

Fim_para

Fim_enquanto

Retorne X

[illegible]



Retorne X





Retorne X

[illegible]

Country	Year	Value
China	2010	1.0
China	2011	1.0
China	2012	1.0
China	2013	1.0
China	2014	1.0
China	2015	1.0
China	2016	1.0
China	2017	1.0
China	2018	1.0
China	2019	1.0
China	2020	1.0
China	2021	1.0
China	2022	1.0
China	2023	1.0
China	2024	1.0
China	2025	1.0
China	2026	1.0
China	2027	1.0
China	2028	1.0
China	2029	1.0
China	2030	1.0
China	2031	1.0
China	2032	1.0
China	2033	1.0
China	2034	1.0
China	2035	1.0
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China	2084	1.0
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China	2116	1.0
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China	2118	1.0
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China	2120	1.0
China	2121	1.0
China	2122	1.0
China	2123	1.0
China	2124	1.0
China	2125	1.0
China	2126	1.0
China	2127	1.0
China	2128	1.0
China	2129	1.0
China	2130	1.0
China	2131	1.0
China	2132	1.0</


$$u \leftarrow Extrai_Min(Q)$$

Para cada $v \in Adj[u]$ faça

$$chave[v] \leftarrow w(u, v)$$

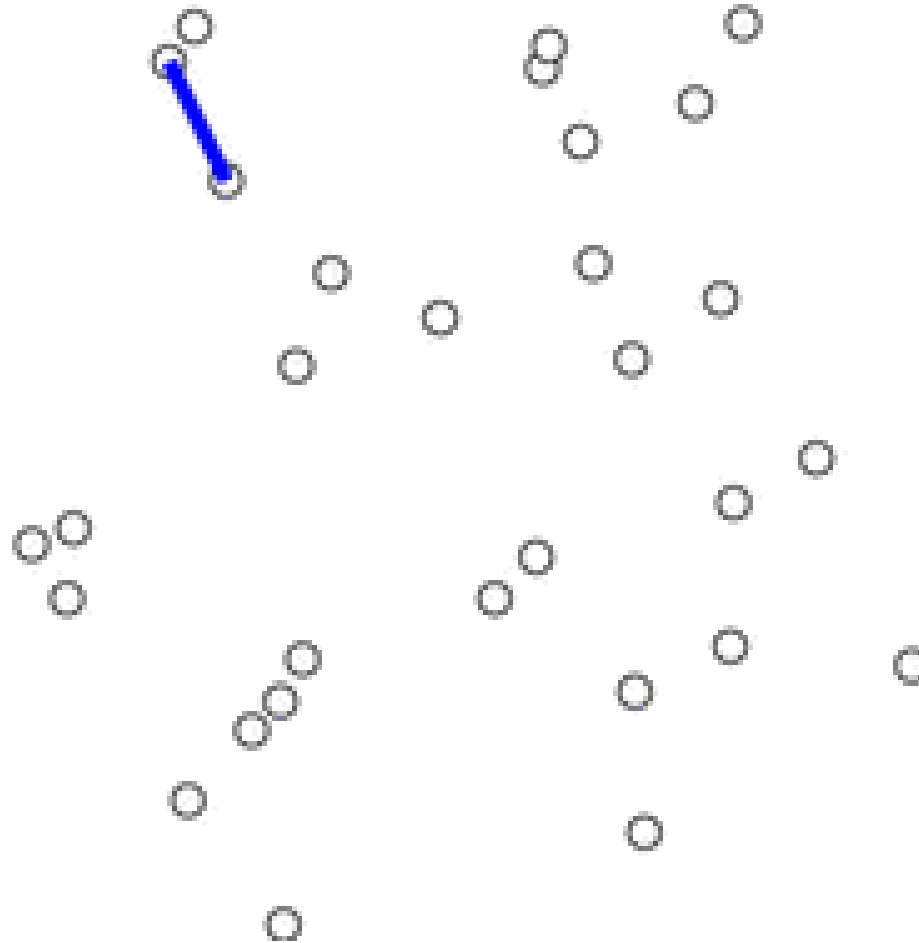
Fim_se

Fim_enquanto

Retorne X

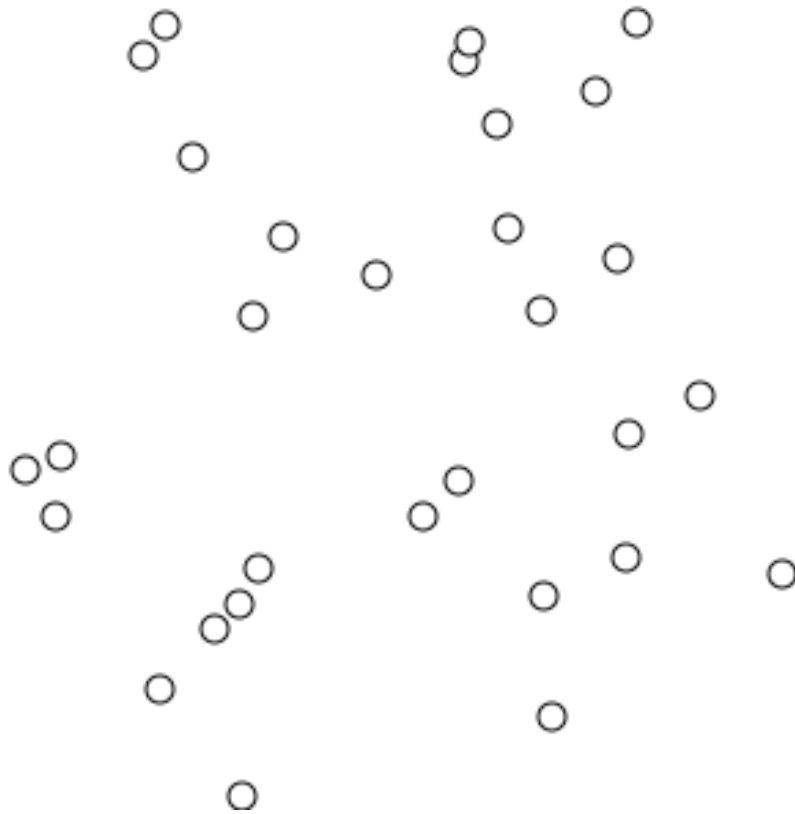
π	NULL	a	f	c	d	g	h	a	c
-------	------	---	---	---	---	---	---	---	---

Algoritmo de Prim - Demo

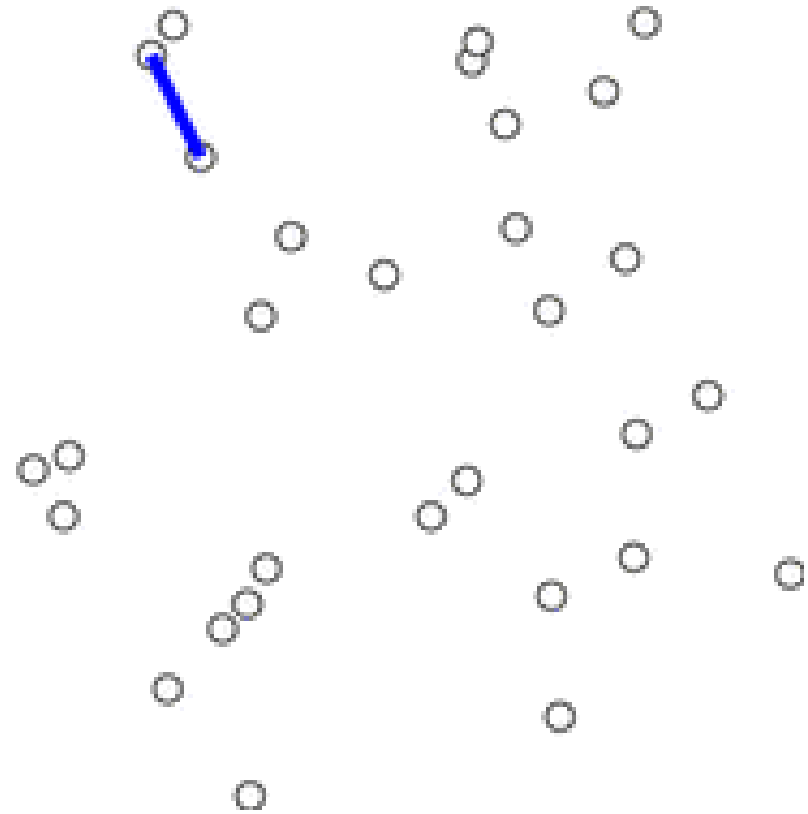


Demonstração do algoritmo de Prim em um gráfico completo com pesos baseados na distância euclidiana (<https://en.wikipedia.org/wiki/File:PrimAlgDemo.gif>)

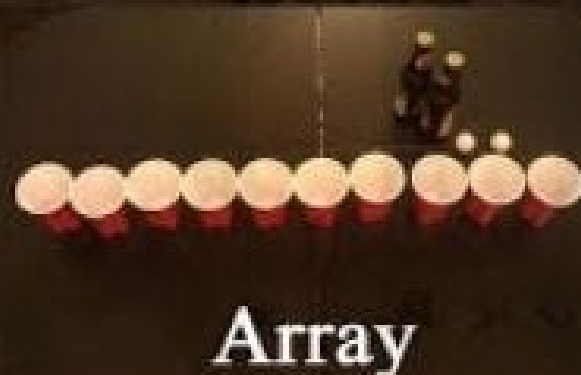
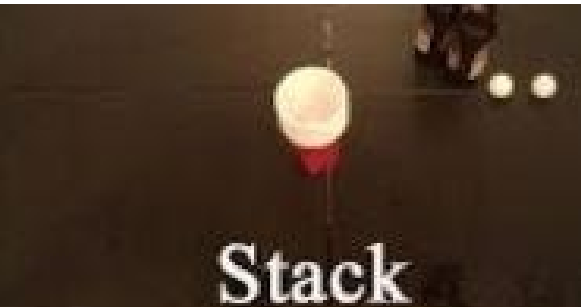
- Kruskal



- Prim



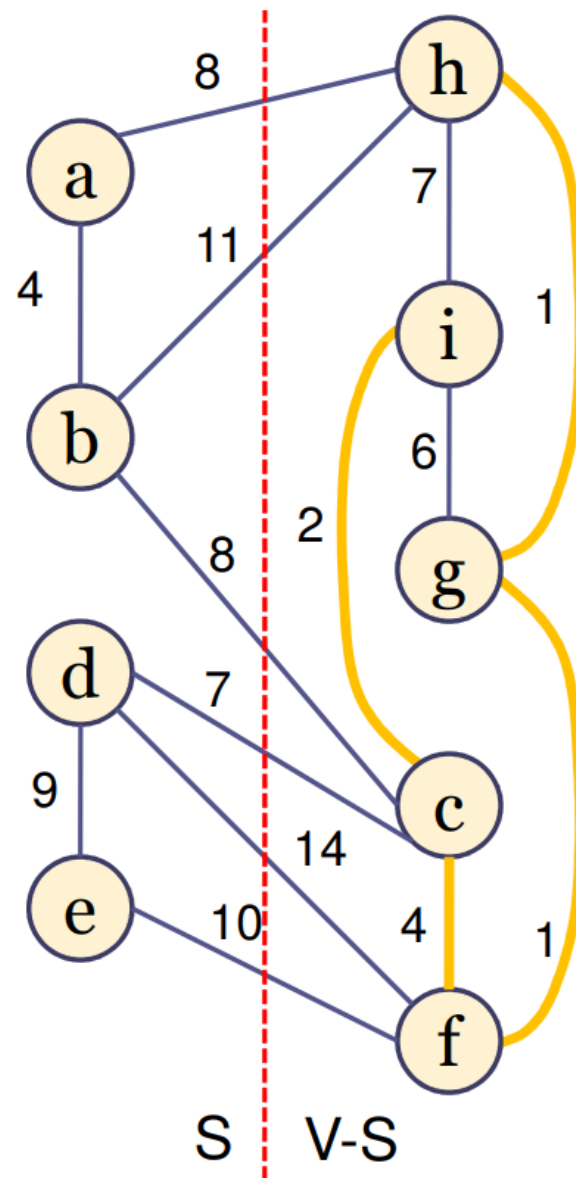
Compreendendo PRIM nos moldes do Algoritmo Genérico para construir a AGM



Árvore Geradora Mínima – Prim

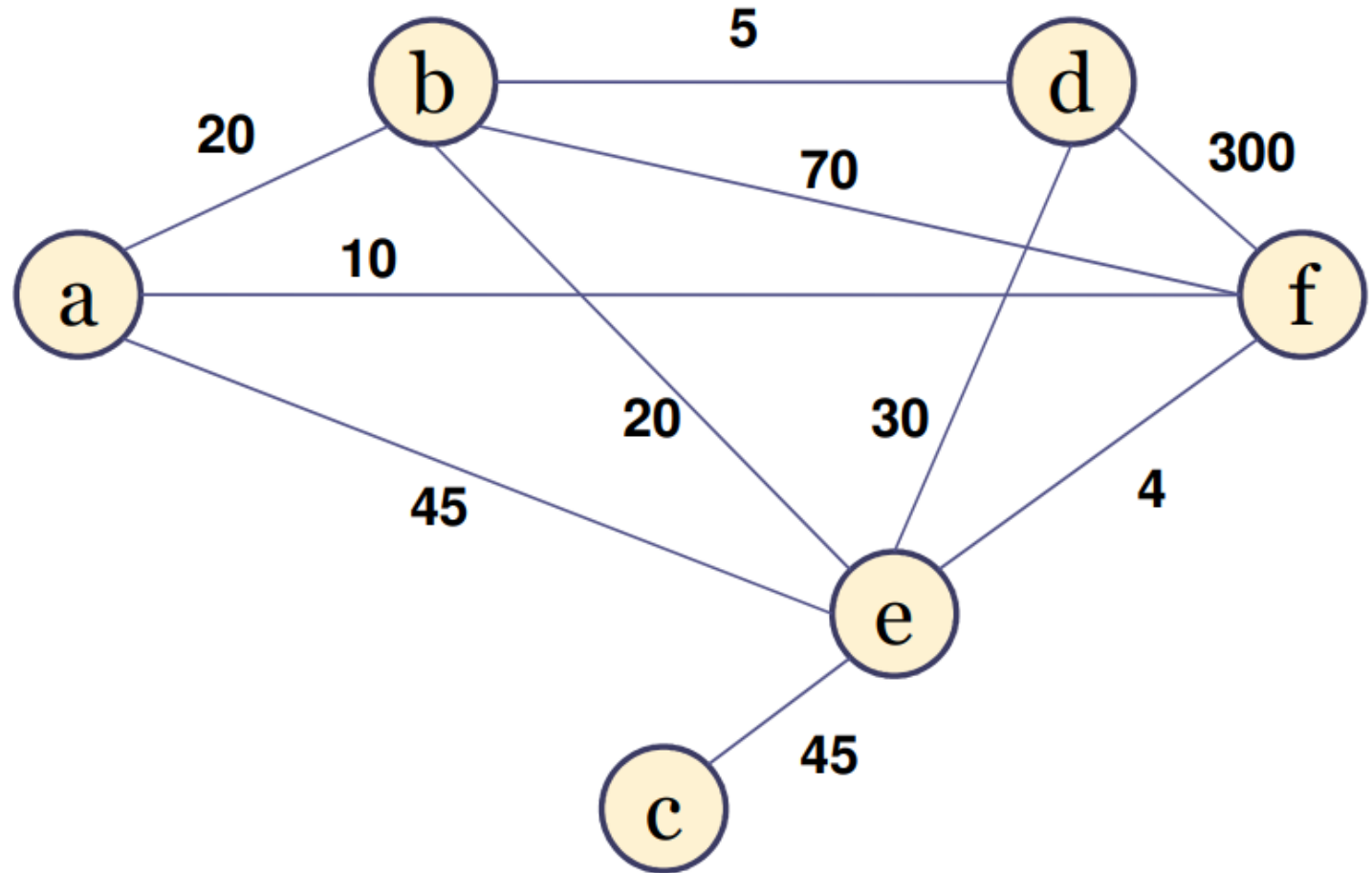
- Ao contrário do algoritmo de Kruskal, no algoritmo de Prim o procedimento de corte em grafo pode ser facilmente visualizado e implementado nos modelos do algoritmo genérico para construir uma AGM

$$S = \{a, b, d, e\}$$
$$V - S = \{h, i, c, g, f\}$$



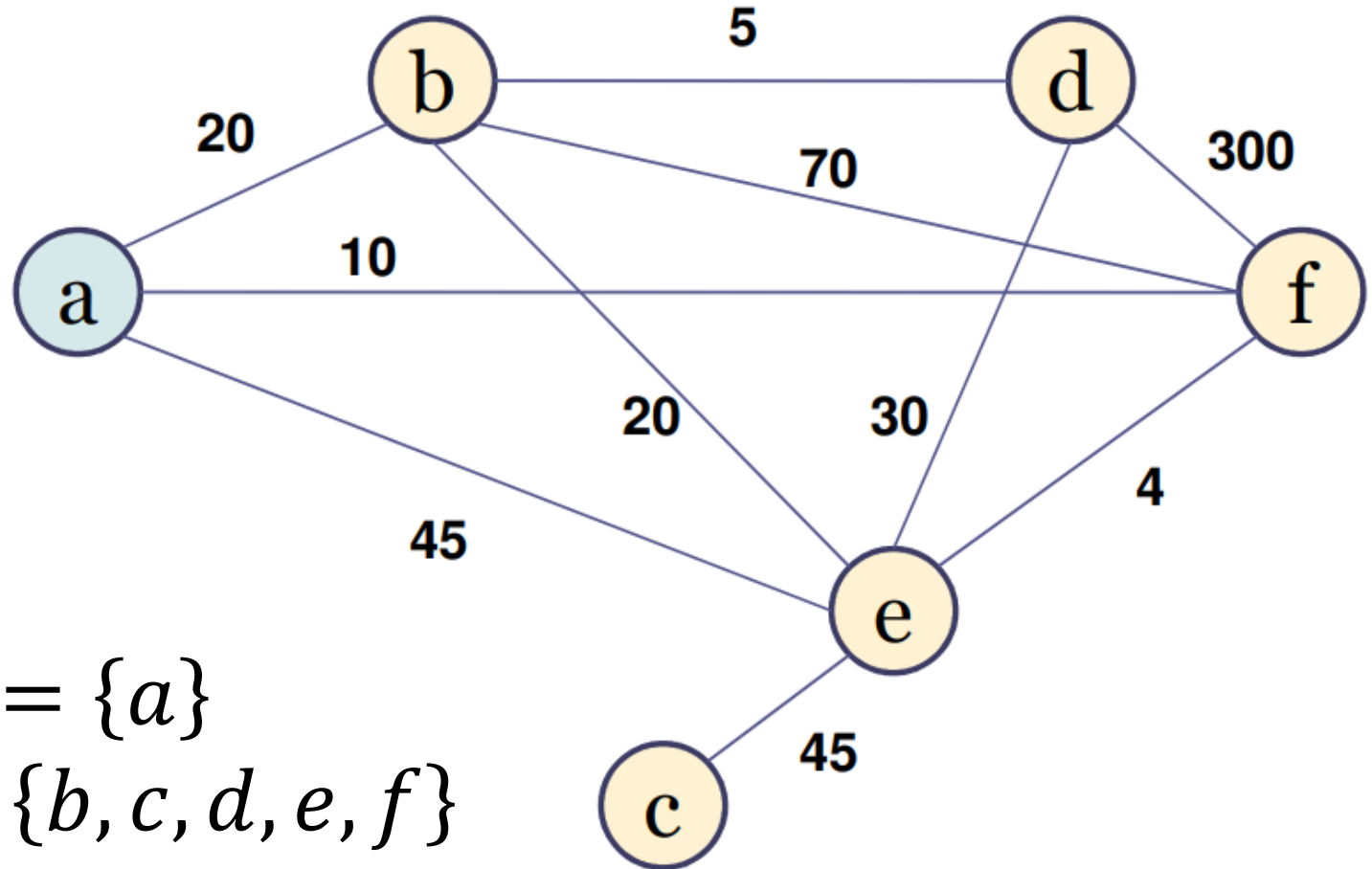
Prim nos models do algoritmo genérico

- Começando de $r = a$



Prim nos models do algoritmo genérico

- Começando de $r = a$



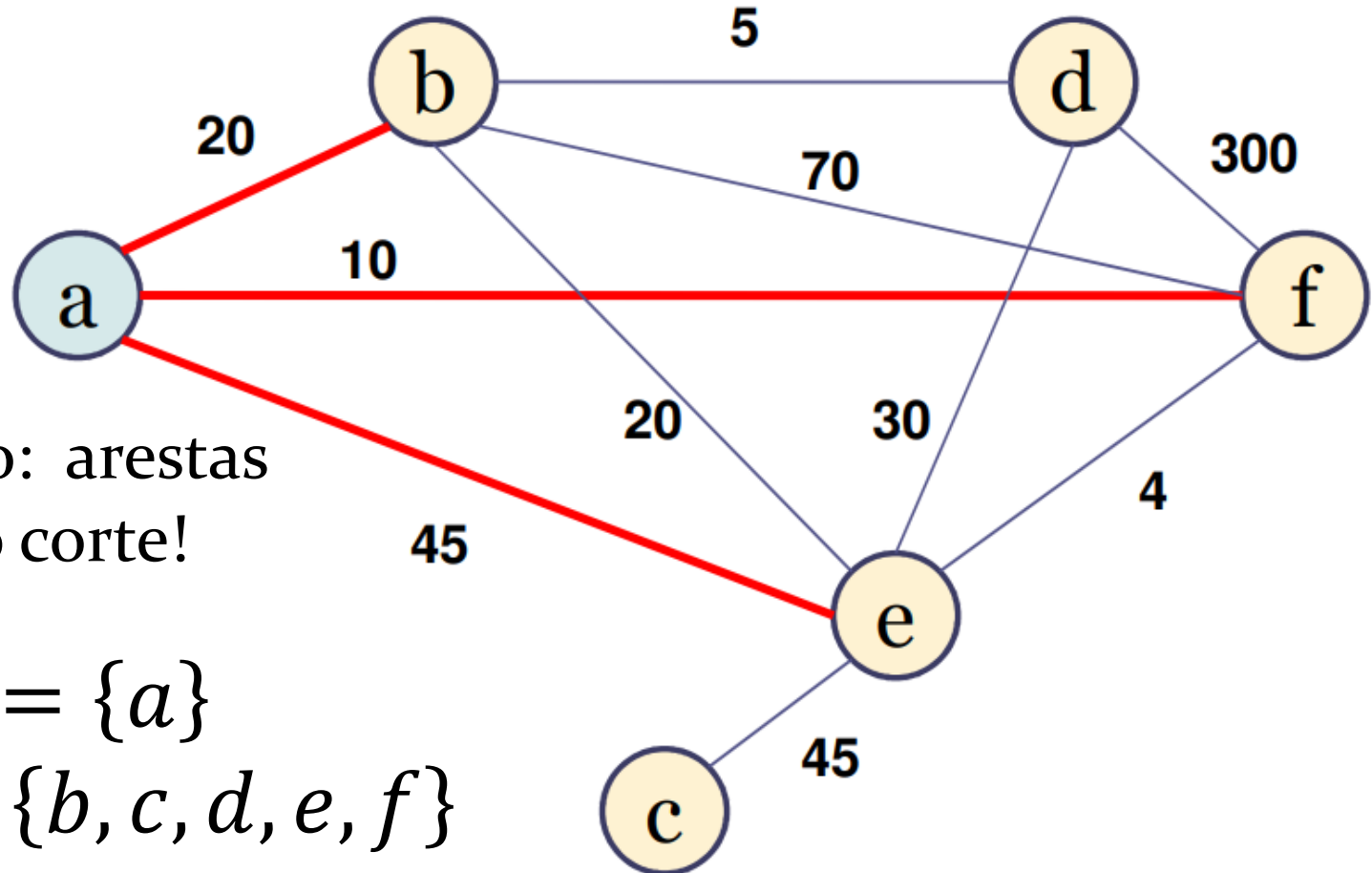
$$S = \{a\}$$

$$V - S = \{b, c, d, e, f\}$$

$$X = \{\}$$

Prim nos models do algoritmo genérico

- Começando de $r = a$



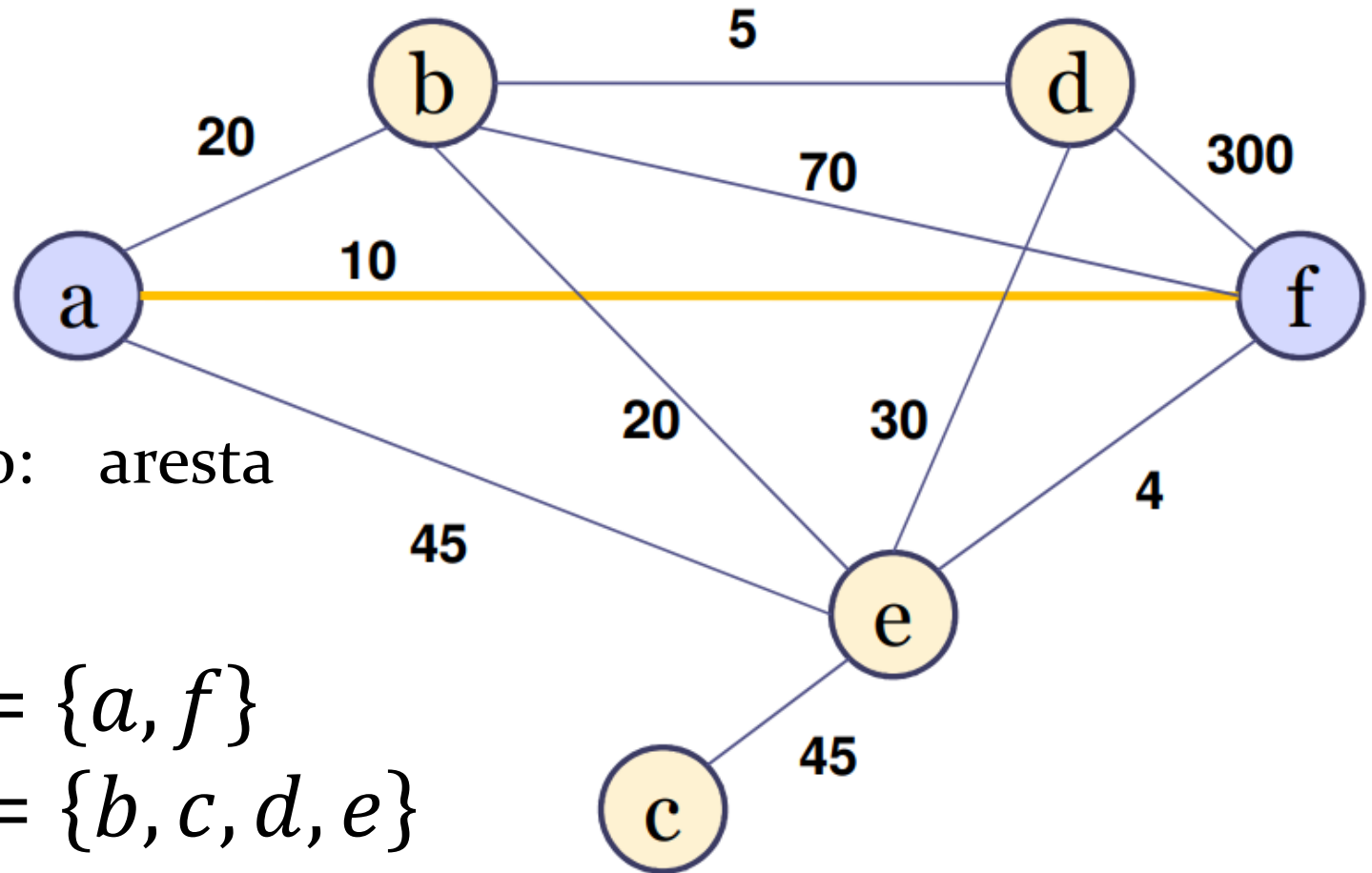
Em vermelho: arestas
que cruzam o corte!

$$S = \{a\}$$

$$V - S = \{b, c, d, e, f\}$$

$$X = \{\}$$

Prim nos models do algoritmo genérico



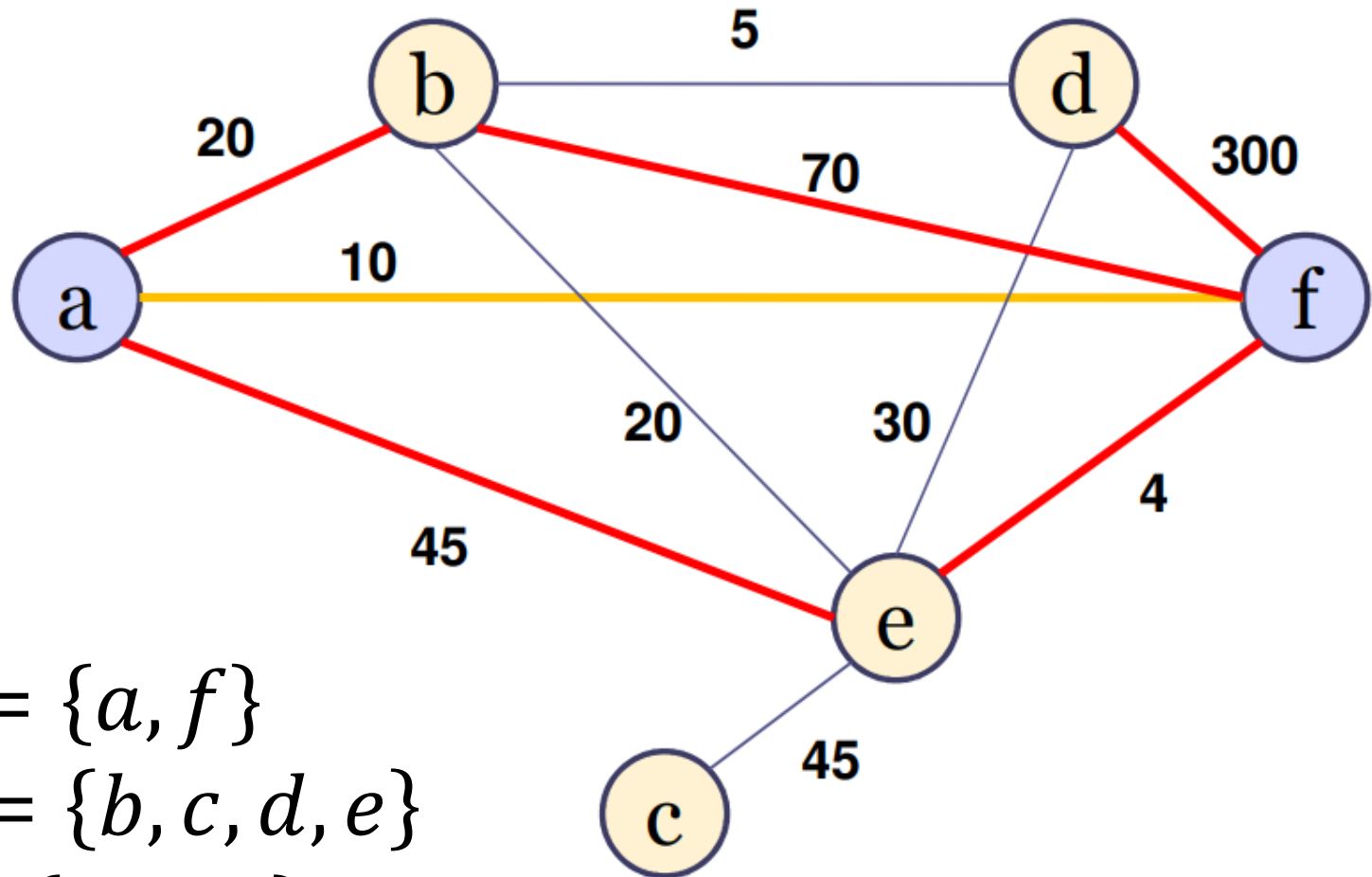
Em amarelo: aresta
leve!

$$S = \{a, f\}$$

$$V - S = \{b, c, d, e\}$$

$$X = \{(a, f)\}$$

Prim nos models do algoritmo genérico

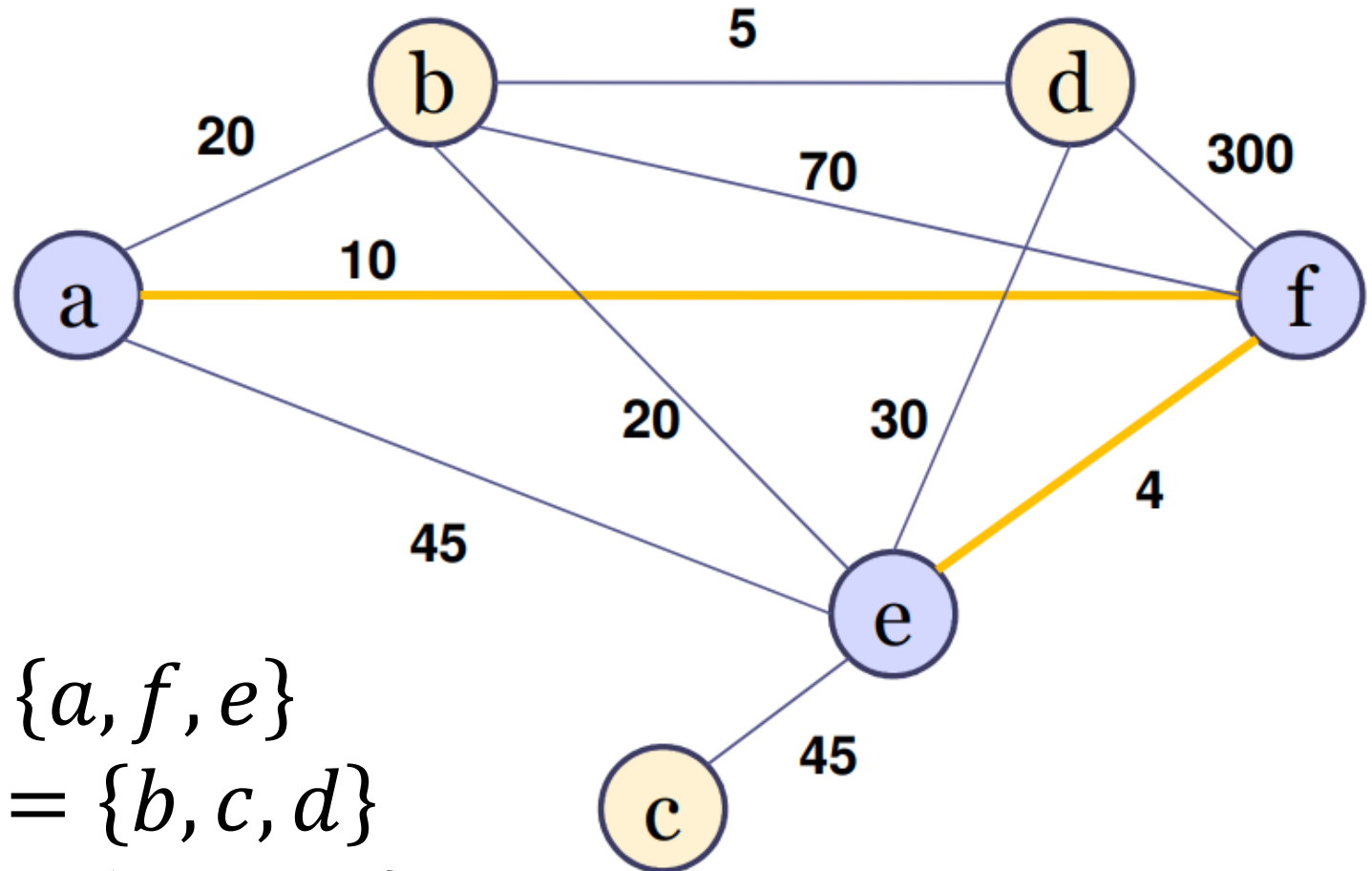


$$S = \{a, f\}$$

$$V - S = \{b, c, d, e\}$$

$$X = \{(a, f)\}$$

Prim nos models do algoritmo genérico

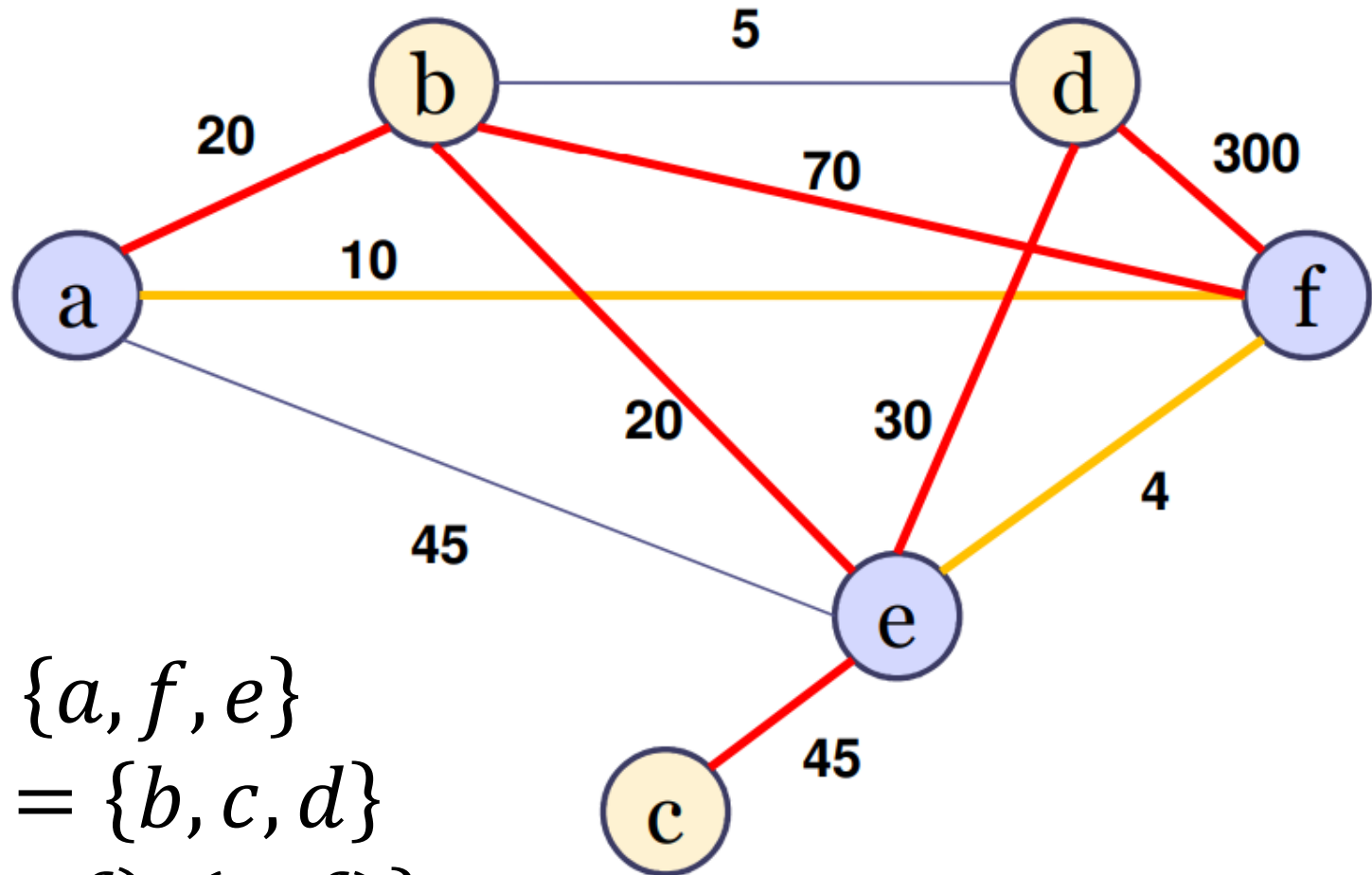


$$S = \{a, f, e\}$$

$$V - S = \{b, c, d\}$$

$$X = \{(a, f), (e, f)\}$$

Prim nos models do algoritmo genérico

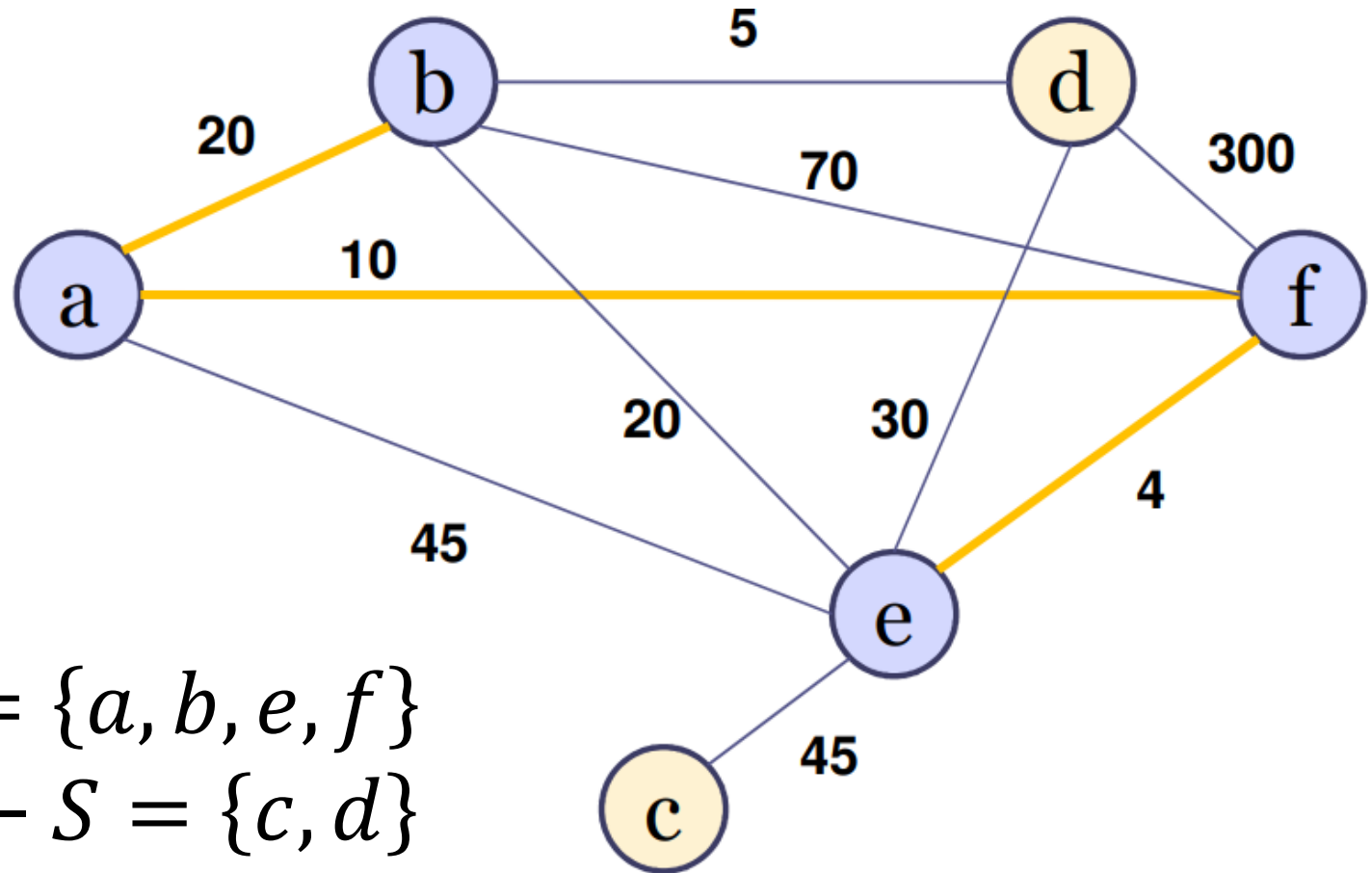


$$S = \{a, f, e\}$$

$$V - S = \{b, c, d\}$$

$$X = \{(a, f), (e, f)\}$$

Prim nos models do algoritmo genérico

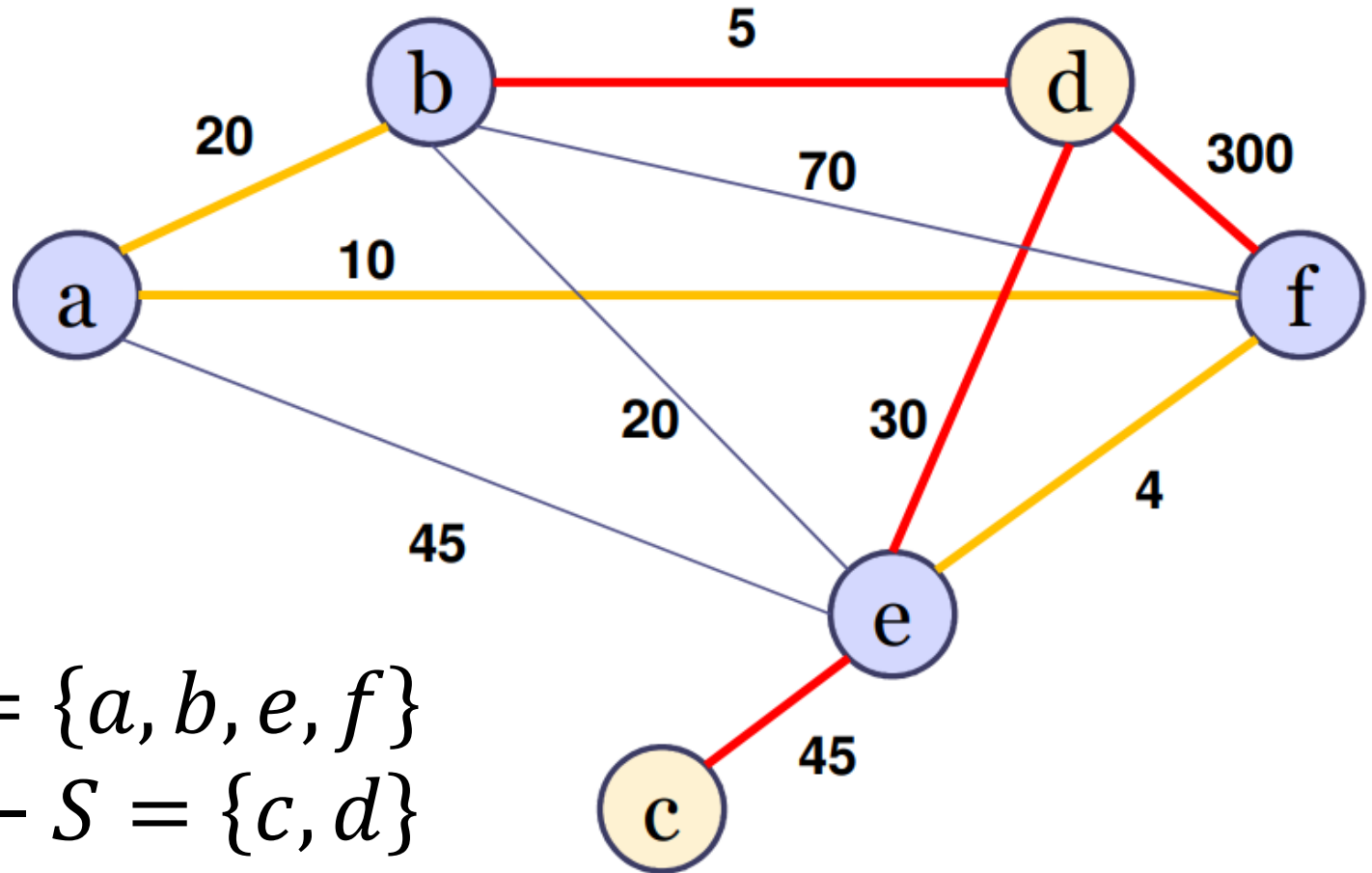


$$S = \{a, b, e, f\}$$

$$V - S = \{c, d\}$$

$$X = \{(a, f), (e, f), (a, b)\}$$

Prim nos models do algoritmo genérico

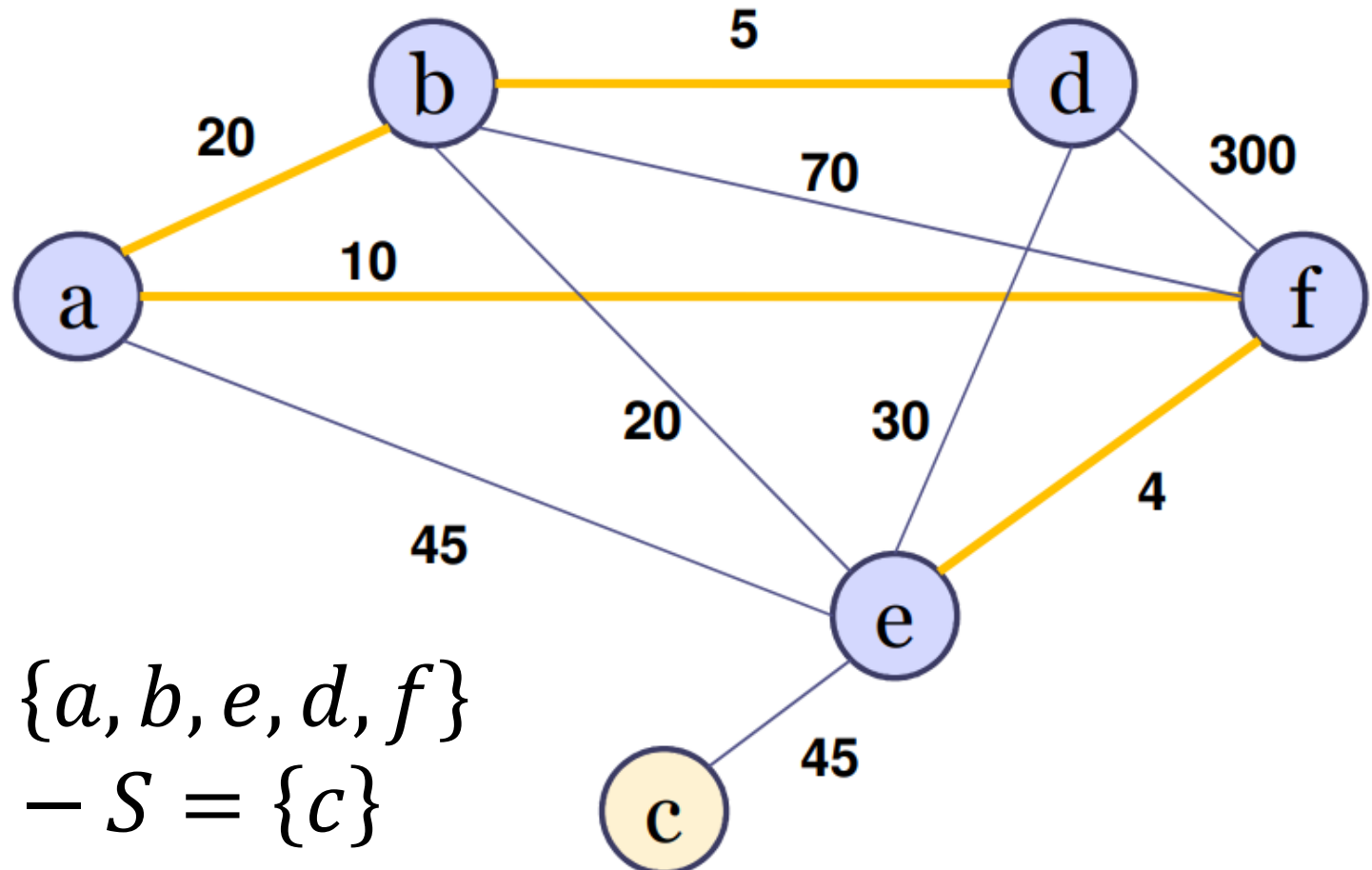


$$S = \{a, b, e, f\}$$

$$V - S = \{c, d\}$$

$$X = \{(a, f), (e, f), (a, b)\}$$

Prim nos models do algoritmo genérico

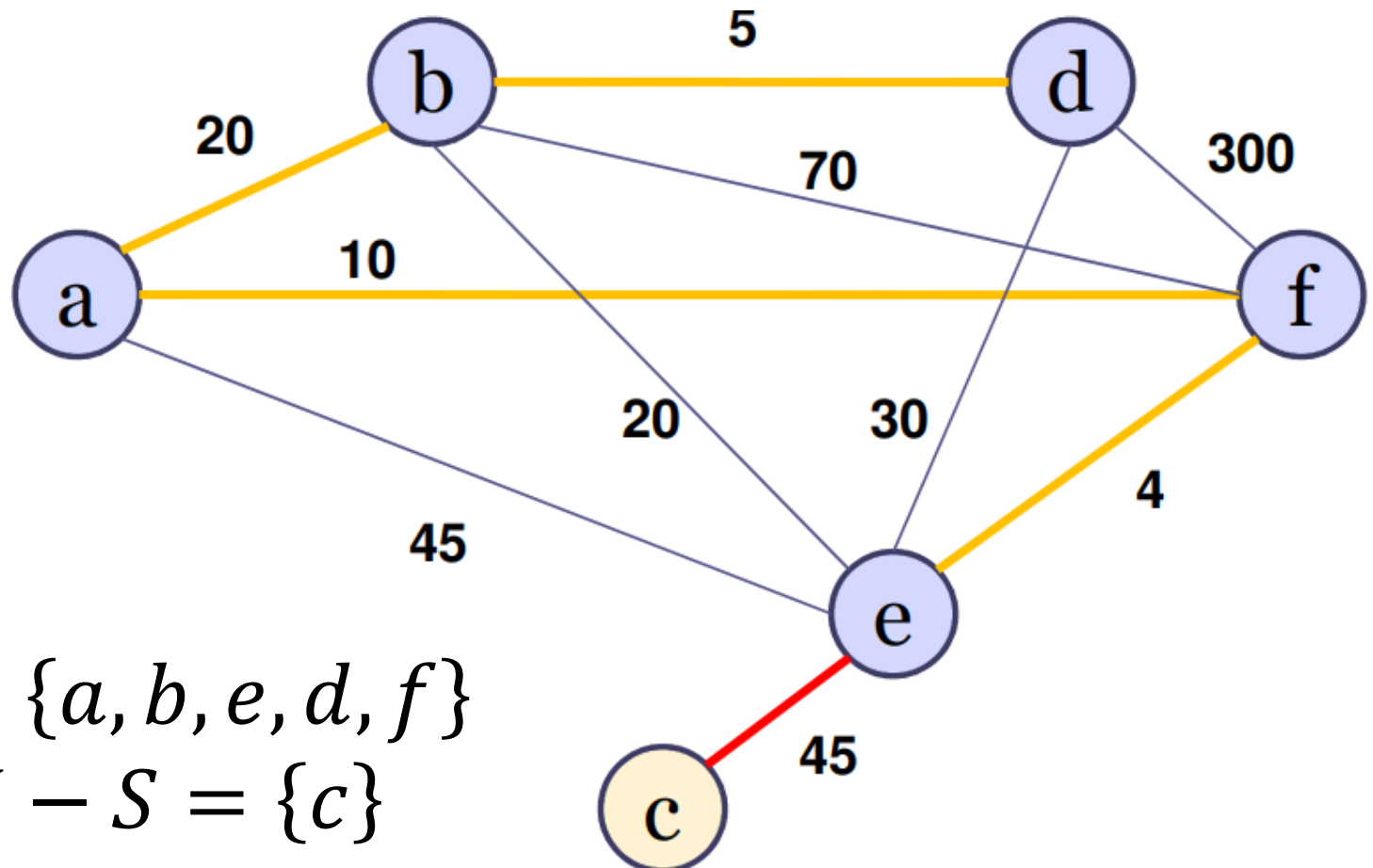


$$S = \{a, b, e, d, f\}$$

$$V - S = \{c\}$$

$$X = \{(a, f), (e, f), (a, b), (b, d)\}$$

Prim nos models do algoritmo genérico



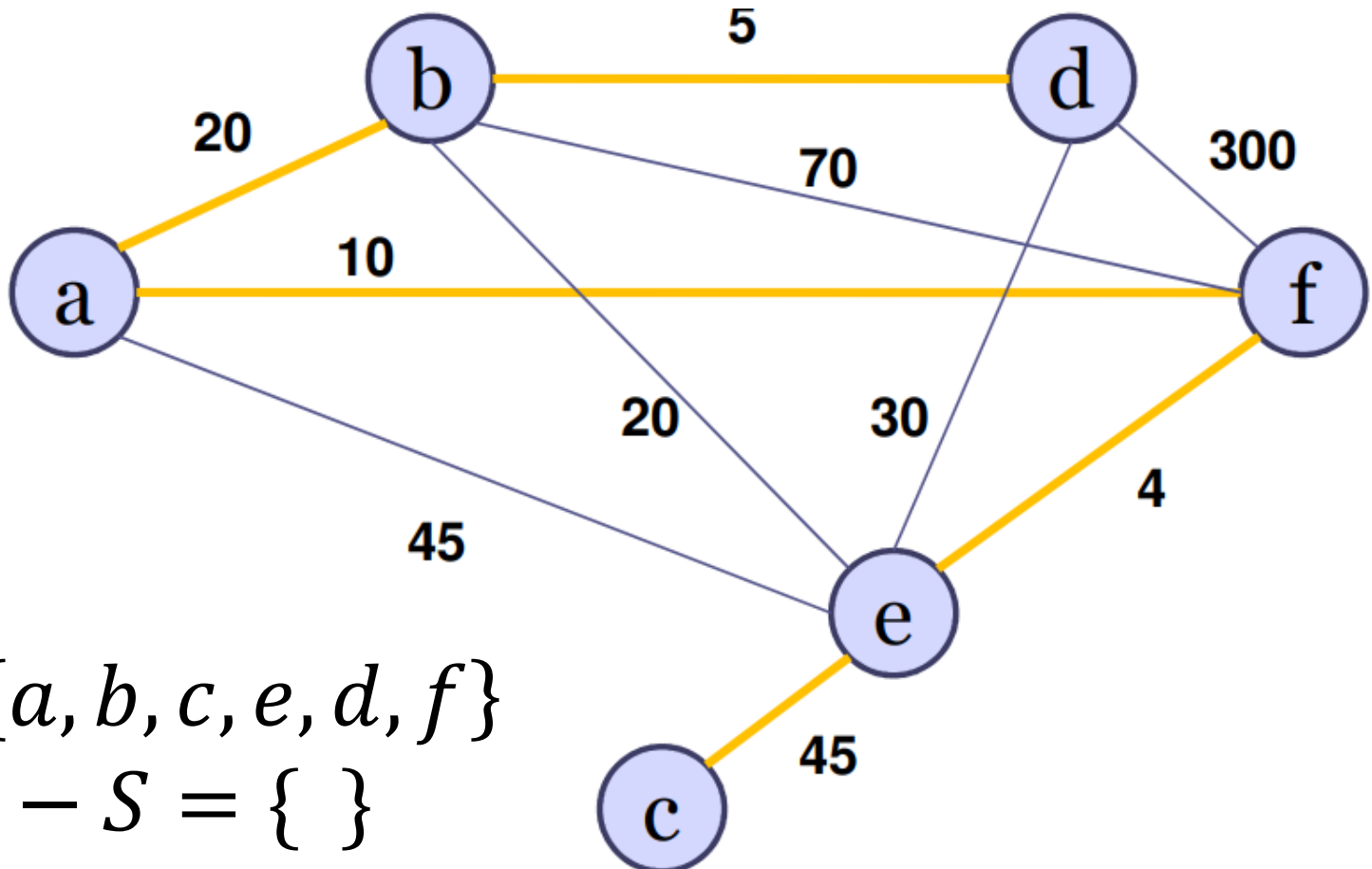
$$S = \{a, b, e, d, f\}$$

$$V - S = \{c\}$$

$$X = \{(a, f), (e, f), (a, b), (b, d)\}$$

Prim nos models do algoritmo genérico

Custo da AGM: 84



$$S = \{a, b, c, e, d, f\}$$

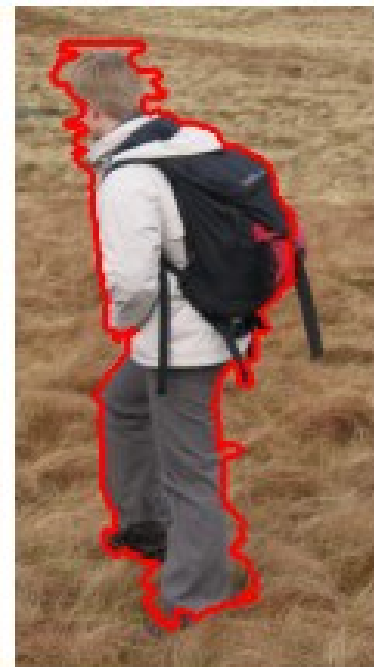
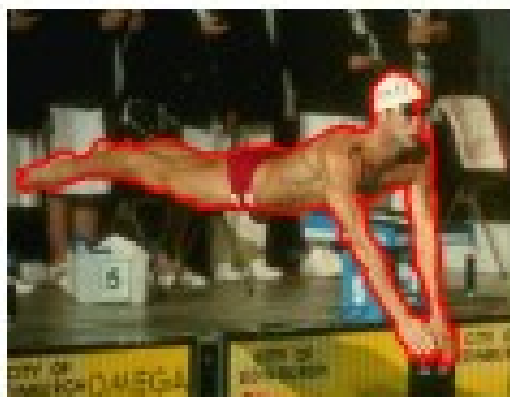
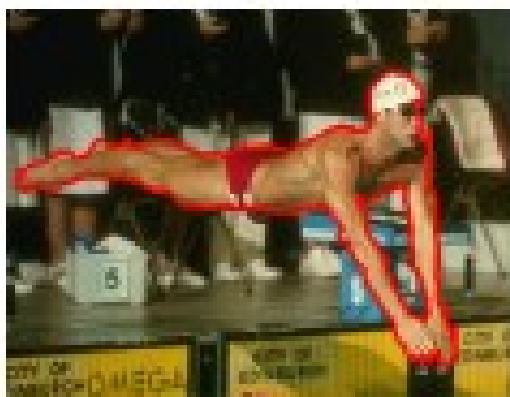
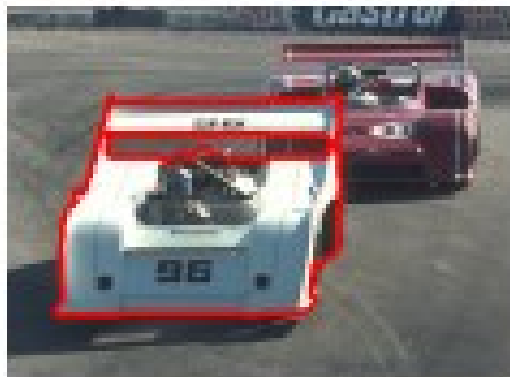
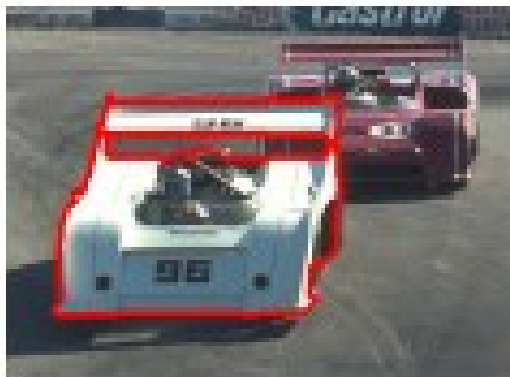
$$V - S = \{ \}$$

$$X = \{(a, f), (e, f), (a, b), (b, d), (c, e)\}$$

Prim e Kruskal – Complexidades vs TADs

- Prim
 - Adicionar uma aresta de menor custo que interliga um novo vértice a cada iteração.
 - $O(|V|^2)$ – Matriz de Adjacência.
 - $O(|E| \log(|V|))$ – Lista de Adjacência.
- Kruskal
 - Adicionar uma aresta de menor peso que interliga árvores distintas a cada iteração.
 - $O(|V|^2 \log(|V|))$ – Matriz de Adjacência.
 - $O(|E| \log(|V|))$ – Lista de Adjacência.

Prim e Kruskal – Árvores Geradoras Mínimas



Kruskal

Prim

Kruskal

Prim