Note:

(2m)

BINARY TO GRAY

1 1 1 0

$\downarrow \quad \oplus \quad \downarrow \quad \oplus$

1 0 0 1

→ 1st no as it is

→ next nos take X-OR

GRAY TO BINARY

1 1 1 0

$\downarrow \quad \oplus \quad \oplus \quad \oplus$

1 0 1 1

→ next no ans value take X-OR.

5. Convert binary 110 1110 ⇒ gray code

1 1 0 1 1 1 0

↓

1 0 1 1 0 0 1 ⇒ gray code.

6. Reduce the boolean expression using boolean law.

A. $(A'+C)(A'+C')(A+B+C'D)$

Using distribution law

$(A'+C)(A'+C') \Rightarrow (A'+CC')$

$(A'+\cancel{CC'}^{\,0})(A+B+C'D)$

$= A'(A+B+C'D)$

$= A'\cancel{A}^{\,0} + A'B + A'C'D$

$= A'B + A'C'D$

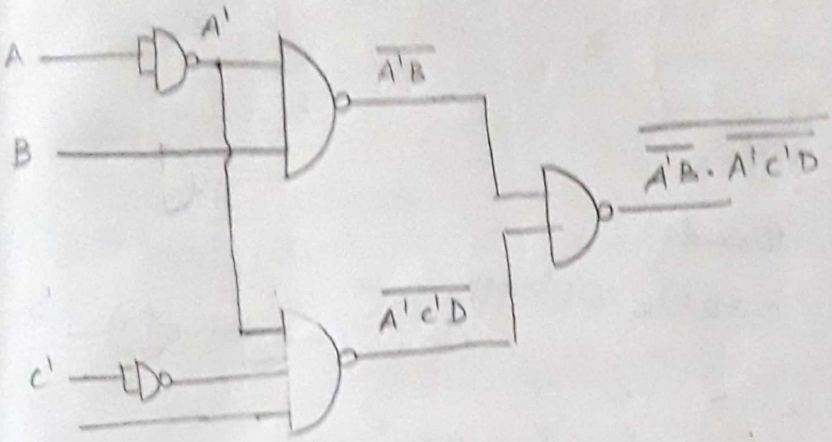
$= A'(B+C'D)$

Implementation using universal logic gates:

ONLY USING $\boxed{\text{NAND}}$ AND $\boxed{\text{NOR}}$

us $Y = A'B + A'C'D$

$\overline{\overline{Y}} = \overline{\overline{A'B} \cdot \overline{A'C'D}}$ ⇒ only using NAND



⇒ only using NOR

$Y = A'B + A'C'D$

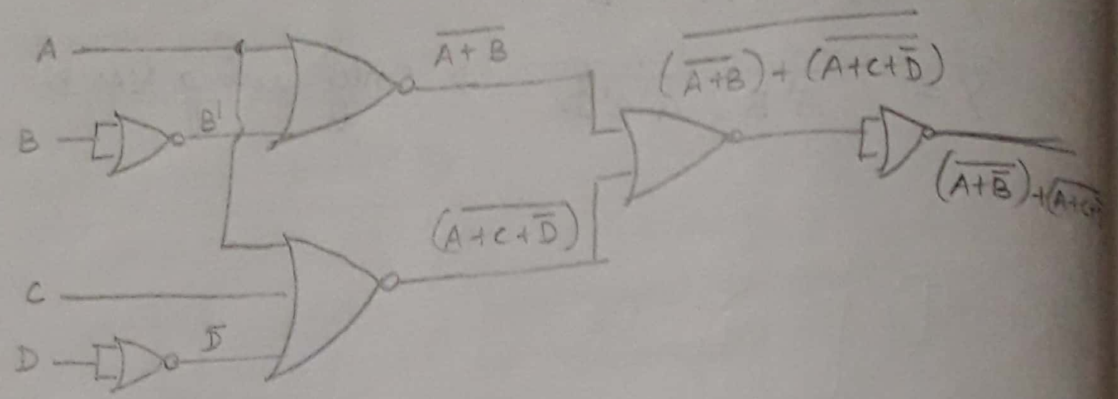
$Y = \overline{\overline{A'B} \cdot \overline{A'C'D}}$

$Y = \overline{(\overline{A'} + \overline{B}) \cdot (\overline{A'} + \overline{C'} + \overline{D})}$

$= \overline{(A + \overline{B}) \cdot (A + C + \overline{D})}$

$= (A + \overline{B}) + \overline{A + C + \overline{D}}$

$$y = \overline{(A+\overline{B})} + (\overline{A+C+\overline{D}})$$



1. (B) continued:

$(C3DF)_{16}$ to binary.

(hexa → decimal)

12 3 13 15

15 → $15 \times 16^0 = 15$

13 → $13 \times 16 = 208$

3 → $3 \times 16^1 = 768$

12 → $12 \times 16^2 = 49,152$

$(50143)_{10}$

(decimal → binary)

- Repeated division by 2

$2 \,\underline{|\, 50143}$

Ans ⇒ $(1100 \; 0011 \; 1101 \; 1111)_2$

decimal to binary

2	50143	
2	25071	- 1
2	12535	- 1
2	6267	- 1
2	3133	- 1
2	1566	- 1
2	783	- 0
2	391	- 1
2	195	- 1
2	97	- 1
2	48	- 1
2	24	- 0
2	12	- 0
2	6	- 0
2	3	- 0
	1	- 1

$\Rightarrow (1100001111011111)_2$

7. Express the function $(cd + b'c + bd')(b+d)$ sum of min terms & pdt of max terms (CANONICAL FORM)

$\Rightarrow (cd + b'c + bd')(b+d)$

Sum of min terms :-

$F = bcd + \cancel{bb'c} + bd' + cd + b'cd + \cancel{bdd'}$

$= bcd + b(c+c')d' + (b+b')cd + b'cd$

$= bcd + bcd' + bc'd' + \cancel{bcd} + b'cd + \cancel{b'cd}$

$\begin{array}{cccc} 111 & 110 & 100 & 011 \\ 7 & 6 & 4 & 3 \end{array}$

$\Sigma m(3, 4, 6, 7)$

Pdt of max terms:-

$= bcd + bcd' + bc'd' + b'cd$

$= cd(b+b') + bd'(c+c')$

$= cd + bd'$

$\Rightarrow bd' + cd$ By distributive law

$= (bd'+c)(bd'+d)$

$= (b+c)(d'+c)(b+d)\cancel{(d+d')}$

$= (b+c+dd')(bb'+c+d')(b+cc'+d)$

$= (b+c+d)\cancel{(b+c+d')}$

$(b+c+d')(b'+c+d')$

$\cancel{(b+c+d)}(b+c'+d)$

$= (b+c+d)(b+c+d')(b'+c+d')(b+c'+d)$

$\quad 000 \qquad 001 \qquad 101 \qquad 010$

$\quad 0 \qquad 1 \qquad 5 \qquad 2$

$= \prod m(0,1,2,5)$

8. Convert

A) $F(x,y,z) = \Sigma(1,3,5)$

$= \Pi(0,2,4,6,7)$

B) $F(A,B,C,D) = \Pi(3,5,8,11)$

$= \Sigma(0,1,2,4,6,7,9,10,12,13,14,15)$

$= \Sigma(0,1,2,4,6,7,9,A,C,D,E,F)$

9. convert:

A. $(u+xw)(x+u'v)$ into sum of pdt and pdt of sum [standard form]

<u>sum of pdt:</u>

$(u+xw)(x+u'v)$

$= \cancel{ux+x'}\; ux + \underbrace{\cancel{uu'}v}_{0} + xwx + xwu'v$

$= ux + xwu'v + xw \Leftarrow$ sum of pdt.

<u>Pdt of sum:</u>   <u>Using distribution law,</u>

$(u+xw)(x+u'v)$

$(u+xwu'v)(x+xwu'v)$

$\Rightarrow (u+x)(u+w)\underbrace{(u+u')}_{1}(u+v)(x+x)$
$(x+w)(x+u')(x+v)$

$\Rightarrow (u+x)(u+w)(u+v)(x+w)(x+u')(x+v)(x+x)$

Pdt of sum

$\Rightarrow (u+x)(u+w)(x+u')(x+v)$

10. <u>Reduced using k-map.</u>

A. $F(w,x,y,z) = \Sigma(2,3,12,13,14,15)$

$wx \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{w}\bar{x}$ 00	0	1	1 (3)	1 (2)
$\bar{w}x$ 01	4	5	7	6
wx 11	1 (12)	1 (13)	1 (15)	1 (14)
$w\bar{x}$ 10	8	9	11	10

$F(w,x,y,z) = wx + \bar{w}\bar{x}y$.

B. $F = w'z + xz + x'y + wx'z$

$wx \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{w}\bar{x}$ 00		1	1	1
$\bar{w}x$ 01		1	1	
wx 11		1	1	
$w\bar{x}$ 10		1	1	1

$x'yz'$

$F(w,x,y,z) = z + x'y$

11. Using tabulation method find the reduced expression:-

A. $F = \Sigma(1, 3, 4, 5, 10, 11, 12, 13, 14, 15)$

	A	B	C	D	
1	0	0	0	1	0
3	0	0	1	1	2
4	0	1	0	0	0
5	0	1	0	1	2
10	1	0	1	0	2
11	1	0	1	1	3
12	1	1	0	0	2
13	1	1	0	1	3
14	1	1	1	0	3
15	1	1	1	1	4

	A	B	C	D		Iteration-I		Iteration-II	
1	0	0	0	1	✓	(1,3)	00-1	(4,12,5,13)	-10-
4	0	1	0	0	✓	(1,5)	0-01	(10,11,14,15)	1-1-
3	0	0	1	1	✓	(4,5)	010- ✓	~~(10,14,11,15)~~	~~1-1-~~
5	0	1	0	1	✓	(4,12)	-100 ✓	~~(4,5,12,13)~~	~~-10-~~
10	1	0	1	0	✓	(3,11)	-011	(12,13,14,15)	11--
12	1	1	0	0	✓	(5,13)	-101 ✓	~~(12,14,13,15)~~	~~11--~~
11	1	0	1	1	✓	(10,11)	101- ✓		
13	1	1	0	1	✓	(10,14)	1-10 ✓		
14	1	1	1	0	✓	(12,13)	110- ✓		
15	1	1	1	1	✓	(12,14)	11-0 ✓		
						(11,15)	1-11 ✓		
						(13,15)	11-1		
						(14,15)	111- ✓		

PRIME IMPLICANTS:

(1,3) – $\bar{A}\bar{B}D$

(1,5) – $\bar{A}\bar{C}D$

(3,11) – $\bar{B}CD$

(4,12,5,13) – $B\bar{C}$

(10,11,14,15) – AC

(12,13,14,15) – AB

Min terms	P.I	1?	3?	4 ✓	5 ✓	10 ✓	11 ✓	12 ✓	13 ✓	14 ✓	15 ✓
✓ (1,3)	$\bar{A}\bar{B}\bar{D}$	X	X								
1,5	$\bar{A}\bar{C}D$	X			X						
3,11	$\bar{B}CD$		X				X				
✓ 4,12,5,13	$B\bar{C}$			X	X			X	X		
✓ 10,11,14,15	AC					X	X			X	X
12,13,14,15	AB							X	X	X	X

$$F = \bar{A}\bar{B}\bar{D} + B\bar{C} + AC.$$

$$F = \bar{A}\bar{B}D + B\bar{C} + AC.$$

Verification using K-map:- $F=\Sigma(1,3,4,5,10,11,12,13,14,15)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	(0)	1 (1)	1 (3)	(2)
$\bar{A}B$ 01	1 (4)	1 (5)	(7)	(6)
AB 11	1 (12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$ 10	(8)	(9)	1 (11)	1 (10)

$$\boxed{F = AB\bar{C} + AC + \bar{A}\bar{B}D}$$

Hence verified.

B. $F = \Sigma(5,6,7,12,14,15)$
$d = \Sigma(3,9,11)$

	A B C D
3	0 0 1 1 •
5	0 1 0 1 •
6	0 1 1 0 •
7	0 1 1 1
9	1 0 0 1 •
11	1 0 1 1
12	1 1 0 0 •
14	1 1 1 0
15	1 1 1 1

	A B C D	Iteration-1	Iteration-2
3	0 0 1 1 ✓	(3,7) 0-11 ✓	(3,7,11,15) --11
5	0 1 0 1 ✓	(3,11) -011 ✓	~~(3,11,7,15) --11~~
6	0 1 1 0 ✓	(5,7) 01-1	(6,7,14,15) -11-
9	1 0 0 1 ✓	(6,7) 011- ✓	(6,14,7,15) -11-
12	1 1 0 0 ✓	(6,14) -110 ✓	
7	0 1 1 1 ✓	(9,11) 10-1	
11	1 0 1 1 ✓	(12,14) 11-0	
14	1 1 1 0 ✓	(7,15) -111 ✓	
15	1 1 1 1 ✓	(11,15) 1-11 ✓	
		(14,15) 111- ✓	

PRIME IMPLICANTS.

(5,7) $\bar{A}BD$

(9,11) $A\bar{B}D$

(12,14) $AB\bar{D}$

(3,7,11,15) → CD

(6,7,14,15) → BC

Min terms	P.I	~~3~~	5	6	7	~~9~~	~~11~~	12	14	15
5,7	$\bar{A}BD$		X		X					
9,11	$A\bar{B}D$									
12,14	$AB\bar{D}$							X	X	
3,7,11,15	CD				X					X
6,7,14,15	BC			X	X				X	X

$$F = \bar{A}BD + AB\bar{D} + BC$$

Verification using K-map.

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	3	2
$\bar{A}B$ 01	4	1 5	1 7	1 6
AB 11	1 12	X 13	1 15	1 14
$A\bar{B}$ 10	8	X 9	X 11	10

$$\boxed{F = BC + \bar{A}BD + AB\bar{D}}$$

Hence Verified.

31.07.18

Unit - II Combinational circuits.

Digital logic circuits

Combinational

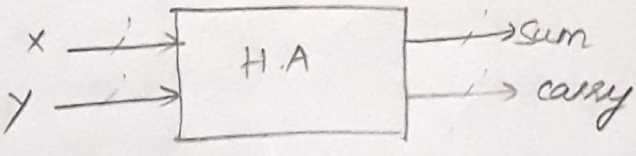
- → Half Adder
- → Full Adder.
- → Ripple carry adder (parallel adder)
- → Mux
- → De Mux
- → Decoder
- → Encoder
- → Comparator

Sequential.

(Flip flops)

- → Register
- → Counter
- → Shift register
- → ALU
- → Memory
- → Control Unit

Eg:- 1. Half Adder.



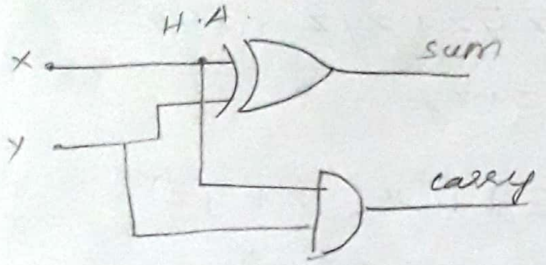
1 bit

x	y	sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

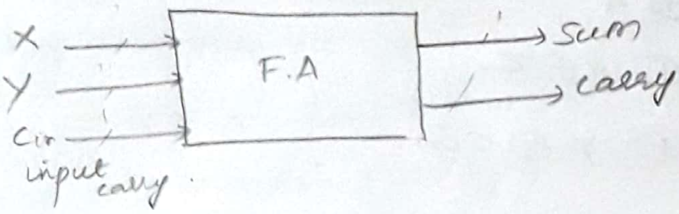
$$\text{Sum} = \overline{x}y + x\overline{y} = x \oplus y$$

$$\text{carry} = xy$$

H·A



Full Adder:



	x y cin	sum	carry.
0	0 0 0	0	0
1	0 0 1	1	0
2	0 1 0	1	0
3	0 1 1	0	1
4	1 0 0	1	0
5	1 0 1	0	1
6	1 1 0	0	1
7	1 1 1	1	1

Sum

$x \backslash yz$	$\bar y\bar z$ 00	$\bar y z$ 01	yz 11	$y\bar z$ 10
$\bar x$ 0		1		1
x 1	1		1	

$F \Rightarrow sum \Rightarrow x\bar y\bar z + \bar x\bar y z + xyz + \bar x y\bar z$

$x \backslash yz$	$\bar y\bar z$ 00	$\bar y z$ 01	yz 11	10 $y\bar z$
$\bar x$ 0			1	
x 1		1	1	1

carry $\Rightarrow$ $\cancel{yz+xz+xy}$

$yz + xz + xy$

sum: $x\bar{y}\bar{z} + \bar{x}\bar{y}z + xyz + \bar{x}y\bar{z}$

carry: $yz + xz + xy$

sum:

$\bar{x}(\bar{y}z + y\bar{z}) + x(yz + \bar{y}\bar{z})$

$\bar{x}(\underbrace{y \oplus z}_{A}) + x(\underbrace{y \odot z}_{\bar{A}})$

$= \bar{x}A + x\bar{A}$

$= x \oplus A$

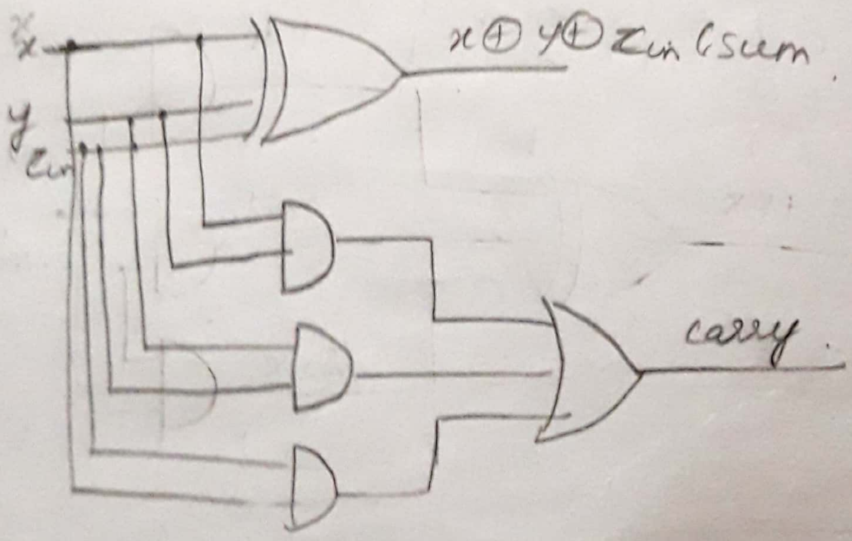
$= x \oplus y \oplus z$

$= x \oplus y \oplus c_{in}.$

carry: $yz + xz + xy.$

$\cancel{x(z+y) + yz}$

$= xy + yc_{in} + xc_{in}.$



Half Adder.

1. Full Adder using 2 half adder & one OR gate.

carry: $xy + yc_{in} + xc_{in}$

$= xy + (x+\bar{x})yc_{in} + x(y+\bar{y})c_{in}$

$= xy + xyc_{in} + \bar{x}yc_{in} + \cancel{xyc_{in}} + x\bar{y}c_{in}$

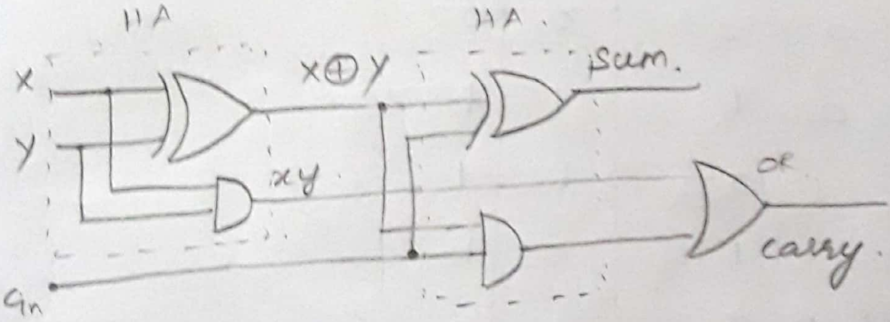
$= \underbrace{xy + xyc_{in}} + \bar{x}yc_{in} + x\bar{y}c_{in}$

Using absorption law: $(A + AB = A)$

$= xy + c_{in}(\bar{x}y + x\bar{y})$

<u>carry</u> $= xy + c_{in}(x \oplus y)$

<u>Sum</u> $= x \oplus y \oplus c_{in}$



2. <u>Half Subtractor.</u>

X	Y	diff (D)	borrow (B)
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

<u>Difference:</u>

$D = \bar{X}Y + X\bar{Y}$

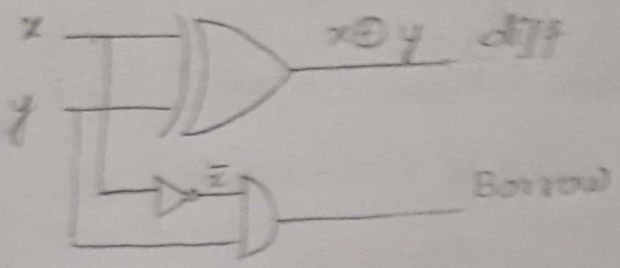
$= X \oplus Y$

<u>Borrow</u>

$B = \bar{X}Y$

Difference: $x \oplus y$

Borrow: $\bar{x}y$.



FULL SUBTRACTOR:

	x	y	C_{in}	D	B
0	0	0	0	0	0
1	0	0	1	1	1
2	0	1	0	1	1
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	0
6	1	1	0	0	0
7	1	1	1	1	1

Difference

x \ $y c_{in}$	$\bar{y}\bar{c}$ 00	$\bar{y}c$ 01	yc 11	$y\bar{c}$ 10
$\bar{x}$ 0	0	1 (1)	3	1 (2)
x 1	1 (4)	5	1 (7)	6

Difference $D = x\bar{y}\overline{c_{in}} + \bar{x}\bar{y}c_{in} + xyc_{in} + \bar{x}y\overline{c_{in}}$

$$\boxed{D = x \oplus y \oplus c_{in}}$$

Borrow

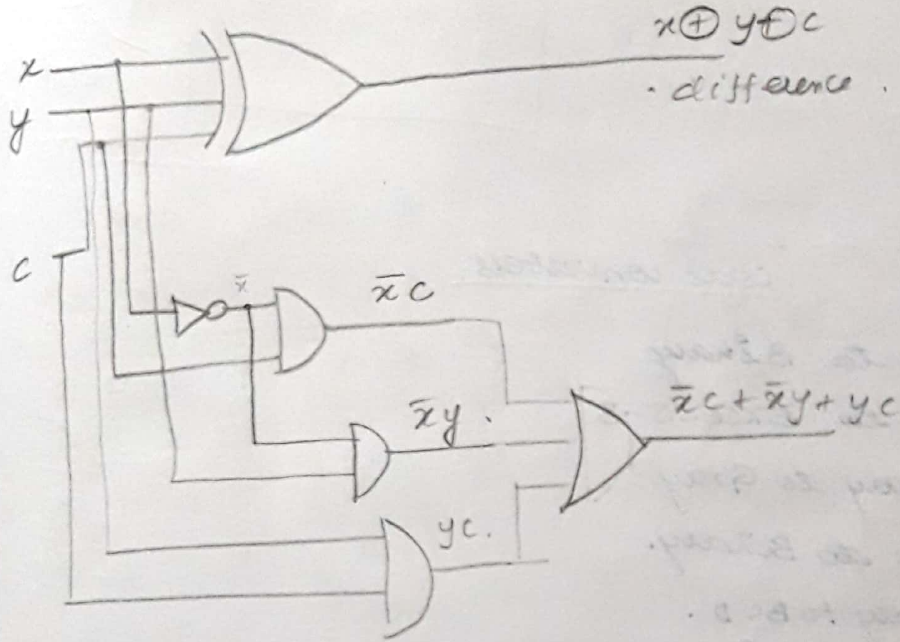
$x \backslash yc_{in}$	$\bar{y}\bar{c}$ 00	$\bar{y}c$ 01	yc 11	$y\bar{c}$ 10
$\bar{x}$ 0	0	1 (1)	1 (3)	1 (2)
x 1	4	5	1 (7)	6

$$B = \bar{x}C_{in} + \bar{x}y + yc$$

$$\boxed{B = \cancel{x}\ \bar{x}c + \bar{x}y + yc}$$

2 half subt + R.

$$= \cancel{\bar{x}y + ct}$$



$$B = \bar{x}c + \bar{x}y + yc.$$

$$= \bar{x}y + \bar{x}(y+\bar{y})c + (x+\bar{x})yc$$

$$= \underbrace{\bar{x}y + \bar{x}yc}\ + \bar{x}\bar{y}c + xyc + \cancel{\bar{x}yc}$$

$$\cancel{\bar{x}y(1+c) = \bar{x}y + c(\bar{x}y + xy) + \bar{x}yc}$$

$$\cancel{= \bar{x}y + c \odot xy + \bar{x}yc}$$

$$= \bar{x}y + \bar{x}\bar{y}c + xyc$$

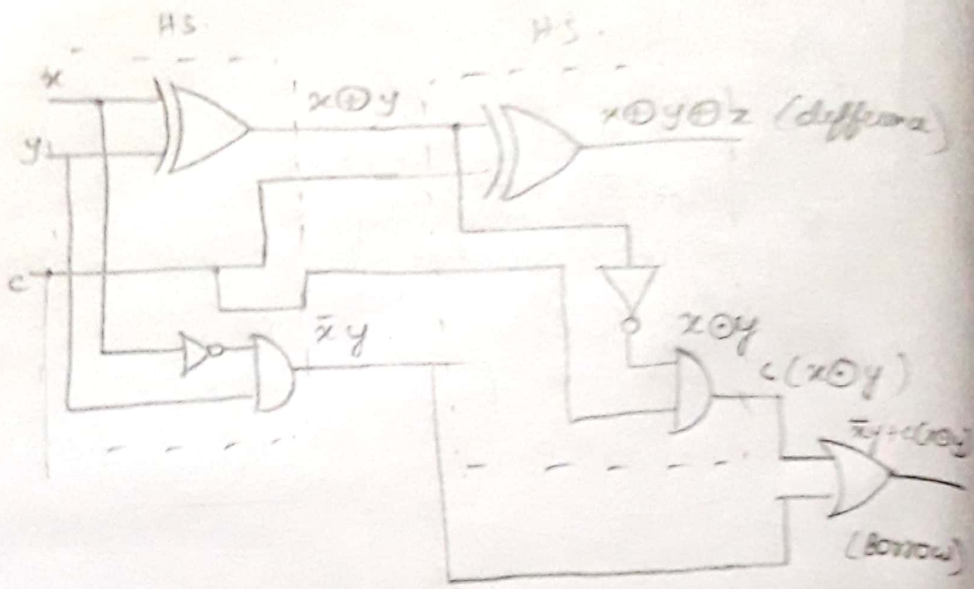
$$= \bar{x}y + c(\bar{x}\bar{y} + xy)$$

$$\cancel{= \bar{x}y + c \odot \bar{x}y}\ \ c(x \odot y)$$

$$B = \bar{x}y + c(x \odot y)$$

$$B = \bar{x}y + c(x \odot y)$$

$$D = x \oplus y \oplus c.$$



07.08.18

Code converters

1. BCD to Binary
2. BCD to EXCESS-3
3. Binary to Gray
4. Gray to Binary.
5. Binary to BCD.
6. Excess 3 to Binary.

I.

	BCD				BINARY			
	A_3	A_2	A_1	A_0	B_3	B_2	B_1	B_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	0
3	0	0	1	1	0	0	1	1
4	0	1	0	0	0	1	0	0
5	0	1	0	1	0	1	0	1
6	0	1	1	0	0	1	1	0
7	0	1	1	1	0	1	1	1
8	1	0	0	0	1	0	0	0
9	1	0	0	1	1	0	0	1
	~~1~~	0	~~1~~	0	Don't care nos.			
	1	0	1	~~1~~				
	~~1~~	~~1~~	0	0				
	1	1	0	1				
	1	1	1	0				
	1	1	1	1				

BCD   BINARY

A_3	A_2	A_1	A_0
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0			

Draw K maps for B_3 B_2 B_1 B_0

For B_3

$A_3A_2 \backslash A_1A_0$	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	x	x	x	x
10	1	1	x	x

$\boxed{B_3 = A_3}$.

BCD   Excess 3.

	A_3	A_2	A_1	A_0	E_3	E_2	E_1	E_0
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0.

K map

E_3

$A_3A_2 \backslash A_1A_0$	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	x	x	x	x
10	1	1	x	x

$E_3 = A_3 + A_2A_0 + A_2A_1$.

$= A_3 + A_2(A_0 + A_1)$

K map for E_2.

$A_3A_2 \backslash A_1A_0$	$\bar{A}_1\bar{A}_0$ 00	$\bar{A}_1A_0$ 01	A_1A_0 11	$A_1\bar{A}_0$ 10
$\bar{A}_3\bar{A}_2$ 00	0 ₀	1 ₁	1 ₃	1 ₂
$\bar{A}_3A_2$ 01	1 ₄	0 ₅	0 ₇	0 ₆
A_3A_2 11	X ₁₂	X ₁₃	X ₁₅	X ₁₄
$A_3\bar{A}_2$ 10	0 ₈	1 ₉	X ₁₁	X ₁₀

$$\boxed{\begin{aligned} E_2 = \cancel{A_3A_0} + \bar{A}_2A_1 + \bar{A}_3\bar{A}_2 \\ A_0\bar{A}_2 + A_2\bar{A}_1\bar{A}_0 \end{aligned}}$$

K map for E_0

$A_3A_2 \backslash A_1A_0$	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	1 (4)	0 (5)	0 (7)	1 (6)
11	X (12)	X (13)	X (15)	X (14)
10	1 (8)	0 (9)	X (11)	X (10)

$$E_0 = \bar{A}_1\bar{A}_0 + A_1\bar{A}_0$$

$$E_0 = \bar{A}_0(\bar{A}_1 + A_1)$$

16, 8, 4, 2

K map for E_1

$A_3A_2 \backslash A_1A_0$	00	01	11	10
00	1 (0)	0 (1)	1 (3)	0 (2)
01	1 (4)	0 (5)	1 (7)	0 (6)
11	X (12)	X (13)	X (15)	X (14)
10	1 (8)	0 (9)	X (11)	X (10)

$$E_1 = \bar{A}_1\bar{A}_0 + A_1A_0$$

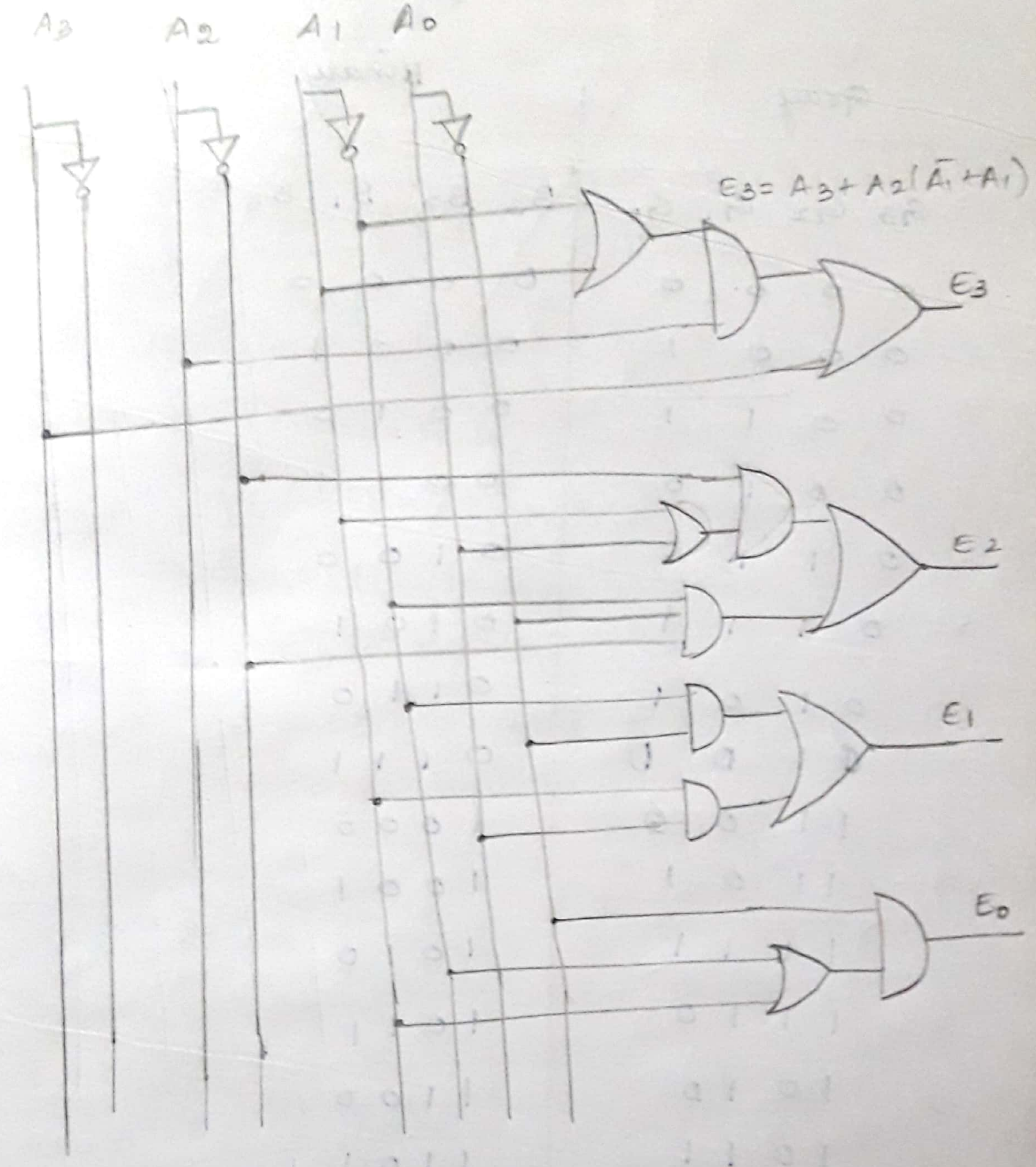
$$E_3 = A_3 + A_2(\bar{A}_1 + A_1)$$

$$E_2 = \bar{A}_2(A_1 + A_0) + \bar{A}_1\bar{A}_0A_2$$

$$E_1 = \bar{A}_1\bar{A}_0 + A_1A_0$$

$$E_0 = \bar{A}_0(\bar{A}_1 + A_1)$$

LOGIC DIAGRAM:



$$E_3 = A_3 + A_2(\bar{A}_1 + A_1)$$

$$E_2 = \bar{A}_2 (A_1 + A_0) + \bar{A}_1 \bar{A}_0 A_2$$

$$E_1 = \bar{A}_1 \bar{A}_0 + A_1 A_0$$

$$E_0 = \bar{A}_0 (\bar{A}_1 + A_1)$$

Using this logic diagram is drawn above

Gray to Binary

	Gray				Binary			
	G_3	G_2	G_1	G_0	B_3	B_2	B_1	B_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	1	0	0	1	0
3	0	0	1	0	0	0	1	1
4	0	1	1	0	0	1	0	0
5	0	1	1	1	0	1	0	1
6	0	1	0	1	0	1	1	0
7	0	1	0	0	0	1	1	1
8	1	1	0	0	1	0	0	0
9	1	1	0	1	1	0	0	1
10	1	1	1	1	1	0	1	0
11	1	1	1	0	1	0	1	1
12	1	0	1	0	1	1	0	0
13	1	0	1	1	1	1	0	1
14	1	0	0	1	1	1	1	0
15	1	0	0	0	1	1	1	1

B_2

G_3G_2 \ G_1G_0	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

$$\boxed{B_3 = G_3}$$

$$B_2 = \overline{G_3} G_2 + G_3 \overline{G_2}$$

$$\boxed{B_2 = G_3 \oplus G_2.}$$

B_1

$G_3G_2 \backslash G_1G_0$	$\bar{G}_1\bar{G}_0$ 00	$\bar{G}_1 G_0$ 01	G_1G_0 11	$G_1\bar{G}_0$ 10
$\bar{G}_3\bar{G}_2$ 00	0	0	1	1
$\bar{G}_3G_2$ 01	1	1	0	0
G_3G_2 11	0	0	1	1
$G_3\bar{G}_2$ 10	1	1	0	0

$$B_1 = G_3\bar{G}_2\bar{G}_1 + G_3G_2G_1 + \bar{G}_3\bar{G}_2G_1 + \bar{G}_3G_2\bar{G}_1$$

$$= G_3(\bar{G}_2\bar{G}_1 + G_2G_1) + \bar{G}_3(\bar{G}_2G_1 + G_2\bar{G}_1)$$

$$= G_3(G_2 \odot G_1) + \bar{G}_3(G_1 \oplus G_2)$$

$$= \bar{G}_3(G_1 \oplus G_2) + G_3(\overline{G_1 \oplus G_2})$$

$$= \bar{G}_3 \oplus G_1 \oplus G_2$$

$$= \cancel{G_3 \oplus G_1 \; G_1 \oplus G_2 \oplus G_3}$$

$$= G_3 \oplus G_2 \oplus G_1$$

B_0

$G_3G_2 \backslash G_1G_0$	$\bar{G}_1\bar{G}_0$ 00	$\bar{G}_1G_0$ 01	G_1G_0 11	$G_1\bar{G}_0$ 10
$\bar{G}_3\bar{G}_2$ 00	0_{0}	1	0_{3}	1_{2}
$\bar{G}_3G_2$ 01	1_{4}	0_{5}	1	0_{6}
G_3G_2 11	0_{12}	1_{13}	0_{15}	1_{14}
$G_3\bar{G}_2$ 10	1_{8}	0_{9}	1_{10}	0_{11}

$$B_0 = \bar{G}_3\bar{G}_2\bar{G}_1G_0 + \bar{G}_3\bar{G}_2G_1\bar{G}_0 + \bar{G}_3G_2\bar{G}_1\bar{G}_0 + \bar{G}_3G_2G_1G_0 + G_3G_2\bar{G}_1G_0 + G_3G_2G_1\bar{G}_0 + G_3\bar{G}_2\bar{G}_1\bar{G}_0 + G_3\bar{G}_2G_1G_0$$

$$= \bar{G}_3\bar{G}_2(\bar{G}_1G_0 + G_1\bar{G}_0) + \bar{G}_3G_2(\bar{G}_1\bar{G}_0 + G_1G_0) + G_3G_2(\bar{G}_1G_0 + G_1\bar{G}_0) + G_3\bar{G}_2(\bar{G}_1\bar{G}_0 + G_1G_0)$$

$$= \bar{G}_3\bar{G}_2(G_1 \oplus G_0) + \bar{G}_3G_2(G_1 \odot G_0) + G_3G_2(G_1 \oplus G_0) + G_3\bar{G}_2(G_1 \odot G_0)$$

$$= (\bar{G}_3\bar{G}_2 + G_3G_2)(G_1 \oplus G_0) + \overline{\oplus}\,(\bar{G}_3G_2 + \bar{G}_2G_3)(G_1 \odot G_0)$$

$$= (G_3 \odot G_2)(G_1 \oplus G_0) + \cancel{G_3G_2}(G_3 \oplus G_2)(G_1 \odot G_0)$$

$$= (G_3 \odot G_2)(G_1 \oplus G_0) + \overline{(G_3 \oplus G_2)}\ \overline{(G_1 \odot G_0)}$$

$$= (G_3 \odot G_2)(G_1 \oplus G_2) + \overline{(G_3 \odot G_2)}(\overline{G_1 \oplus G_0})$$

$$\cancel{A\,B + \bar{A}\,\bar{B}}$$

$$= \cancel{G_3 \odot G_2 \odot G_1}$$

$$= \underbrace{(G_3 \odot G_2)}_{A}\underbrace{(G_1 \oplus G_0)}_{B} + \underbrace{\overline{(G_3 \odot G_2)}}_{\bar{A}}\underbrace{(\overline{G_0 \oplus G_1})}_{\bar{B}}$$

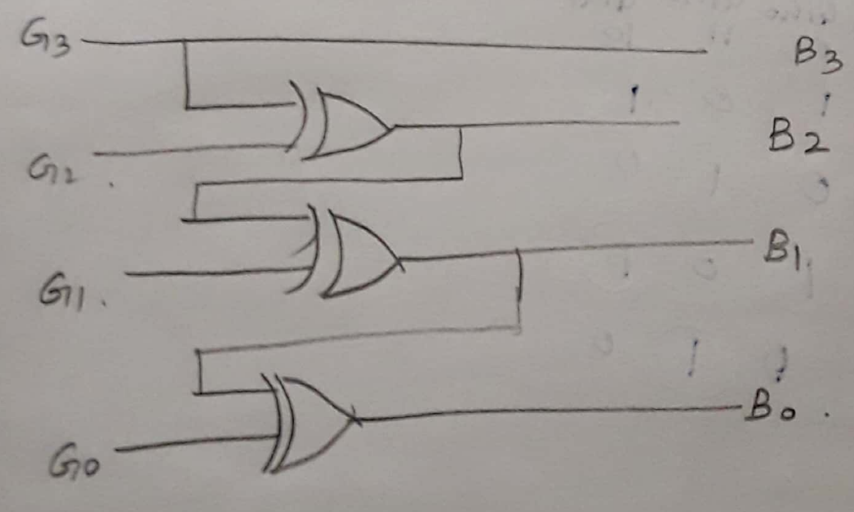
$$\cancel{(G_3 \odot G_2) \odot G_1 \oplus G_0}$$

$$= \underbrace{(\overline{G_3 \oplus G_2})}_{\bar{A}}\underbrace{(G_1 \oplus G_0)}_{B} + \underbrace{(\overline{G_0 \oplus G_1})}_{\bar{B}}\underbrace{(G_2 \oplus G_3)}_{A}$$

$$= G_3 \oplus G_2 \oplus G_1 \oplus G_0$$

$$B_0 = G_0 \oplus B_2 \oplus G_1$$

Design:-



10.8.18

Binary parallel Adders.

Half adders – 2 bits } each input
Full adder – 3 bits } 1 bits.

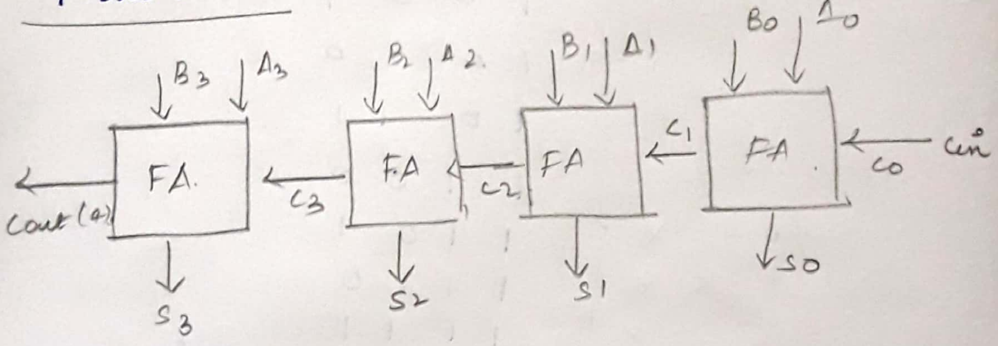
1. Ripple carry Adder (carry propagate Adder)
2. carry look ahead adder.

1. Ripple carry Adder:

(made of full adders)

If n bits are required, the n-number of full adders are required.

4-bit RCA.



$A = A_3A_2A_1A_0 = 1101$

$B = B_3B_2B_1B_0 = 1001$

Initially i/p carry is zero.

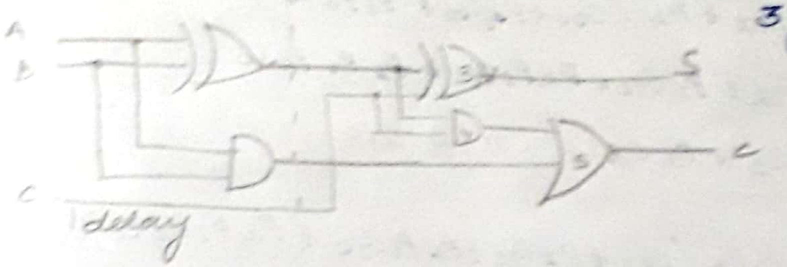
	C_3	C_2	C_1	$C_0 = 0$	
$A =$	A_3	A_2	A_1	A_0	3 inputs are added
$B =$	B_3	B_2	B_1	B_0	∴ full adders are used
C_4 Cout	S_3	S_2	S_1	S_0	

Propagation delay: (Generally in nano second)

Time Delay between the input arrivals to the gate & the generation of carry.

Propagation delay depends upon the gate.

3 gate delay



In full adder → 2 gate delay. (in general)

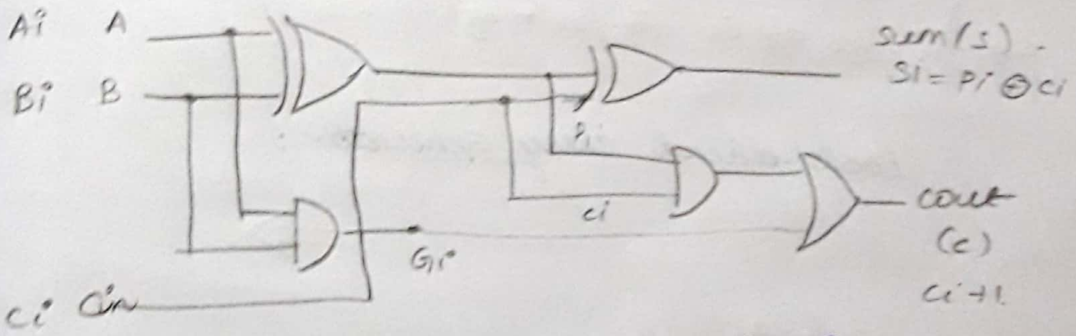
Disadvantage:

Longest propagation delay.

2. Carry Look ahead adder: (high speed parallel adder)

All the carry are generated simultaneously.

We modify based on full adder
[consider it as single bit]



Assume $A \to A_i$, $B \to B_i$, $c_{in} = c_i$

$P_i \to$ Propagate function $= A_i \oplus B_i$ (for i/p)

$G_i \to$ Generate function $= A_i B_i$. (And combination of inputs)

$$c_{i+1} = G_i + P_i C_i$$

apply $i = 0$

$\rightarrow C_1 = G_0 + P_0 C_0$, where $C_0 \to$ input carry.

when $i = 2$

$\rightarrow C_2 = G_1 + P_1 C_1$

sub c_1 in c_2

$\rightarrow C_2 = G_1 + P_1 (G_0 + P_0 C_0)$

$= G_1 + P_1 G_0 + P_1 P_0 C_0$

$$C_3 = G_2 + P_2 C_2$$
$$= G_2 + P_2 (G_1 + P_0 G_0 + P_1 P_0 C_0)$$
$$= G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0 .$$

$$C_4 = C_3 + P_3 C_3$$ (for 4 bit CLA, C_4 is the final carry)

$$= C_3 + P_3 (G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0)$$

$$= C_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 C_0$$

$$C_n = G_{n-1} + P_{n-1} G_{n-2} + P_{n-1} P_{n-2} G_{n-3} + \dots \dots + (P_{n-1} P_{n-2} \dots P_1 P_0 C_0)$$

For 8 bit CLA:

$$C_{out} = C_7 = G_7 + P_7 G_6 + P_7 P_6 G_5 + P_7 P_6 P_5 G_4 + P_7 P_6 P_5 P_4 G_3 + P_7 P_6 P_5 P_4 P_3 G_2 + P_7 P_6 P_5 P_4 P_3 P_2 G_1 + P_7 P_6 P_5 P_4 P_3 P_2 P_1 G_0 + P_7 P_6 P_5 P_4 P_3 P_2 P_1 P_0 \underline{\underline{C_0}}$$

↑ input carry

→ Final o/p is in form of P & G

Look ahead carry Generator:

$\Longrightarrow$

For 4 bit CLA;

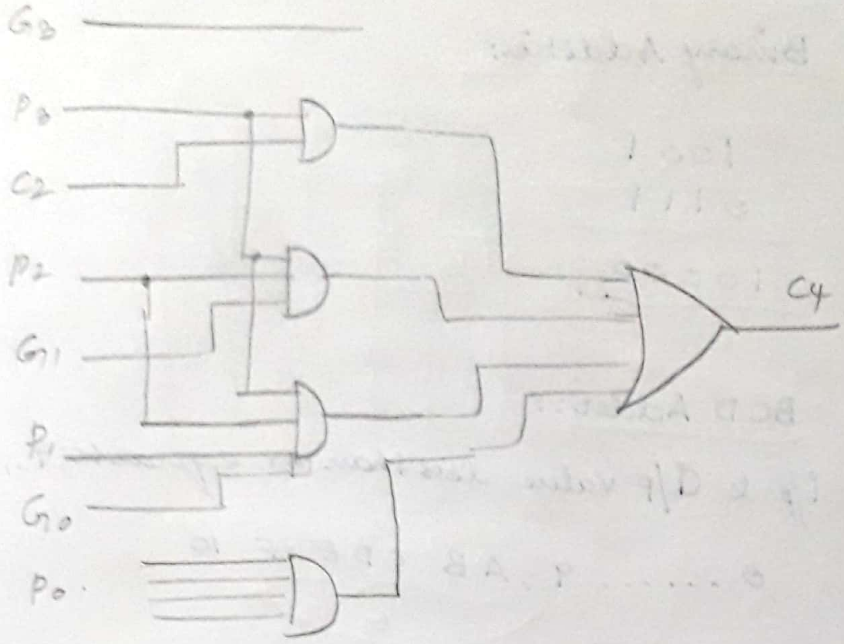
we get 4 gate delay (fixed) from i/p to output sum.

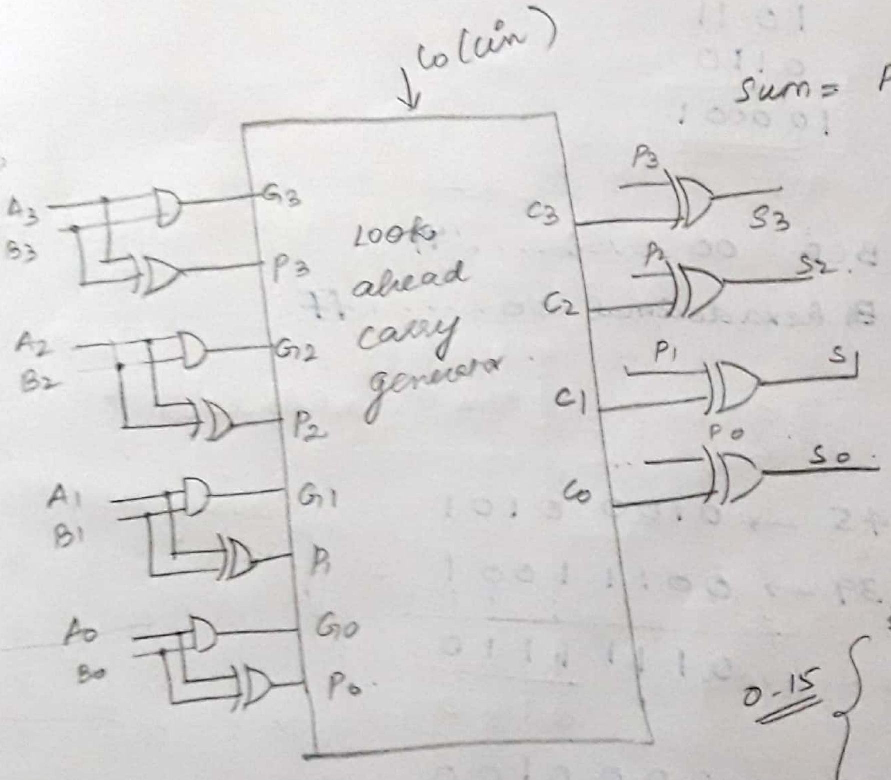
(Very high speed parallel adder)

to generate G, P → 1 gate

to generate sum = 1 gate.

Look ahead carry generator





$sum = P_i \oplus C_i$

Binary add

0-15
```
 1001
 0111
-----
10000
```
(16)

BCD addition

0 to 9

From 10 it repeats.

BCD Adder. (4 bit)

Binary Addition:

```
  1001
  0111
 ------
 10000  -> 16
```

BCD Adder:

i/p & o/p value less than or equal to 9.

0 ...... 9 , $\underbrace{\text{A B C D E F}}_{6}$ 10

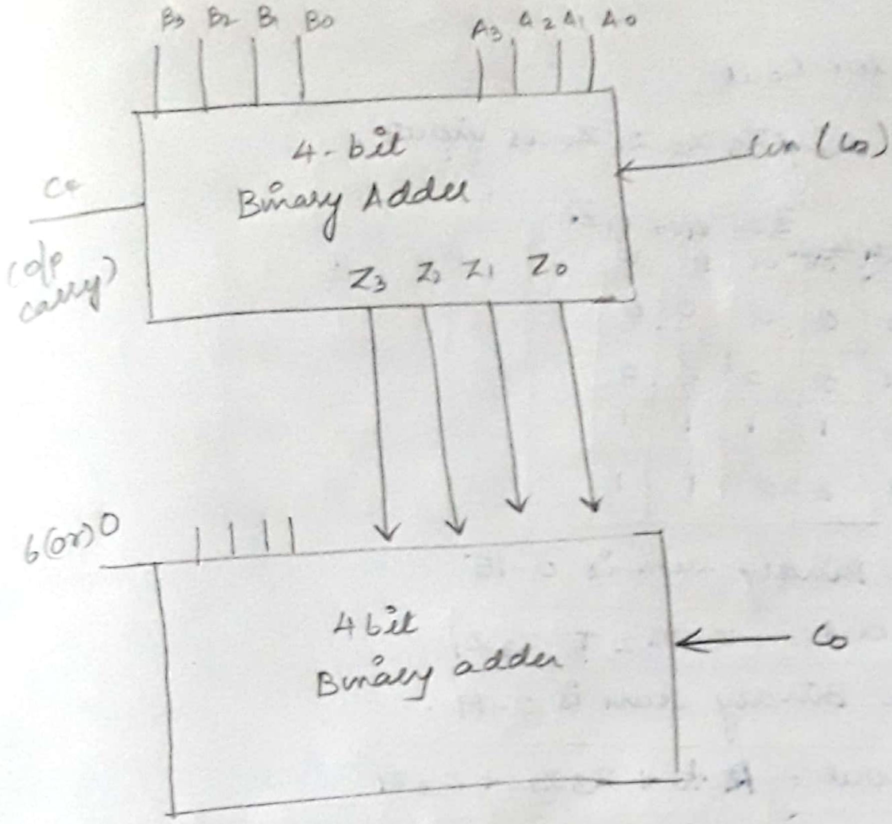
add 6

```
   1011
   0110
 -------
 100001
```

In BCD 00 .... 99 } highest value
In Bi hexadecimal 00 ..... FF

If o/p is greater than 9, add 6 to it.

```
45 -> 0100 0101
39 -> 0011 1001 (+)  } binary addition
      ----------
      0111 1110
           0110
      ----------
84 -> 1000 0100
```



DECIMAL	Binary sum					BCD sum				
	K	Z3	Z2	Z1	Z0	Cout	S3	S2	S1	S0
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	1
2	0	0	0	1	0	0	0	0	1	0
3	0	0	0	1	1	0	0	0	1	1
4	0	0	1	0	0	0	0	1	0	0
5	0	0	1	0	1	0	0	1	0	1
6	0	0	1	1	0	0	0	1	1	0
7	0	0	1	1	1	0	0	1	1	1
8	0	1	0	0	0	0	1	0	0	0
9	0	1	0	0	1	0	1	0	0	1
10	0	1	0	1	0	1	0	0	0	0
11	0	1	0	1	1	1	0	0	0	1
12	0	1	1	0	0	1				
13	0	1	1	0	1	1				
14	0	1	1	1	0	1				
15	0	1	1	1	1	1				
⋮										
19										

K map for Cout

By taking $Z_3\, Z_2\, Z_1\, Z_0$ as inputs.

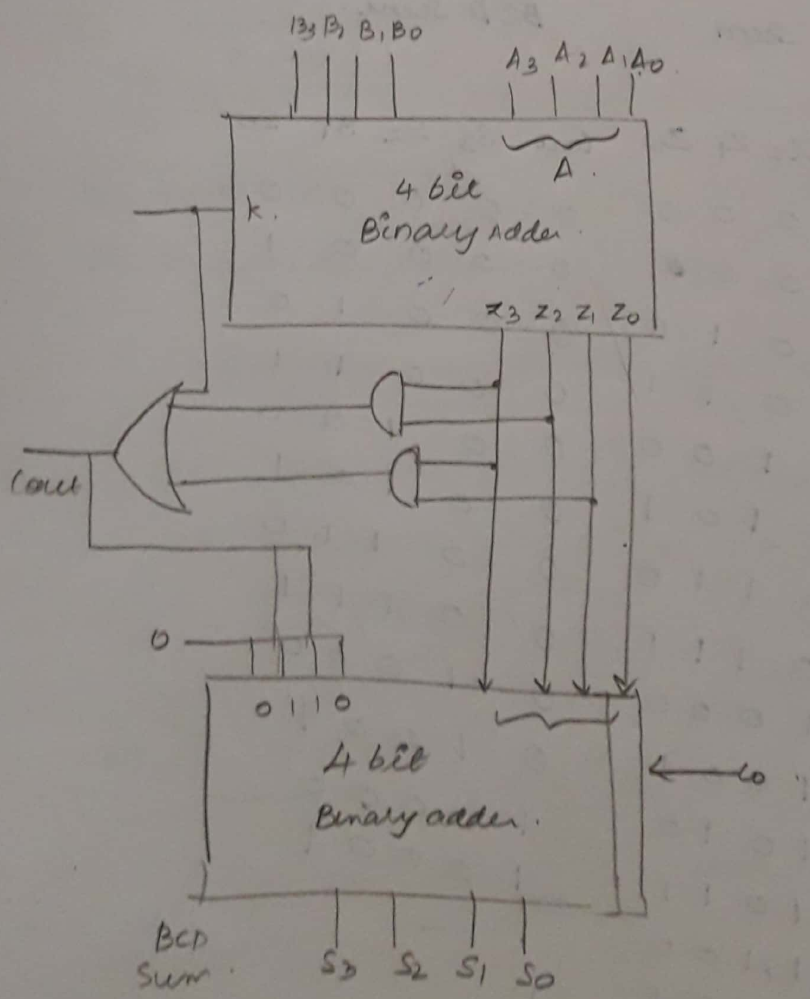
$Z_3 Z_2$ \ $Z_1 Z_0$	$\bar{Z}_1\bar{Z}_0$ 00	$\bar{Z}_1 Z_0$ 01	$Z_1 Z_0$ 11	$Z_1 \bar{Z}_0$ 10
$\bar{Z}_3\bar{Z}_2$ 00	0	0	0	0
$\bar{Z}_3 Z_2$ 01	0	0	0	0
$Z_3 Z_2$ 11	1	1	1	1
$Z_3 \bar{Z}_2$ 10	0	0	1	1

When Binary sum is 0-15

$$\boxed{\text{cout} = Z_3 Z_2 + Z_3 Z_1}$$

When Binary sum is 0-19.

$$\boxed{\text{cout} = K + Z_3 Z_2 + Z_3 Z_1}$$



Magnitude comparator

Inputs

A		B		Outputs			
A_1	A_0	B_1	B_0	$A>B$	$A=B$	$A<B$	
0	0	0	0	0	1	0	0
0	0	0	1	0	0	1	1
0	0	1	0	0	0	1	2
0	0	1	1	0	0	1	3
0	1	0	0	1	0	0	4
0	1	0	1	0	1	0	5
0	1	1	0	0	0	1	6
0	1	1	1	0	0	1	7
1	0	0	0	1	0	0	8
1	0	0	1	1	0	0	9
1	0	1	0	0	1	0	10
1	0	1	1	0	0	1	11
1	1	0	0	1	0	0	12
1	1	0	1	1	0	0	13
1	1	1	0	1	0	0	14
1	1	1	1	0	1	0	15

K map

$A>B$

$A_1A_0 \backslash B_1B_0$	$\bar{B_1}\bar{B_0}$ 00	$\bar{B_1}B_0$ 01	B_1B_0 11	$B_1\bar{B_0}$ 10
$\bar{A_1}\bar{A_0}$ 00	0	0	0	0
$\bar{A_1}A_0$ 01	1	0	0	0
A_1A_0 11	1	1	0	1
$A_1\bar{A_0}$ 10	1	1	0	0

$\boxed{A>B}$

$$= A_0\bar{B_1}\bar{B_0} + A_1A_0\bar{B_0} + A_1\bar{B_1}$$

(ii) A = B

A = B

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00	1	0	0	0
01	0	1	0	0
11	0	0	1	0
10	0	0	0	1

$$\boxed{A=B} = \bar{A}_1\bar{A}_0\bar{B}_1\bar{B}_0 + \bar{A}_1A_0\bar{B}_1B_0 + A_1A_0B_1B_0 + A_1\bar{A}_0B_1\bar{B}_0$$

(iii) A < B

$A_1A_0 \backslash B_1B_0$	$\bar{B}_1\bar{B}_0$ 00	$\bar{B}_1B_0$ 01	B_1B_0 11	$B_1\bar{B}_0$ 10
$\bar{A}_1\bar{A}_0$ 00	0	1	1	1
$\bar{A}_1A_0$ 01	0	0	1	1
A_1A_0 11	0	0	0	0
$A_1\bar{A}_0$ 10	0	0	1	0

$$\boxed{A<B} = \bar{A}_1B_1 + \bar{A}_1\bar{A}_0B_0 + \bar{A}_0B_1B_0$$

Case(i) : $\boxed{A>B}$

$$A>B = A_1\bar{B}_1 + A_0\bar{B}_1\bar{B}_0 + A_1A_0\bar{B}_0$$

$$= A_1\bar{B}_1 + (A_1+\bar{A}_1)A_0\bar{B}_1\bar{B}_0 + A_1A_0(B_1+\bar{B}_1)\bar{B}_0$$

$$= A_1\bar{B}_1 + A_1A_0\bar{B}_1\bar{B}_0 + \bar{A}_1A_0\bar{B}_1\bar{B}_0 + A_1A_0B_1\bar{B}_0 + \cancel{A_1A_0\bar{B}_1\bar{B}_0}$$

$$= A_1\bar{B}_1 + A_1A_0\bar{B}_1\bar{B}_0 + A_0\bar{B}_0(\bar{A}_1\bar{B}_1 + A_1B_1)$$

$$= \underbrace{A_1\bar{B}_1 + A_1A_0\bar{B}_1\bar{B}_0}_{\text{use adsorption law}} + (A_1 \odot B_1)A_0\bar{B}_0$$

$$= A_1\bar{B}_1(1 + A_0\bar{B}_0) + (A_1 \odot B_1)A_0\bar{B}_0$$

$$\boxed{A>B = A_1\bar{B}_1 + (A_1 \odot B_1)A_0\bar{B}_0}$$

From (i), (ii) and (iii) cases:-

1101
1101

$$(A=B) = x_1x_0$$

$$(A>B) = A_1\bar{B}_1 + x_1A_0\bar{B}_0$$

$$(A<B) = \bar{A}_1B_1 + x_1\bar{A}_0B_0$$

Scanned by CamScanner

case (iii): $\boxed{A < B}$

$$A<B = \bar{A_1}B_1 + \bar{A_1}\bar{A_0}B_0(B_1+\bar{B_1}) + \bar{A_0}B_1B_0(A_1+\bar{A_1})$$

$$= \bar{A_1}B_1 + \bar{A_1}\bar{A_0}B_0B_1 + \bar{A_1}\bar{A_0}\bar{B_1}B_0 + A_1\bar{A_0}B_1B_0 + \bar{A_1}\bar{A_0}B_1B_0$$

$$= \bar{A_1}B_1(1+\bar{A_0}B_0) + \bar{A_0}B_0(\bar{A_1}\bar{B_1}+A_1B_1)$$

$$= \bar{A_1}B_1 + \bar{A_0}B_0\underbrace{(A_1 \odot B_1)}_{X_1} = \bar{A_1}B_1 + \bar{A_0}B_0X_1$$

case (ii) $\boxed{A=B}$:-

From ①

$$= \bar{A_1}\bar{A_0}\bar{B_1}\bar{B_0} + \bar{A_1}A_0\bar{B_1}B_0 + A_1A_0B_1B_0 + A_1\bar{A_0}B_1\bar{B_0}$$

$$= \bar{A_1}\bar{B_1}(\bar{A_0}\bar{B_0} + A_0B_0) + A_1B_1(A_0B_0 + \bar{A_0}\bar{B_0})$$

$$= \bar{A_1}\bar{B_1}(A_0 \odot B_0) + A_1B_1(A_0 \odot B_0)$$

$$= (A_0 \odot B_0)(\bar{A_1}\bar{B_1} + A_1B_1)$$

$$= (A_0 \odot B_0)(A_1 \odot B_1)$$

$$= \cancel{X_0 X_0 +} X_1X_0$$

Let $X_1 = A_1 \odot B_1$
$X_0 = A_0 \odot B_0$ } → ①

similarly $\boxed{X_i = A_i \odot B_i}$

While comparing: LHS → RHS
left to Right.

eg:- $X > Y$

$X = 5\,3\,4\,3\,2\,1\,7\,8$

$Y = 5\,3\,4\,2\,1\,7\,8\,7$

1101
1101

4-bit

$A = A_3\ A_2\ A_1\ A_0$

$B = B_3\ B_2\ B_1\ B_0$

$$(A > B) = A_3 \overline{B_3} + X_3 A_2 \overline{B_2} + X_3 X_2 A_1 \overline{B_1} + X_3 X_2 X_1 A_0 \overline{B_0}$$

$$(A < B) = \overline{A_3} B_3 + X_3 \overline{A_2} B_2 + X_3 X_2 \overline{A_1} B_1 + X_3 X_2 X_1 \overline{A_0}$$

$$(A = B) = X_3\ X_2\ X_1\ X_0$$