

Unit : II Combinational circuits

Digital Logic circuits

Combinational

- output depends only on present input
- Half adder
- Full adder
- Ripple carry adder
- Carry look ahead adder
- Multiplexer
- Demultiplexer
- Coder
- Decoder
- Comparator

Sequential

Flip flops

Register

Counter

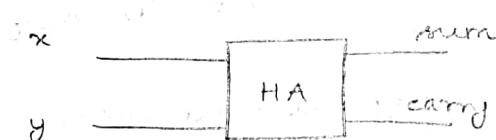
Shift register

ALU

Memory

Control unit

Half adder:

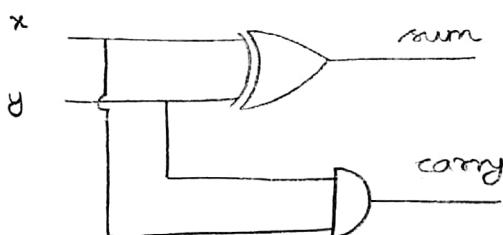


x	y	sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

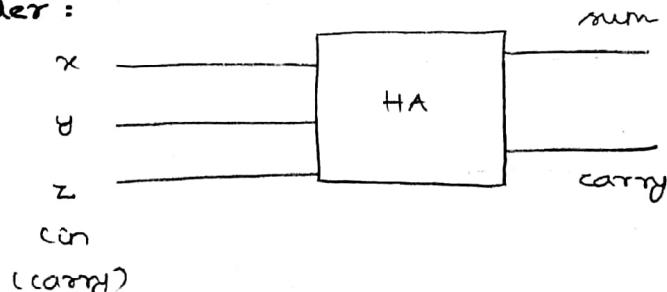
$$\text{sum} = \overline{x}\overline{y} + \overline{x}y + x\overline{y}$$

sum of minterms

$$\text{carry} = xy$$



Full adder:



$$\text{sum} = \bar{x}\bar{y}\text{con} + \bar{x}y\bar{\text{con}} + x\bar{y}\text{con} + xy\bar{\text{con}}$$

$$x\bar{y}\bar{\text{con}} + y\bar{x}\bar{\text{con}}$$

$$\text{carry} = x\text{con} + \bar{x}y + y\text{con}$$

	$y\text{con}$	x	num	
x	00	01	11	10
0	0	1	3	2
1	1	4	5	6

	$y\text{con}$	x	num	carry
x	00	01	11	10
0	0	1	1	3
1	1	4	5	7

$$\text{sum} = \bar{x}y(1 + \text{con}) + \bar{x}\bar{y}\text{con} +$$

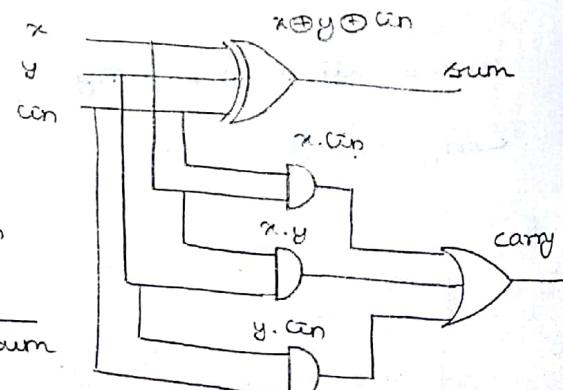
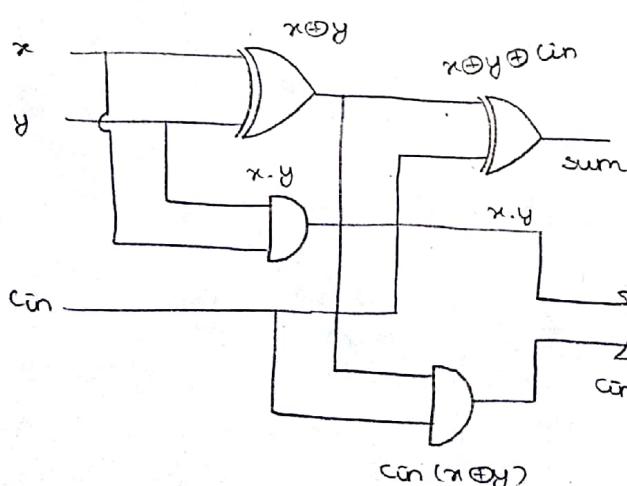
$$= \bar{\text{con}}(\bar{x}y + \bar{x}\bar{y}) + \text{con}(xy + \bar{x}\bar{y})$$

$$= \bar{\text{con}}(x \oplus y) + \text{con}(xy)$$

$$= \bar{\text{con}}(A) + \text{con}(\bar{A})$$

$$= \bar{\text{con}} \oplus A$$

$$\text{sum} = \bar{\text{con}} \oplus x \oplus y$$



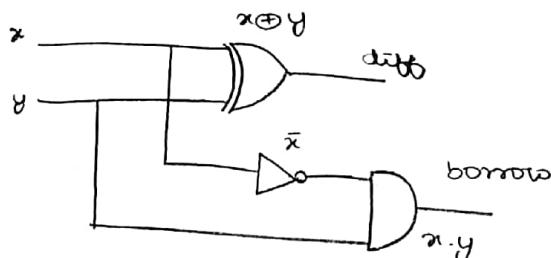
$$\text{con}(x \oplus y)$$

Half subtractor :

x	y	$A \oplus Y$	diff	borrow
0	0	0	0	0
0	1	1	1	0
1	0	1	0	0
1	1	0	0	0

$$\text{sum} = \bar{x}y + \bar{y}x = x \oplus y$$

$$\text{borrow} = \bar{x}y$$



x	y	cin	diff	borrow
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

Full subtractor :

$$\text{difference} = \bar{x}\bar{y}\text{cin} + \bar{x}y\text{cin} + \\ x\bar{y}\text{cin} + xy\text{cin}$$

$$\text{borrow} = \bar{x}\text{cin} + \bar{x}y + y\text{cin}$$

$$\text{difference} = \text{cin}(\bar{x}y + \bar{x}\bar{y}) + \\ \text{cin}(\bar{x}y + x\bar{y})$$

$$= \text{cin}(\bar{x}y) + \text{cin}(x\bar{y})$$

$$= \text{cin} \bar{x} + \text{cin} A$$

$$= \text{cin} \oplus x \oplus y$$

		difference			
		00	01	11	10
		x	y	cin	
0	0	0	1	0	1
1	0	1	0	0	2
0	1	4	5	1	7
1	1	5	6	1	6

		borrow			
		00	01	11	10
		x	y	cin	
0	0	0	1	0	1
1	0	1	0	0	2
0	1	4	5	1	7
1	1	5	6	1	6

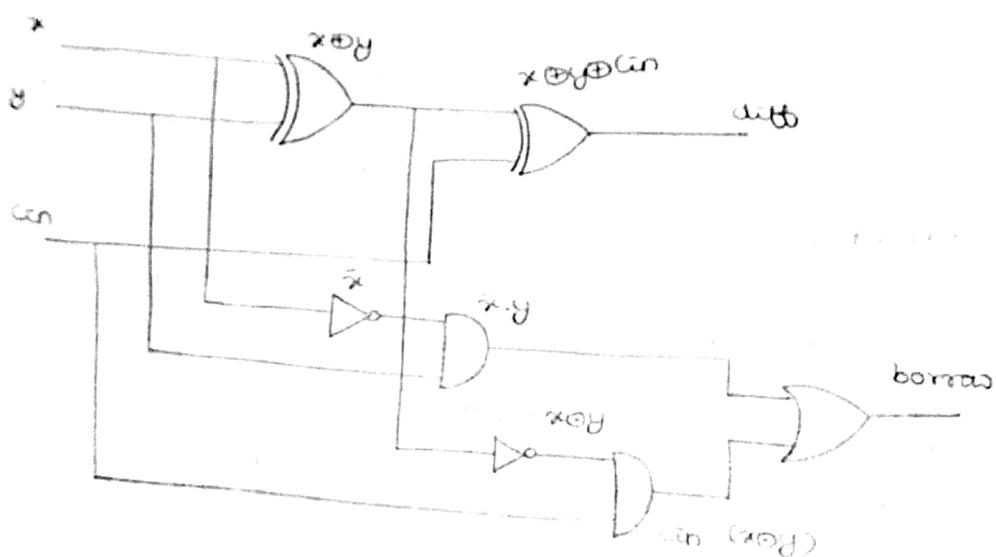
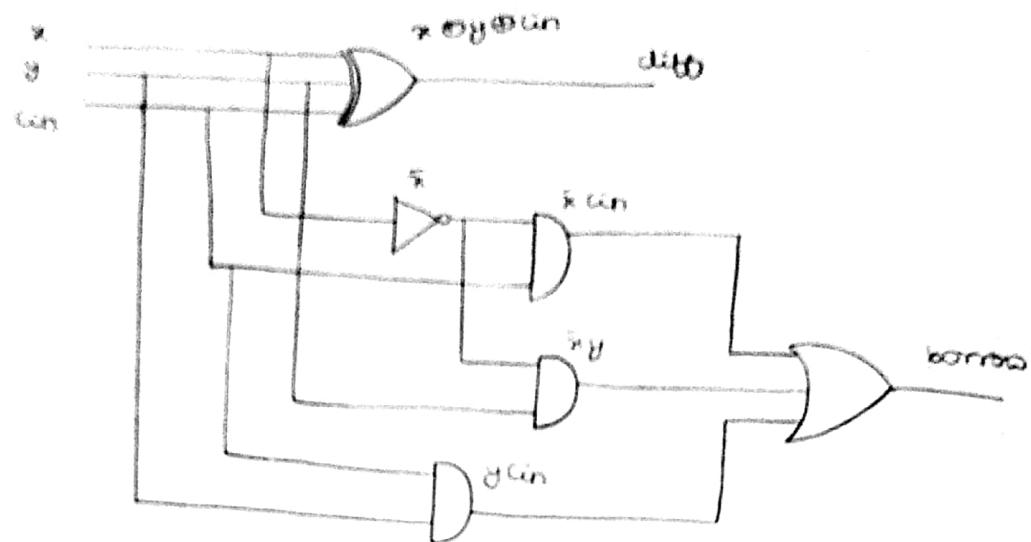
$$\text{borrow} = \bar{x}\text{cin} + \bar{x}y + y\text{cin}$$

$$= \bar{x}y + \bar{x}(\text{cin})(y + \bar{y}) + y\text{cin}(\bar{x} + x)$$

$$= \bar{x}y + \bar{x}y\text{cin} + \bar{x}\bar{y}\text{cin} + xy\text{cin} + \bar{y}y\text{cin}$$

$$= \bar{x}y + \text{cin}(\bar{x}\bar{y} + xy)$$

$$= \overline{R_x} + \text{com}(R_y)$$



Code converters

1. BCD to Binary
2. BCD to Excess - 3
3. Binary to Gray
4. Gray to binary
5. Binary to BCD

BCD

BINARY

A_3	A_2	A_1	A_0	B_3	B_2	B_1	B_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	X	0	0	1	0
0	0	1	1	0	0	1	1
0	1	0	0	0	1	0	0
0	0	1	1	0	1	0	1
0	1	1	0	0	1	1	0
0	1	1	1	0	1	1	1
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	1

 B_3

A_3	A_2	$A_1 A_0$
00	01	00 01 11 10
0	0	0 0 0 0
0	1	0 1 0 2
1	0	4 5 7 6
1	1	X 12 13 15 14
10	0	1 1 X X 11 10

$$B_3 = A_3$$

BCD

Excess - 3

A_3	A_2	$A_1 A_0$
00	01	00 01 11 10
0	0	0 0 0 0
0	1	0 1 0 2
1	0	4 5 7 6
1	1	X 12 13 15 14
10	0	1 1 X X 11 10

 E_3

A_3	A_2	$A_1 A_0$
00	01	00 01 11 10
0	0	0 0 0 0
0	1	0 1 0 2
1	0	4 5 7 6
1	1	X 12 13 15 14
10	0	1 1 X X 11 10

$$E_3 = A_3 + A_2 A_0 + A_2 A_1$$

BCD

Excess - 3

A_3	A_2	A_1	A_0	E_3	E_2	E_1	E_0
0	0	0	0	0	0	1	1
1	0	0	1	0	1	0	0
2	0	1	0	0	1	0	1
3	0	0	1	0	1	1	0
0	1	0	0	0	1	1	1
0	0	1	0	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	0	1	1

A_3	A_2	$A_1 A_0$
00	01	00 01 11 10
0	0	0 0 0 0
0	1	0 1 0 2
1	0	4 5 7 6
1	1	X 12 13 15 14
10	0	1 1 X X 11 10

 E_2

A_3	A_2	$A_1 A_0$
00	01	00 01 11 10
0	0	0 0 0 0
0	1	0 1 0 2
1	0	4 5 7 6
1	1	X 12 13 15 14
10	0	1 1 X X 11 10

$$\begin{aligned} E_2 &= A_3 A_0 + \bar{A}_2 A_1 \\ &= A_2 \bar{A}_1 \bar{A}_0 + A_3 A_0 \\ &= A_3 A_0 + \bar{A}_2 A_1 + \\ &\quad A_2 \bar{A}_1 \bar{A}_0 \end{aligned}$$

		E ₁				E ₀					
		A ₁ A ₀	00	01	11	10	A ₁ A ₀	00	01	11	10
A ₃ A ₂		00	1 0	0 1	1 0	0 2	00	1 0	0 1	0 3	1 2
		01	1 4	0 5	1 7	0 6	01	1 4	0 5	0 7	1 6
		11	X 12	X 13	X 15	X 14	11	X 12	X 13	X 15	X 14
		10	1 8	0 9	X 11	X 10	10	1 8	0 9	X 11	X 10

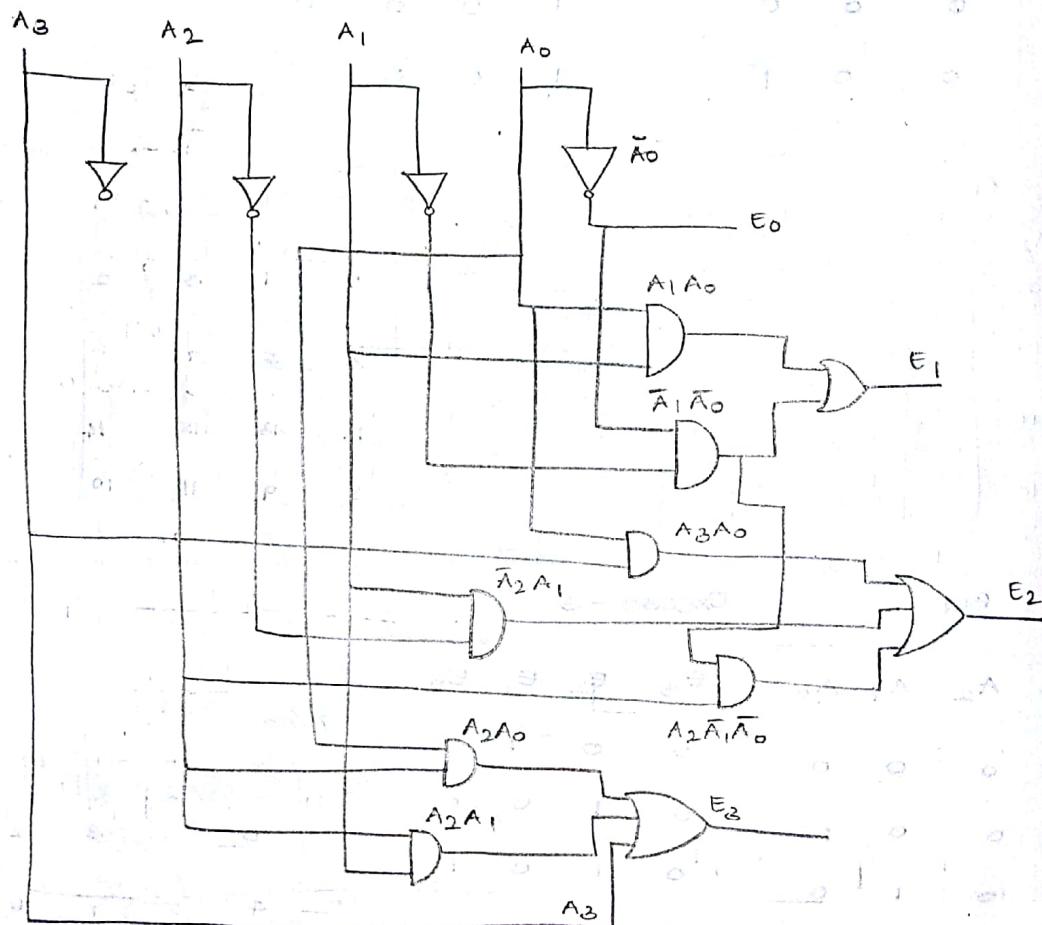
$$E_1 = \bar{A}_1 \bar{A}_0 + A_1 A_0$$

$$= A_1 \odot A_0$$

$$E_0 = \bar{A}_0$$

$$E_3 = A_3 + A_2 A_0 + A_2 A_1$$

$$E_2 = A_3 A_0 + \bar{A}_2 A_1 + A_2 \bar{A}_1 \bar{A}_0$$



$$B_0 = \bar{G}_3 \bar{G}_2 (G_1 \oplus G_0) + \bar{G}_3 G_2 (G_1 \odot G_0) + G_3 G_2 (G_1 \oplus G_0) +$$

$$= (G_1 \oplus G_0) (\bar{G}_3 \bar{G}_2 + G_3 G_2) + (G_1 \odot G_0) (\bar{G}_3 G_2 + G_3 \bar{G}_2)$$

$$= (G_1 \oplus G_0) (G_3 \odot G_2) + (G_1 \odot G_0) (G_3 \oplus G_2)$$

$$= G_1 \oplus G_0 \oplus G_3 \oplus G_2 = G_0' + B_1$$

Gray

Binary

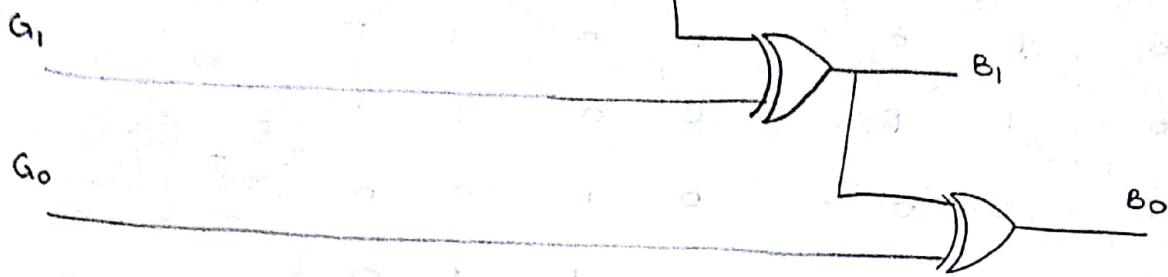
G_3	G_2	G_1	G_0	B_3	B_2	B_1	B_0
0	0	0	0	0	0	0	$B_1 = \bar{G}_3 \bar{G}_2 G_1 +$
0	0	0	1	0	0	0	$\bar{G}_3 G_2 \bar{G}_1 + G_3 G_2 G_1 +$
0	0	1	0	0	0	1	$G_3 \bar{G}_2 \bar{G}_1$
0	0	1	0	0	0	1	$= \bar{G}_2 (G_1 \oplus G_3) +$
0	1	0	0	0	1	0	$G_2 (G_1 \oplus \bar{G}_3)$
0	1	0	1	0	1	0	$= G_2 \oplus G_1 \oplus G_3$
0	0	1	0	$\cancel{0}$	1	0	$= B_2 \oplus G_1$
0	1	1	1	$\cancel{0}$	$\cancel{1}$	$\cancel{1}$	$B_2 = \bar{G}_3$
0	1	0	1	$\cancel{0}$	$\cancel{1}$	$\cancel{1}$	$B_2 = \bar{G}_3 G_2 + G_3 \bar{G}_2 = G_3 \oplus G_2$
0	1	0	1	0	1	1	$B_0 = \bar{G}_3 \bar{G}_2 \bar{G}_1 G_0 +$
0	1	0	0	0	1	1	$\bar{G}_3 \bar{G}_2 G_1 \bar{G}_0 +$
$\cancel{0}$	1	0	0	$\cancel{0}$	0	0	$\bar{G}_3 G_2 \bar{G}_1 \bar{G}_0 +$
1	1	0	1	1	0	0	$\bar{G}_3 G_2 G_1 G_0 +$
1	1	1	1	1	0	0	$G_3 G_2 \bar{G}_1 G_0 +$
1	1	1	1	0	1	1	$G_3 G_2 G_1 \bar{G}_0 +$
1	0	1	0	1	0	0	$G_3 \bar{G}_2 G_1 \bar{G}_0 +$
1	0	1	1	1	0	1	$G_3 \bar{G}_2 \bar{G}_1 \bar{G}_0 +$
1	0	0	1	1	1	0	$G_3 \bar{G}_2 G_1 G_0$
1	0	0	0	1	1	1	
1	0	0	0	0	1	1	

$A_3 A_2$		B_3	
$A_1 A_0$			
00	01	11	10
0	0	0	2
0	4	5	6
1	12	13	14
1	8	9	11

$A_3 A_2$		B_1	
$A_1 A_0$			
00	01	11	10
0	1	1	2
1	4	5	6
0	12	13	14
1	8	9	11

$A_3 A_2$		B_2	
$A_1 A_0$			
00	01	11	10
0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

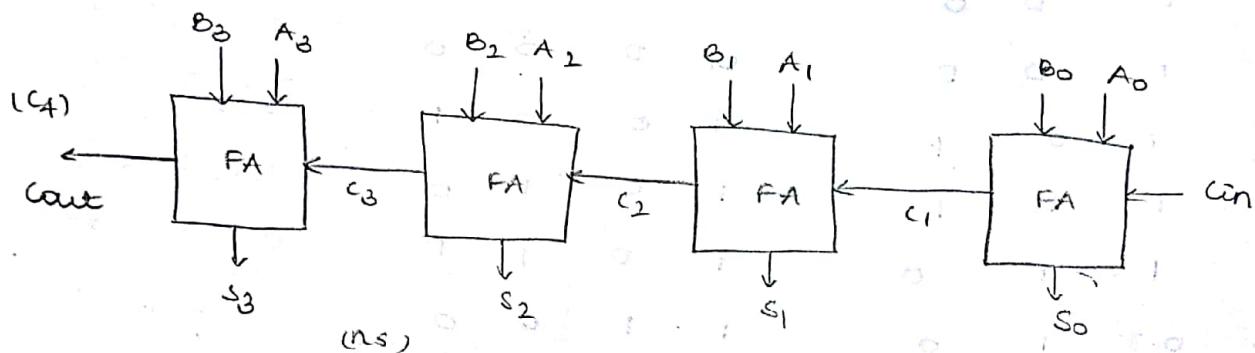
$A_3 A_2$		B_0	
$A_1 A_0$			
00	01	11	10
0	1	D ₃	D ₂
1	4	D ₅	D ₇
0	12	D ₁₀	D ₁₅
1	8	D ₉	D ₁₁



Binary parallel adders

1. Ripple carry adder (carry propagate adder)
2. carry look ahead adder

RCA :

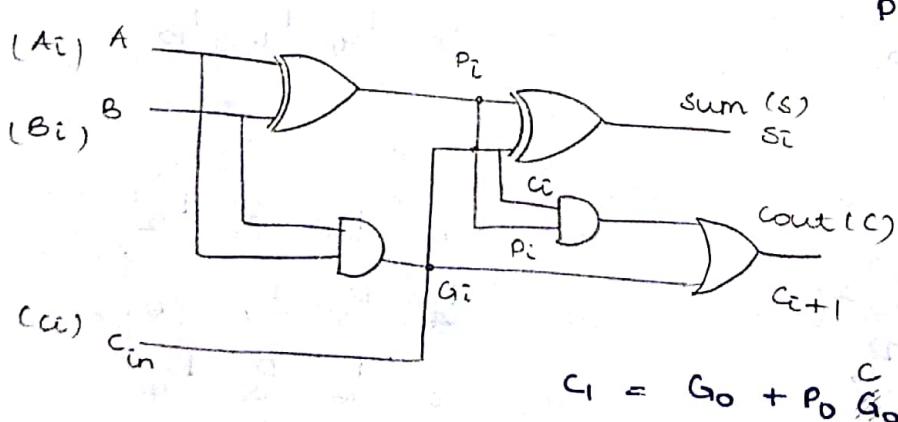


Propagation delay : Time interval b/w input reaching input terminal and producing output at output terminal.

Longest path of carry generation

↓
Disadvantage

Carry look ahead adder :



$P_i \Rightarrow$ propagate fun.

$$= A_i \oplus B_i$$

$G_i \Rightarrow$ generate fun.

$$= A_i \cdot B_i$$

$$C_{i+1} = C_i P_i + G_i$$

$$C_1 = G_0 + P_0 G_0$$

C_0 - input carry

$$C_2 = G_1 + P_1 G_1$$

$$C_2 = G_1 + P_1 (G_0 + P_0 G_0)$$

$$= G_1 + P_1 G_0 + P_1 P_0 G_0$$

$$C_3 = C_2 P_2 + G_2$$

$$= G_2 + P_2 G_1 + P_1 P_2 G_0 + P_1 P_2 P_0 G_0 C_0$$

$$C_4 = G_3 + C_3 P_3$$

$$C_4 = G_3 + G_2 P_3 + P_2 P_3 G_1 + P_1 P_2 P_3 G_0 + P_1 P_2 P_0 P_3 G_0 C_0$$

8-bit : CLA

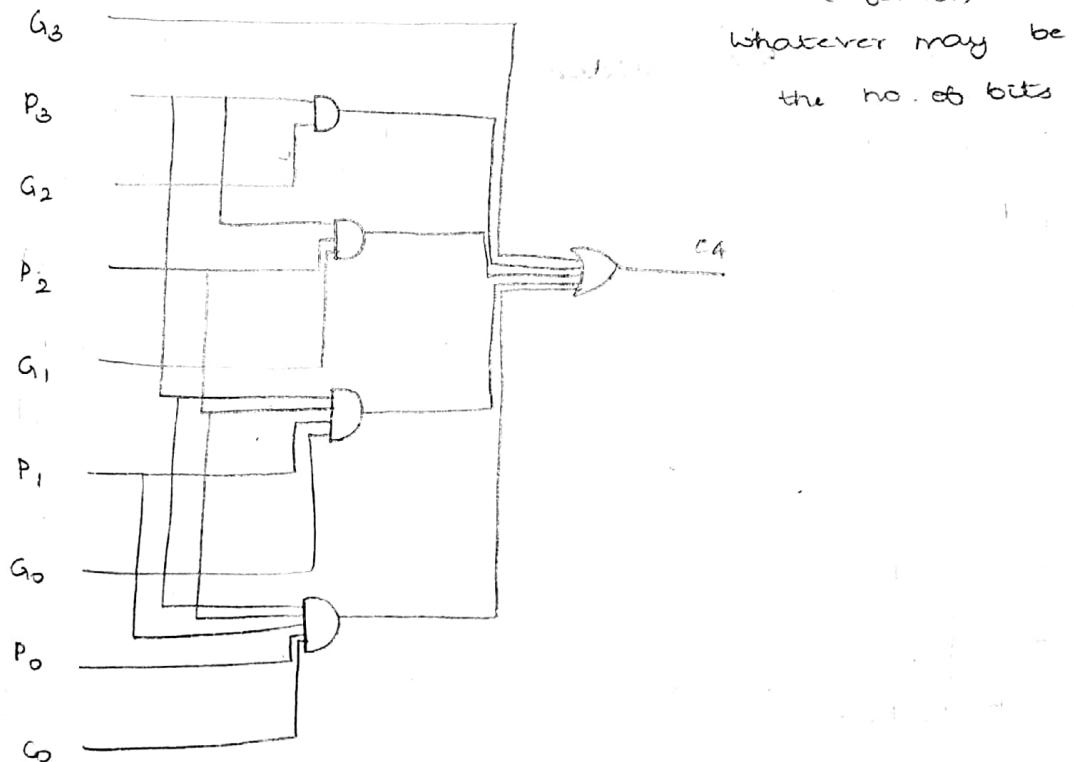
$$(C_n = G_{n-1} + P_{n-1} G_{n-2} + P_{n-1} P_{n-2} G_{n-3} + \dots + P_{n-1} P_{n-2} \dots P_1 P_0 C_0)$$

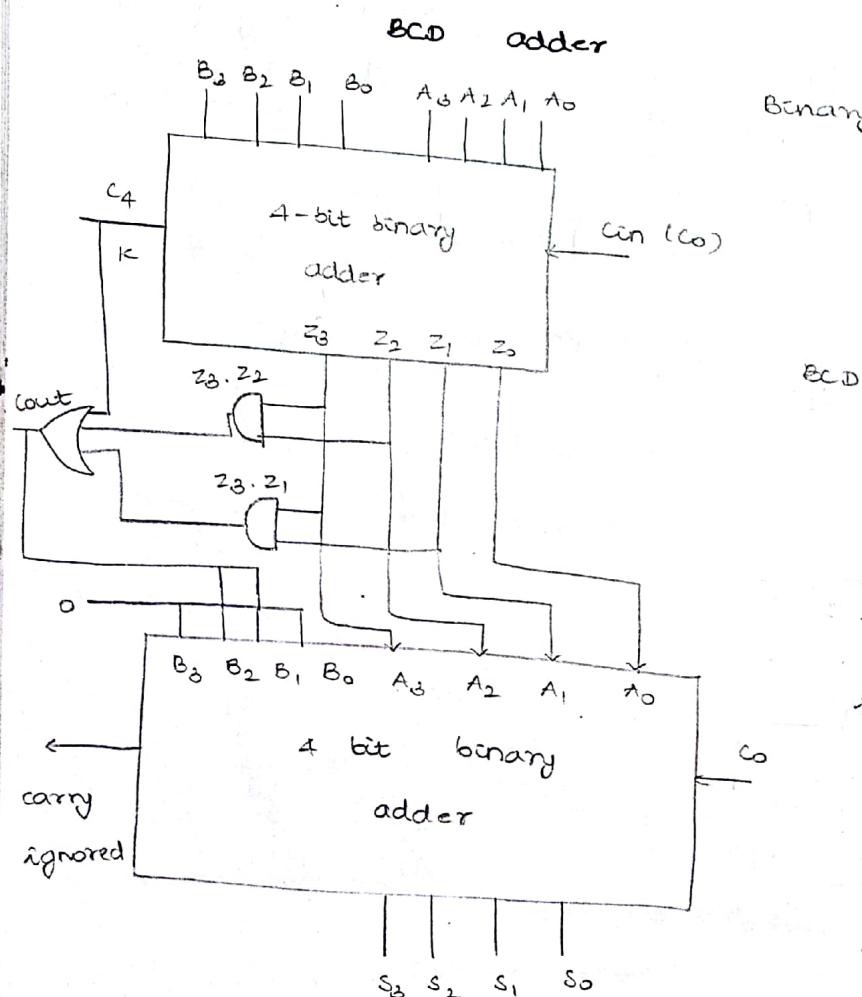
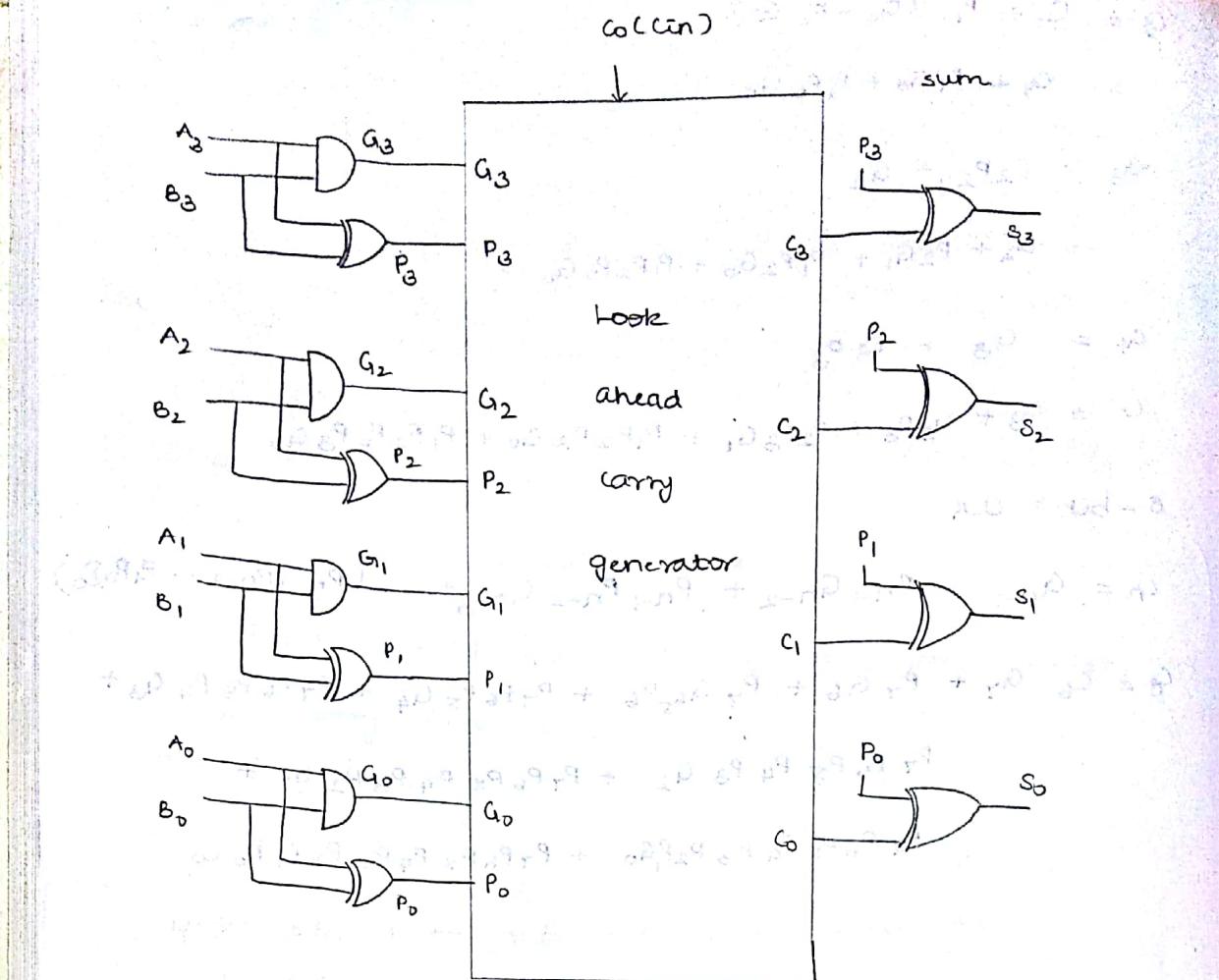
$$C_8 = C_6 G_7 + P_7 G_6 + P_7 G_5 P_6 + P_7 P_6 P_5 G_4 + P_7 P_6 P_5 P_4 G_3 +$$

$$P_7 P_6 P_5 P_4 P_3 G_2 + P_7 P_6 P_5 P_4 P_3 P_2 G_1 +$$

$$P_7 P_6 P_5 P_4 P_3 P_2 P_1 G_0 + P_7 P_6 P_5 P_4 P_3 P_2 P_1 P_0 C_0$$

Lookahead carry generator \rightarrow 4 gate delay
(fixed)





$$\begin{array}{r}
 111 \\
 1001 \\
 0111 \\
 \hline
 10000
 \end{array}$$

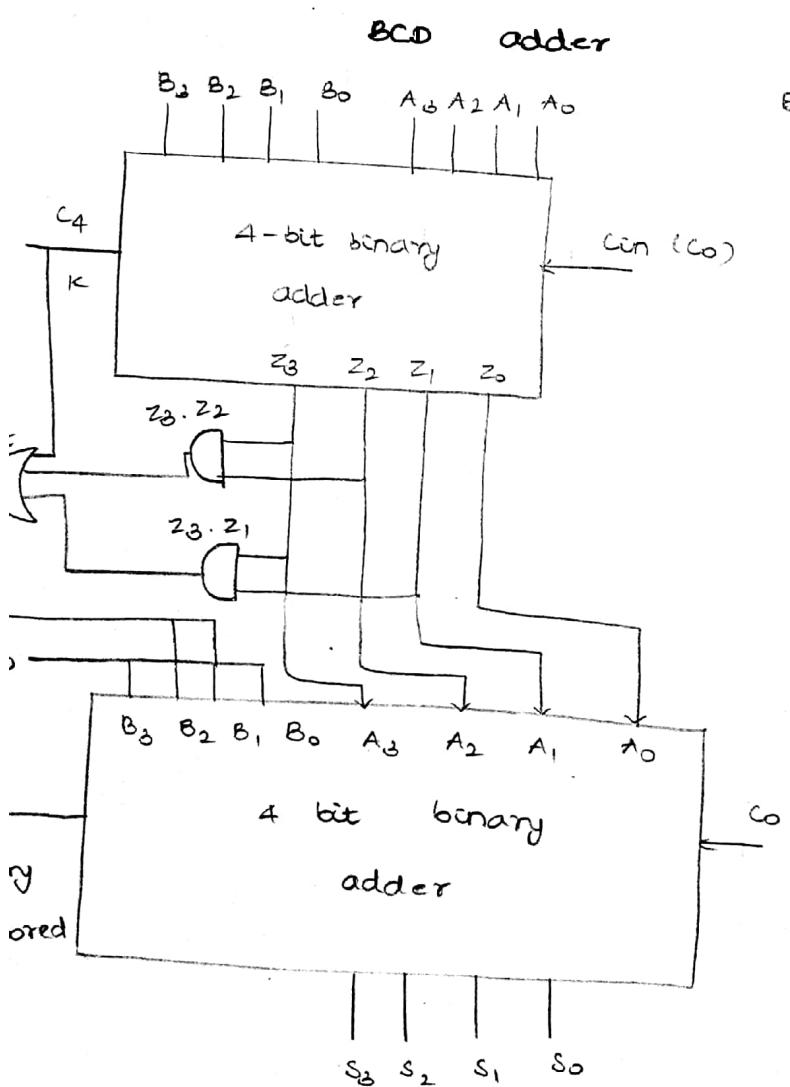
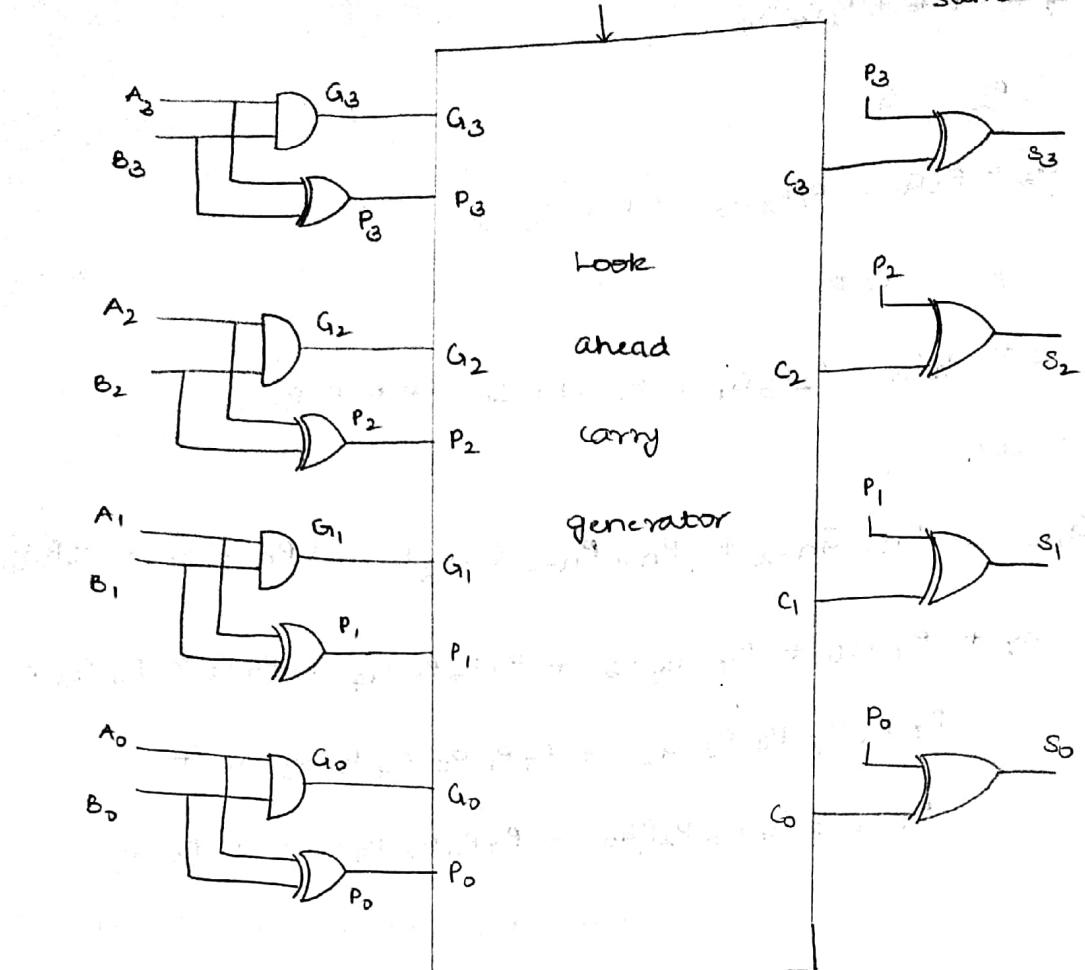
Binary

$$\begin{array}{r}
 1001 \\
 0111 \\
 \hline
 10000
 \end{array}$$

BCD

$$\begin{array}{r}
 10110 \\
 \hline
 1 \ 6
 \end{array}$$

$0111110 \rightarrow$ will check
 ↓
 ib > 9
 next
 check
 this
 ib this
 too > 9 add 6 to
 previous result



$$\begin{array}{r}
 111 \\
 1001 \\
 0111 \\
 \hline
 10000 \\
 16
 \end{array}$$

Binary

$$\begin{array}{r}
 1001 \\
 0111 \\
 \hline
 10000 \\
 0110 \\
 \hline
 10110 \\
 \hline
 16
 \end{array}$$

BCD

$0111\ 1110 \rightarrow$ will check
 ↓
 if > 9
 next
 check this
 ↓
 if this
 too > 9 add 6 to
 previous result

Decimal

Binary num

BCD num

	K	z_3	z_2	z_1	z_0	Cout	s_3	s_2	s_1	s_0
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	1
2	0	0	0	1	0	0	0	0	1	0
3	0	0	0	1	1	0	0	0	1	1
4	0	0	1	0	0	0	0	1	0	0
5	0	0	1	0	1	0	0	1	0	1
6	0	0	1	1	0	0	0	1	1	0
7	0	0	1	1	1	0	0	0	1	1
8	0	1	0	0	0	0	1	0	0	0
9	0	1	0	0	1	0	1	0	0	1
10	0	1	0	1	0	1	0	0	0	1
11	0	1	0	1	1	1	0	0	1	0
12	0	1	1	0	0	1	0	0	1	1
13	0	1	1	0	1	1	0	1	0	0
14	0	1	1	1	0	1	0	1	0	1
15	0	1	1	1	1	1	0	1	1	0
16	1	0	0	0	0	1	0	1	1	1
17	1	0	0	0	1	1	1	0	0	0
18	1	0	0	1	1	1	1	0	0	1
19	1	0	0	1	1	1	1	1	0	0

Cout

		$z_1 z_0$	00	01	11	10
$z_3 z_2$		00	0	0	0	0
	00	00	0	1	0	2
	01	00	0	0	0	0
	01	01	4	5	7	6
	11	11	1	1	1	1
	11	11	12	13	15	14
	10	10	8	9	14	10

When binary sum is 0-15

$$\text{cout} = z_3 z_2 + z_3 z_1$$

When binary sum is 0-19

$$\text{cout} = K + z_3 z_2 + z_3 z_1$$

Magnitude comparator

Input

Output

A

B

A > B

A = B A < B

A_1

A_0

B_1

B_0

0

0

0

0

0

1

0

0

0

0

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0

1

A > B

		00	01	11	10
		0	1	3	2
$A_1 A_0$	$B_1 B_0$	00	1	4	5
		01	4	12	13
$A_1 A_0$	$B_1 B_0$	11	13	15	14
		10	18	19	11
					10

		00	01	11	10
		1	0	3	2
$A_1 A_0$	$B_1 B_0$	00	4	5	7
		01	12	13	15
$A_1 A_0$	$B_1 B_0$	11	1	15	14
		10	6	9	11
					10

$$F = A_1 \bar{B}_1 + \bar{B}_1 \bar{B}_0 A_0 + A_1 A_0 \bar{B}_0$$

$$F = \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 +$$

$$\bar{A}_1 A_0 \bar{B}_1 B_0 + A_1 A_0 B_1 B_0 +$$

$$A_1 \bar{A}_0 B_1 \bar{B}_0$$

A_1	B_0	00	01	11	10
A_0	0	1	1	1	2
00	0	4	5	1	1
01	12	12	15	14	
11	8	9	11		10

$A \leq B$

$$A = B$$

$$\begin{aligned}
 F &= \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 B_1 B_0 + \\
 &\quad \bar{A}_1 \bar{B}_1 A_0 B_0 + \bar{A}_0 \bar{B}_0 A_1 B_1, \\
 &= \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 B_1 B_0 + \\
 &\quad \bar{A}_1 \bar{B}_1 \oplus A_0 B_0
 \end{aligned}$$

$$F = \bar{A}_1 B_1 + \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_0 B_1 B_0$$

$$\begin{aligned}
 A > B &= A_1 \bar{B}_1 + B_1 \bar{B}_0 A_0 + A_1 A_0 \bar{B}_0 \\
 &= A_1 \bar{B}_1 + A_0 A_1 \bar{B}_1 \bar{B}_0 + \bar{A}_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 \bar{B}_0 B_1 + A_1 A_0 \bar{B}_0 \bar{B}_1 \\
 &= A_1 \bar{B}_1 + A_0 A_1 \bar{B}_1 \bar{B}_0 + \bar{A}_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 \bar{B}_0 B_1 \\
 &= A_1 \bar{B}_1 + A_0 A_1 \bar{B}_1 \bar{B}_0 + A_0 \bar{B}_0 (\bar{A}_1 \bar{B}_1 + A_1 B_1) \\
 &= A_1 \bar{B}_1 + A_0 \overbrace{A_1 \bar{B}_1}^x \bar{B}_0 + A_0 \bar{B}_0 (\bar{A}_1 \bar{B}_1) \quad \text{absorption law} \\
 A > B &= A_1 \bar{B}_1 + A_0 \bar{B}_0 (\bar{A}_1 \bar{B}_1) = A_1 \bar{B}_1 + x_1 A_0 \bar{B}_0 \\
 &\quad \bar{A}_0 B_0 \bar{A}_1 B_1 \quad \bar{A}_0 \bar{B}_0 A_1 B_1 \\
 A \leq B &= \bar{A}_1 B_1 + \bar{A}_1 \bar{A}_0 B_0 B_1 + \bar{A}_1 \bar{A}_0 B_0 \bar{B}_1 + \bar{A}_0 B_1 B_0 A_1 + \\
 &\quad \bar{A}_0 B_1 B_0 \bar{A}_1 \\
 &= \bar{A}_1 B_1 + \bar{A}_1 \bar{A}_0 B_0 B_1 + \bar{A}_0 B_0 (\bar{A}_1 \bar{B}_1 + A_1 B_1) \\
 &= \bar{A}_1 B_1 + \bar{A}_1 \bar{A}_0 B_0 B_1 + \bar{A}_0 B_0 (\bar{A}_1 \bar{B}_1) \\
 &= \bar{A}_1 B_1 + \bar{A}_0 B_0 (\bar{A}_1 \bar{B}_1) = \bar{A}_1 B_1 + \bar{A}_0 B_0 x_1
 \end{aligned}$$

$$F = \bar{A}_1 \bar{B}_1 (A_0 B_0 + \bar{A}_0 \bar{B}_0) + A_1 B_1 (A_0 B_0 + \bar{A}_0 \bar{B}_0)$$

$$= \bar{A}_1 \bar{B}_1 (A_0 \bar{B}_0) + A_1 B_1 (A_0 \bar{B}_0)$$

$$= (A_0 \bar{B}_0) (\bar{A}_1 \bar{B}_1 + A_1 B_1)$$

$$= (A_0 \bar{B}_0) (A_1 \bar{B}_1) = x_0 \cdot x_1$$

$$\text{Let } x_1 = A_1 \oplus B_1$$

$$x_0 = A_0 \oplus B_0$$

$$x_1 = A_1 \oplus B_1$$

4-bit :

$$A \geq B = A_3 \bar{B}_3 + x_3 A_2 \bar{B}_2 + x_3 x_2 A_1 \bar{B}_1 + x_3 x_2 x_1 A_0 \bar{B}_0$$

$$A \leq B = \bar{A}_3 B_3 + x_3 \bar{A}_2 B_2 + x_3 x_2 \bar{A}_1 B_1 + x_3 x_2 x_1 \bar{A}_0$$

$$A = B = x_3 x_2 x_1 x_0$$

8-bit :

$$A \geq B = x_7 A_7 \oplus B_7 + A_7 B_7 A_6 \oplus B_6 + A_7 A_6 B_6 \oplus B_7 B_6 + \dots$$

$$A \geq B = A_7 \bar{B}_7 + x_7 A_6 \bar{B}_6 + x_7 x_6 A_5 \bar{B}_5 + x_7 x_6 x_5 A_4 \bar{B}_4 +$$

$$x_7 x_6 x_5 x_4 A_3 \bar{B}_3 + x_7 x_6 x_5 x_4 x_3 A_2 \bar{B}_2 + x_7 x_6 x_5 x_4 x_3 x_2$$

$$A \geq B = A_7 \bar{B}_7 + x_7 \bar{A}_6 B_6 + x_7 x_6 \bar{A}_5 B_5 + x_7 x_6 x_5 \bar{A}_4 B_4 +$$

$$x_7 x_6 x_5 x_4 \bar{A}_3 B_3 + x_7 x_6 x_5 x_4 x_3 \bar{A}_2 B_2 + x_7 x_6 x_5 x_4 x_3 x_2 \bar{A}_1 B_1$$

$$A = B = x_7 x_6 x_5 x_4 x_3 x_2 x_1 x_0$$

Design a combinational circuit with 3 input
1 output π_0 output is 1 then binary value of $A \geq B$
is 1-3 if 0 otherwise.

	A	B	C	D
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

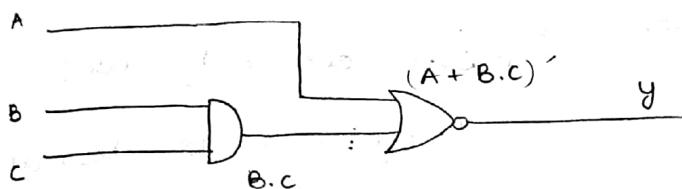
	BC	00	01	11	10
A	0	1	1	0	1
D	1	0	1	0	0

$$F = \bar{A}\bar{B} + \bar{A}\bar{C}$$

$$= \bar{A}(\bar{B} + \bar{C})$$

$$= \overline{\bar{A} \cdot \bar{B} \cdot \bar{C}}$$

$$Y = \overline{(A + BC)}$$



3 input one output , O/P is 1 when binary value of I/P is even number otherwise 0

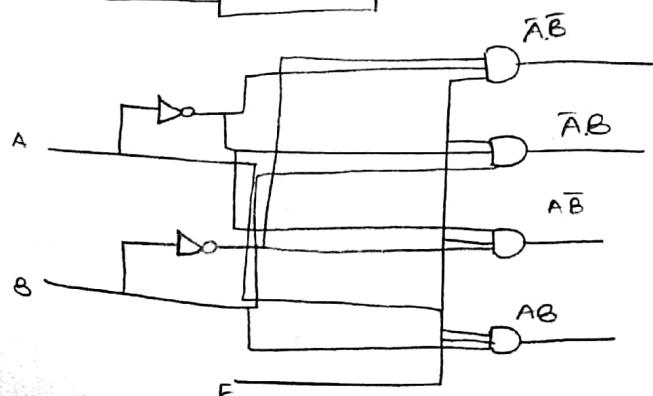
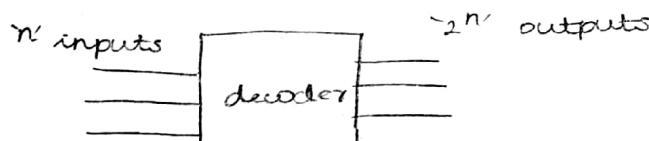
	BC	00	01	11	10
A	0	1	0	1	0
D	1	0	1	0	1



$$Y = \bar{C}$$

Decoder

$$\frac{2 \text{ to } 4}{2 = 4}$$



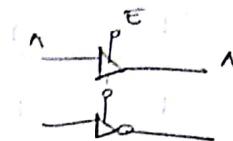
AB	D ₀	D ₁	D ₂	D ₃
00	1	0	0	0
01	0	1	0	0
10	0	0	1	0
11	0	0	0	1

E	A	B	D ₀	D ₁	D ₂	D ₃	Tristate device
0	X	X	0	0	0	0	
1	0	0	1	0	0	0	low 0
1	0	1	0	1	0	0	high 1
1	1	0	0	0	1	0	high impedance
1	1	1	0	0	0	1	

Buffer → holds the input and creates a delay.

O/P follows I/P

active low



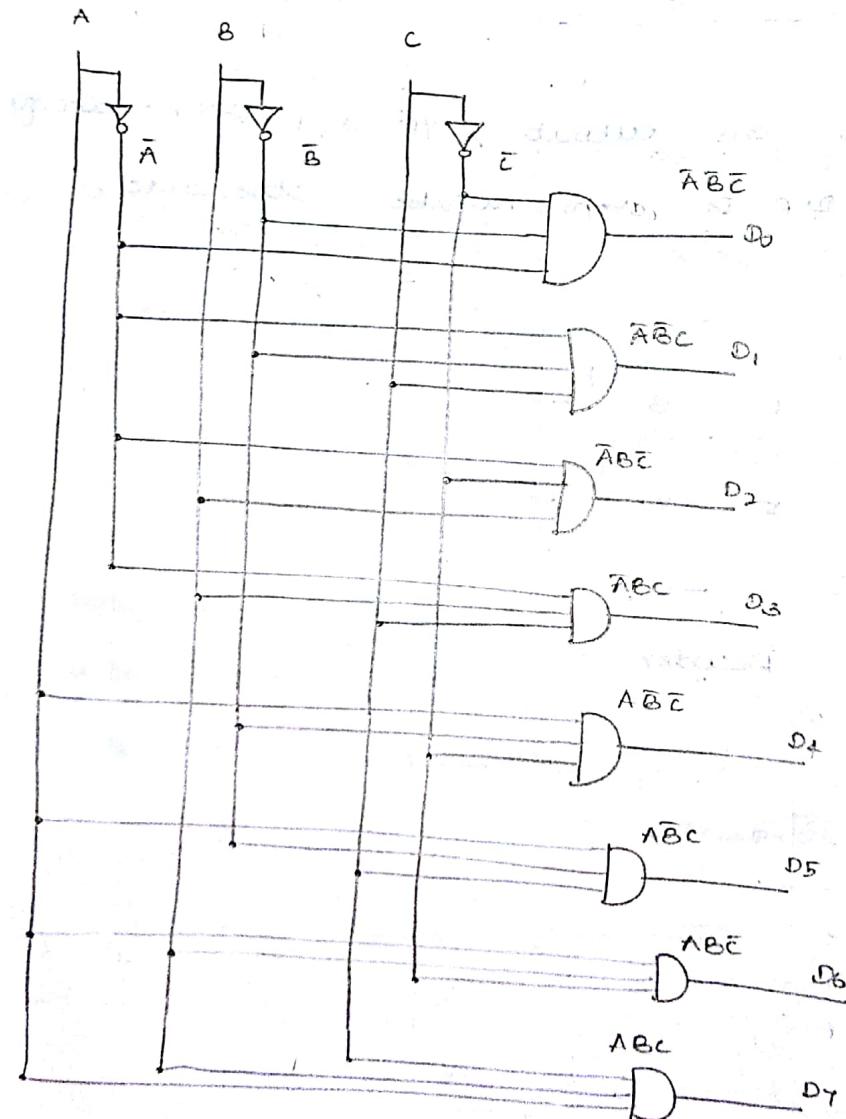
3 to 8

74LS128

4 to 16

} most commonly used

Eg: BCD to decimal

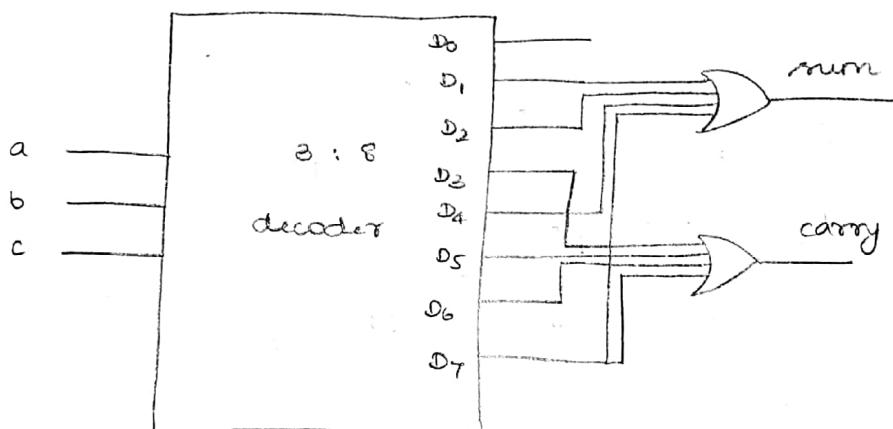


A	B	C	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	0	1
1	1	1	0	0	0	0	0	0	0	1

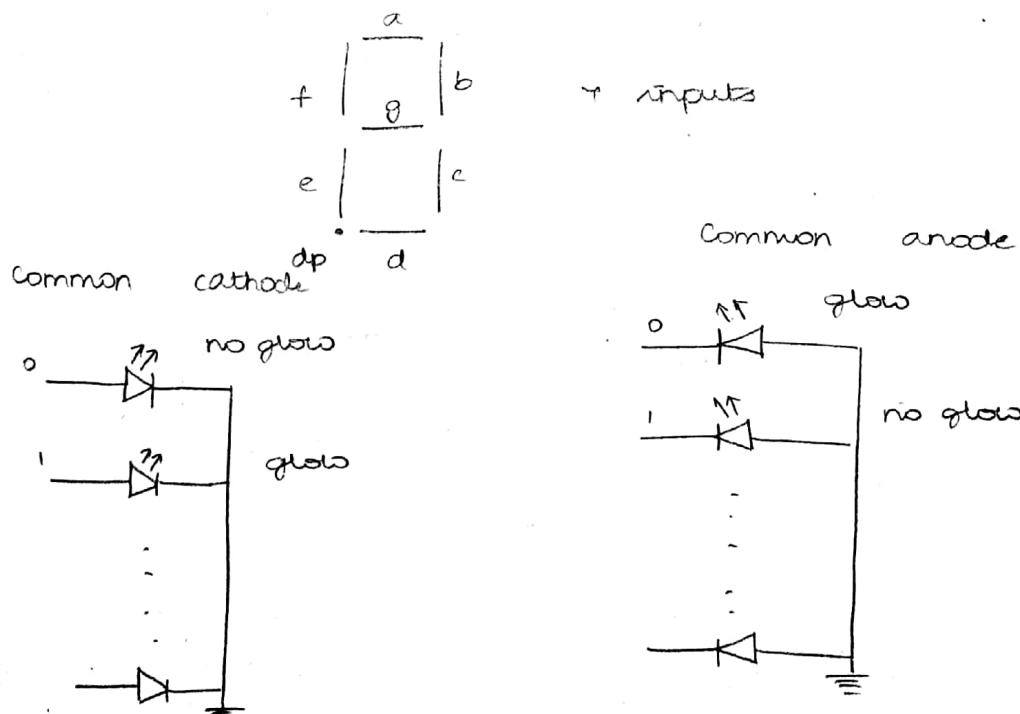
Full adder using decoder

$$\text{sum} = \Sigma (1, 2, 4, 7)$$

$$\text{cout} = \Sigma (3, 5, 6, 7)$$



Seven segment

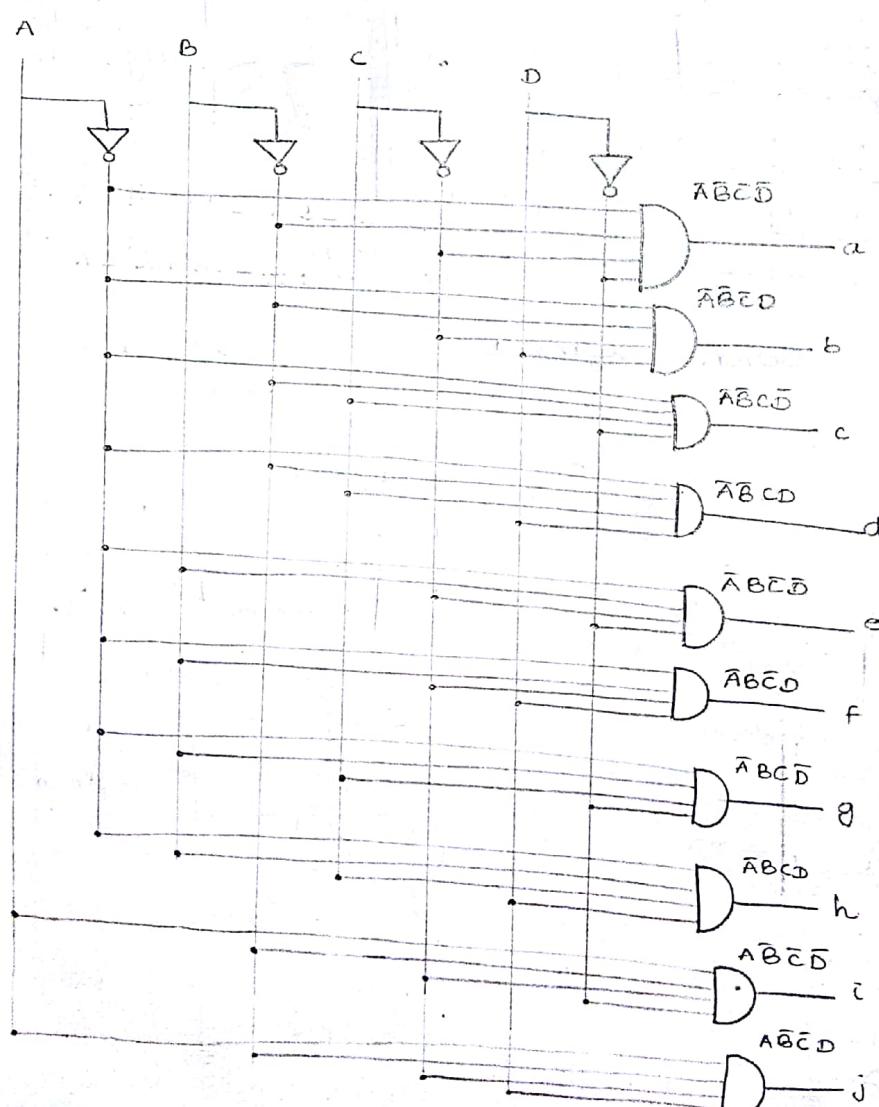


BCD to seven segment

BCD

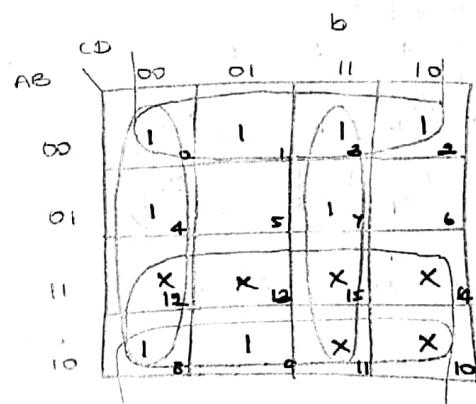
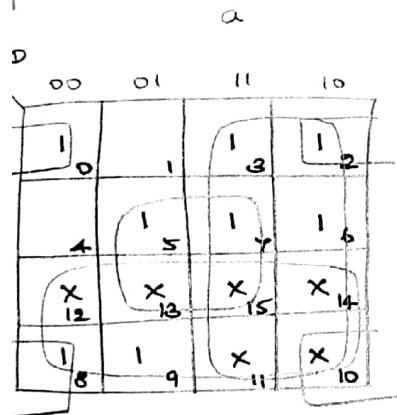
seven segment

	A	B	C	D	a	b	c	d	e	f	g	h	i	j
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	1	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	1	0	0	0	0	0	0	0
3	0	0	1	1	0	0	0	1	0	0	0	0	0	0
4	0	1	0	0	0	0	0	0	1	0	0	0	0	0
5	0	1	0	1	0	0	0	0	0	1	0	0	0	0
6	0	1	1	0	0	0	0	0	0	0	0	0	0	0
7	0	1	1	1	0	0	0	0	0	0	0	1	0	0
8	1	0	0	0	0	0	0	0	0	0	0	0	1	0
9	1	0	0	1	0	0	0	0	0	0	0	0	0	1



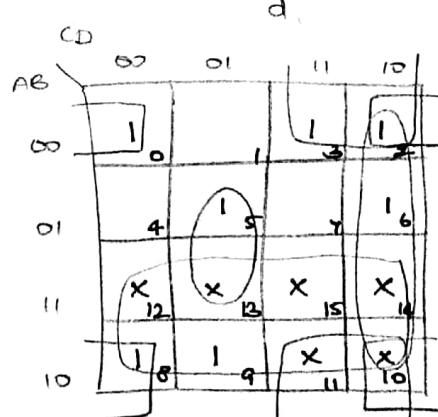
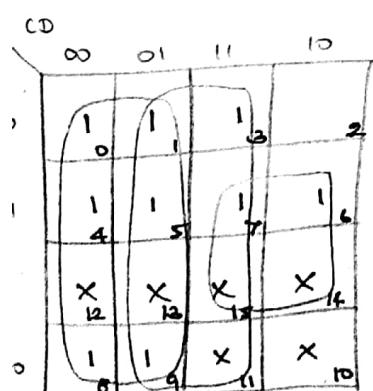
BCD to seven segment

BCD				seven segment						
A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	0	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1



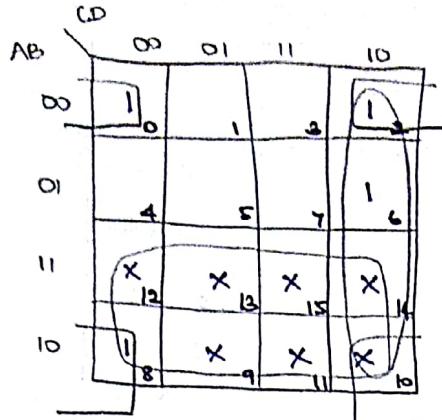
$$\begin{aligned}
 a &= C + A + \bar{B}\bar{D} + BD \\
 &= C + A + B\bar{D} \\
 &\quad \vdots \\
 &= C
 \end{aligned}$$

$$\begin{aligned}
 b &= \dots + \bar{B} + CD + \bar{C}\bar{D} \\
 &= \dots + \bar{B} + C\bar{D}
 \end{aligned}$$

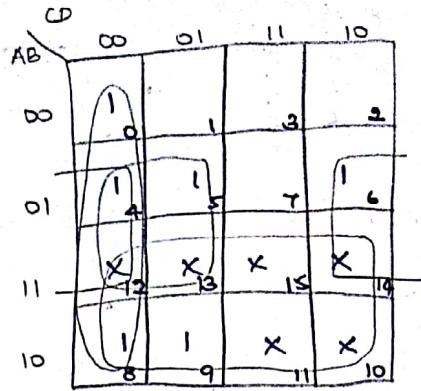


$$c = \bar{C} + D + BC$$

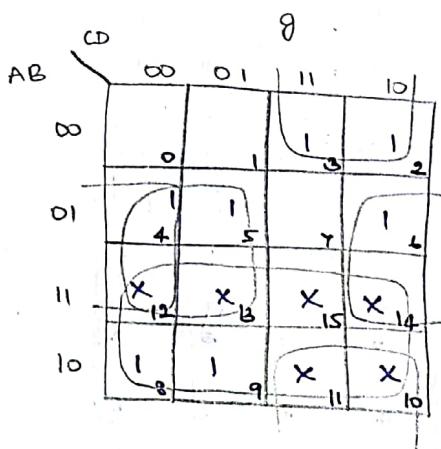
$$\begin{aligned}
 d &= \bar{B}\bar{D} + A + B\bar{C}D + \bar{C}\bar{D} + \bar{B}C \\
 &= \bar{D}(\bar{B} + c) + \bar{B}C + B\bar{C}D + A
 \end{aligned}$$



$$e = A + \overline{B} \overline{D} + C \overline{D}$$

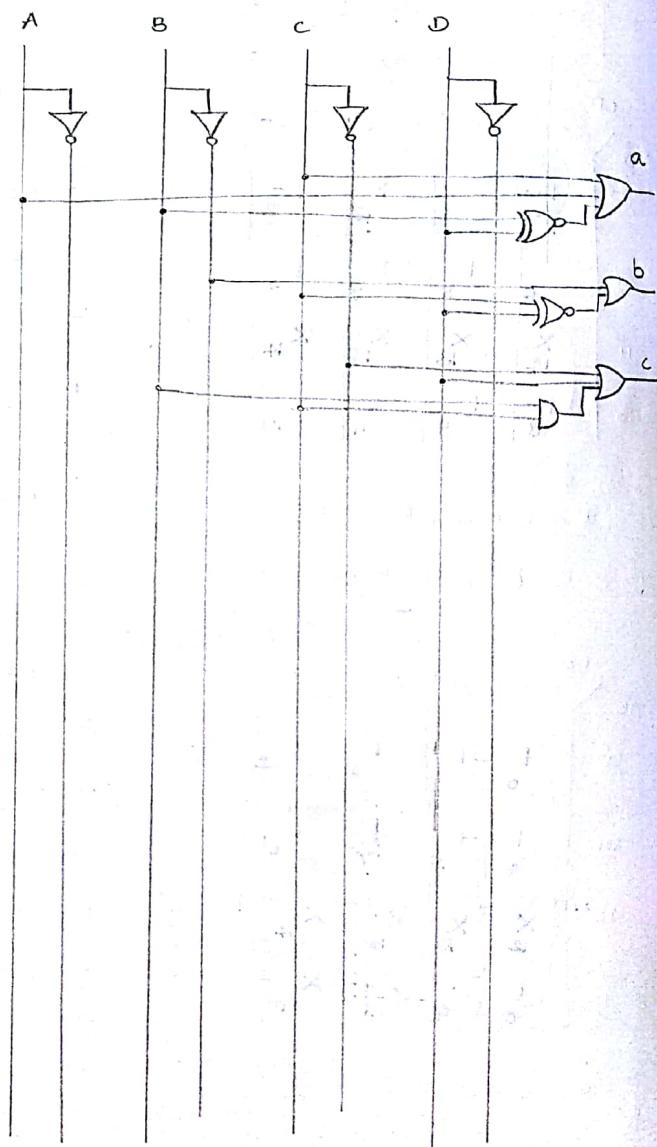


$$f = A + \overline{C} \overline{D} + B \overline{C} + B \overline{D}$$

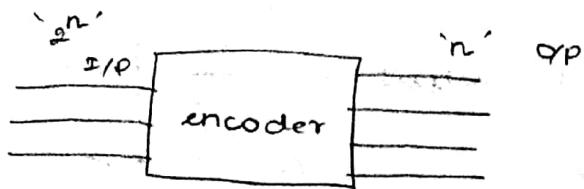


$$g = A + B \overline{C} + B \overline{D} + \overline{B} C$$

$$= A + B \overline{D} + B \oplus C$$

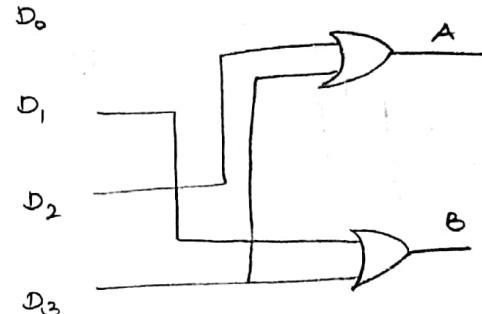


ENCODER



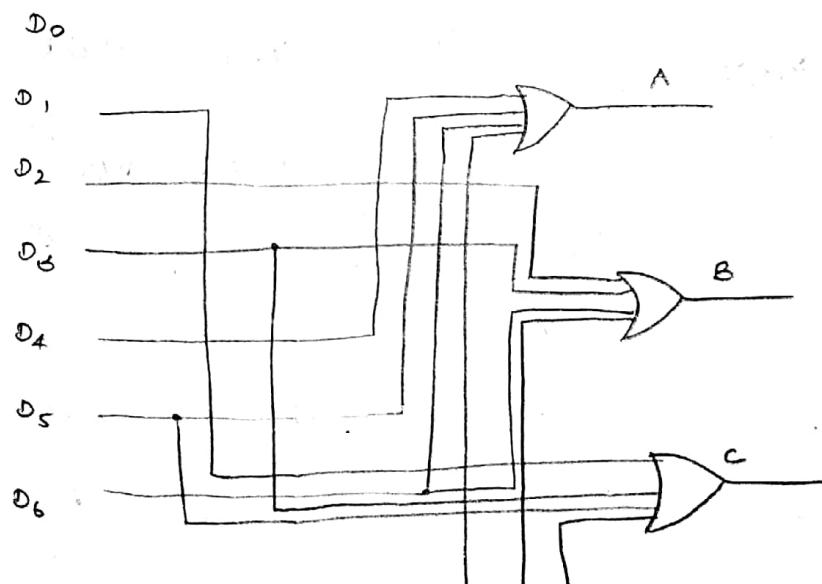
4:2 Encoder

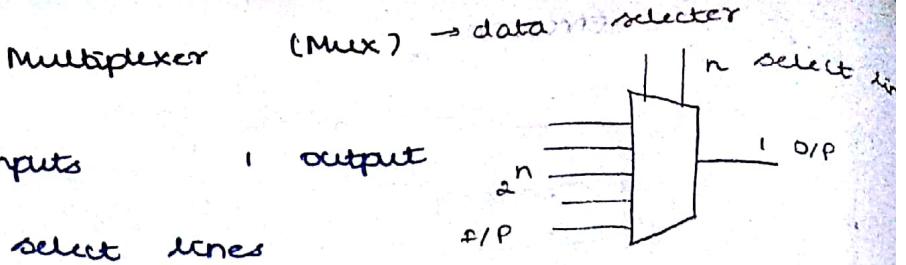
D_0	D_1	D_2	D_3	A	B
1	0	0	0	1	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1



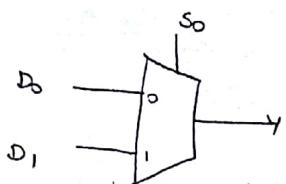
8:3 Encoder

D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	A	B	C
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	0	0
0	0	0	0	0	0	0	1	1	1	1



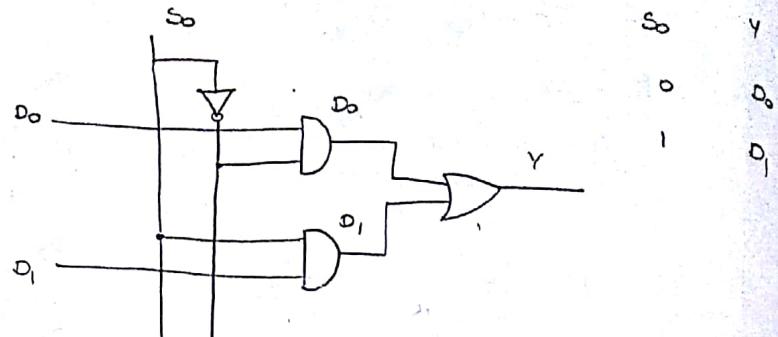


$2:1$ MUX

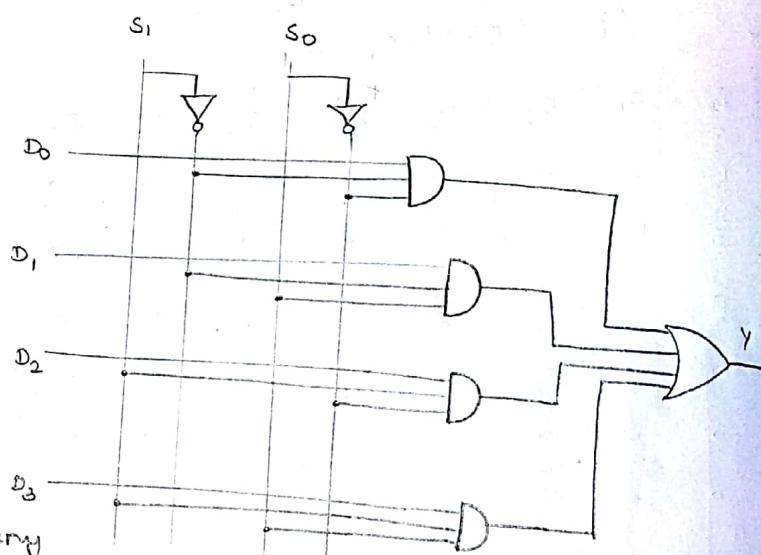
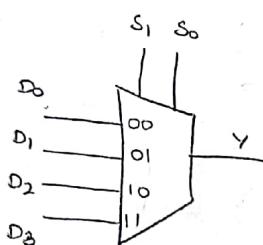


$$S_0 = 0 \quad Y = D_0$$

$$S_0 = 1 \quad Y = D_1$$



$4:1$ MUX



Using multiplexer
any digital circuit can be constructed

Design a full adder

using multiplexer
to design a full adder
no. of inputs 'n'

$$\text{sum} = \Sigma (1, 2, 4, 7)$$

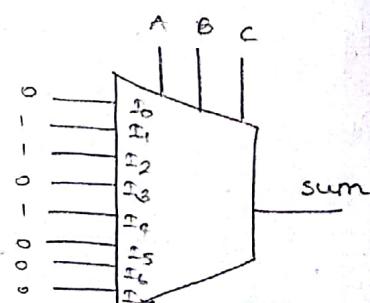
$$\text{carry} = \Sigma (3, 5, 6, 7)$$

choose ' $n-1$ ' select line

either $4:1$ or $8:1$

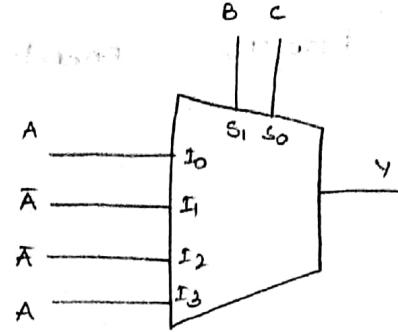
A	B	C	sum	carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

$8:1$ MUX



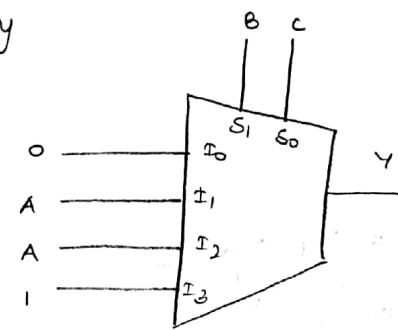
Implementation table for sum

	I_0	I_1	I_2	I_3
\bar{A}	0	1	2	3
A	4	5	6	7
	A	\bar{A}	\bar{A}	A



Implementation table for carry

	I_0	I_1	I_2	I_3
\bar{A}	0	1	2	3
A	4	5	6	7
	0	A	A	1



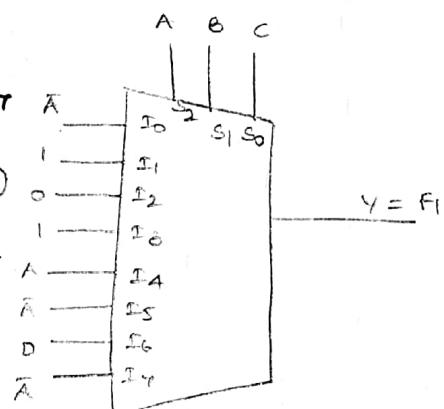
$$F_1 = \Sigma (0, 1, 3, 5, 7, 9, 11, 12)$$

$$F_2 = \Sigma (1, 5, 7, 11, 12, 13, 15)$$

↓
4 inputs \rightarrow 3 select lines
↓
8:1 MUX

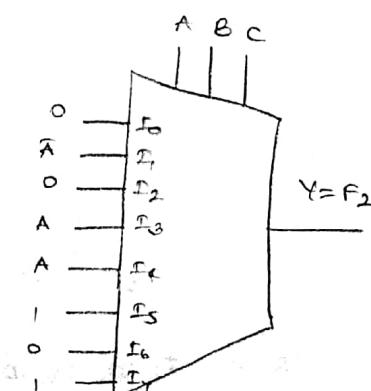
Implementation table for F_1

	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
\bar{A}	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
	A	\bar{A}	\bar{A}	A	A	\bar{A}	0	\bar{A}



Implementation table for F_2

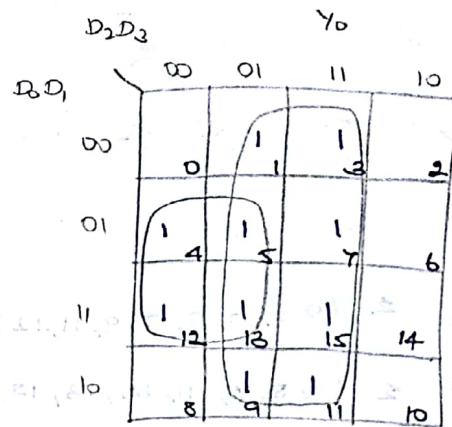
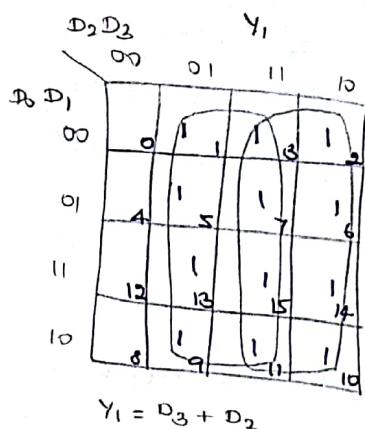
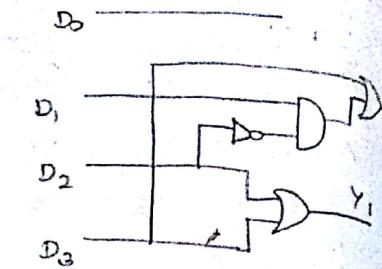
	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
\bar{A}	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
	0	\bar{A}	0	A	A	1	0	1



Priority Encoder

4 = 2

	D_0	D_1	D_2	D_3	Y_1	Y_0
8	1	0	0	0	0	0
4, 12	X	1	0	0	0	1
2, 6, 10, 14	X	X	1	0	1	0
1, 3, 5, 7, 9, 11, 13, 15	X	X	X	1	1	1



$$Y_0 = D_3 + D_1 \bar{D}_2$$

	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	Y_2	Y_1	Y_0
128	1	0	0	0	0	0	0	0	0	0	0
64	X	1	0	0	0	0	0	0	0	0	1
32	X	X	1	0	0	0	0	0	0	0	0
16	X	X	X	1	0	0	0	0	0	0	0
8	X	X	X	X	1	0	0	0	1	0	0
4	X	X	X	X	X	1	0	0	1	0	1
2	X	X	X	X	X	X	1	0	1	1	0
1	X	X	X	X	X	X	X	1	1	1	1

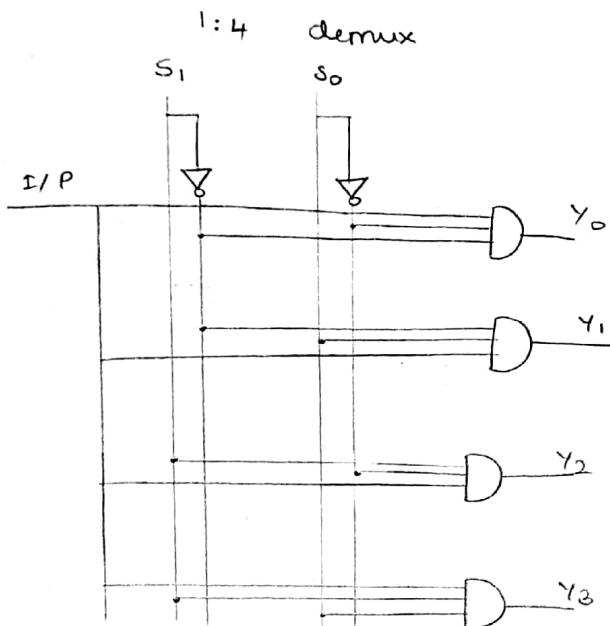
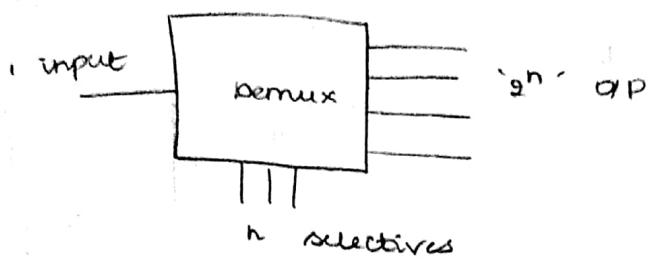
$$Y_2 = \sum (4, 5, 6, 7)$$

$$Y_2 = D_4 \bar{D}_5 \bar{D}_6 \bar{D}_7 + D_7 + D_6 \bar{D}_7 + D_5 \bar{D}_6 \bar{D}_7$$

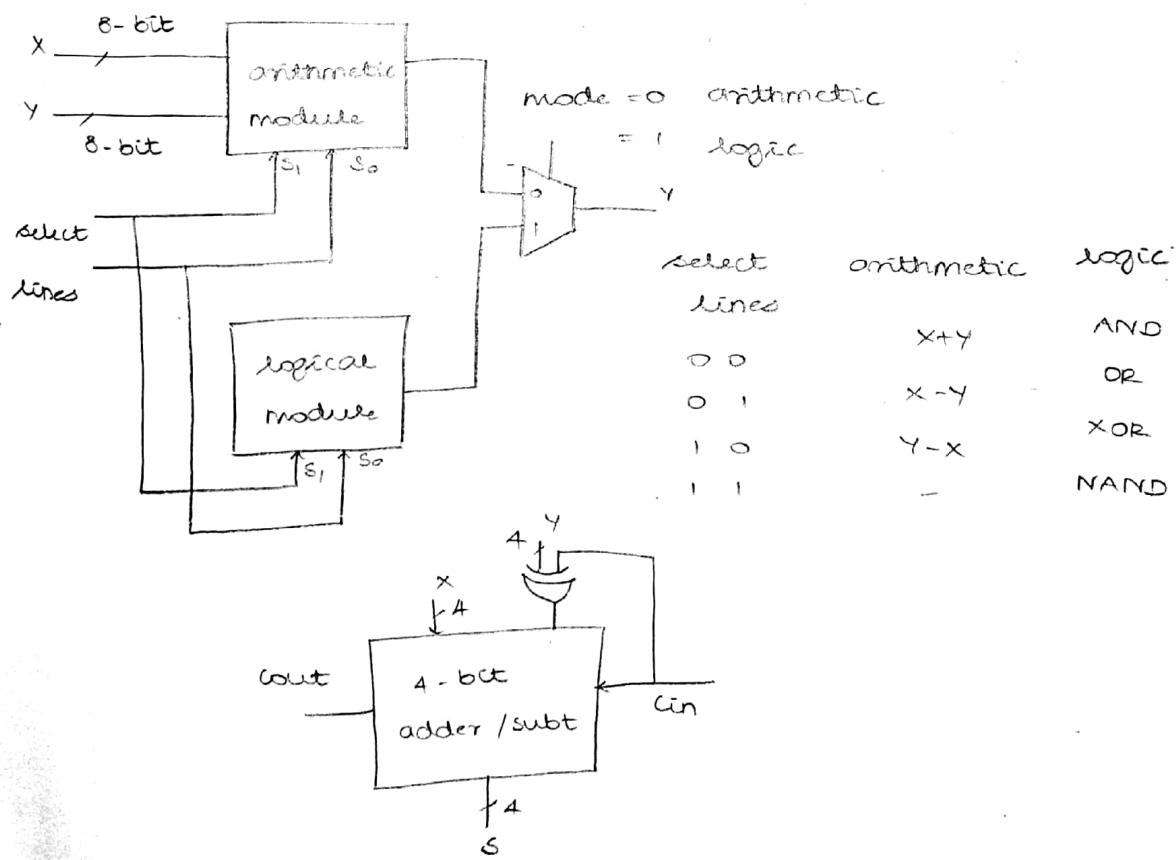
$$Y_1 = D_7 + D_6 \bar{D}_7 + D_5 \bar{D}_4 \bar{D}_5 \bar{D}_6 \bar{D}_7 + D_2 \bar{D}_3 \bar{D}_4 \bar{D}_5 \bar{D}_6 \bar{D}_7$$

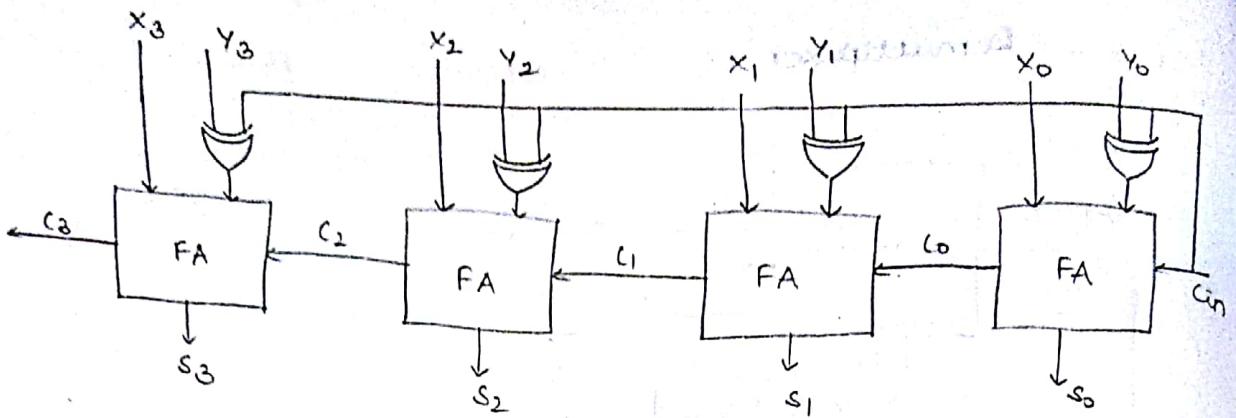
$$Y_0 = D_7 + D_5 \bar{D}_6 \bar{D}_7 + D_3 \bar{D}_4 \bar{D}_5 \bar{D}_6 \bar{D}_7 + D_1 \bar{D}_2 \bar{D}_3 \dots \bar{D}_7$$

Demultiplexer



8-bit ALU





$x+y$ and $x-y$

$$\begin{array}{c} 0 \\ 0 \\ \oplus \\ 1 \\ 1 \end{array}$$

$$0 \quad 0 \rightarrow y \Rightarrow x + y + 0$$

$$0 \quad 1 \rightarrow \bar{y} \Rightarrow x + (\bar{y} + 1)$$

$y \rightarrow \text{adder}$

$\bar{y} \rightarrow \text{subtractor}$

\downarrow

1's complement

\downarrow

2's complement