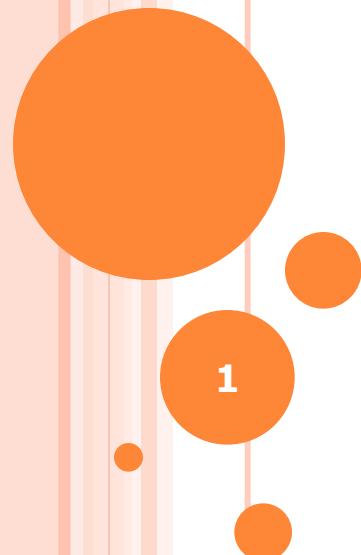


UNIT III SYNCHRONOUS SEQUENTIAL LOGIC



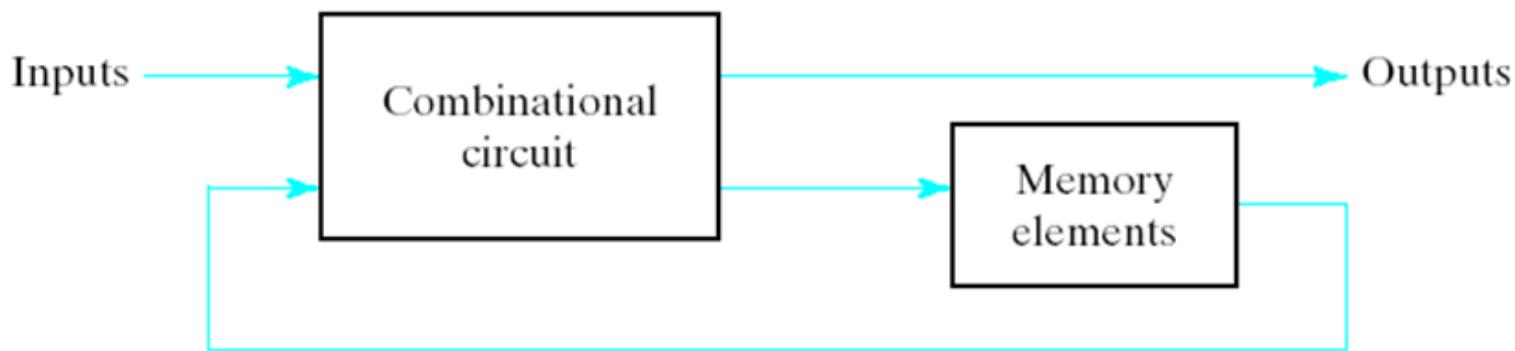
DIVYA ARIVALAGAN
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Anna University

SEQUENTIAL CIRCUITS

- Storage Elements: Latches , Flip-Flops
- Analysis of Clocked Sequential Circuits
- State Reduction and Assignment
- Design Procedure
- Registers
- Counters
- HDL Models of Sequential Circuits.

SEQUENTIAL CIRCUITS

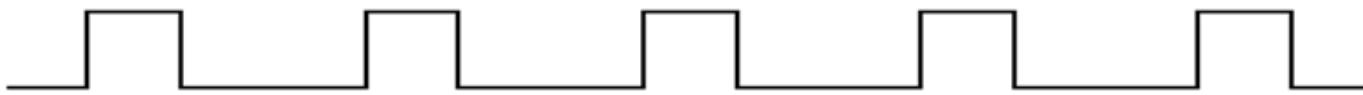
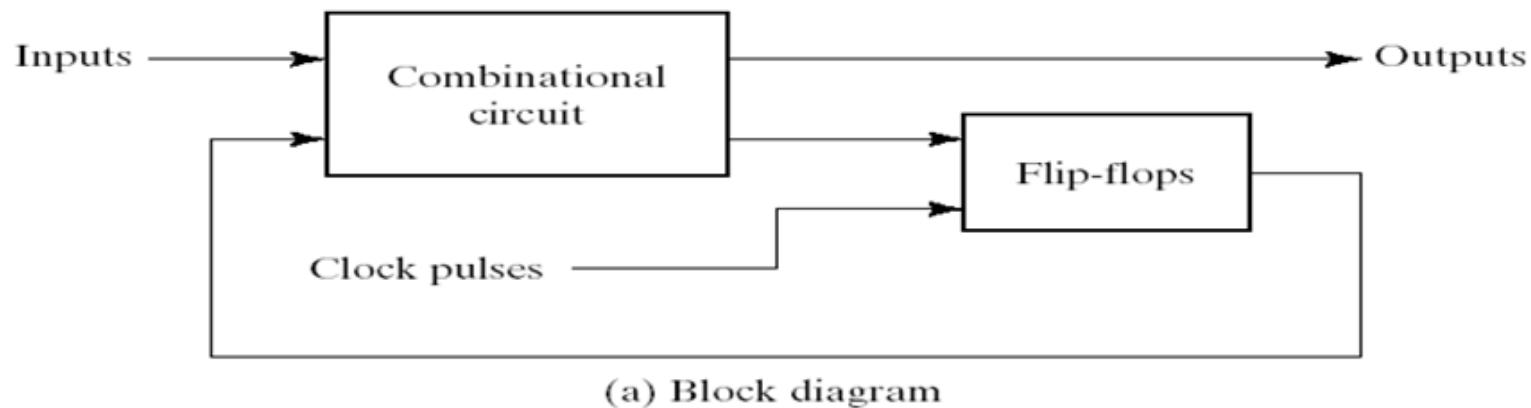
Every digital system is likely to have combinational circuits, most systems encountered in practice also include storage elements, which require that the system be described in term of **sequential logic**.



Block Diagram of Sequential Circuit

SYNCHRONOUS CLOCKED SEQUENTIAL CIRCUIT

A sequential circuit may use many **flip-flops** to store as many bits as necessary. The outputs can come either from the combinational circuit or from the flip-flops or both.



(b) Timing diagram of clock pulses

Synchronous Clocked Sequential Circuit

LATCHES

- Latches are basic storage elements
- Storage elements that operate with signal levels are referred to as latches.
- These are controlled by a clock transition are flip flops.

FLIP FLOP (CLOCKED LATCHES)

- A flip flop is a binary storage device capable of storing one bit information either 0 or 1
- 2 stable states HIGH and LOW
- It has the property to remain in one state indefinitely until it is directed by an input signal to switch over to the other state .It is also called as Bistable multivibrator.
- The basic formation of flip flop is storage of data
- The memory element used in the clocked sequential circuit are called Flip Flops.

FLIP-FLOPS

The state of a latch or flip-flop is switched by a change in the control input.

This momentary change is called a **trigger** and the transition it causes is said to trigger the flip-flop.

The D latch with pulses in its control input is essentially a flip-flop that is triggered every time the pulse goes to the **logic 1 level**.

As long as the pulse input remains in the level, any changes in the data input will change the output and the state of the latch.

APPLICATIONS OF FLIP FLOP

- Registers
- Counters/timers
- As a delay element
- As a memory element

TYPES OF FLIP FLOP

- SR Flip flop
- JK Flip flop
- D Flip flop
- T Flip flop

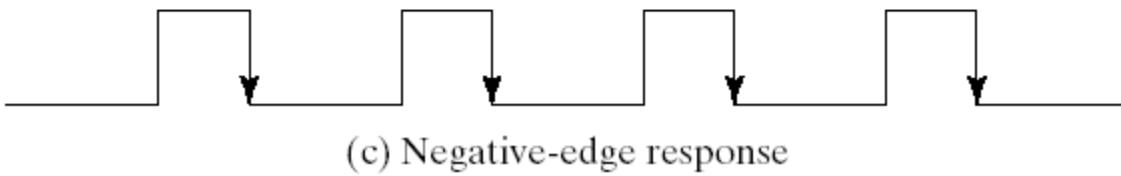
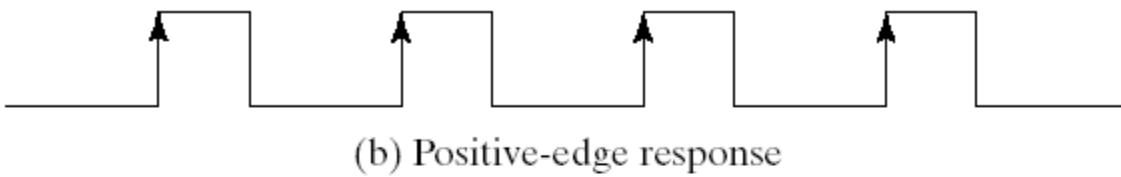
CLOCK RESPONSE IN LATCH

In Fig (a) a positive level response in the control input allows changes, in the output when the D input changes while the clock pulse stays at logic 1.

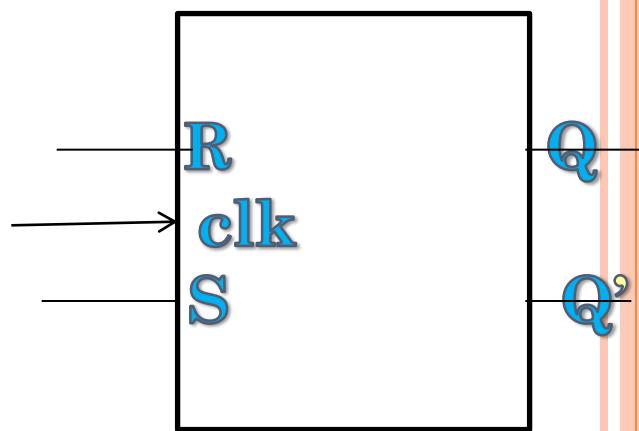
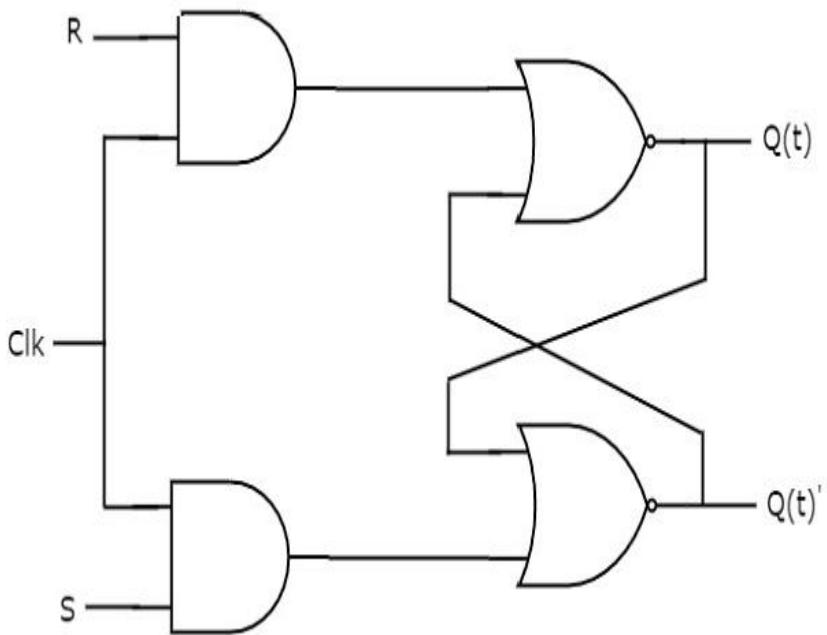


(a) Response to positive level

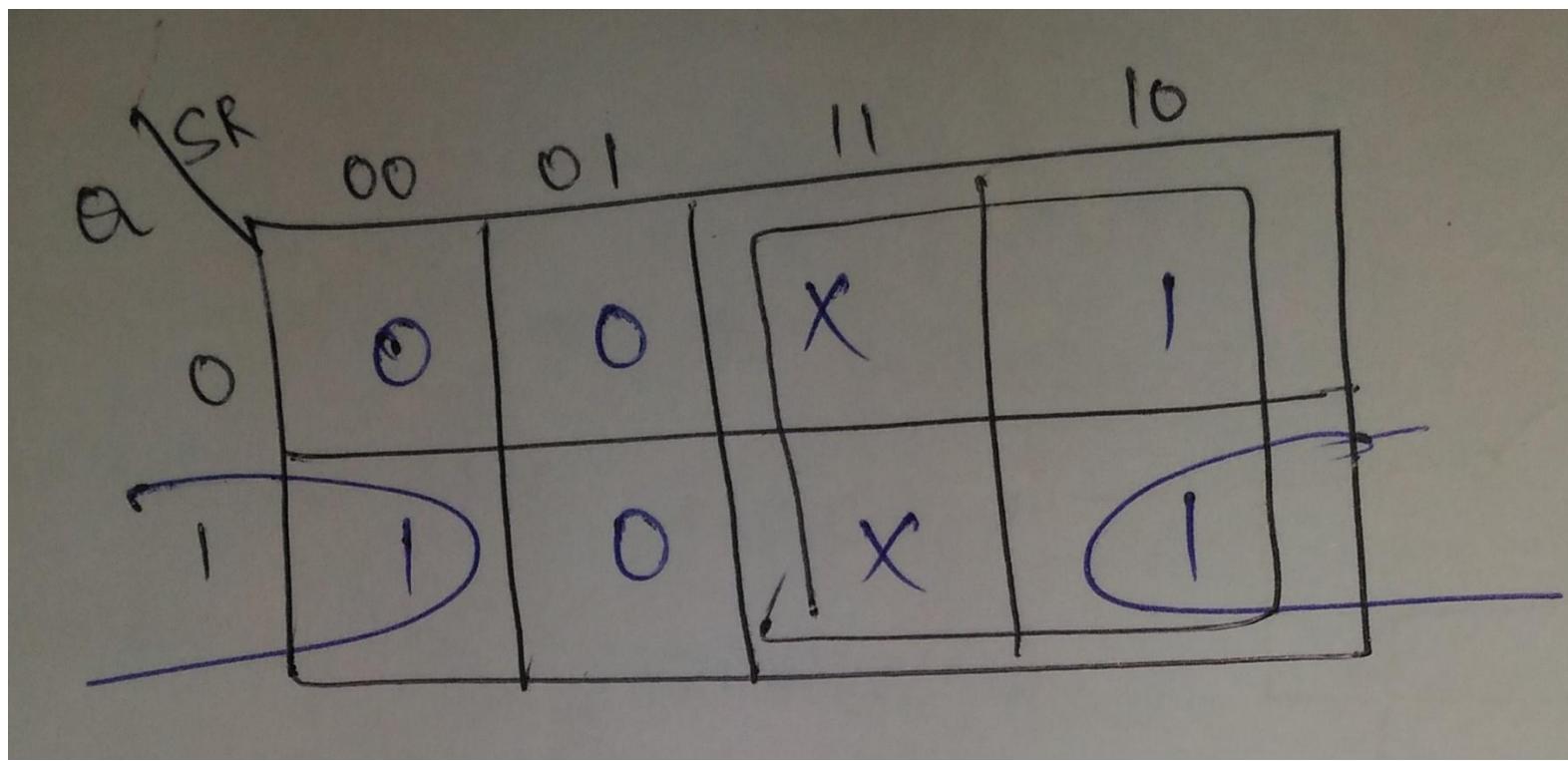
CLOCK RESPONSE IN FLIP-FLOP



SR FLIP FLOP



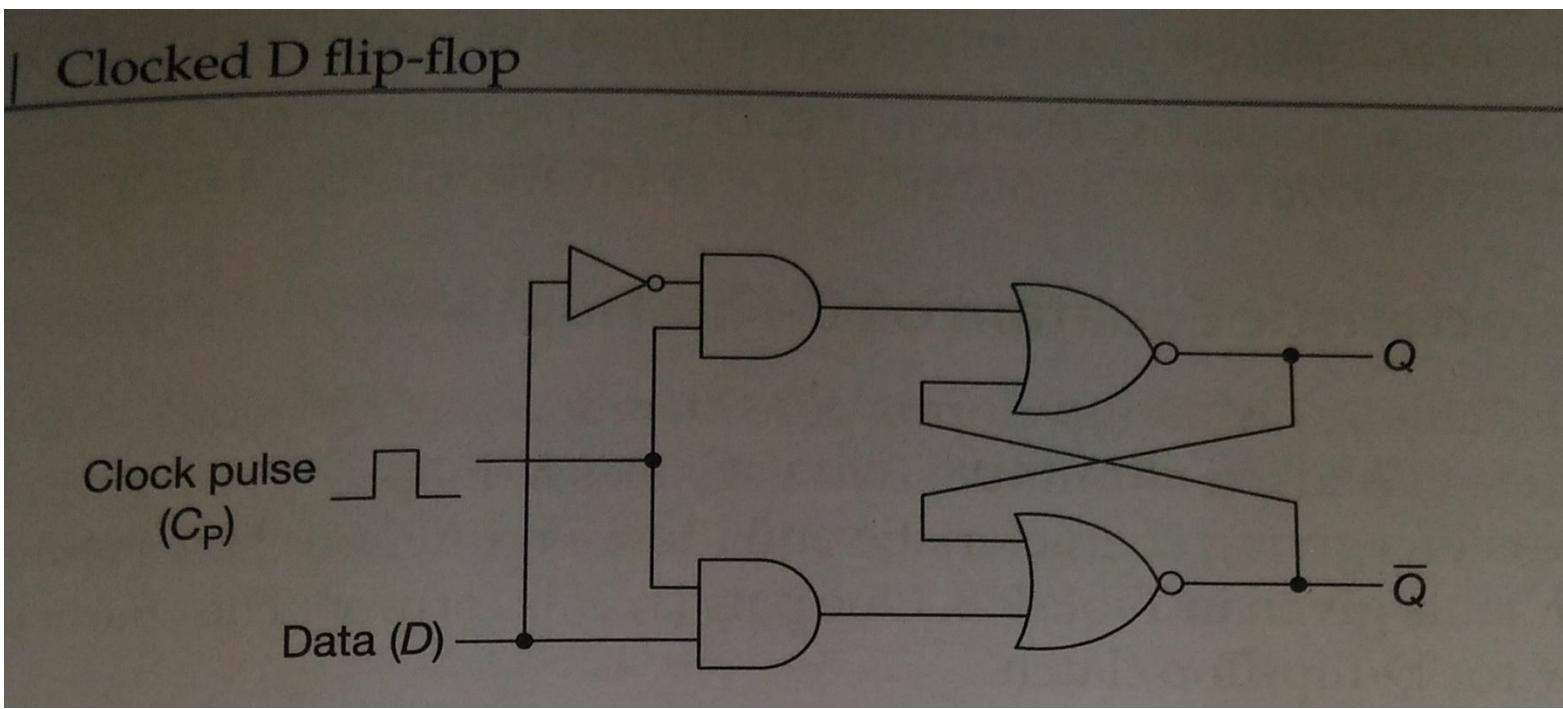
Q	S	R	Q(t+1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	X
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	X

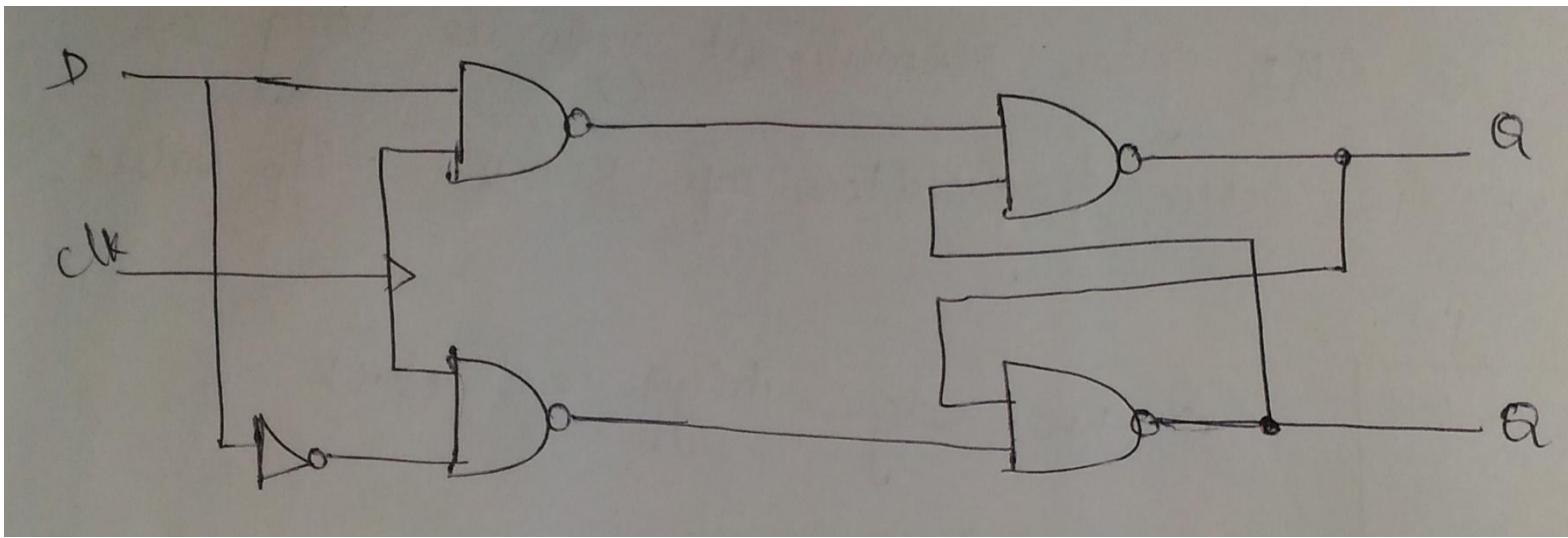


$$Q(t+1) = S + Q R'$$

* S and R cannot be
simultaneously.

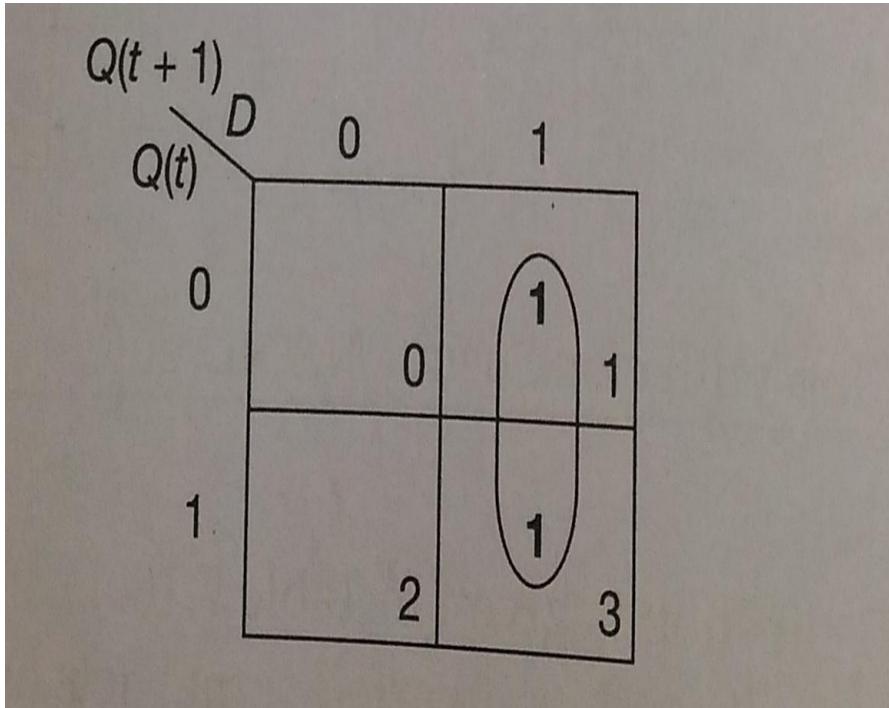
D FLIP FLOP





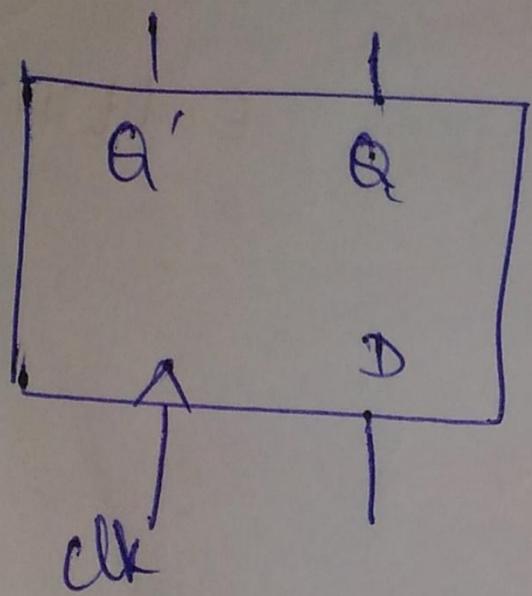
CHARACTERISTICS OF D FLIP FLOP

Inputs		Outputs	
$Q(t)$	D	$Q(t + 1)$	$\bar{Q}(t + 1)$
0	0	0	1
0	1	1	0
1	0	0	1
1	1	1	0

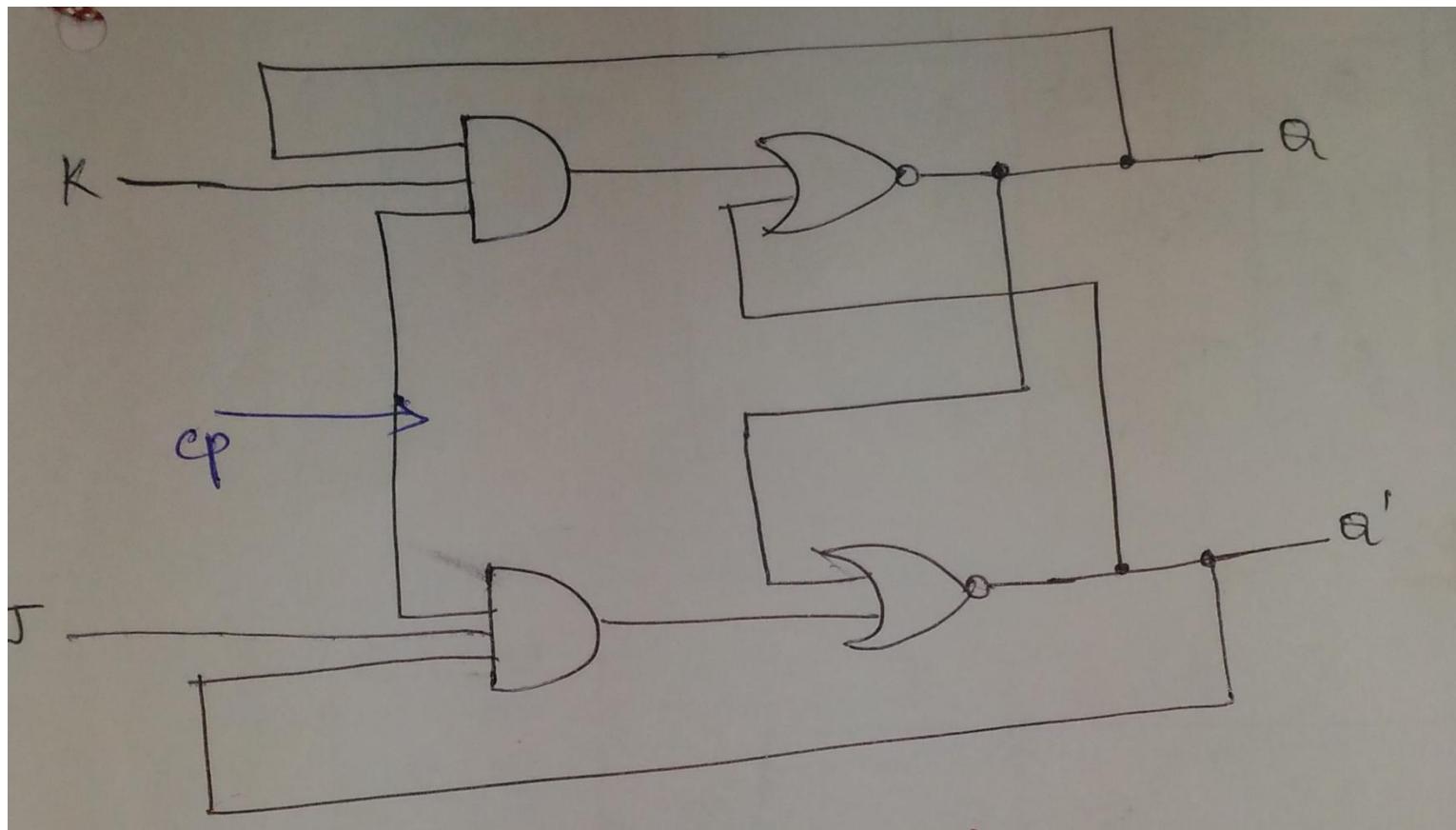


$$Q(t+1) = D$$

logic Representation of D - flip flop

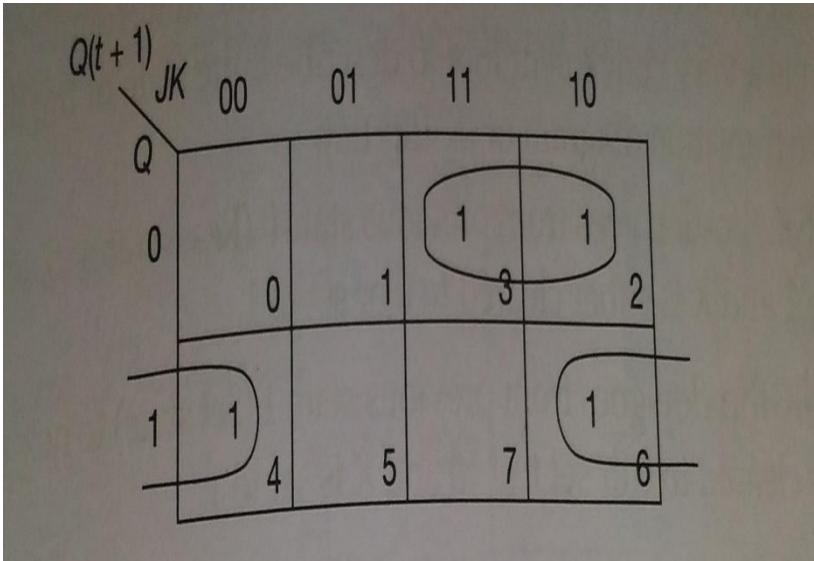


JK FLIP FLOP



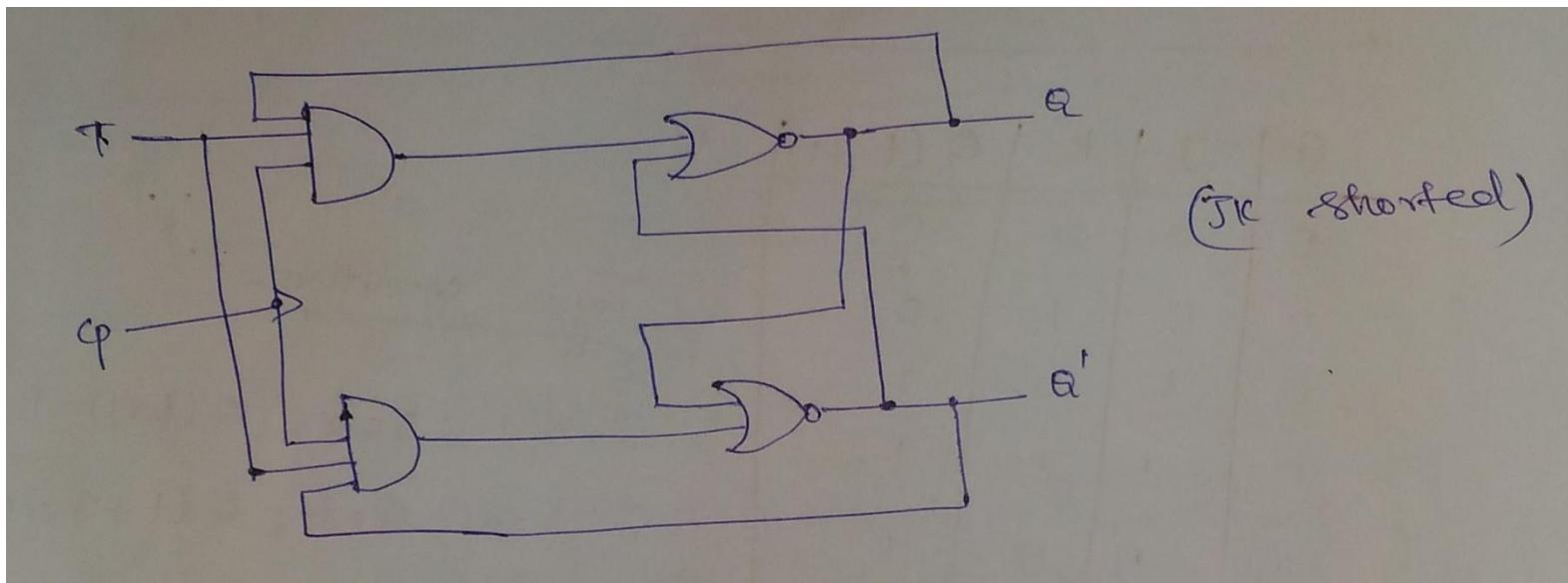
J	K	Q	comment
0	0	0	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'	Toggle

Q	J	K	Q(t+1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0



$$Q(t+1) = J\bar{Q}(t) + \bar{K}Q(t)$$

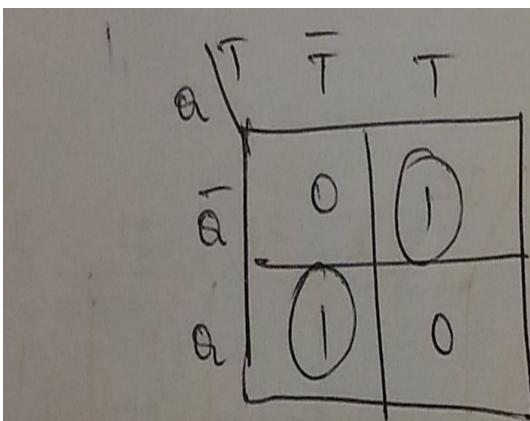
T FLIP FLOP



$$T = 1 \quad Q = 0$$

$$T = 0 \quad Q = 0.$$

a	T	$Q(t+1)$
0	0	0
0	1	1
1	0	1
1	1	0



$$Q(t+1) = \bar{a}T + a\bar{T}$$

$$Q(t+1) = a \oplus T$$

CHARACTERISTIC TABLE

S	R	$Q(t+1)$	T	K	$Q(t+1)$
0	0	$Q(t)$	0	0	Q , no change
0	1	0, Reset	0	1	0, Reset
1	0	1, Set	1	0	1, Set
1	1	?	1	1	Q' , toggle

T	$Q(t+1)$
0	$Q(t)$
1	Toggle, $Q'(t)$

D	$Q(t+1)$
0	0
1	1

CHARACTERISTIC EQUATION

Characteristics equation of flip flops

$$1) S-R \Rightarrow Q(t+1) = S + \bar{R} Q(t)$$

$$2) D \Rightarrow Q(t+1) = D$$

$$3) T \Rightarrow Q(t+1) = T \cdot \bar{Q}(t) + \bar{T} Q(t)$$

$$4) J-K \Rightarrow Q(t+1) = T \oplus Q(t).$$

EXCITATION TABLE

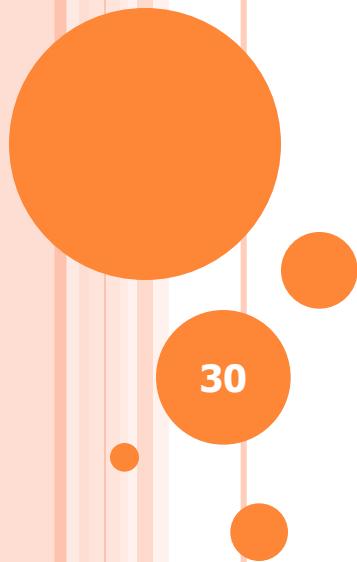
$Q(t)$	$Q(t+1)$	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

$Q(t)$	$Q(t+1)$	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

$Q(t)$	$Q(t+1)$	D
0	0	0
0	1	1
1	0	0
1	1	1

$Q(t)$	$Q(t+1)$	T
0	0	0
0	1	1
1	0	1
1	1	0

Conversion of Flip Flops



SR TO JK

TABLE 7.20 | CONVERSION OF SR-FLIP-FLOP

Minterm	Input of J-K-flip-flop		Output of J-K-flip-flop		Input of SR-flip-flop	
	J	K	$Q(t)$	$Q(t + 1)$	S	R
m_0	0	0	0	0	0	X
m_1	0	0	1	1	X	0
m_2	0	1	0	0	0	X
m_3	0	1	1	0	0	1
m_4	1	0	0	1	1	0
m_5	1	0	1	1	X	0
m_6	1	1	0	1	1	0
m_7	1	1	1	0	0	1

$$S(J, K, Q(t)) = \sum m(4, 6) + \sum d(1, 5)$$

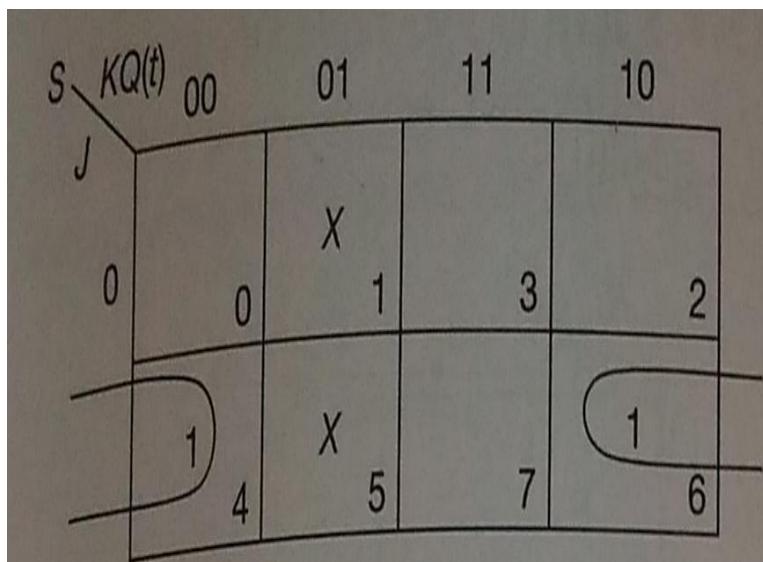


FIGURE 7.51 | K-map for S

$$R(J, K, Q(t)) = \sum m(3, 7) + \sum d(0, 2)$$

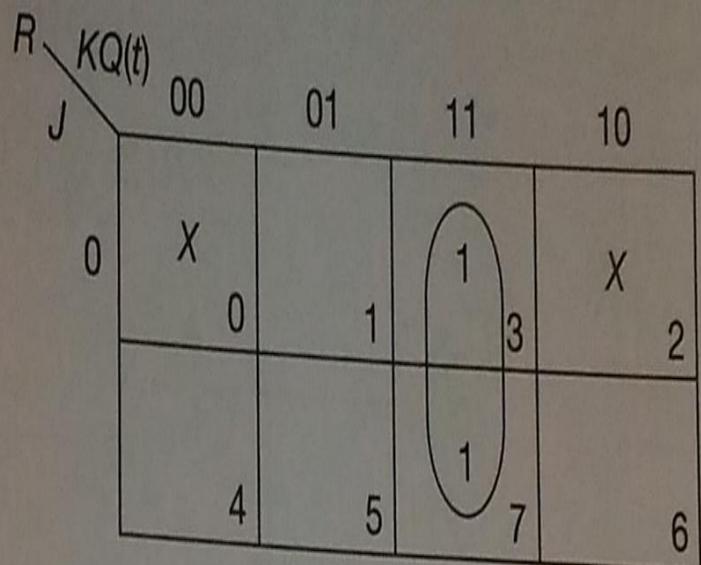
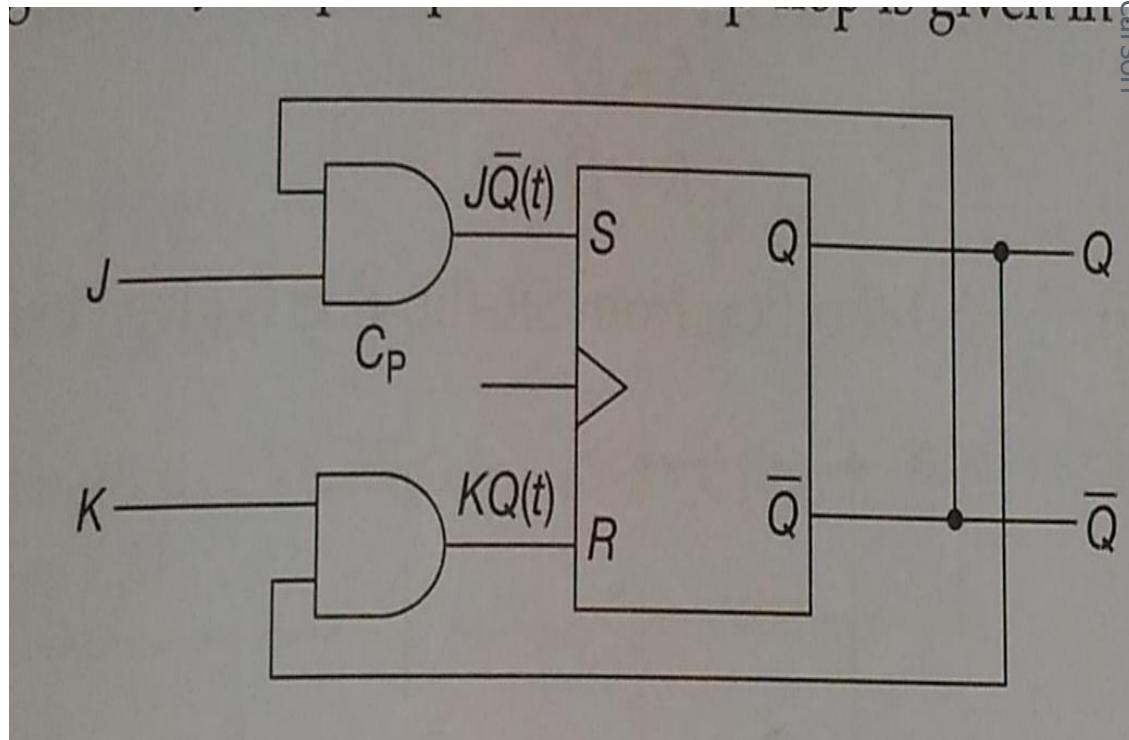


FIGURE 7.52 | K-map for R

$$S = J\bar{Q}(t)$$

$$R = KQ(t)$$

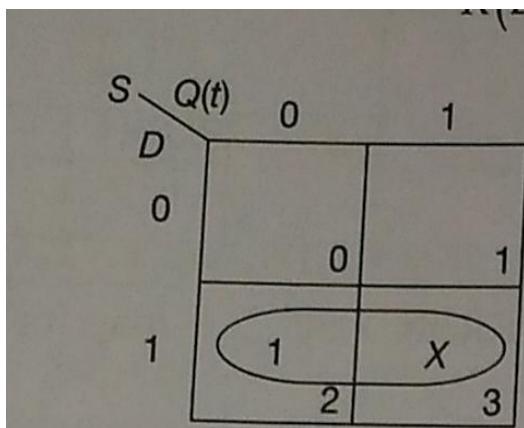
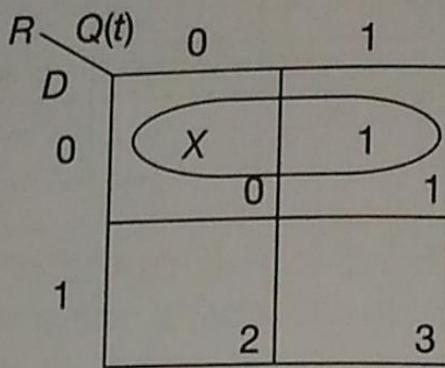


SR TO D

Minterm	Input of D-flip-flop		Output of SR-flip-flop		Input of SR-flip-flop	
	D	Q(t)	Q(t + 1)	S	R	
m_0	0	0	0	0	X	
m_1	0	1	0	0	1	
m_2	1	0	1	1	0	
m_3	1	1	1	X	0	

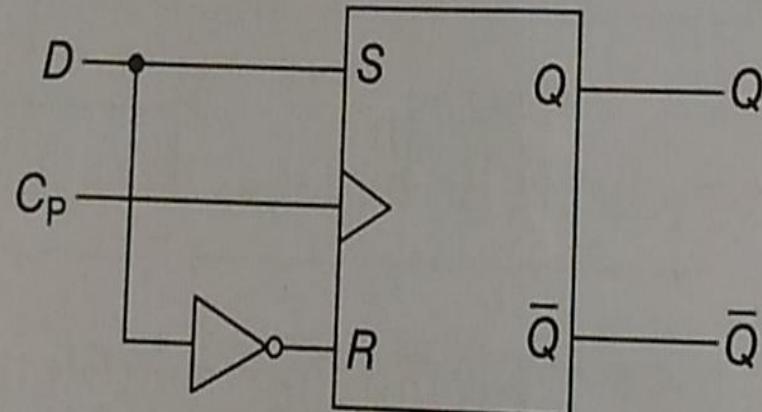
$$S(D, Q(t)) = \sum m(2) + \sum d(3)$$

$$R(D, Q(t)) = \sum m(1) + \sum d(0)$$

**FIGURE 7.54** | K-map for S **FIGURE 7.55** | K-map for R

$$S = D$$

$$R = \bar{D}$$

**FIGURE 7.56** | D flip-flop from SR flip-flop

JK TO T

Minterm	Conversion of J-K-flip-flop to T-flip-flop				Input of J-K-flip-flop
	T	Output of T-flip-flop		J	
		Q(t)	Q(t + 1)		
m_0	0	0	0	0	X
m_1	0	1	1	X	0
m_2	1	0	1	1	X
m_3	1	1	0	X	1

$$J(T, Q) = \sum m(2) + \sum d(1, 3)$$

$$K(T, Q) = \sum m(3) + \sum d(0, 2)$$

f $J(T, Q)$ and $K(T, Q)$

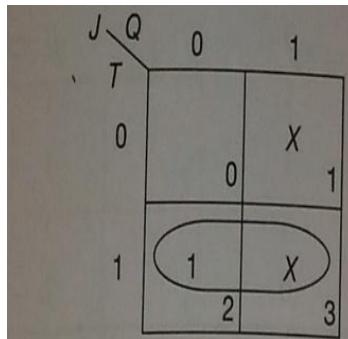


FIGURE 7.57 | K-map for J

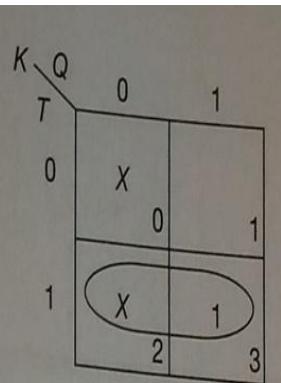
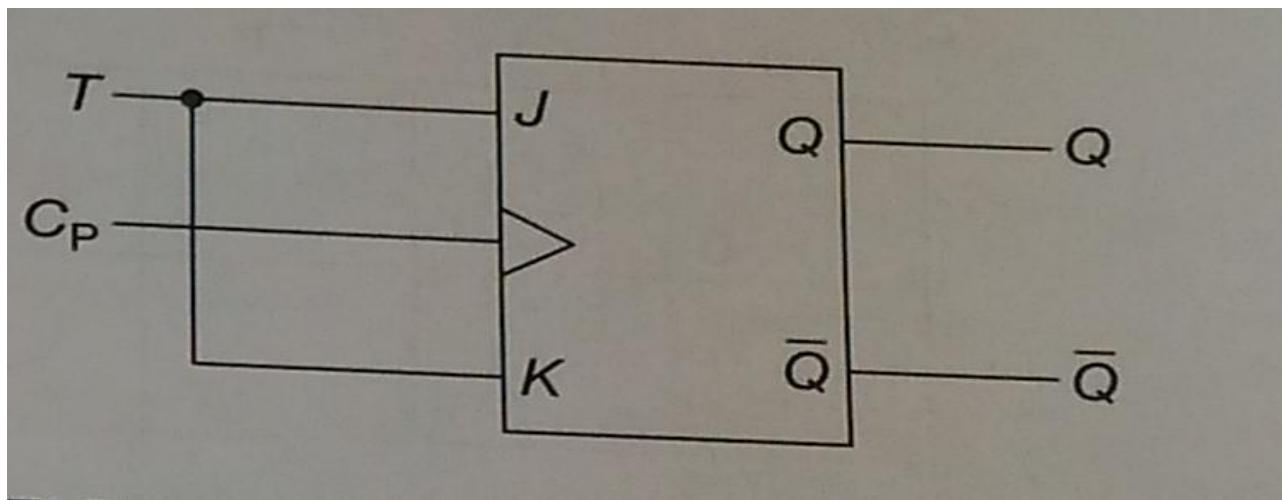


FIGURE 7.58 | K-map for K

$$J = T$$

$$K = T$$



D TO JK

Minterm	Input of J-K-flip-flop		Output of J-K-flip-flop		Input of D-flip-flop
	J	K	$Q(t)$	$Q(t + 1)$	
m_0	0	0	0	0	0
m_1	0	0	1	1	1
m_2	0	1	0	0	0
m_3	0	1	1	0	0
m_4	1	0	0	1	1
m_5	1	0	1	1	1
m_6	1	1	0	1	1
m_7	1	1	1	0	0

$$D(J, K, Q(t)) = \sum m(1, 4, 5, 6)$$

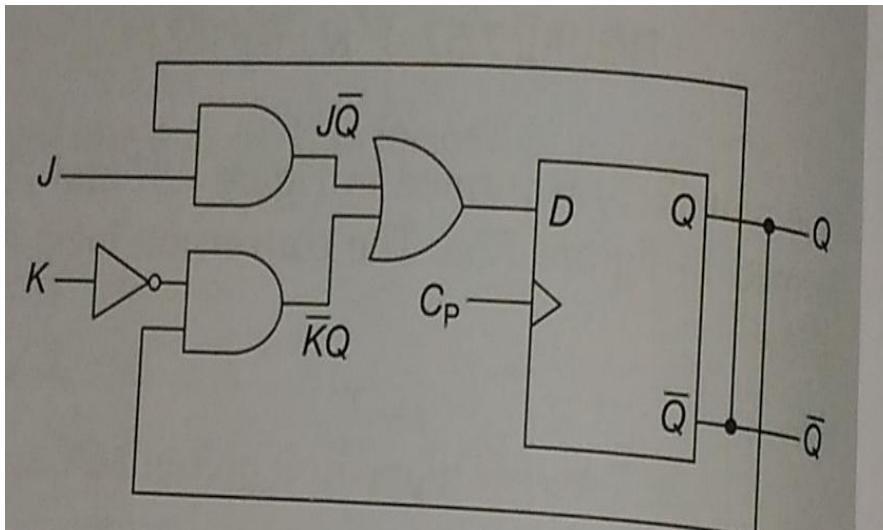
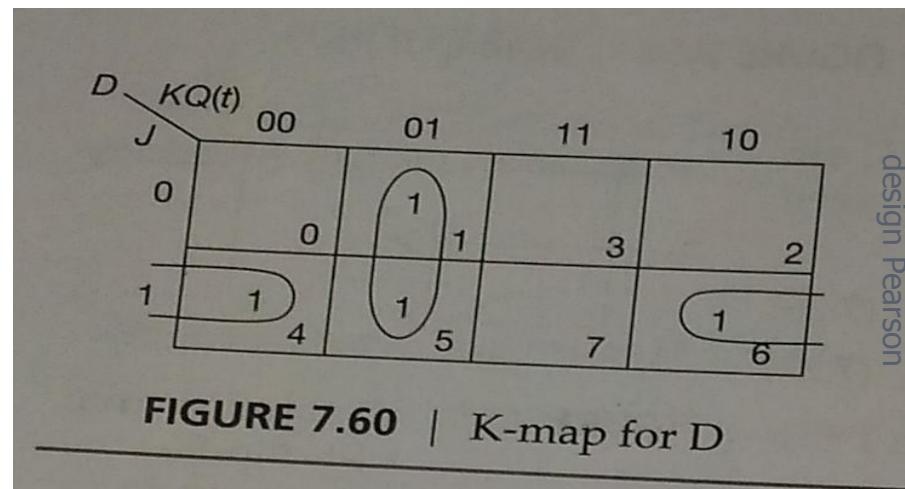


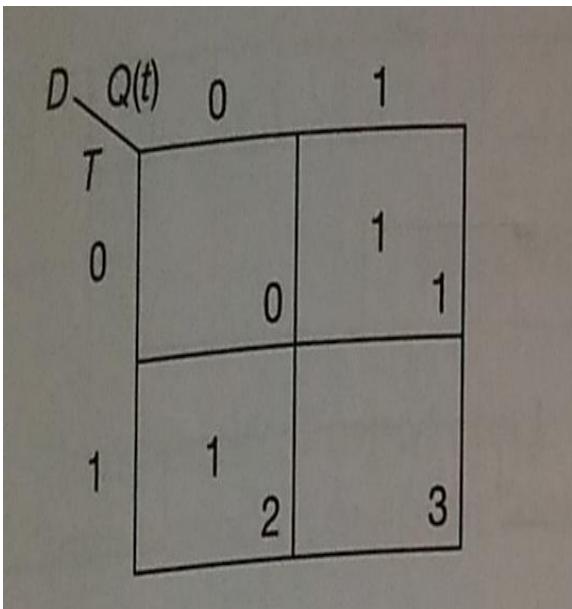
FIGURE 7.61 | J-K-flip-flop from D-flip-flop

$$D = \overline{JQ(t)} + \overline{K}Q(t)$$

T TO D

Minterm	Conversion of D-flip-flop to T-flip-flop				Input of T-flip-flop
	D	Output of D-flip-flop		T	
		Q(t)	Q(t + 1)		
m_0	0	0	0	0	0
m_1	0	1	0	1	1
m_2	1	0	1	1	0
m_3	1	1	1	0	

$$T(D, Q(t)) = \sum m(1, 2)$$



$$T = D\overline{Q(t)} + \bar{D}Q(t)$$

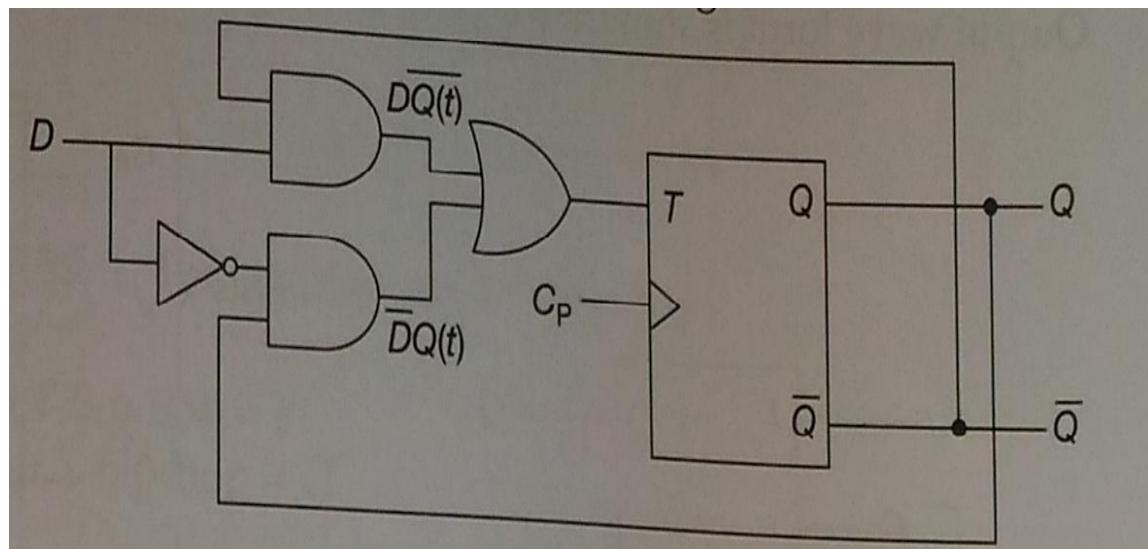
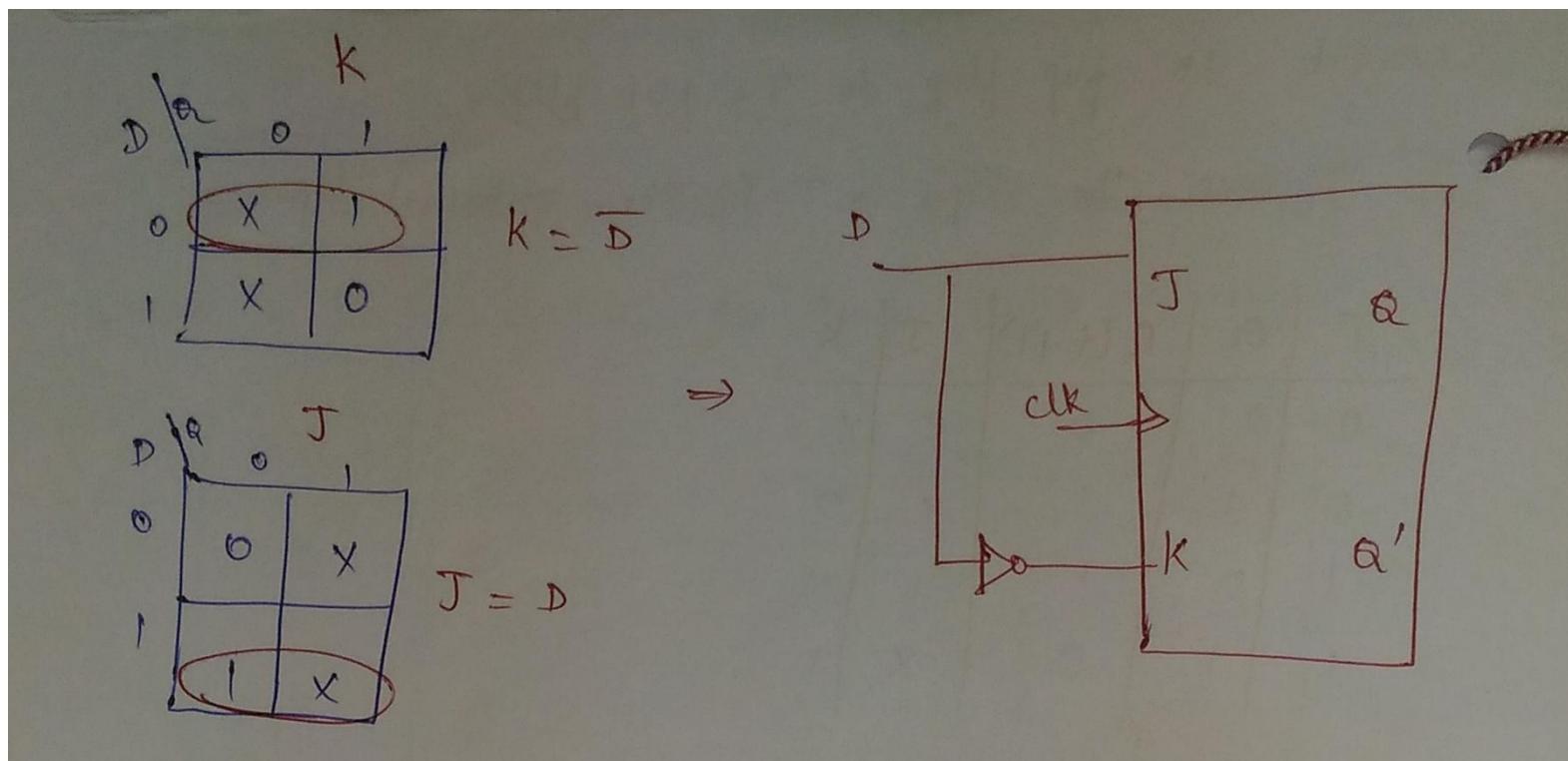


FIGURE 7.62 | T flip-flop

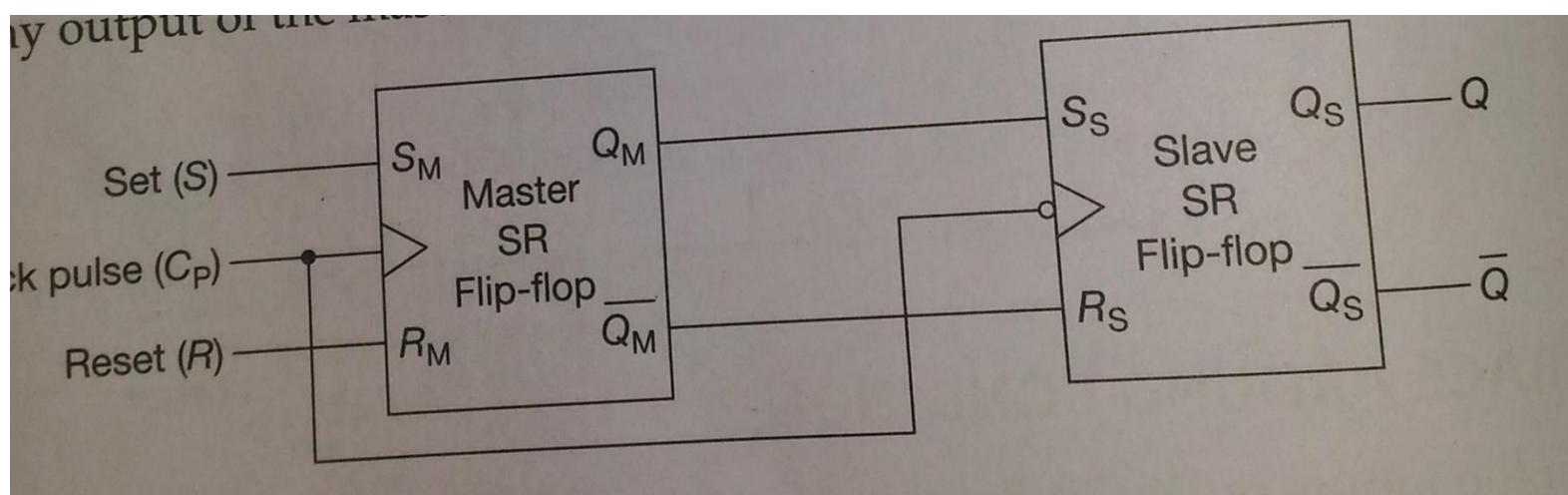
JK TO D

D	Q	(Q(t+1))	J	K
0	0	0	0	X
0	1	1	X	-
1	0	1	-	X
1	1	0	X	0

Excitation table

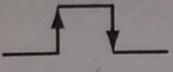
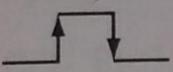
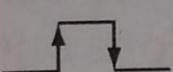
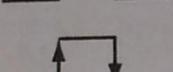


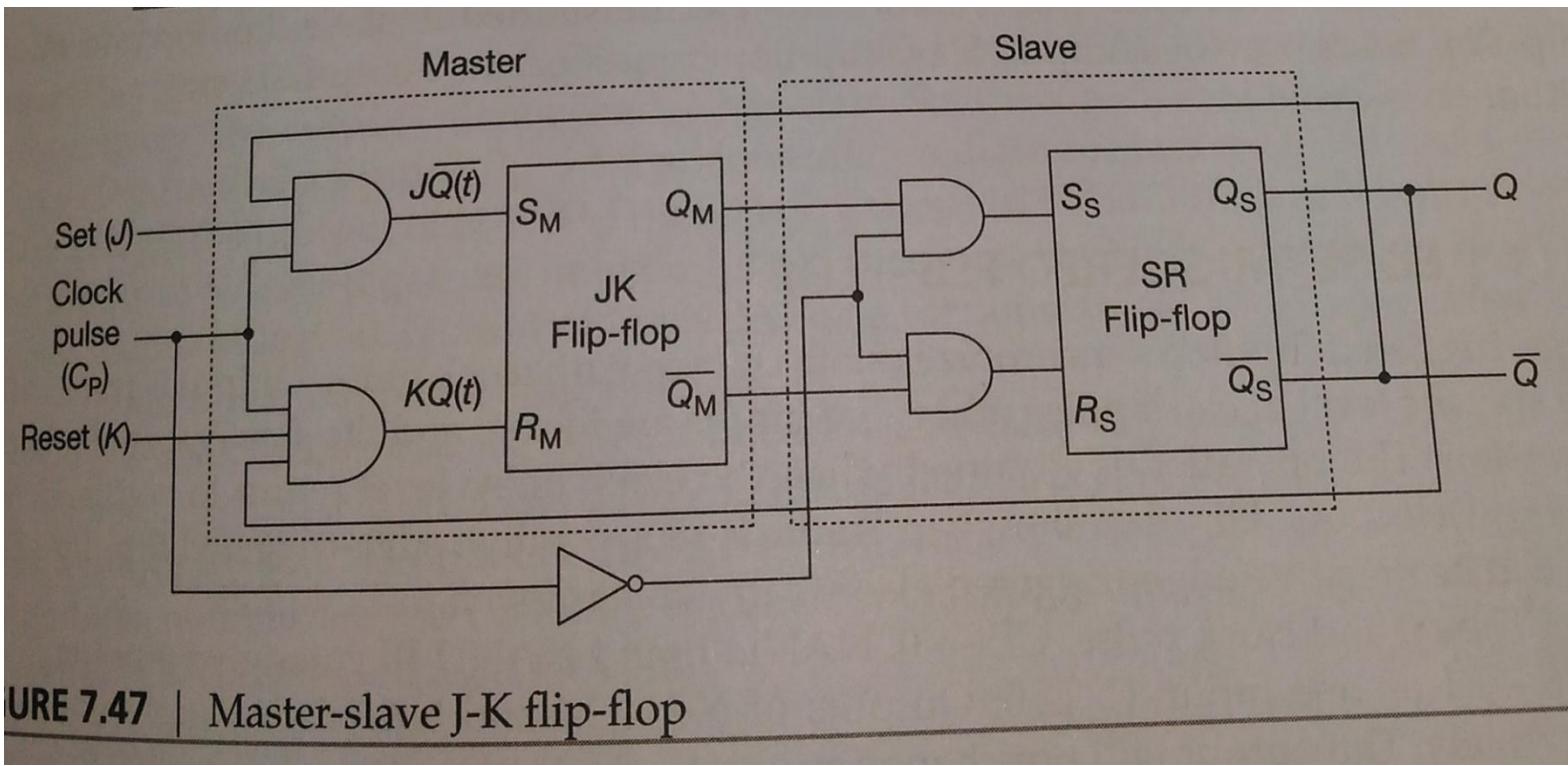
MASTER-SLAVE FLIP FLOP



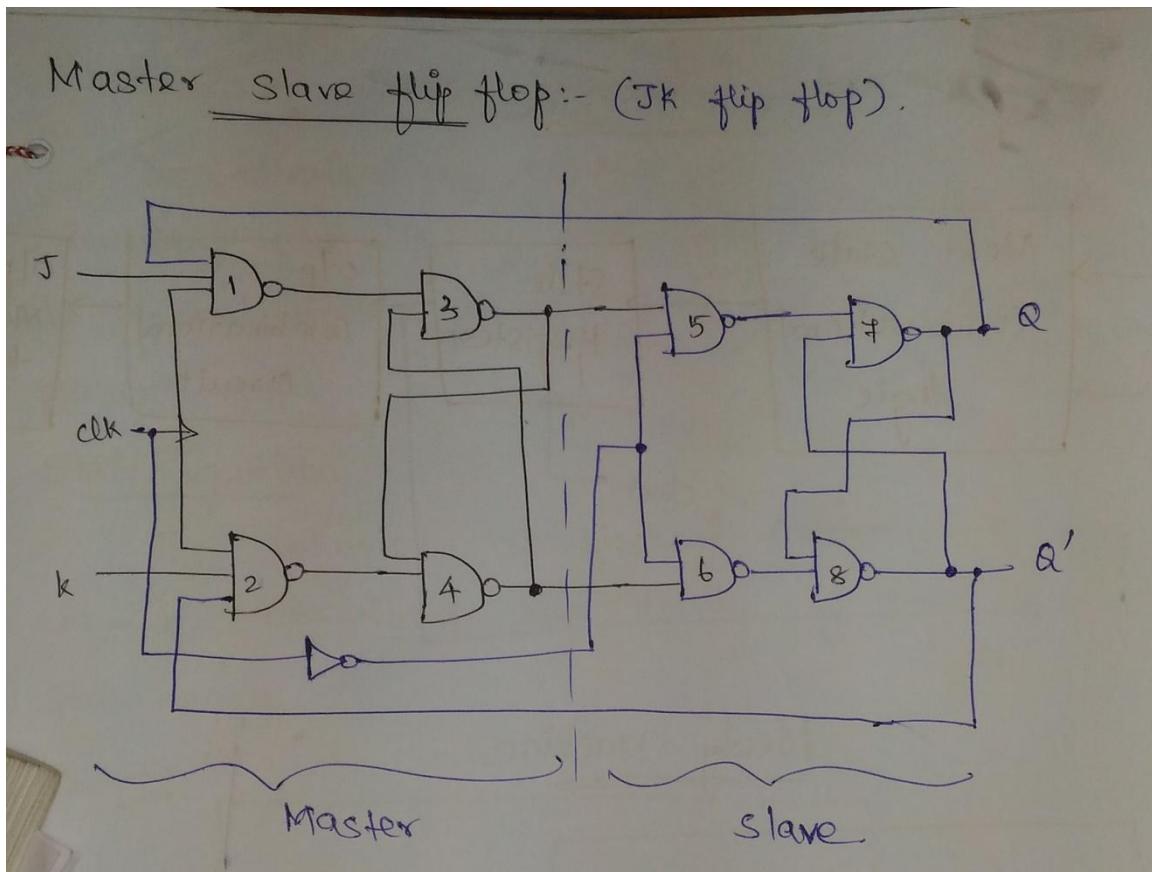
- Master flip flop performs the operation during positive level or edge triggering
- Slave flip flop performs the operation during negative level or edge triggering while sharing the same clock pulse.

TABLE 7.23 | Characteristics of master and slave SR-flip-flop

Inputs			Outputs	Remarks
S (Set)	R (Reset)	CP	$Q(t + 1)$	
0	0		$Q(t)$	Previous state
0	1		0	Set
1	0		1	Clear (Reset)
1	1		?	Indeterminate state



URE 7.47 | Master-slave J-K flip-flop



MEALY AND MOORE MODELS (1)

- The most general model of a sequential circuit has inputs, outputs, and internal states. It is customary to distinguish between two models of sequential circuits:
 - the Mealy model and the Moore model
- They differ in the way the output is generated.
 - In the Mealy model, the output is a function of both the present state and input.
 - In the Moore model, the output is a function of the present state only.

MEALY AND MOORE MODELS (2)

When dealing with the two models, some books and other technical sources refer to a sequential circuit as a **finite state machine** abbreviated **FSM**.

- The Mealy model of a sequential circuit is referred to as a Mealy FSM or Mealy machine.
- The Moore model is referred to as a Moore FSM or Moore machine.

MOORE/MELAY MODELS

Moore / Melay Models:-

There are 2 models developed for representing Synchronous Sequential circuits based on the way of o/p is generated.

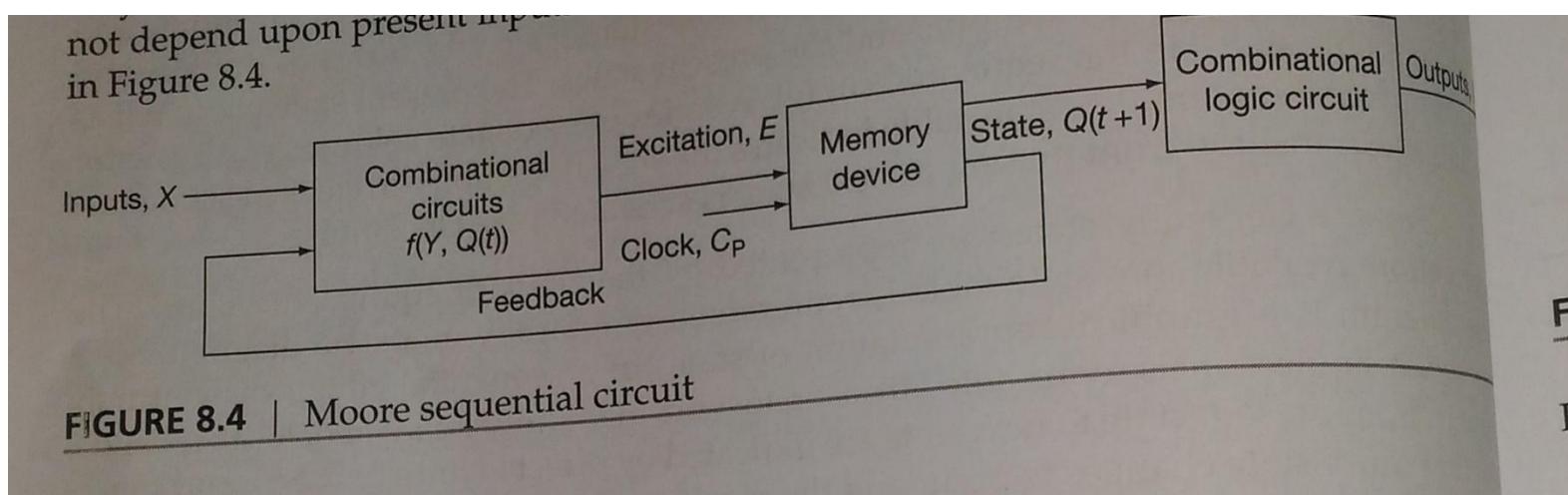
1). Moore ckt / Moore State machine

* o/p is a func of present state only.

2). Melay ckt / Melay State Machine.

* o/p is a func of present state as well as i/p.

MOORE MODEL



Excitation is given by

$$E = f(Q(t), X)$$

Next state is determined by

$$Q(t+1) = f(E, Q(t))$$

Output is given by

$$Y = f(Q(t))$$

where X is the input variables

E is excitation for the memory device

$Q(t)$ is the present output state (at time t) of memory device.

Y is the output variables

MELAY MODEL

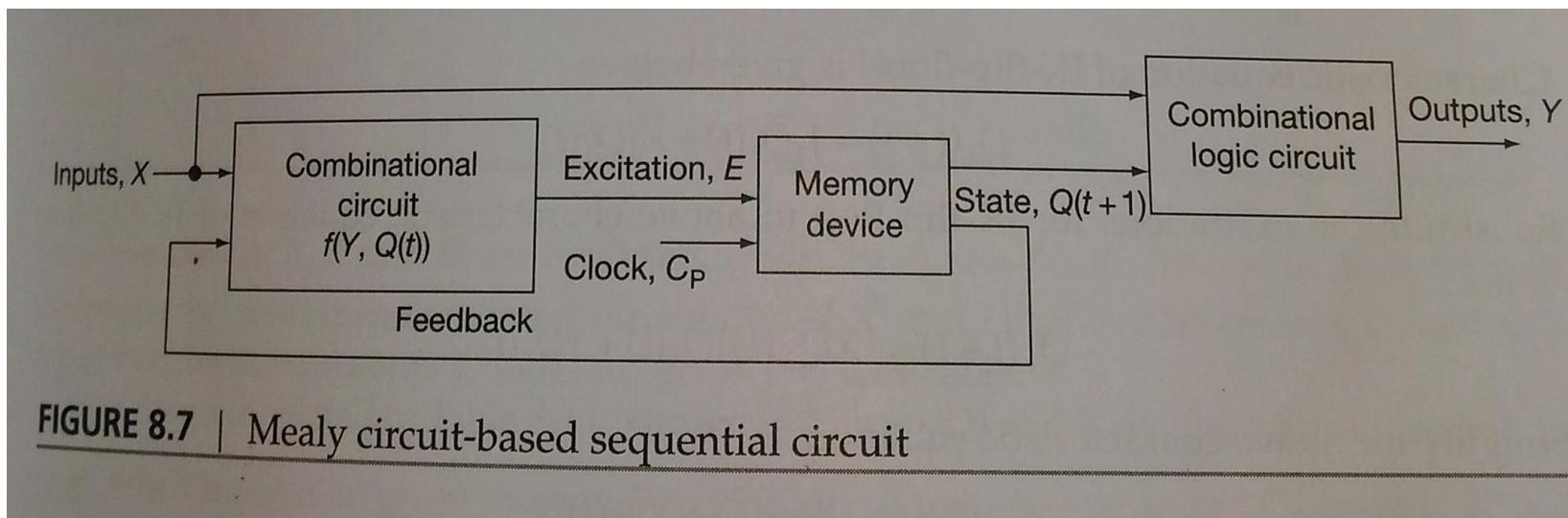


FIGURE 8.7 | Mealy circuit-based sequential circuit

Excitation is given by

$$E = f(Q(t), X)$$

Next state is determined as

$$Q(t+1) = f(E, Q(t))$$

Output is given by

$$Y = f(X, Q(t))$$

where X is the input variables

E is excitation for the memory device

$Q(t)$ is the present output state (at time t) of memory device.

Y is the output variables

Analysis of clocked sequential circuits



ANALYSIS OF CLOCKED SEQUENTIAL CIRCUITS

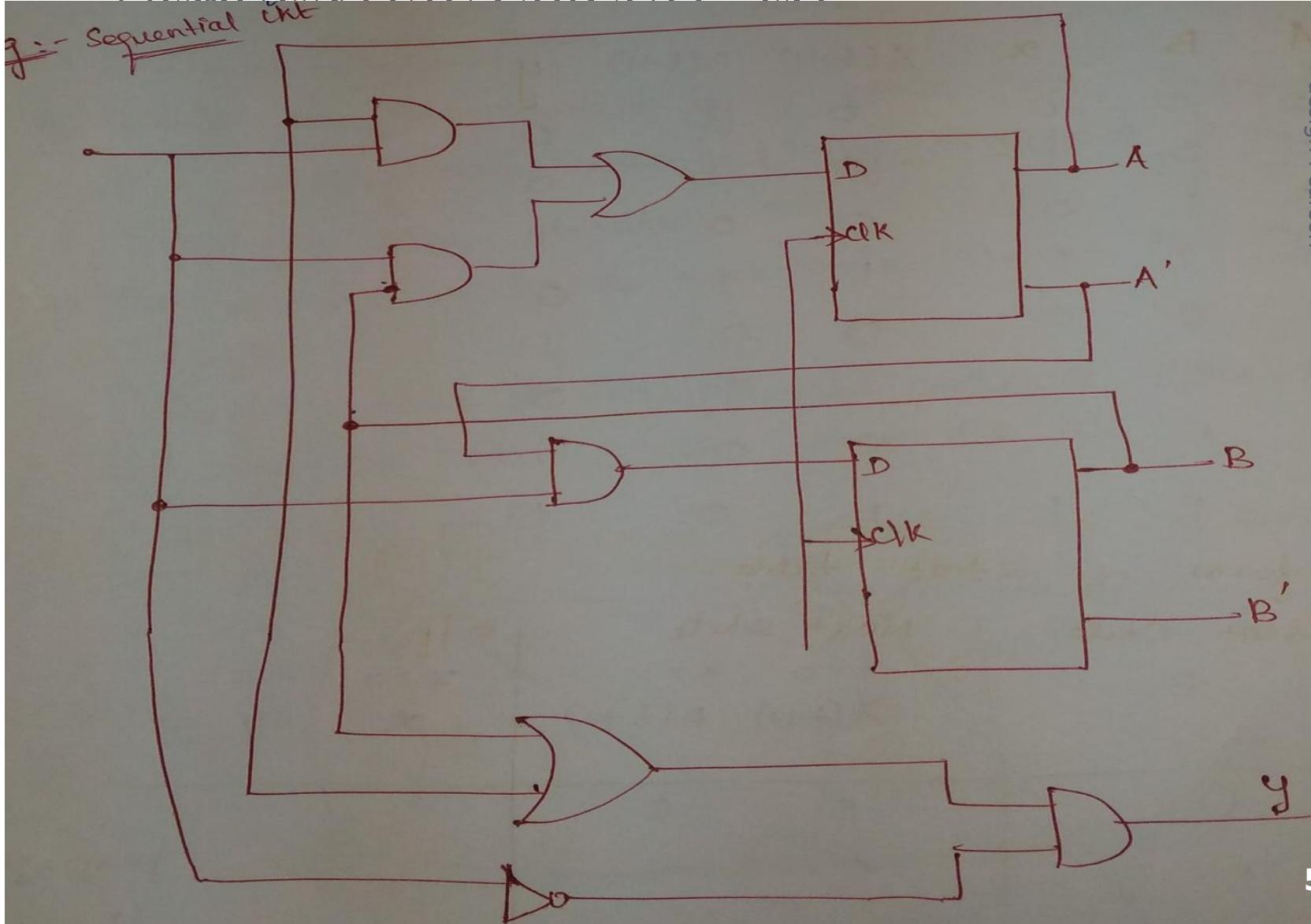
Analysis of clocked sequential circuits:

The behaviour of clocked sequential circuit is determined from ilp, olp and state of its flip flop the olp and next state are functions of ilp and present state.

State equation:-

The behaviour of clocked sequential circuit can be described algebraically by mean of State eqn.

SEQUENTIAL CIRCUIT - EG



$$A(t+1) = A(t) \cdot x(t) + B(t) \cdot x(t) = Ax + Bx$$

$$B(t+1) = A'(t) \cdot x(t) = A'x$$

$$y(t+1) = [A(t) + B(t)] \cdot x'(t)$$

$$= (A + B)x'$$

$$= Ax' + Bx'$$

State Table:-						
Present state			Input	Next State	Output	
A	B	x		A(t+1)	B(t+1)	y
0	0	0		0	0	0
0	0	1		0	1	0
0	1	0		0	0	1
0	1	1		1	1	0
1	0	0		0	0	1
1	0	1		1	0	0
1	1	0		0	0	1
1	1	1		1	0	0

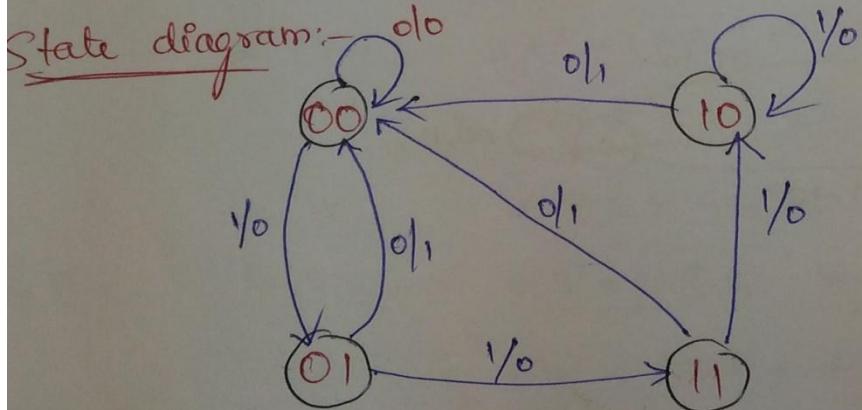
2nd

2nd form of state table :-

Present State		Next State				I.P.		x = 0 x = 1	
A	B	x = 0	x = 0	x = 0	x = 1	A(t+1)	B(t+1)	A(t+1)	B(t+1)
0	0	0	0	0	1	0	0	0	0
0	1	0	0	1	1	1	1	0	0
1	0	0	0	1	0	1	1	0	0
1	1	0	0	1	0	1	1	0	0

State Diagram Rules

- 1). State is represented by a circle and transitions between states are indicated by directed lines connecting the circles.
- 2) The binary no. inside each circle shows the state of flip flop.
- 3) The ip value during present state is labelled 1st and the no. after slash gives the op.



ANALYSIS WITH D FLIP FLOP

Analysis with D-flip flops:-

Ckt diagram:-

The circuit diagram illustrates a digital logic system. On the left, two input lines, labeled x and y , enter a two-input OR gate. The output of this OR gate is connected to one input of a second OR gate. The other input of this second OR gate is also connected to the D input of a D flip-flop. The clk (clock) input of the D flip-flop is connected to a square pulse waveform labeled "clock". The output of the D flip-flop is labeled A . A feedback line connects the output A back to the D input of the D flip-flop.

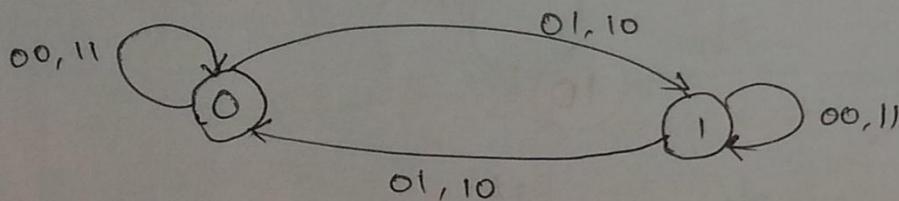
(i) State equation:-

$$D_A = A \oplus x \oplus y$$
$$A(t+1) = A \oplus x \oplus y.$$

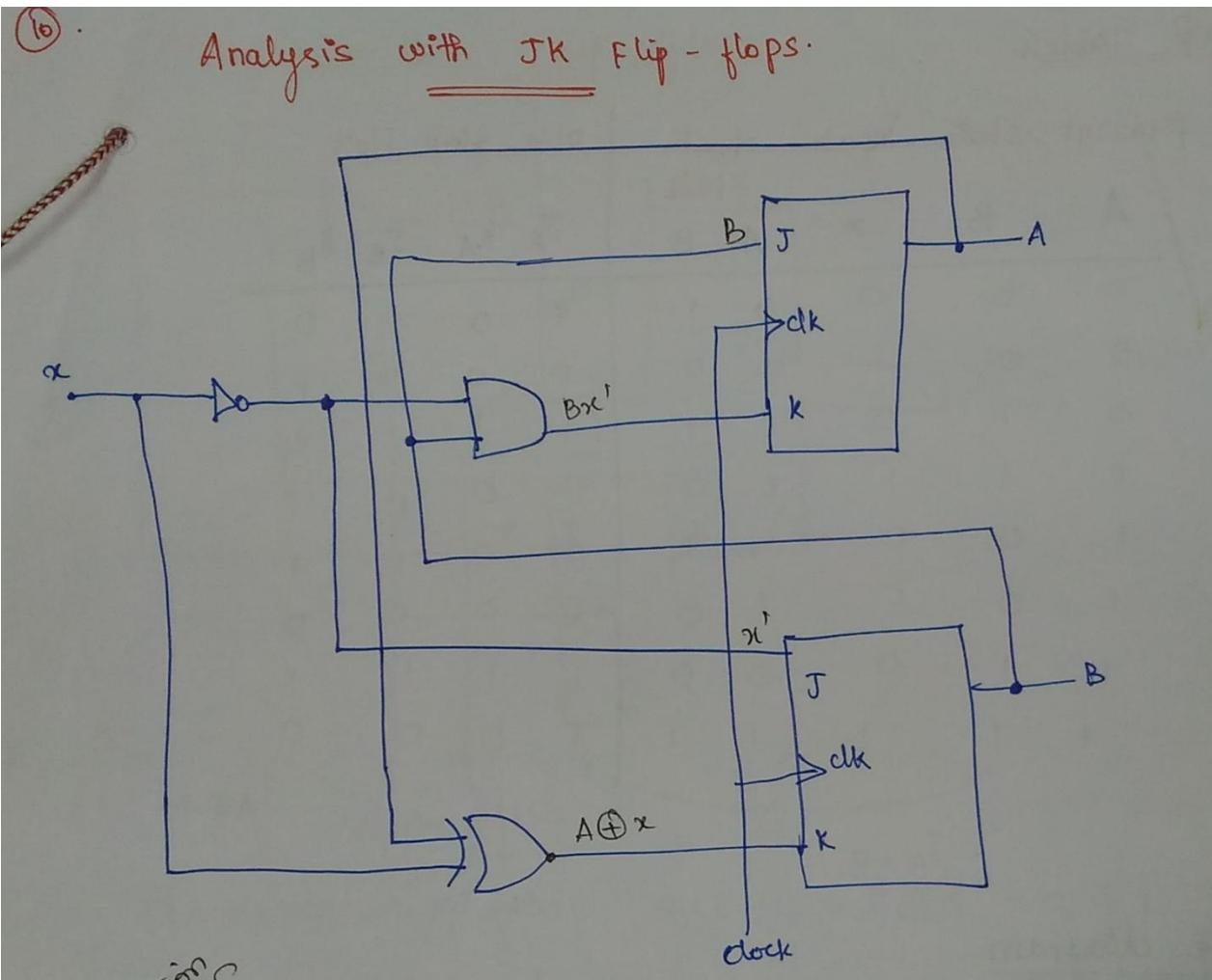
state table :-

Present State A	Inputs x y		Next state A
	0	1	
0	0	0	0
0	0	1	1
0	1	0	1
1	0	0	0
1	1	0	1
1	1	1	1

State diagram:-



ANALYSIS WITH JK FLIP FLOP



clock

$$\left. \begin{array}{l} \text{if equation} \\ J_A = B \\ J_B = x' \end{array} \right\} \quad \begin{array}{l} K_A = Bx' \\ K_B = A'x + Ax' = A \oplus x \end{array}$$

State equation:-

$$A(t+1) = JA' + K'A$$

$$B(t+1) = JB' + K'B$$

$$A(t+1) = BA' + (Bx')'A = A'B + \cancel{A'B'}(B' + x)A$$

$$= \underline{BA'} + \underline{B'A} + xA.$$

$$= A \oplus B + Ax$$

$$B(t+1) = x'B' + (A \oplus x)'B$$

$$= B'x' + ABx + A'Bx'.$$

State Table:-

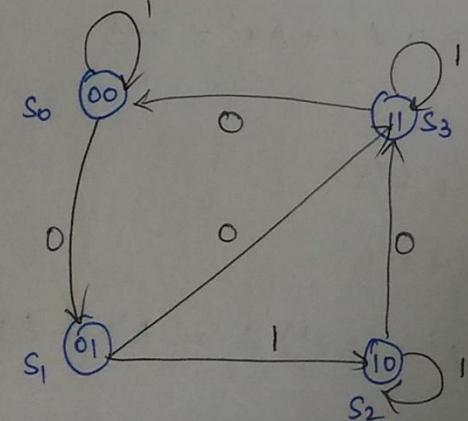
Present State		Input	Next State		flip-flop ilp's			
A	B	x	A	B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

$\therefore J_A = B$

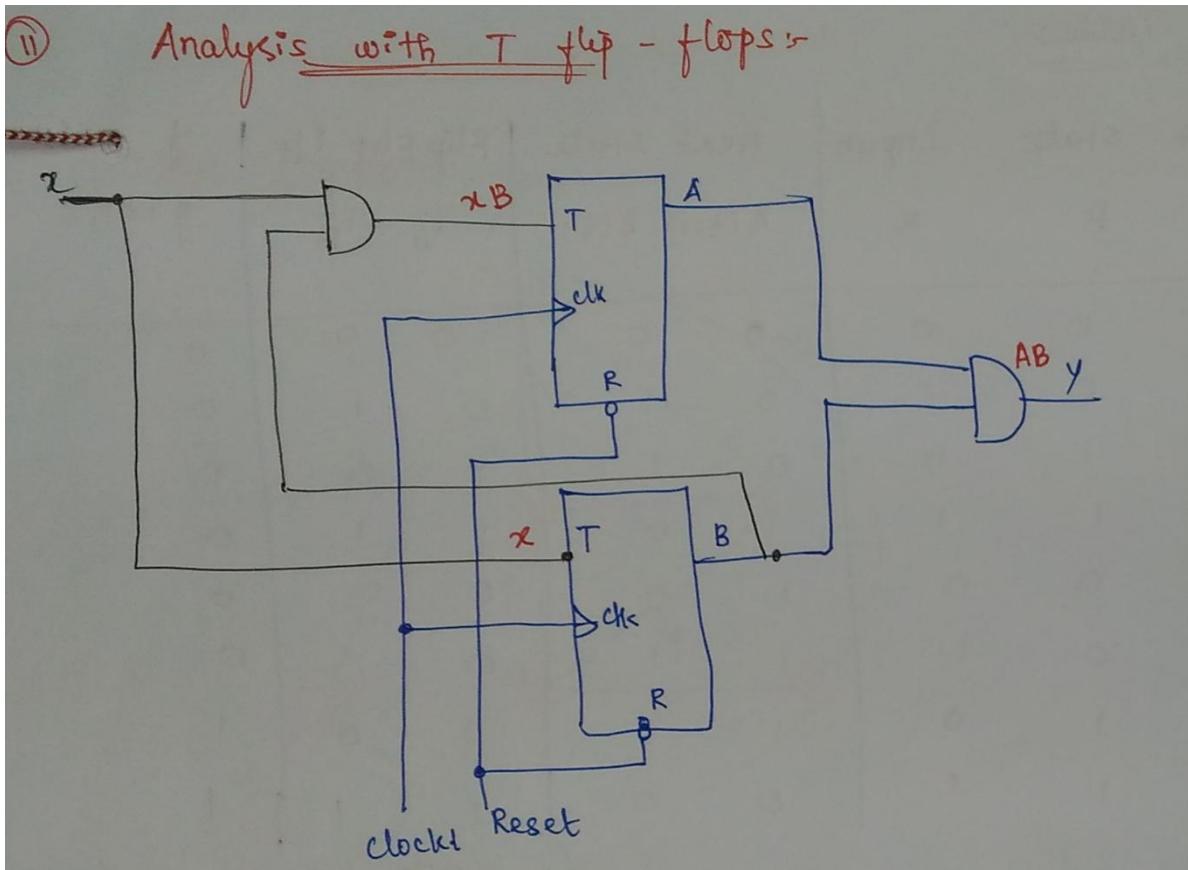
$K_A = Bx'$

$x' \rightarrow A \oplus x$

State diagram:-



ANALYSIS WITH T FLIP FLOP



Ques
characteristic eqn :- $Q(t+1) = T \oplus Q = T'Q + TQ'$.

State equation :- $T_A = xB$

$$T_B = x$$

$$y = AB.$$

Since, $Q(t+1) = T'Q + TQ'$

$$A(t+1) = T_A' A + T_A A'$$

$$= (xB)' \cdot A + A' (xB)$$

$$= A \oplus xB.$$

$$B(t+1) = T_B' B + T_B B'$$

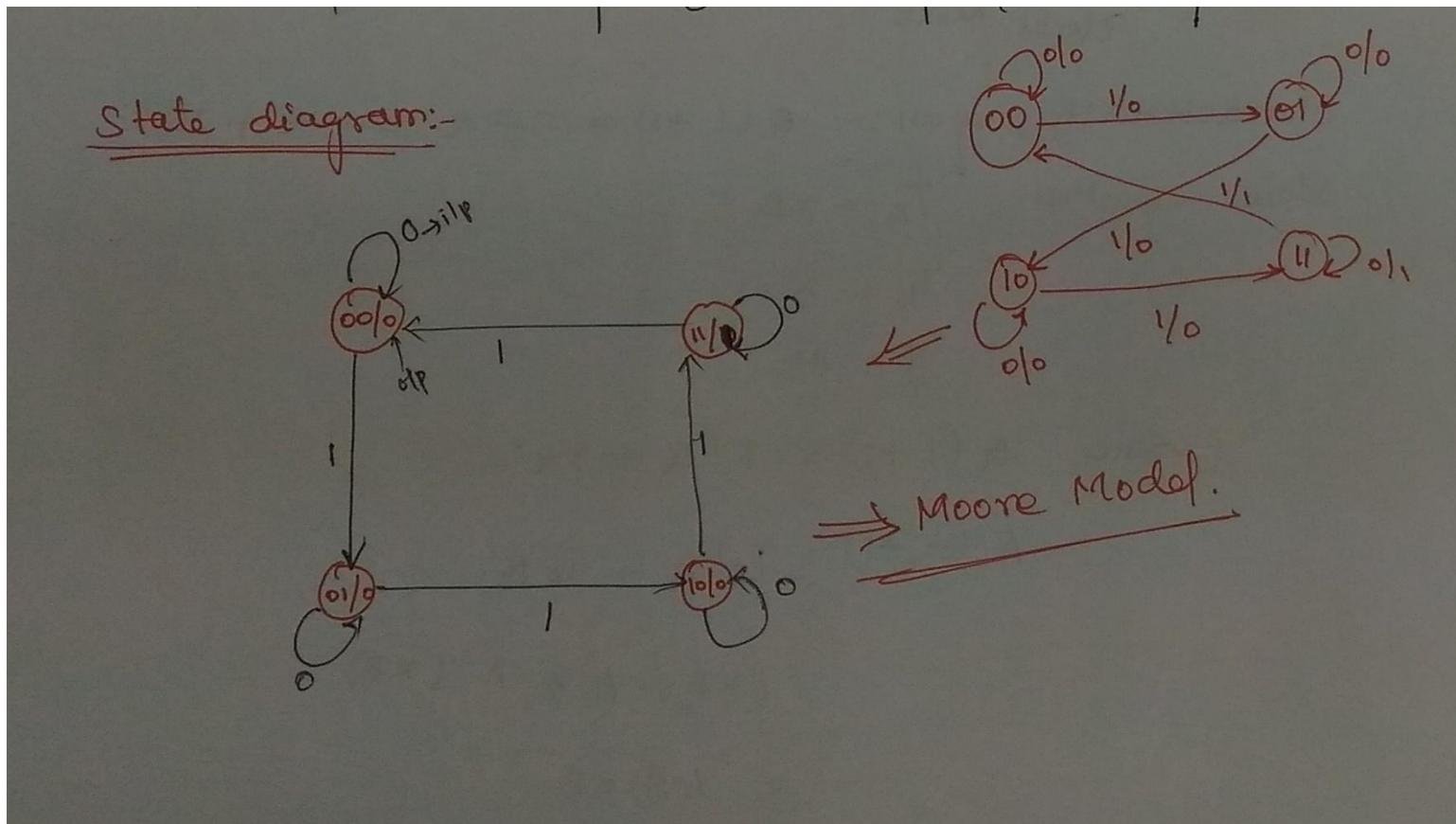
$$= x'B + x \cdot B'$$

$$= x \oplus B.$$

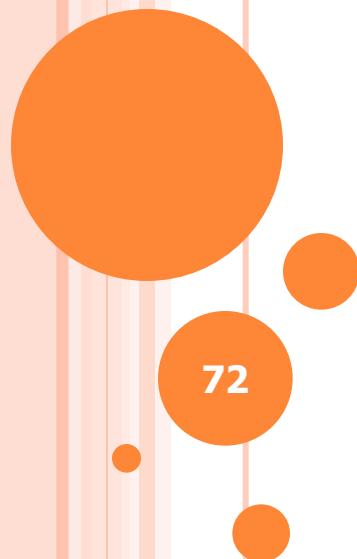
$$y = AB.$$

State Table:-

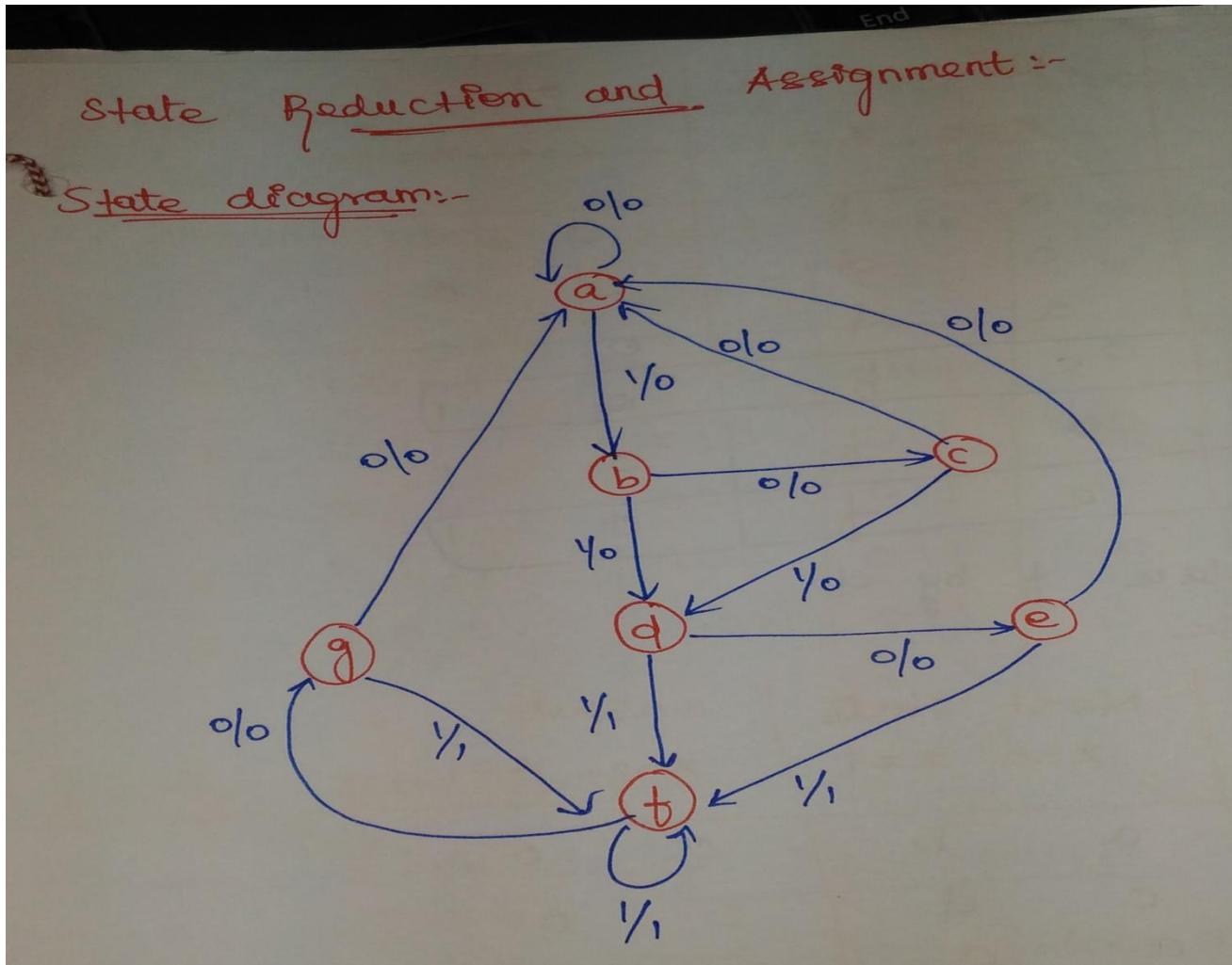
Present State			Input x	Next State		Flip Flop IP		y $y = AB$
A	B	x		$A(t+1)$	$B(t+1)$	T_A	T_B	
0	0	0		0	0	0	0	0
0	0	1		0	1	0	-1	0
0	1	0		0	1	0	0	0
0	1	1		1	0	-1	-1	0
1	0	0		1	0	0	0	0
1	0	1		1	1	0	0	-1
1	1	0		1	1	0	0	1
1	1	1		0	0	1	1	1



State Reduction and Assignment



STATE REDUCTION AND ASSIGNMENT



1/1

State Table :-

Present state	Next state		output $x=0 \quad x=1$
	$x=0$	$x=1$	
a	b		0 0 0 -
b	c	d	0 0 0 0
c	a	d	0 0 0 0
d	e	t	0 0 0 0
e	f	t	0 0 0 0
f	t	t	0 0 0 0
g	t		0 0 0 0
<u>a</u>		<u>t</u>	<u>1</u>
<u>a</u>		<u>t</u>	<u>1</u>
<u>a</u>		<u>t</u>	<u>1</u>

* Reduce the no. of flip flop.

* Replace g by e

Reducing the state table
b) present state

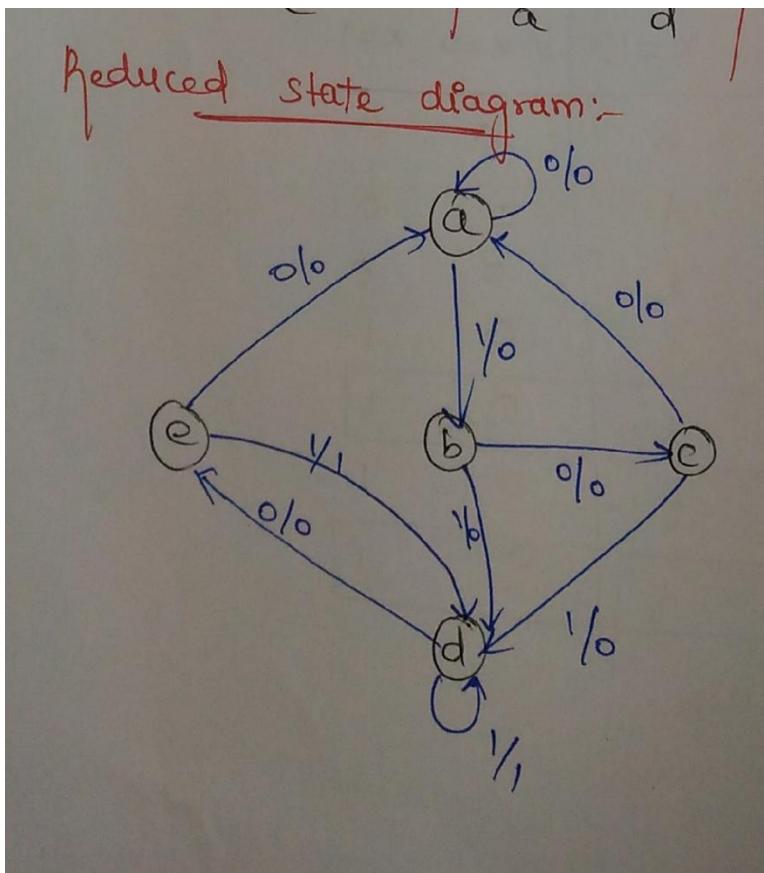
	Next state		off	
	$x=0$	$x=1$	$x=0$	$x=1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	e	f	0	1

c). Replace f by d.

c). Replace f by a.

Reduced state table

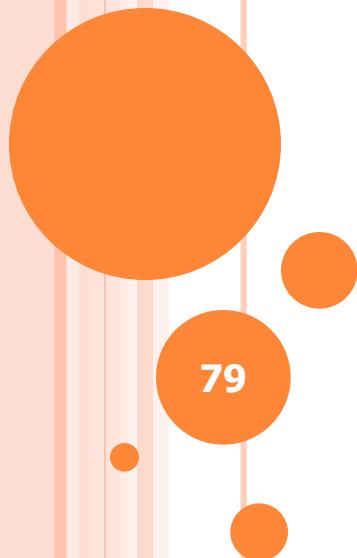
Present state	Next state		Output	
	$x=0$	$x=1$	$x=0$	$x=1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	d	0	1
e	a	d	0	1



Reduced state table with binary assignment :-

Present state	Next state		Output	
	$x=0$	$x=1$	$x=0$	$x=1$
$a \rightarrow 000$	000	001	0	0
$b \rightarrow 001$	010	011	0	0
$c \rightarrow 010$	000	011	0	0
$d \rightarrow 011$	100	011	0	1
$e \rightarrow 100$	000	011	0	1

IMPLICATION CHART



EXAMPLE 8.8 Reduce the states given in the state Table 8.21 using implication chart.

TABLE 8.21 | State table

Present State	Next State		Output, Z	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
A	A	B	0	0
B	C	D	0	0
C	E	F	0	0
D	B	A	0	1
E	C	D	0	0
F	B	A	0	1

SET UP IMPLICATION CHART

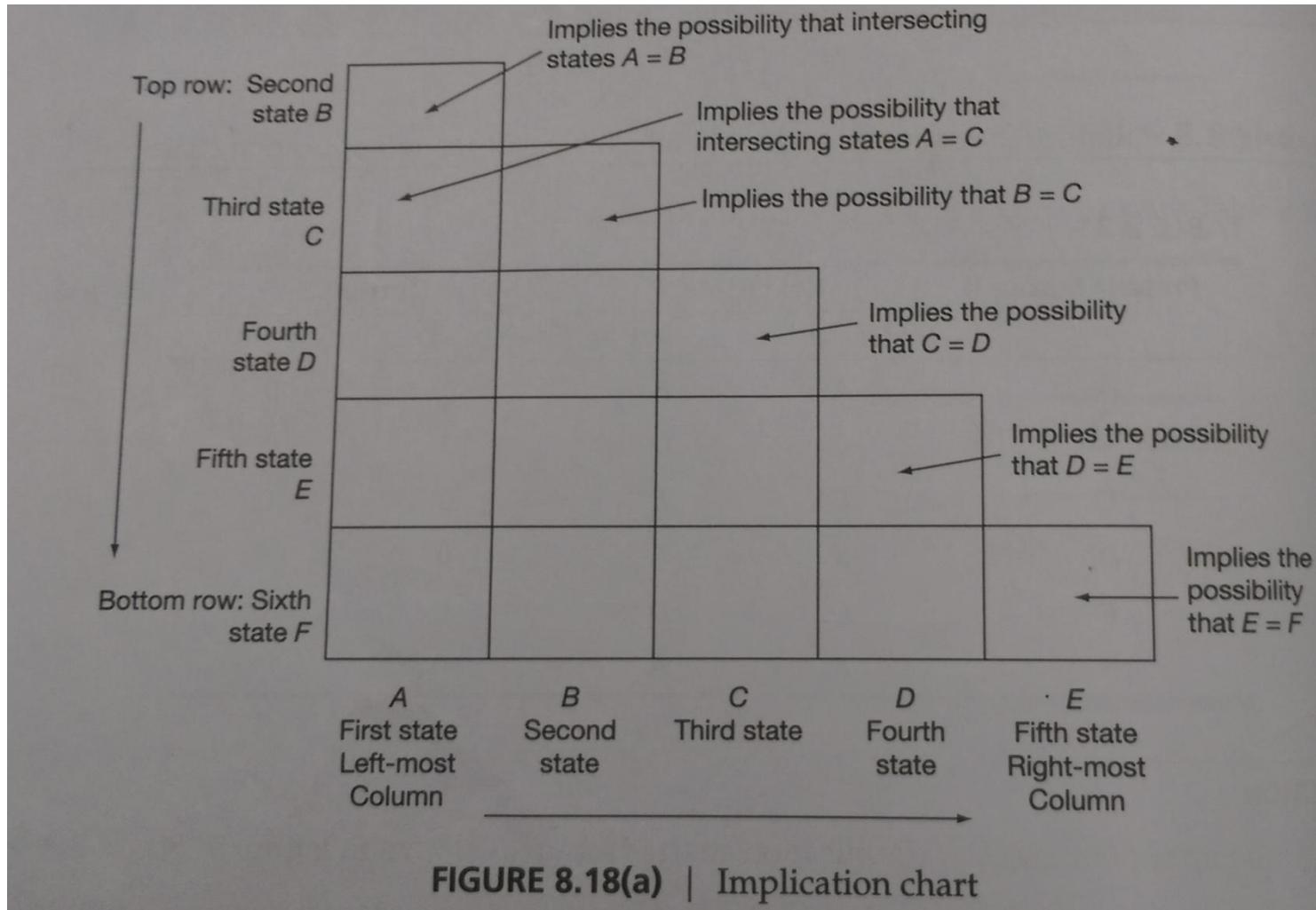
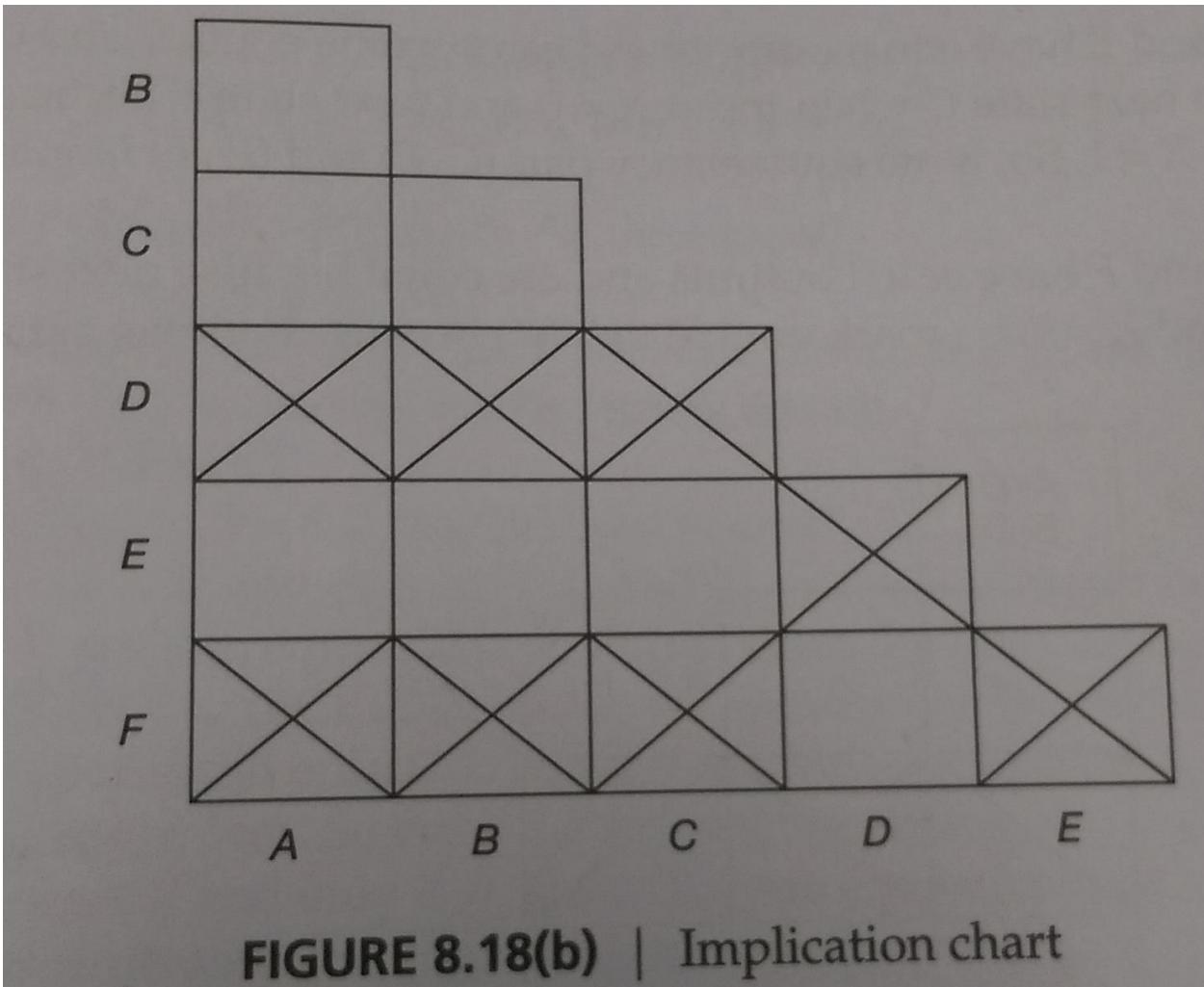
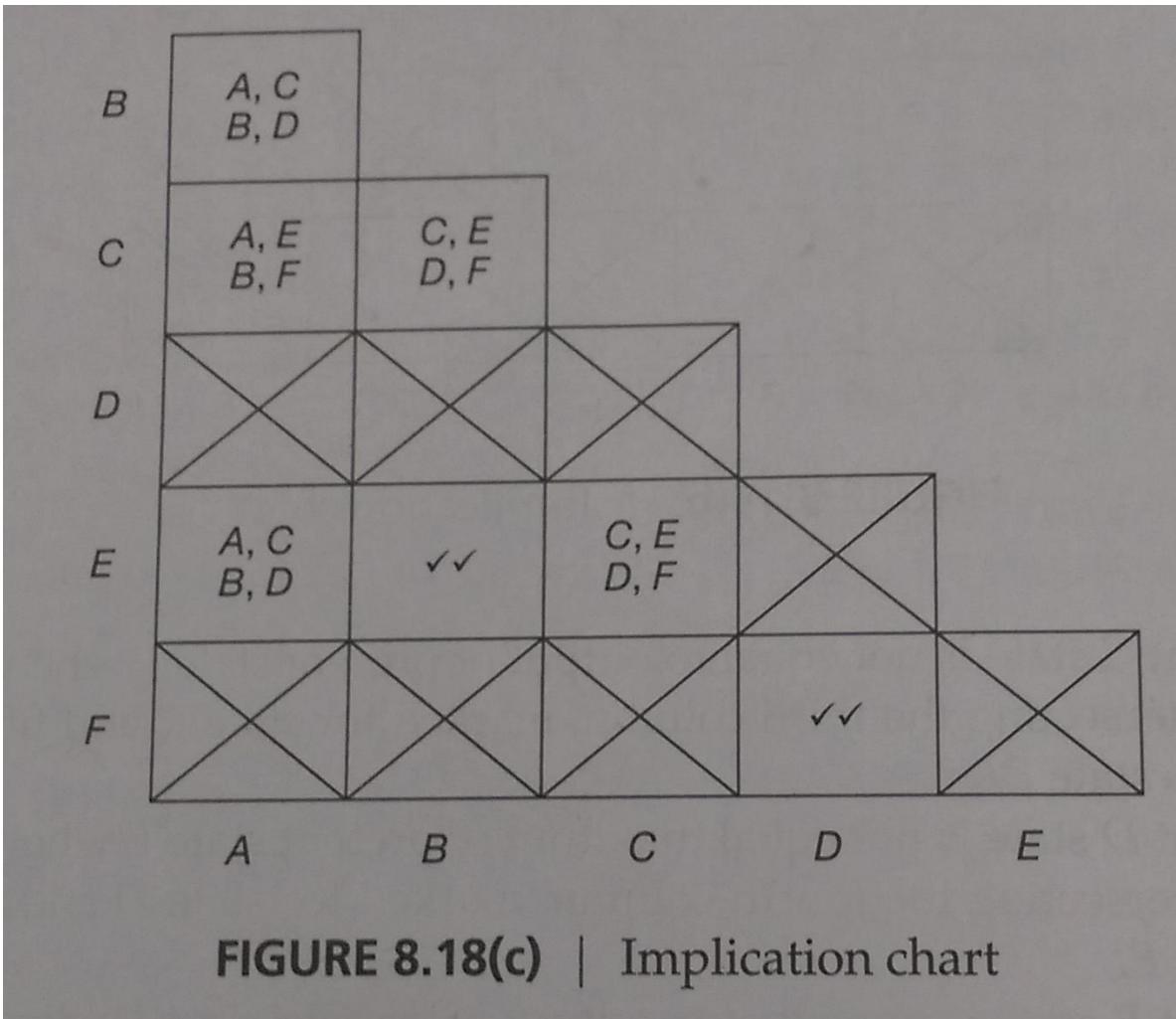


FIGURE 8.18(a) | Implication chart

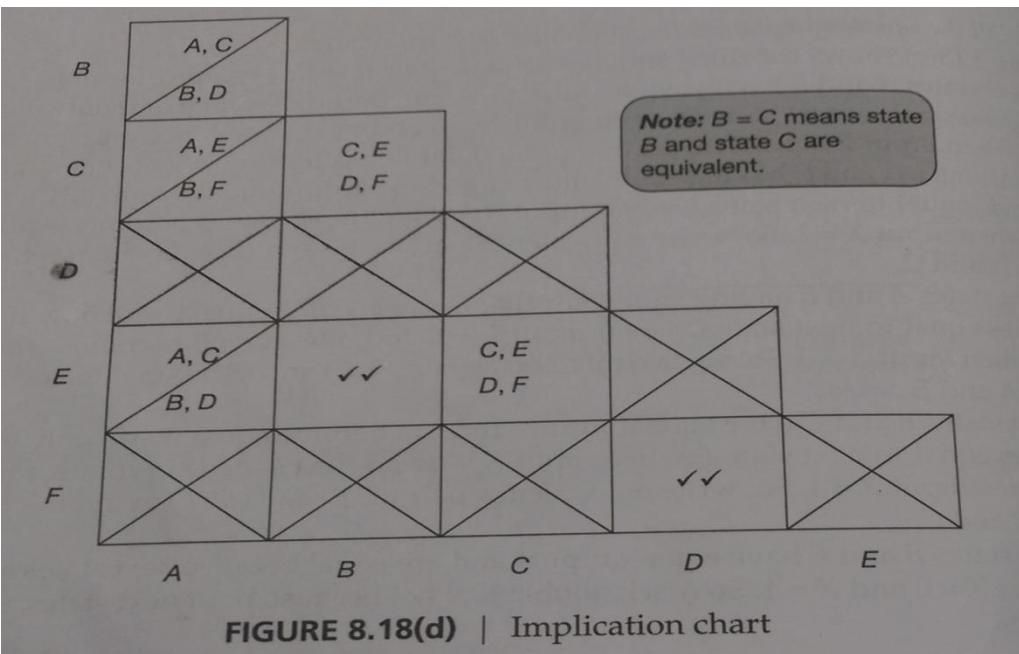
FILL THE SQUARE FOR STATE PAIRS WHICH HAVE DIFFERENT OUTPUTS



FILL THE SQUARES FOR STATE PAIRS WHICH MAY HAVE SAME OUTPUTS



APPLY EQUIVALENCY TEST OF EACH SQUARE



$$B=C=E$$

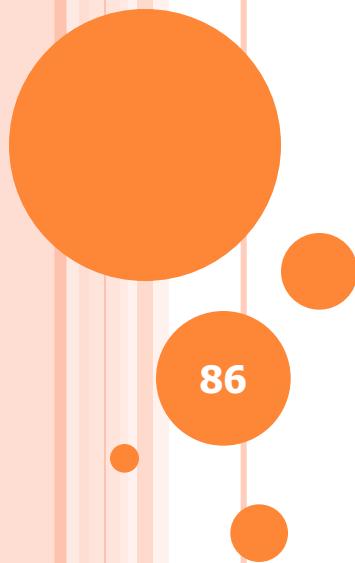
$$C=E$$

$$D=F$$

TABLE 8.22 | Reduced state table

Present State	Next State		Output, Z	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
A	A	B	0	0
B	B	D	0	0
D	B	A	0	1

Design procedure

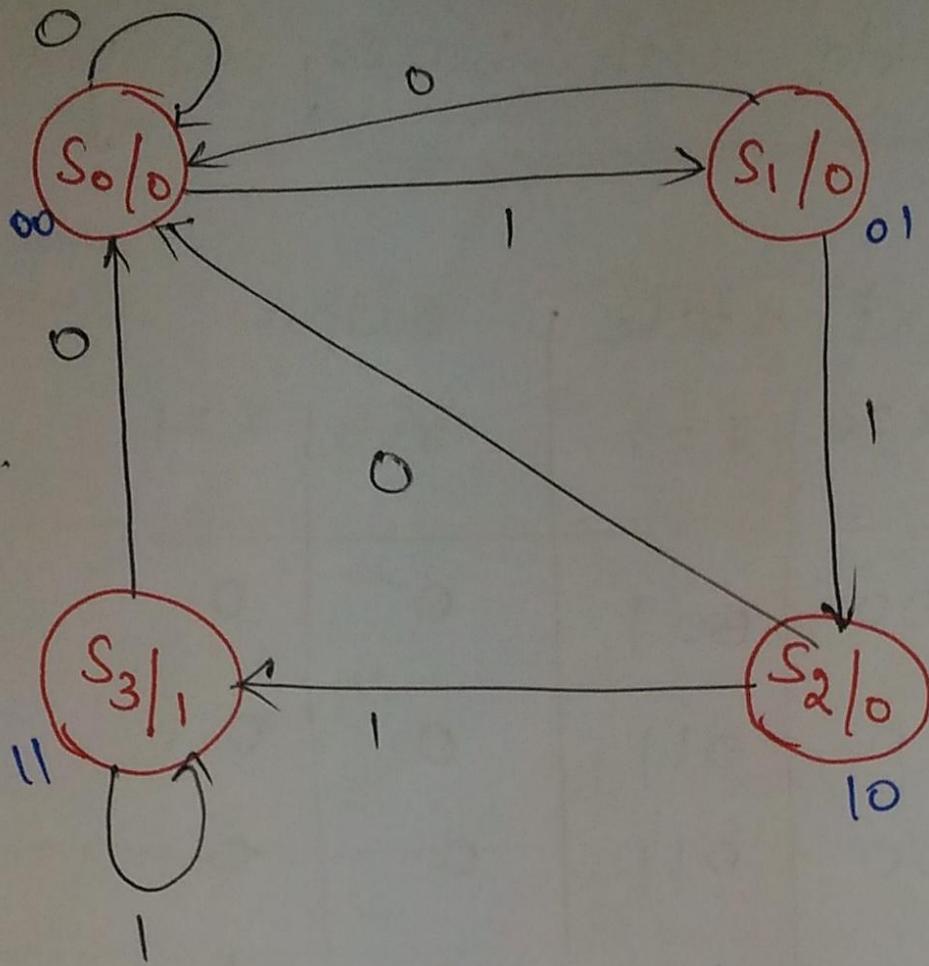


PROCEDURE TO DESIGN SYNCHRONOUS SEQUENTIAL CIRCUIT

1. From the word description and specifications of the desired operation, derive a state diagram for the circuit.
2. Reduce the number of states if necessary
3. Assign binary values to the state
4. Obtain the binary coded stable table
5. Choose the type of flip flop to be used
6. Derive the simplified flip flop input and output equations
7. Draw the logic diagram

①.

State
diagram.



State table:-

Present state		x	Next state		Old y
A	B		A (t+1)	B (t+1)	
0	0	0	0	0	0 0 0 0 0 0
0	0	0	1	0	1
0	0	1	0	0	0
0	0	1	0	0	0
1	0	0	0	0	0
1	0	0	1	1	1
1	1	1	0	0	0
1	1	1	1	1	1

$$A(t+1) = D_A(A, B, x) = \Sigma(3, 5, 7)$$

$$B(t+1) = D_B(A, B, x) = \Sigma(1, 5, 7)$$

$$y(A, B, x) = \Sigma(6, 7).$$

	Bx	00	01	11	10
A	0	-	-	-	-
	1	-	-	-	-

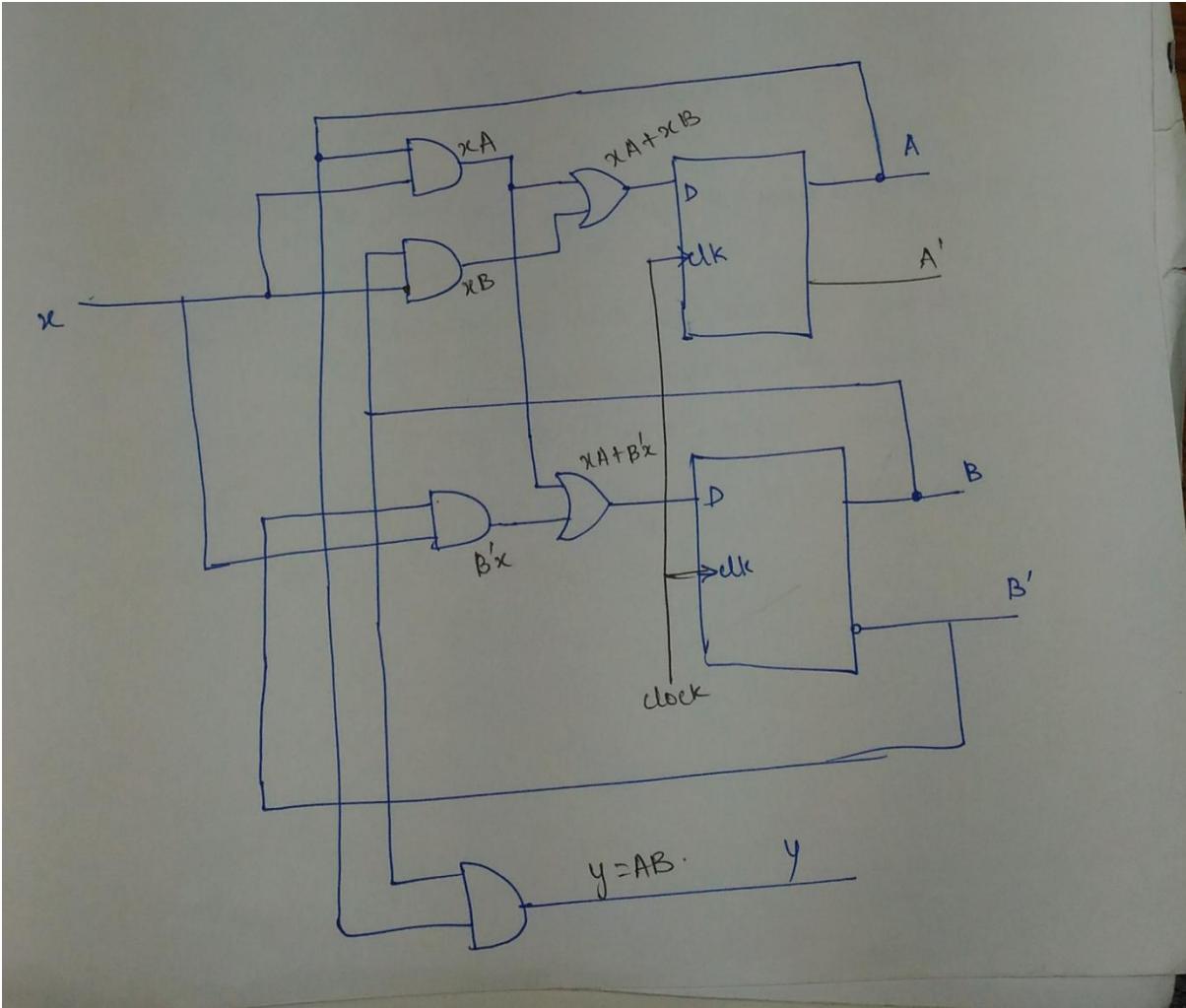
$$D_A = A\bar{x} + Bx \\ = \bar{x}(A + B)$$

	Bx	00	01	11	10
A	0	-	-	-	-
	1	-	-	-	-

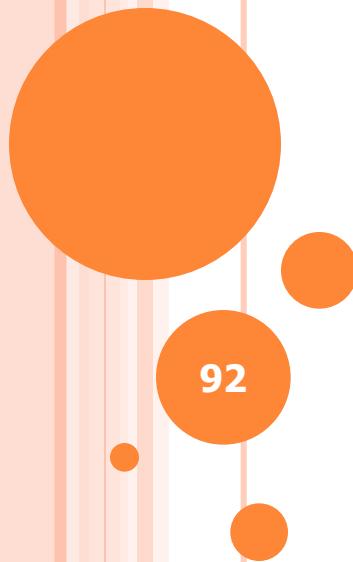
$$D_B = A\bar{x} + B'\bar{x} \\ = \bar{x}(A + B')$$

	Bx	00	01	11	10
A	0	-	-	-	-
	1	-	-	-	-

$$y = AB.$$



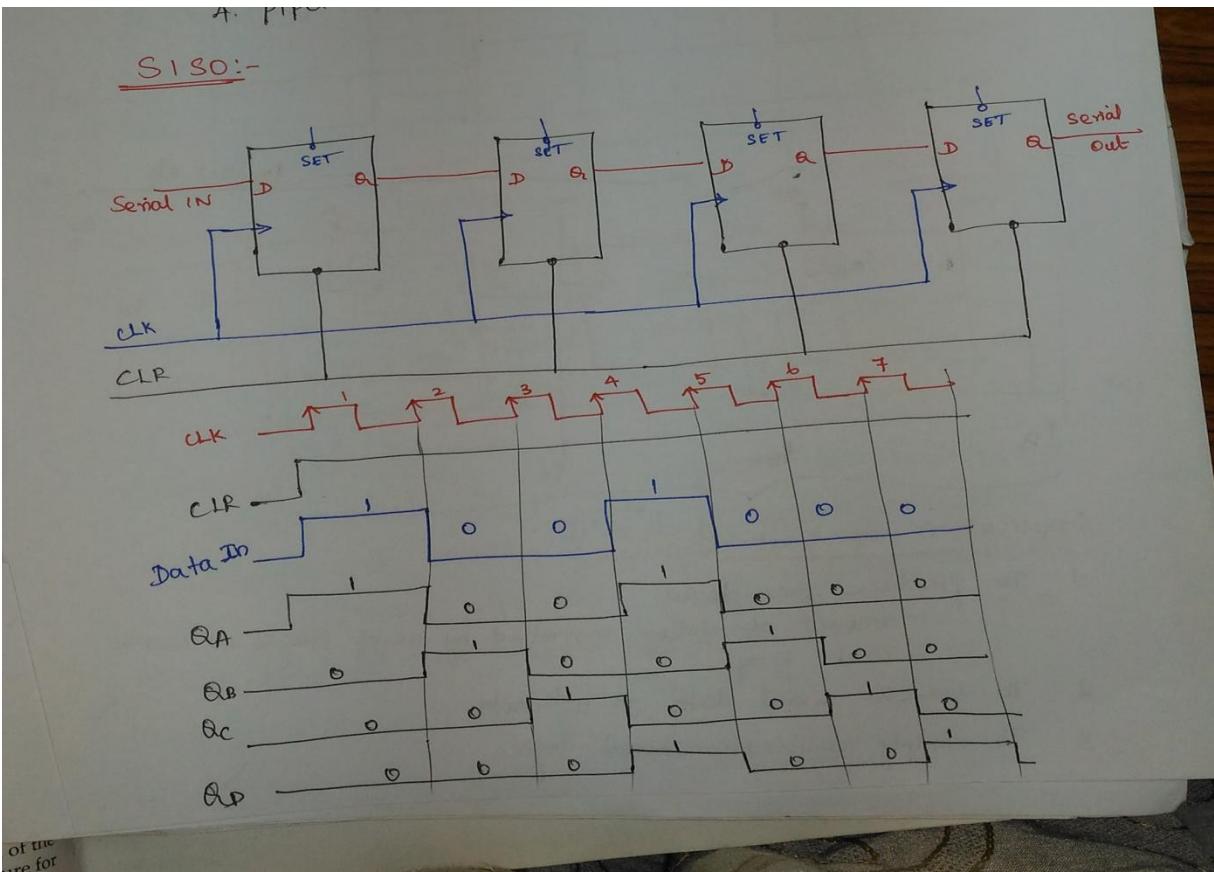
Shift Registers



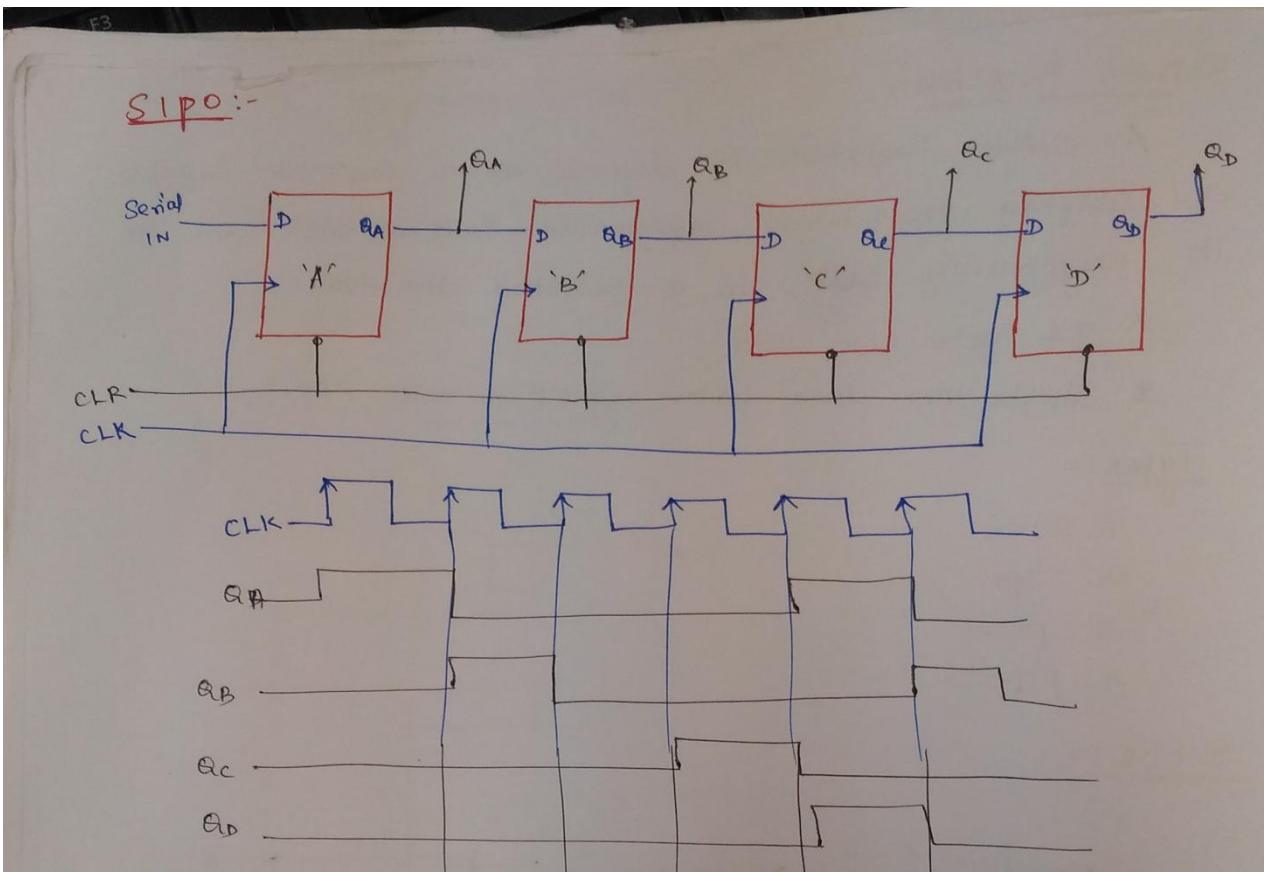
SHIFT REGISTERS

- One flip-flop can store one-bit of information. In order to store multiple bits of information, we require multiple flip-flops.
- The group of flip-flops, which are used to hold (store) the binary data is known as **register**.
- If the register is capable of shifting bits either towards right hand side or towards left hand side is known as **shift register**.
- An ‘N’ bit shift register contains ‘N’ flip-flops.
- Four types of shift registers based on applying inputs and accessing of outputs.
 - Serial In - Serial Out shift register
 - Serial In - Parallel Out shift register
 - Parallel In - Serial Out shift register
 - Parallel In - Parallel Out shift register

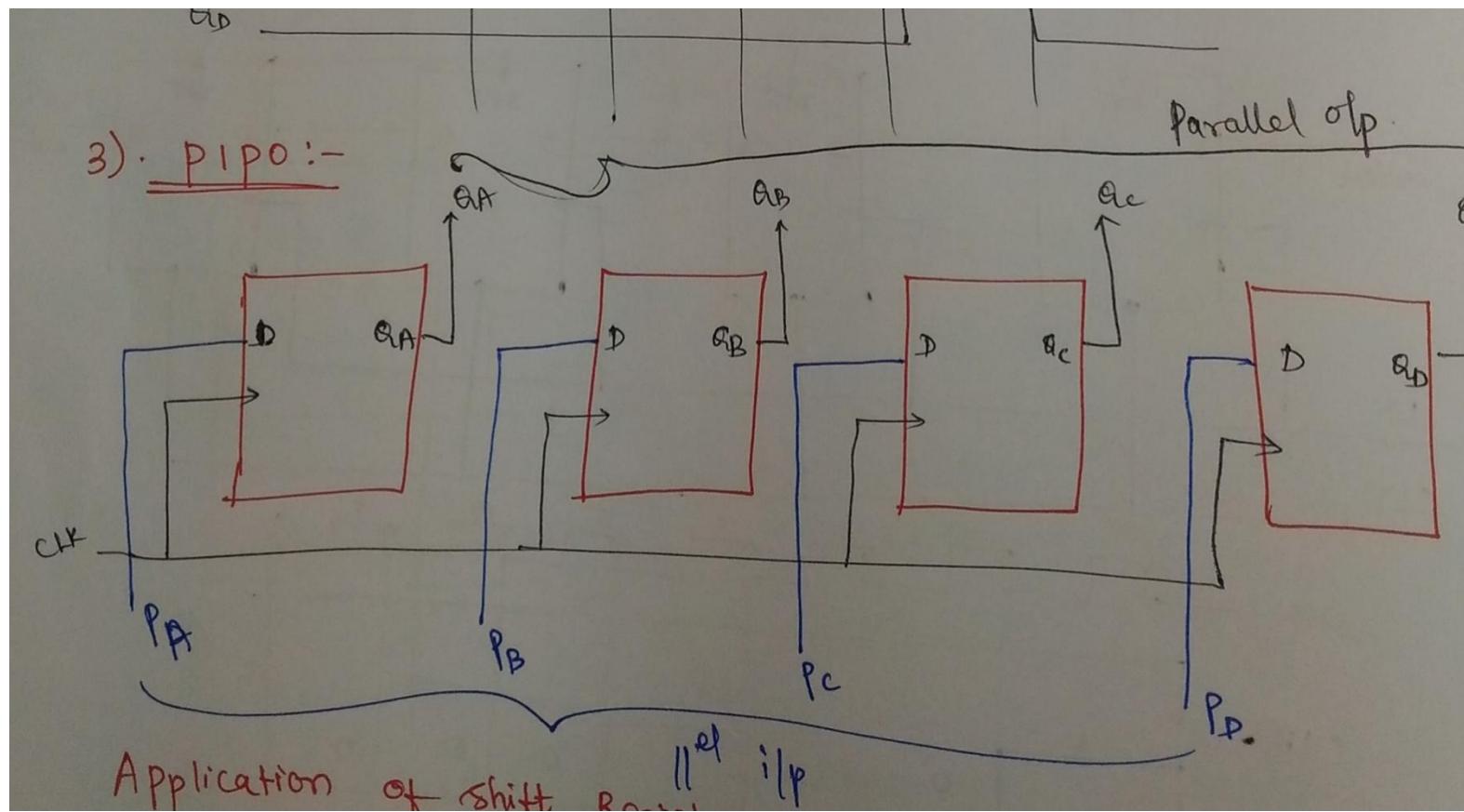
SISO



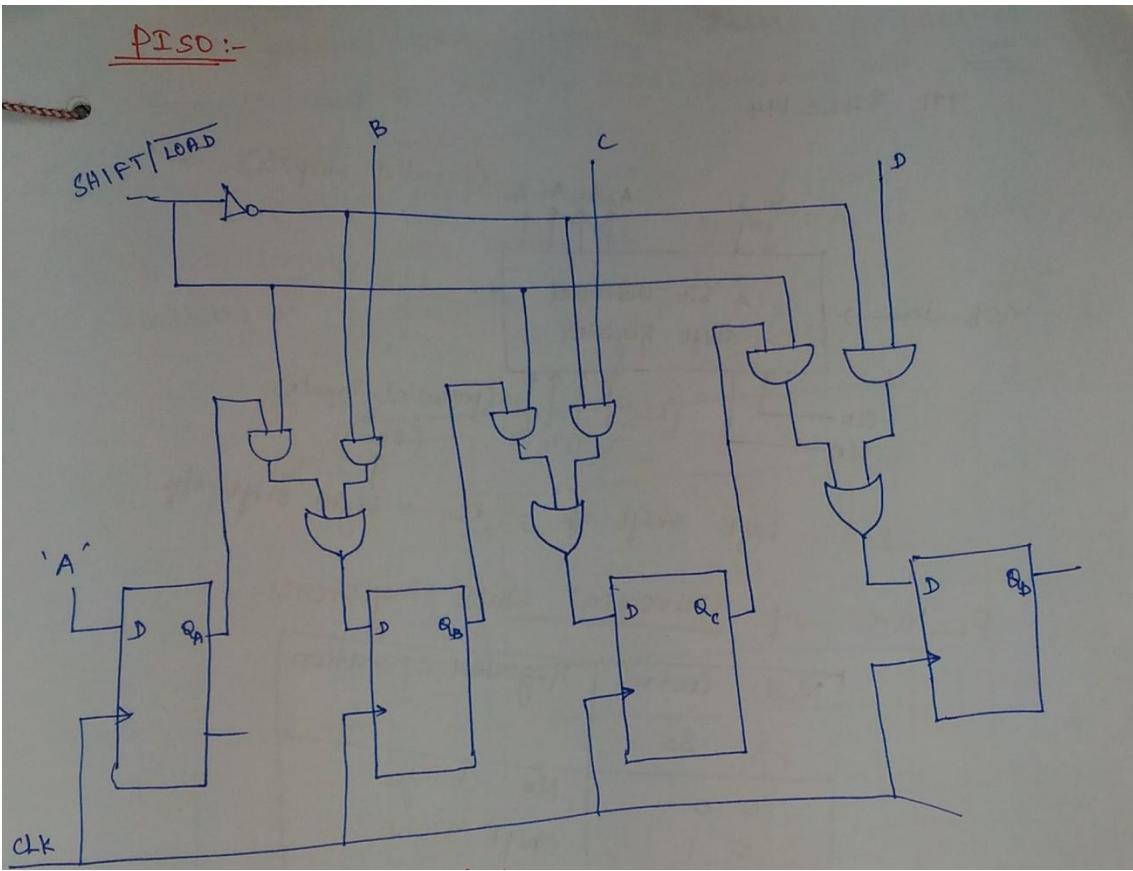
SIPO



PIPO



PISO



UNIVERSAL SHIFT REGISTER

- ***Universal Shift Register*** is a register which can be configured to load and/or retrieve the data in any mode (either serial or parallel) by shifting it either towards right or towards left.
- In other words, a combined design of unidirectional (either right- or left-shift of data bits as in case of SISO, SIPO, PISO, PIPO) and bidirectional shift register along with parallel load provision is referred to as **universal shift register**.
- Such a shift register capable of storing n input bits .

Features of Universal Shift Registers:-

Mode Control		Registered operation
S_1	S_0	
0	0	No change
0	1	shift Right
1	0	shift Left
1	1	parallel Load.

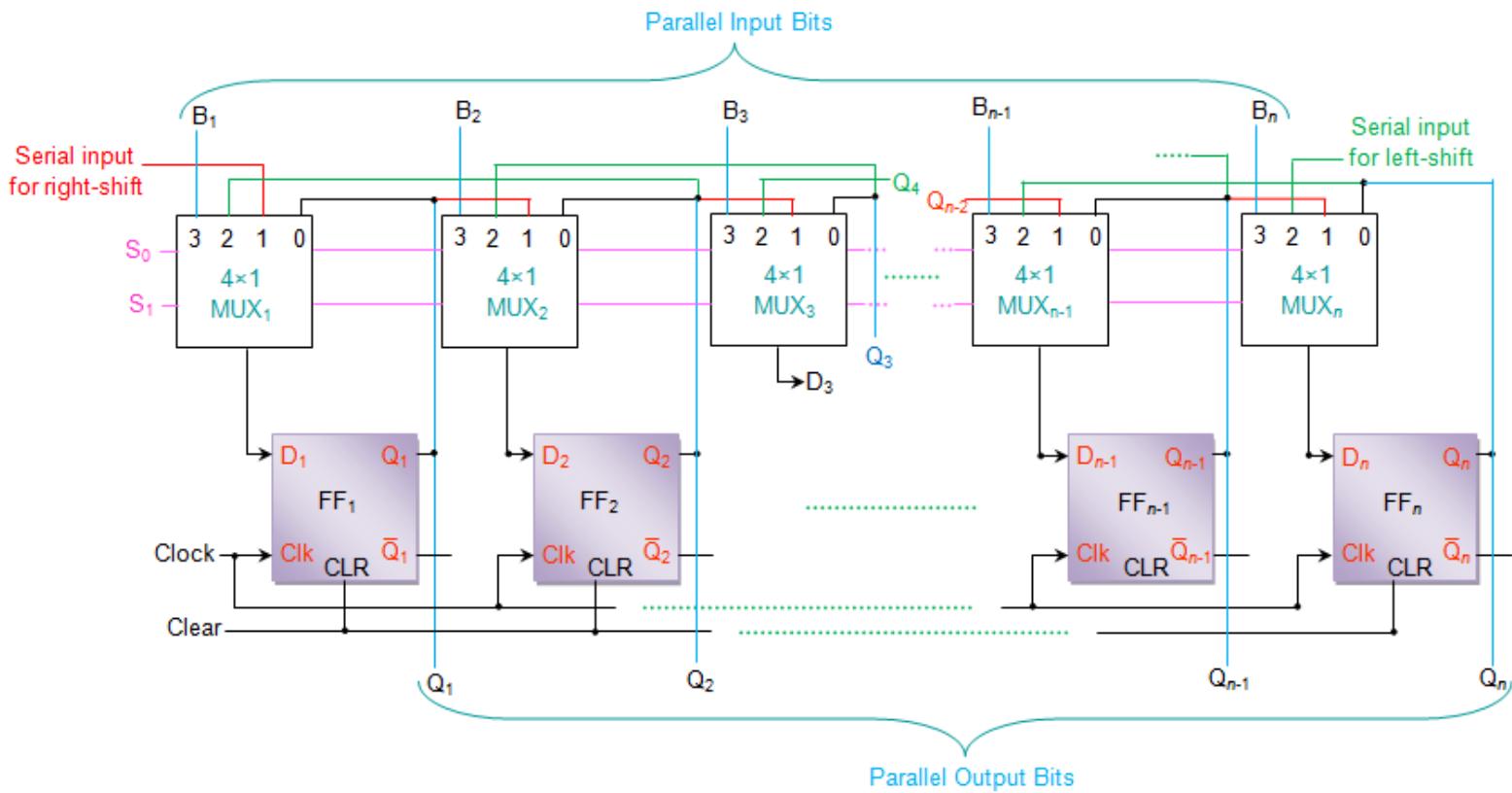
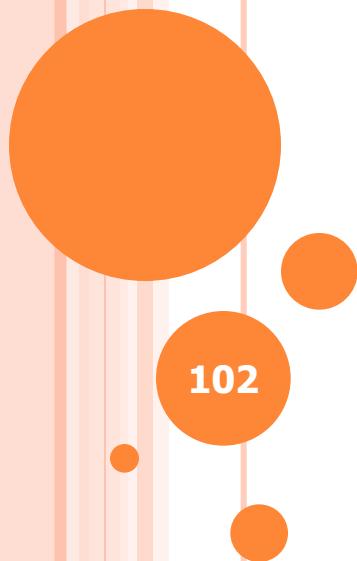


Figure 1 n -bit Universal Shift Register

APPLICATIONS OF SHIFT REGISTERS

- **Delay Lines of Shift Registers**
- Shift registers of Serial In Serial Out (SISO) kind can be used to delay the digital signals by a definite period time.
- The time delay introduced by the n-bit shift register is equal to n times the inverse of the clock frequency driving the shift register.
- Very long shift registers of this kind can be used as delay line memory in computer systems to store the temporary data.
- Further, multiple bidirectional shift registers connected in parallel can be used as a stack.
- To convert serial data into parallel data
- To simplify the combinational logic

Counters



- Counter is an electronic circuit used to count the number of times an event occurs.
- In digital electronics counters are constructed using series of flip-flops.

Types:-

1. Ripple counter (or) Asynchronous counter.

→ no common clock.

→ 1st FF gets clock, and proceeding FF's get clock from previous FF o/p.

→ drawback: long propagation delay.

2. Synchronous counter.

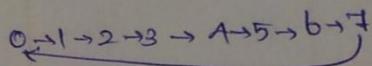
→ All FF having common clock.

→ Reduce propagation delay.

Other types:-

* Modulo N counter

* Circular counter

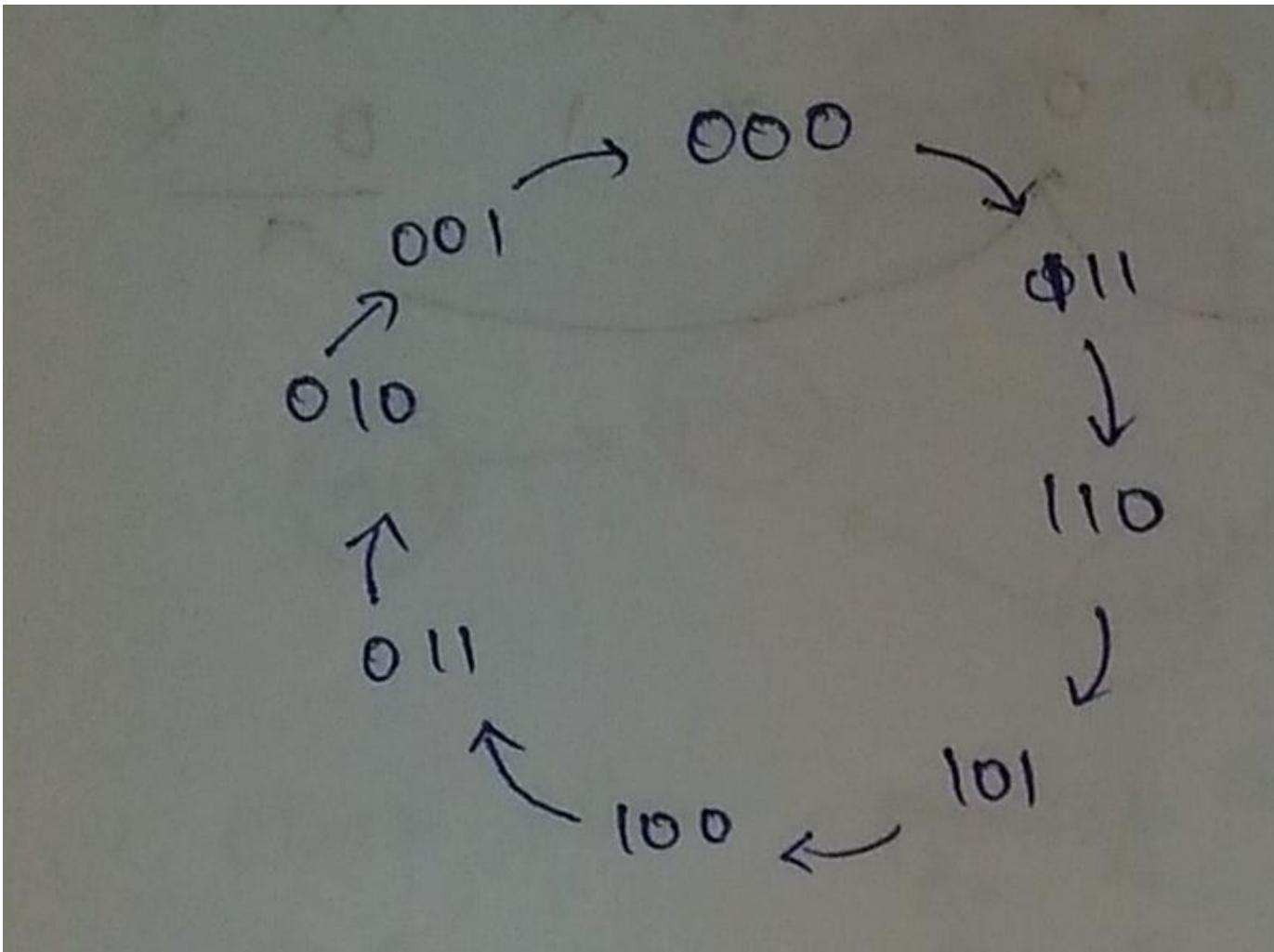


MOD-8 UP COUNTER

Counters:-

① Mod- 8 Synchronous up counter :

$$2^3 = 8 ; 2^n = 2^3 \quad n=3 .$$



up Counter:

$n=3$.

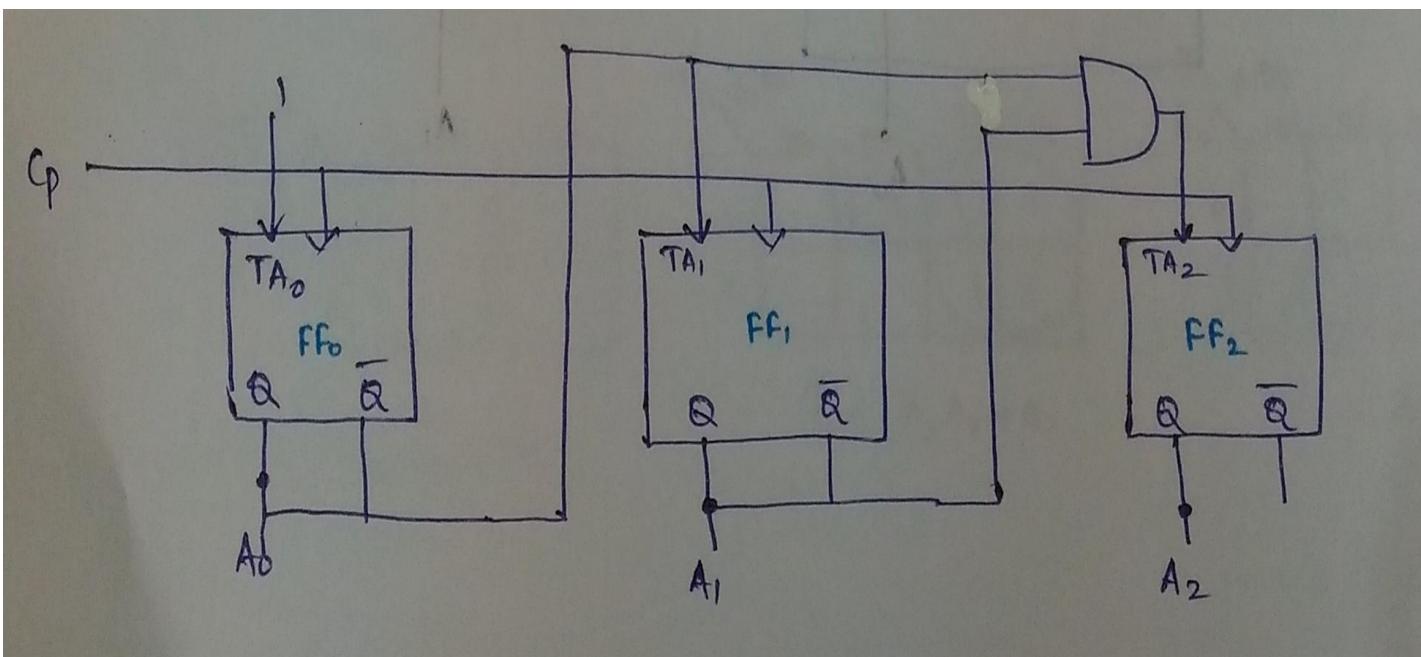
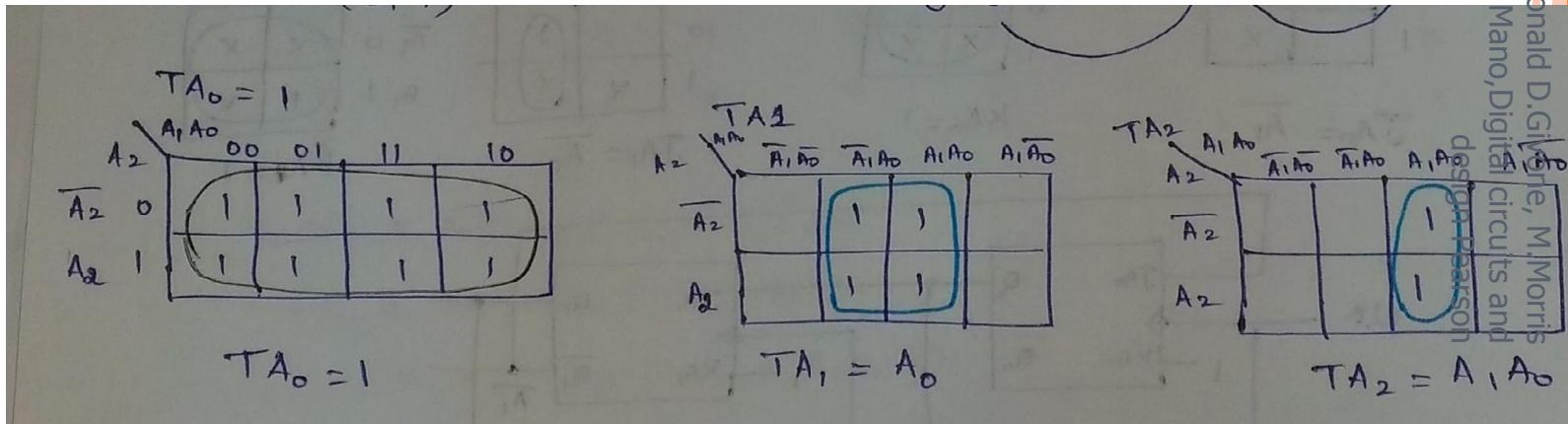
clock pulse	A ₂	A ₁	A ₀	Count seq.	Next	T _{FF}
0	0	0	0	0 0 1	0 1 0	0 0 \rightarrow 0
1	0	0	1	0 1 0	0 0 1	0 1 \rightarrow 1
2	0	1	0	0 1 1	0 0 0	FF
3	0	1	1	1 0 0	1 1 1	TA ₂
4	1	0	0	1 0 1	0 0 1	TA ₁
5	1	0	1	1 1 0	0 0 0	TA ₀
6	1	1	0	1 1 1	0 0 1	
7	1	1	1	0 0 0	1 1 1	
	0	0	0			

Excitation of FF

$$TA_0 = \sum m(0, 1, 2, 3, 4, 5, 6, 7)$$

$$TA_1 = \sum m(1, 3, 5, 7)$$

$$TA_2 = \sum m(3, 7)$$



MOD-8 DOWN COUNTER

10.5.5 | Modulus-8 Synchronous Down Counter

Modulus 8 (MOD-8) synchronous up counters require three flip-flops.

$$2^3 = 8 \Rightarrow 2^n = 2^3, \text{ so } n = 3.$$

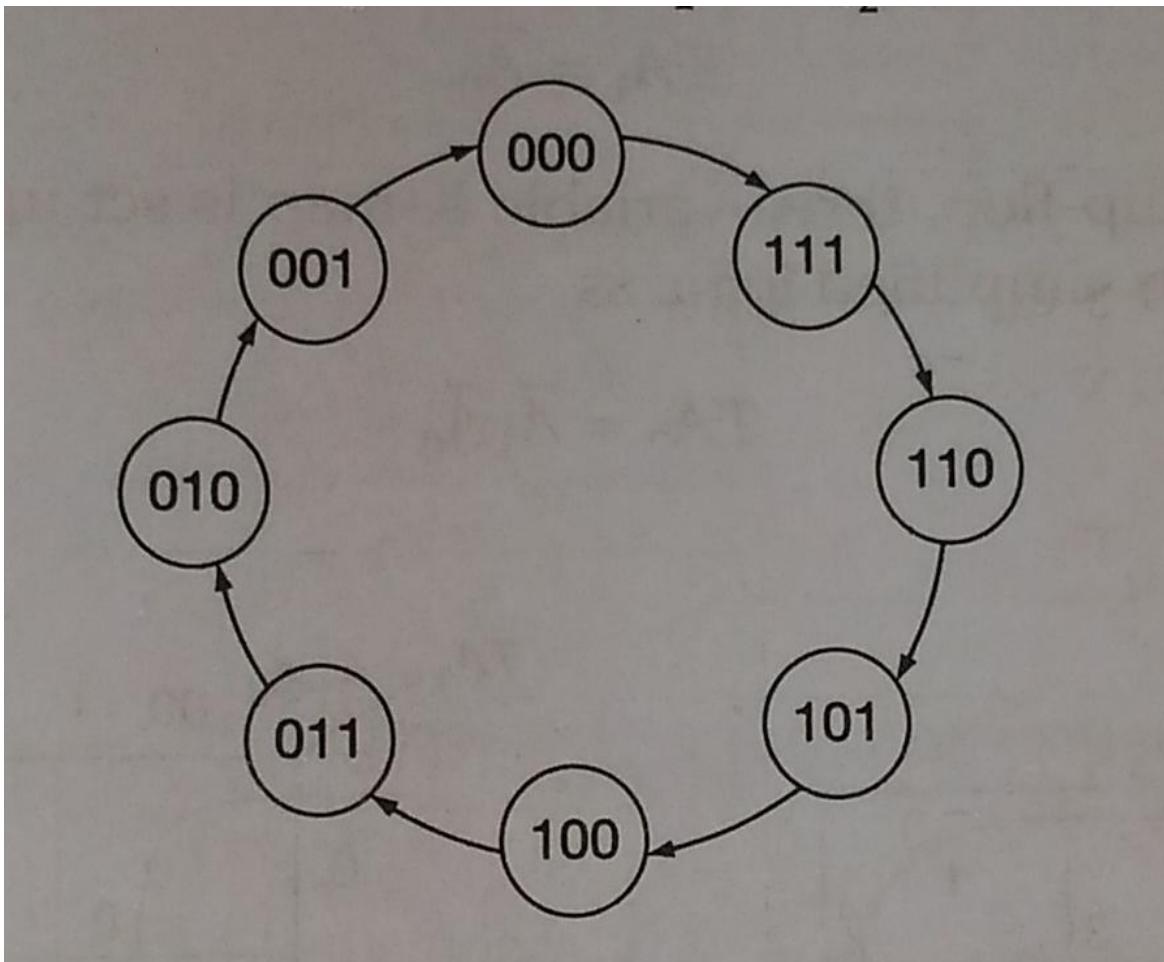


TABLE 10.21 | Down synchronous MOD-8 counter

Clock Pulse Count	Countdown Sequence			Excitations of Flip-flop		
	A_2	A_1	A_0	TA_2	TA_1	TA_0
7	1	1	1	0	0	1
6	1	1	0	0	1	1
5	1	0	1	0	0	1
4	1	0	0	1	1	1
3	0	1	1	0	0	1
2	0	1	0	0	1	1
1	0	0	1	0	0	1
0	0	0	0	1	1	1
7	1	1	1			

$$TA_0(A_2, A_1, A_0) = \sum m(0, 1, 2, 3, 4, 5, 6, 7)$$

$$TA_1(A_2, A_1, A_0) = \sum m(0, 2, 4, 6)$$

$$TA_2(A_2, A_1, A_0) = \sum m(0, 2)$$

$$TA_0 = 1$$

$$TA_1 = \bar{A}_0$$

$$TA_2 = \bar{A}_1 \bar{A}_0$$

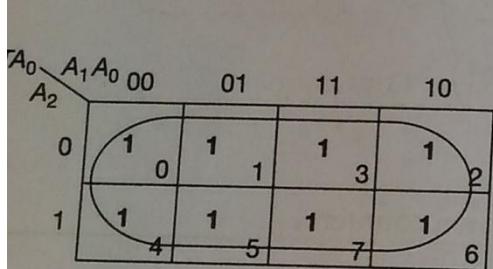


FIGURE 10.60 | K-map for TA_0

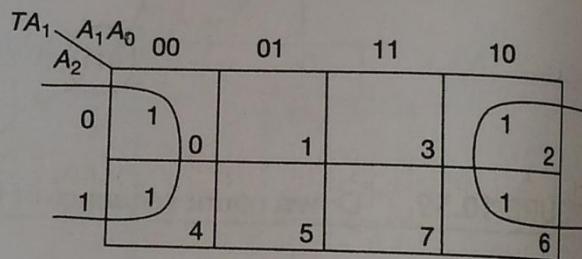


FIGURE 10.61 | K-map for TA_1

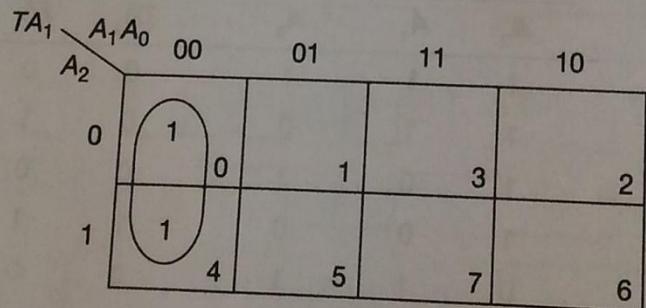
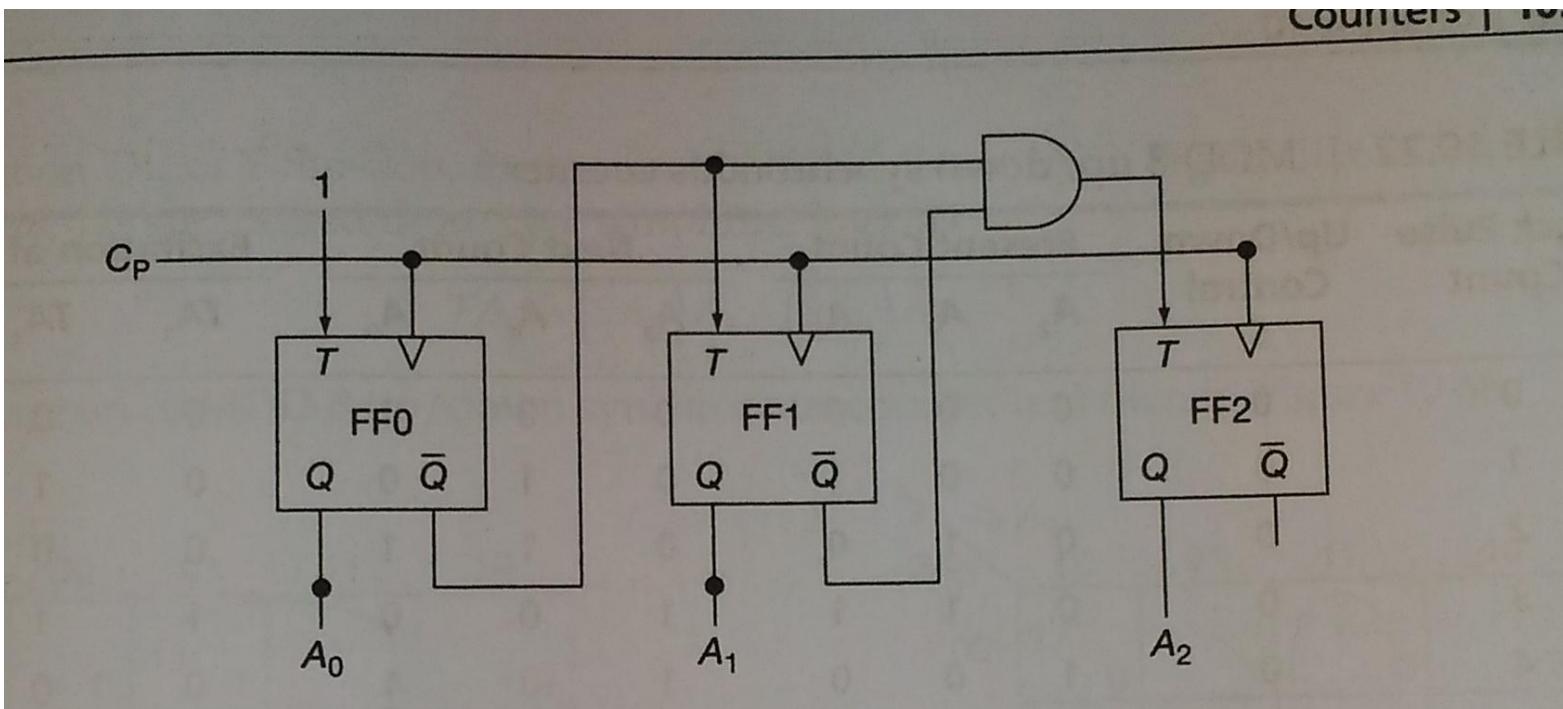


FIGURE 10.62 | K-map for TA_2



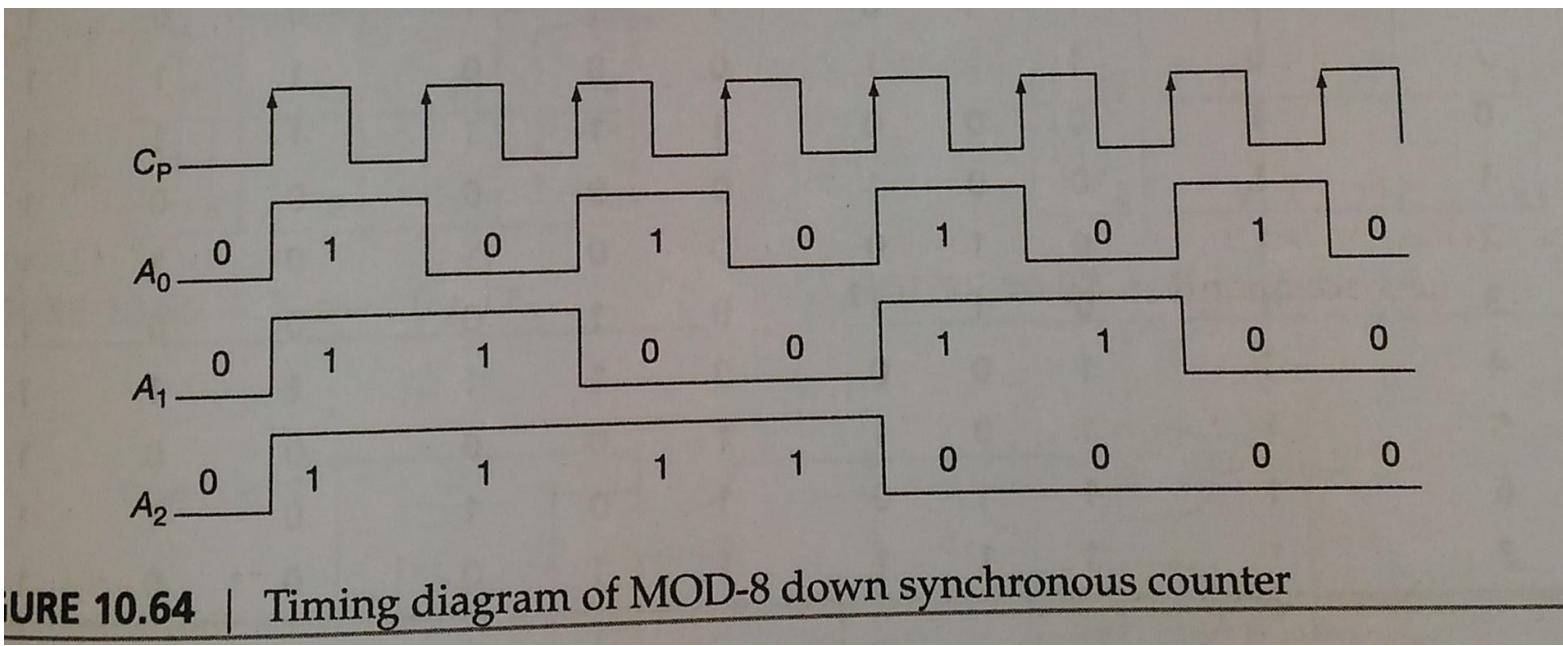


FIGURE 10.64 | Timing diagram of MOD-8 down synchronous counter

Design a modulus-3 (MOD-3) up synchronous counter using JK-flip-flop.

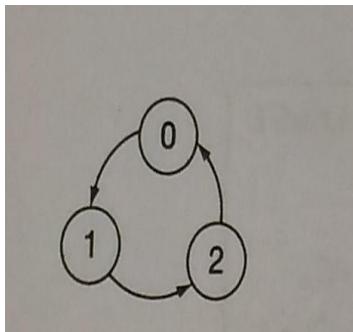


TABLE 10.25 | MOD-3 synchronous counter

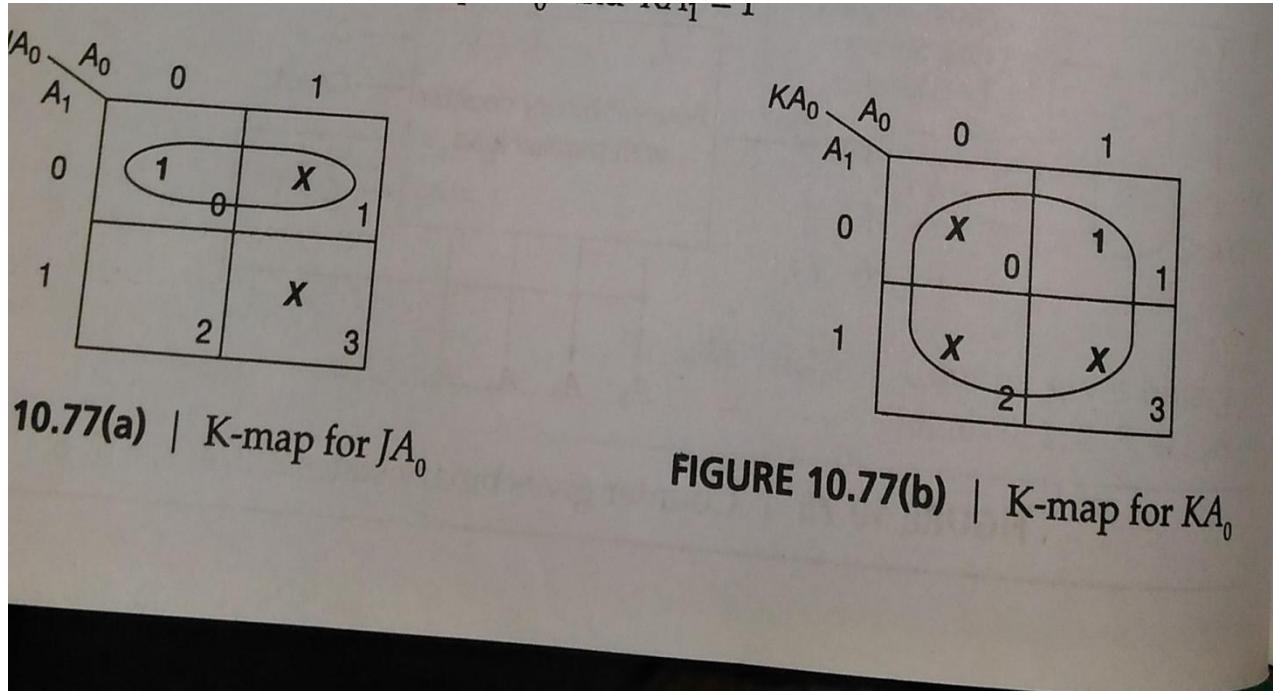
Clock Pulse	Sequence up Count		Excitation			
	A ₁	A ₀	JA ₁	KA ₁	JA ₀	KA ₀
0	0	0	0	X	1	X
1	0	1	1	X	X	1
2	1	0	X	1	0	X
	0	0	-	-	-	-

$$JA_0(A_1, A_0) = \sum m(0) + \sum m(1, 3)$$

$$KA_0(A_1, A_0) = \sum m(1) + \sum m(0, 2, 3)$$

$$JA_1(A_1, A_0) = \sum m(1) + \sum d(2, 3)$$

$$KA_1(A_1, A_0) = \sum m(2) + \sum d(0, 1, 3)$$



$$JA_0 = \bar{A}_1 \text{ and } KA_0 = 1$$

Given

$$JA_1 = A_0 \text{ and } KA_1 = 1$$

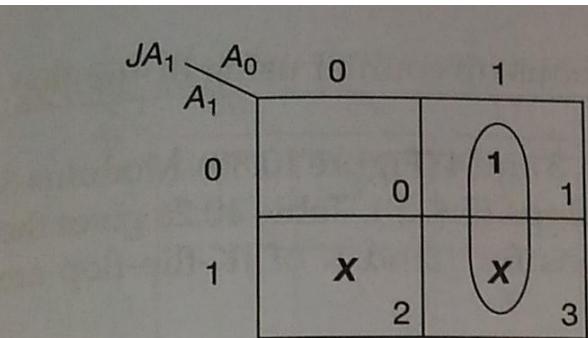


FIGURE 10.77(c) | K-map for $J A_1$

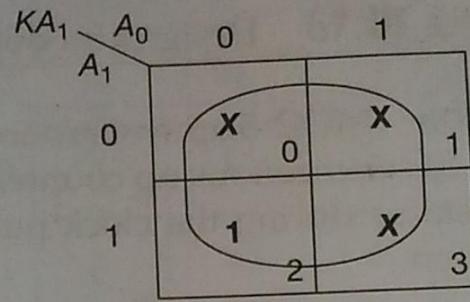


FIGURE 10.77(d) | K-map for $K A_1$

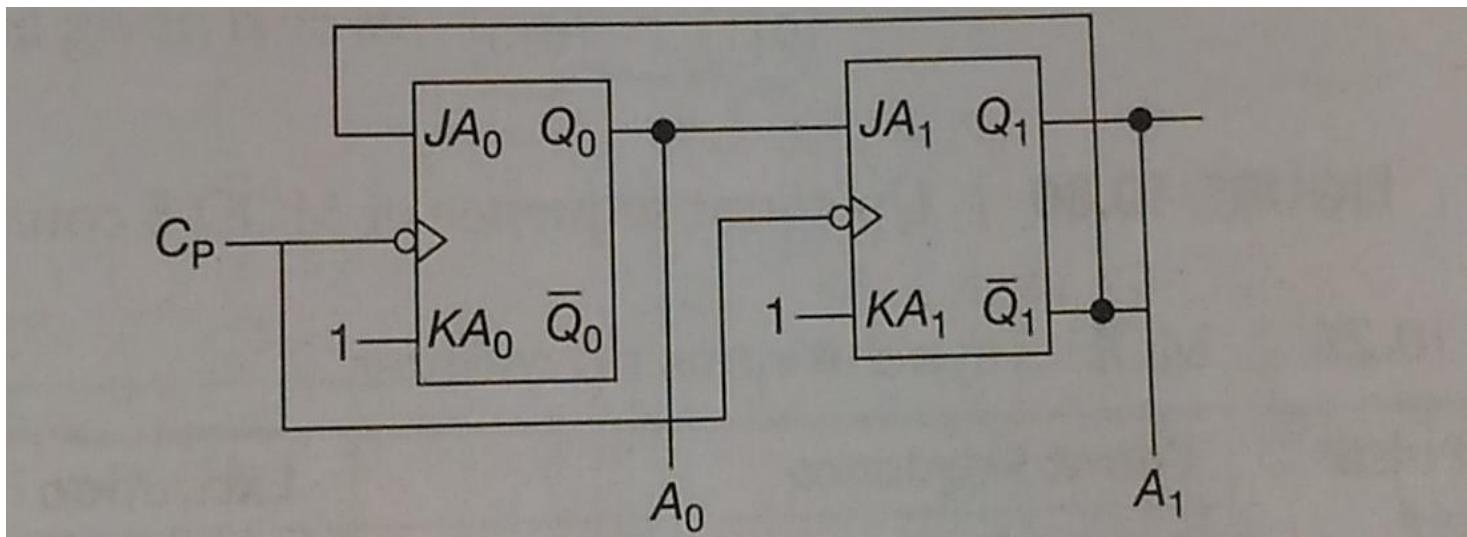


FIGURE 10.78 | MOD-3 synchronous up counter

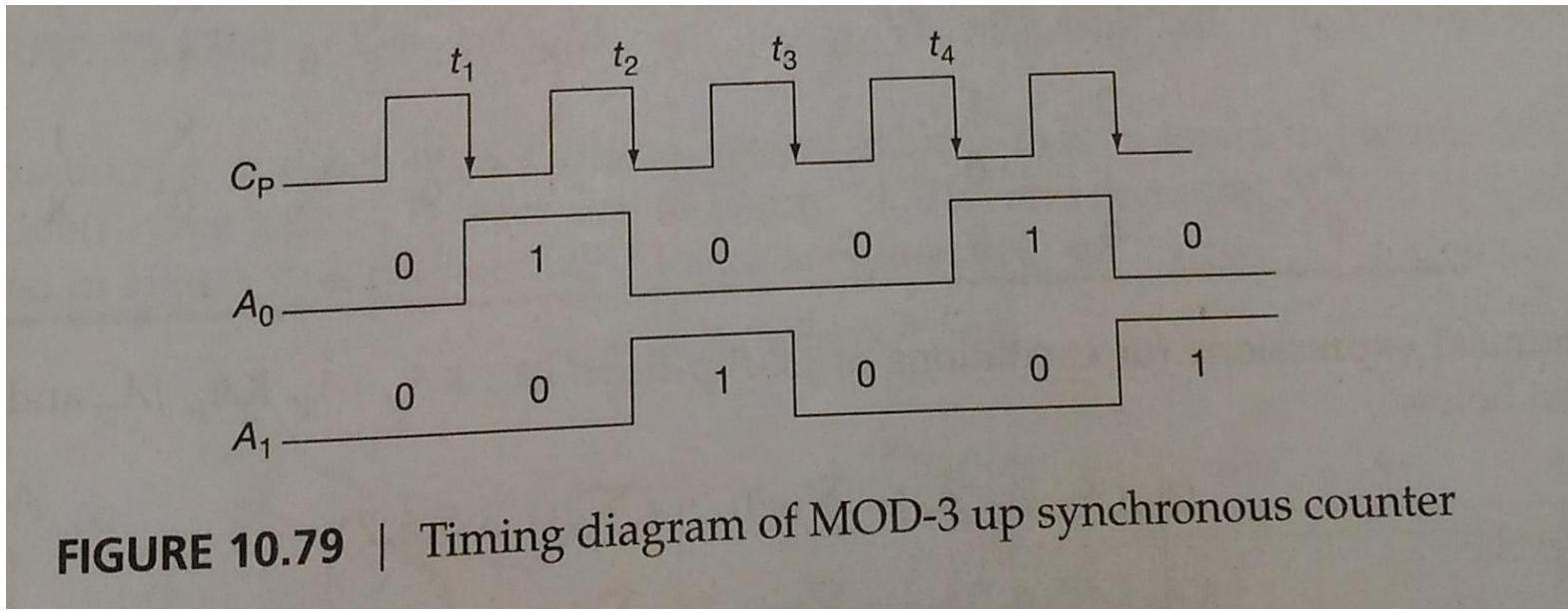


FIGURE 10.79 | Timing diagram of MOD-3 up synchronous counter

BCD TO GRAY CODE

TABLE 10.27 | Excitation table

Clock Pulse Count	Present Count				Next Count				Excitation of Flip-flop			
	A_2	A_1	A_0		g_3	g_2	g_1	g_0	T_3	T_2	T_1	T_0
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1	0	0	0	0
2	0	0	1	0	0	0	1	1	0	0	0	1
3	0	0	1	1	0	0	1	0	0	0	1	0
4	0	1	0	0	0	1	1	0	0	0	1	0
5	0	1	0	1	0	1	1	1	0	0	1	1
6	0	1	1	0	0	1	0	0	0	0	1	1
7	0	1	1	1	0	1	0	0	0	1	0	0
8	1	0	0	0	1	1	0	0	0	1	0	0
9	1	0	0	1	1	1	0	1	0	1	0	0

$$T_0(A_3, A_2, A_1, A_0) = \sum m(2, 3, 6, 7) + \sum d(10, 11, 12, 13, 14, 15)$$

$$T_1(A_3, A_2, A_1, A_0) = \sum (4, 5, 6, 7) + \sum d(10, 11, 12, 13, 14, 15)$$

$$T_2(A_3, A_2, A_1, A_0) = \sum (8, 9) + \sum d(10, 11, 12, 13, 14, 15)$$

$$T_3(A_3, A_2, A_1, A_0) = 0$$

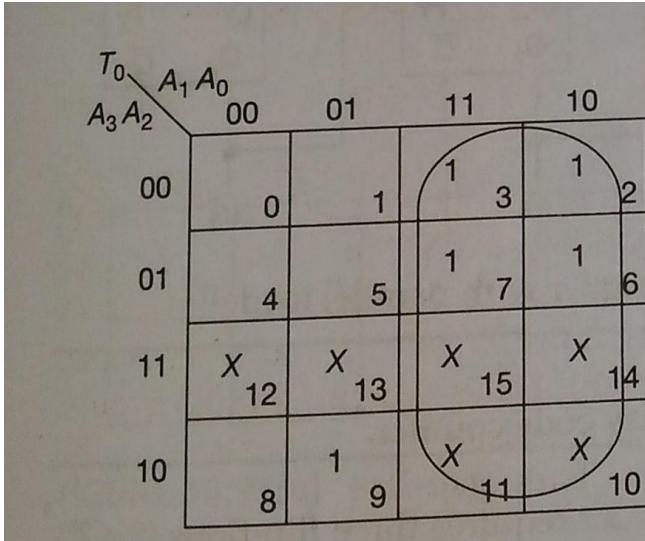


FIGURE 10.93(a) | K-map for T_0

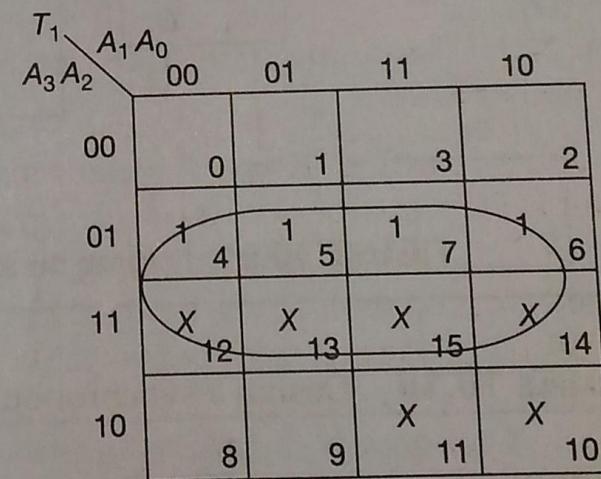


FIGURE 10.93(b) | K-map for T_1

$$T_0 = A_1$$

$$T_2 = A_3$$

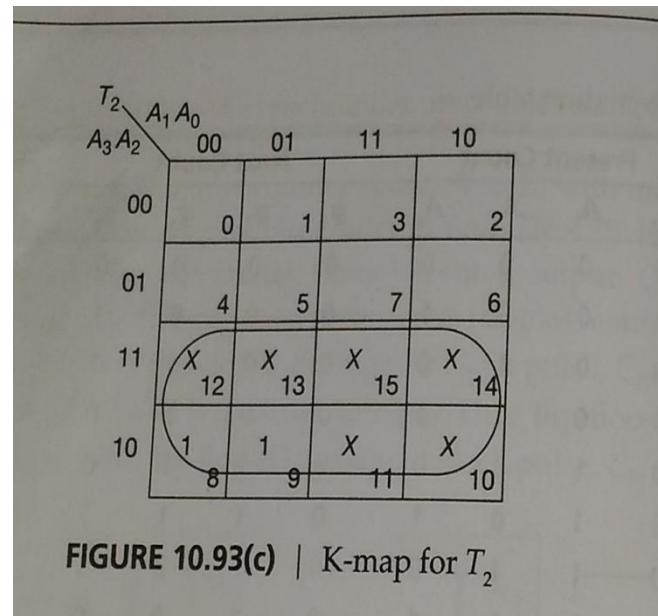
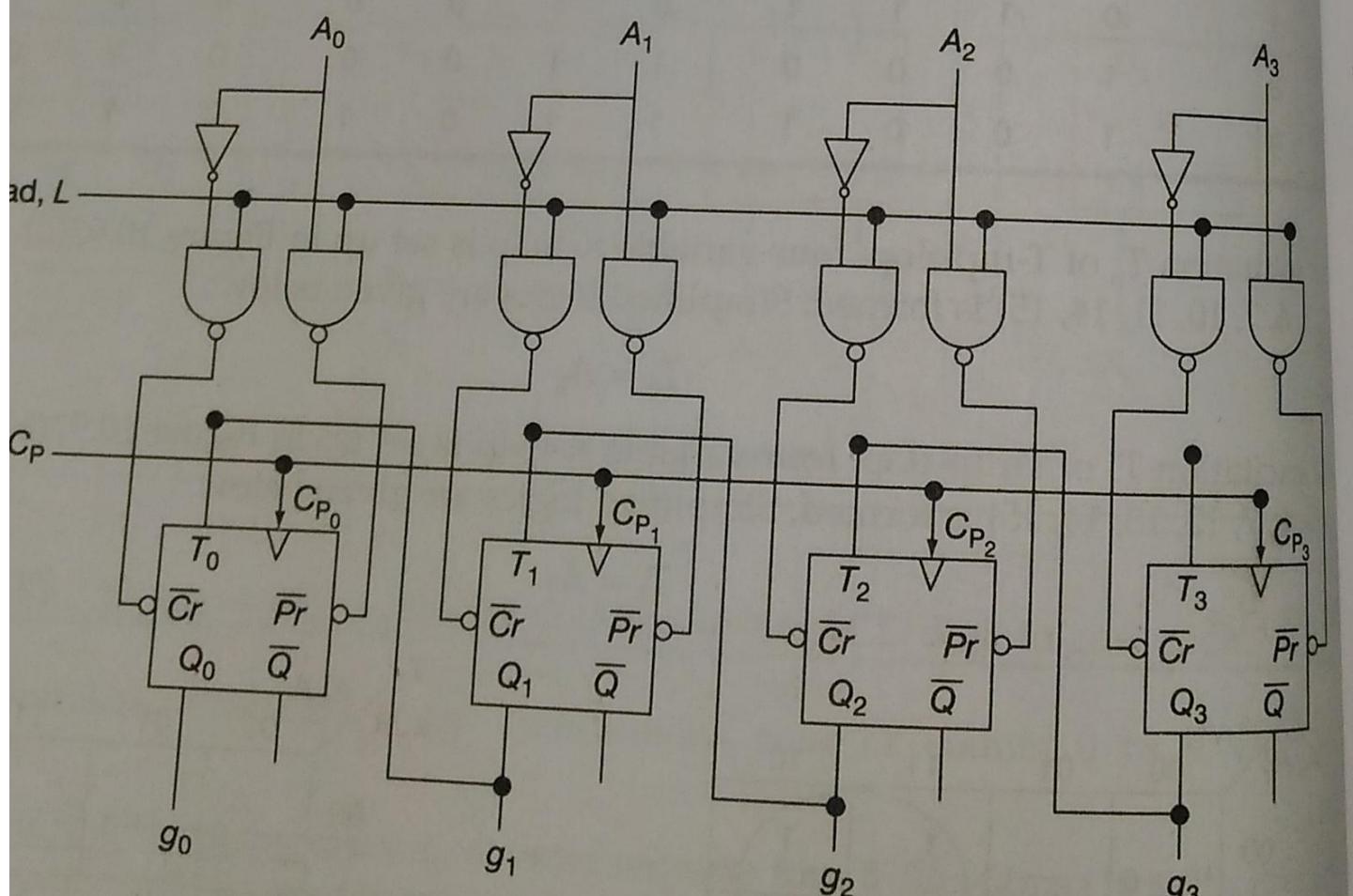


FIGURE 10.93(c) | K-map for T_2

FIGURE 10.55(c) | K-map for T_2



EXAMPLE 10.17 Design a synchronous counter that goes through 0, 1, 2, 4, 0. Unused states must go to zero or to next state on next clock pulse.

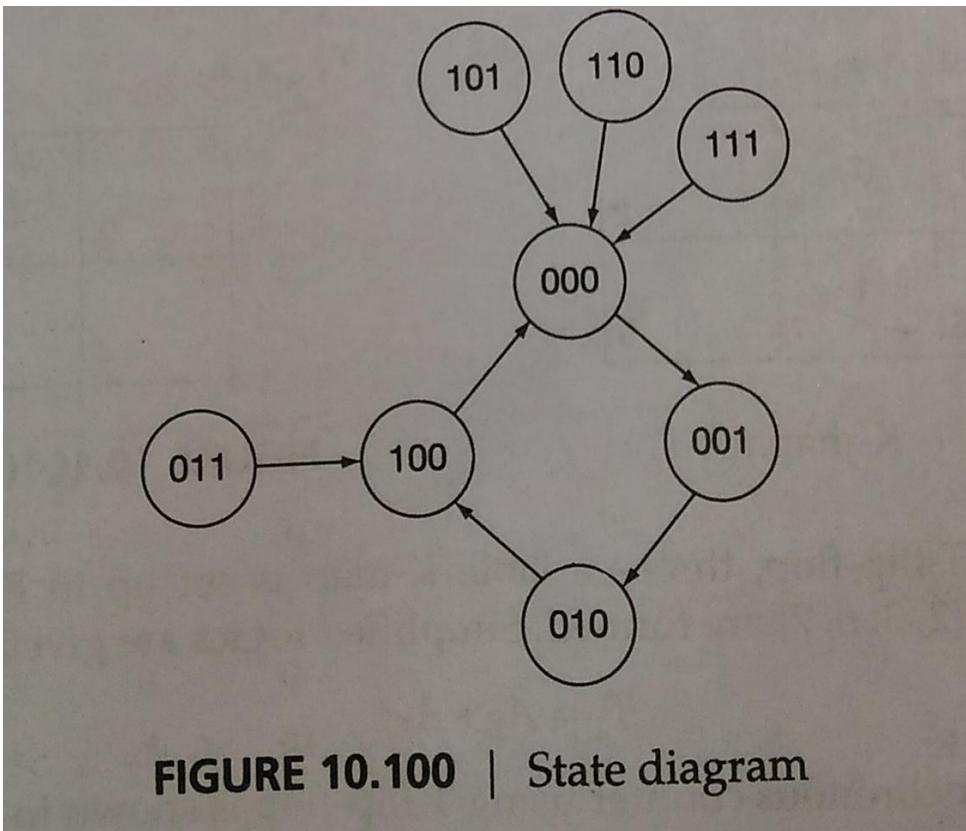


FIGURE 10.100 | State diagram

TABLE 10.29 | MOD-8 synchronous counter

Clock Pulse Count	Present Count			Next Count			Excitation of Flip-flop		
	A_2	A_1	A_0	A_2	A_1	A_0	T_2	T_1	T_0
0	0	0	0	0	0	1	0	0	1
1	0	0	1	0	1	0	0	1	1
2	0	1	0	1	0	0	1	1	0
3	0	1	1	1	0	0	1	1	1
4	1	0	0	0	0	0	1	0	0
5	1	0	1	0	0	0	1	0	1
6	1	1	0	0	0	0	1	1	0
7	1	1	1	0	0	0	1	1	1

$$T_0(A_2, A_1, A_0) = \sum m(0, 1, 3, 5, 7)$$

$$T_1(A_2, A_1, A_0) = \sum m(1, 2, 3, 6, 7)$$

$$T_2(A_2, A_1, A_0) = \sum m(2, 3, 4, 5, 6, 7)$$

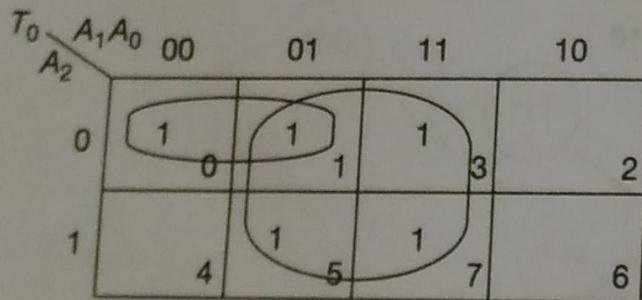


FIGURE 10.101(a) | K-map for T_0

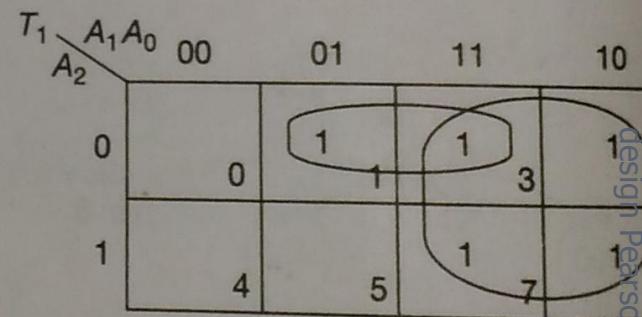


FIGURE 10.101(b) | K-map for T_1

$$T_0 = \bar{A}_2 \bar{A}_1 + A_0$$

$$T_1 = \bar{A}_2 A_0 + A_1$$

$$T_2 = A_1 + A_2$$

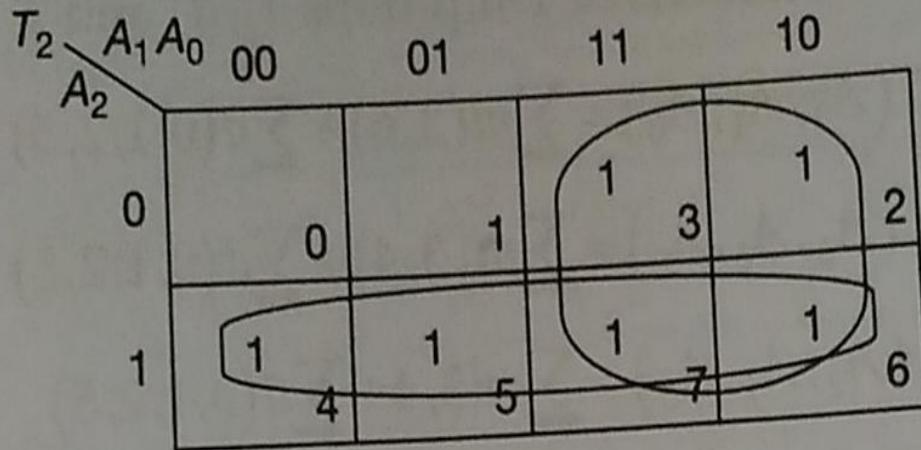


FIGURE 10.101(c) | K-map for T_2

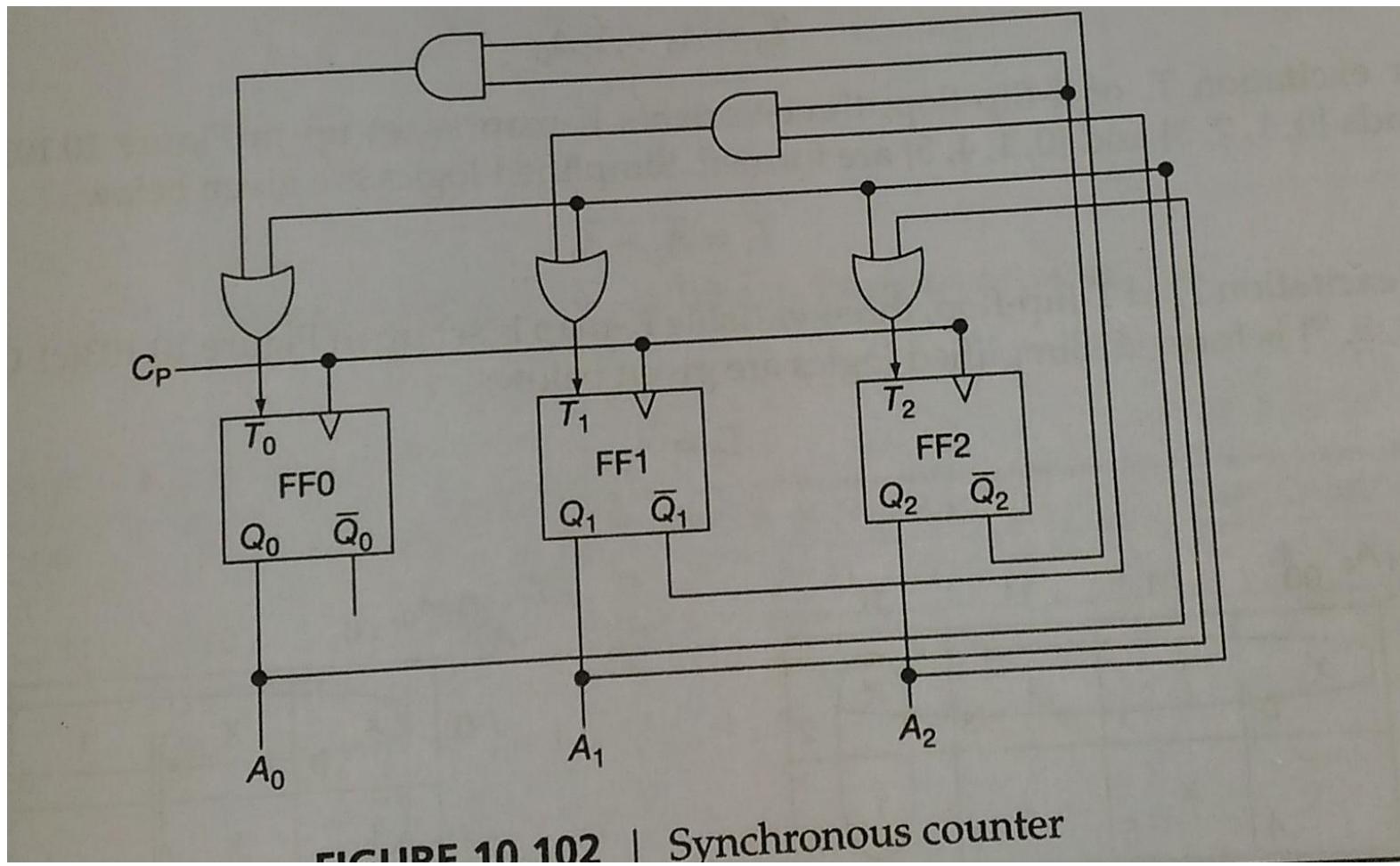


FIGURE 10.102 | Synchronous counter

RING COUNTER

- Counter is nothing but a digital device meant to count. These are usually built using bi-stable devices called flip-flops.
- Generally either D or JK type flip-flops are used to design the counters, no matter which type they are of applies even for **ring counter**.
- In fact, the way of connection which exists between the flip-flops is the factor which determines the kind of counter designed. Now, let us assume that we have arranged all the flip-flops in series such that the output of the preceding flip-flop is fed as an input to the immediate next flip-flop. Further, let us connect the output of the last flip-flop as an input to the first flip-flop in the chain.

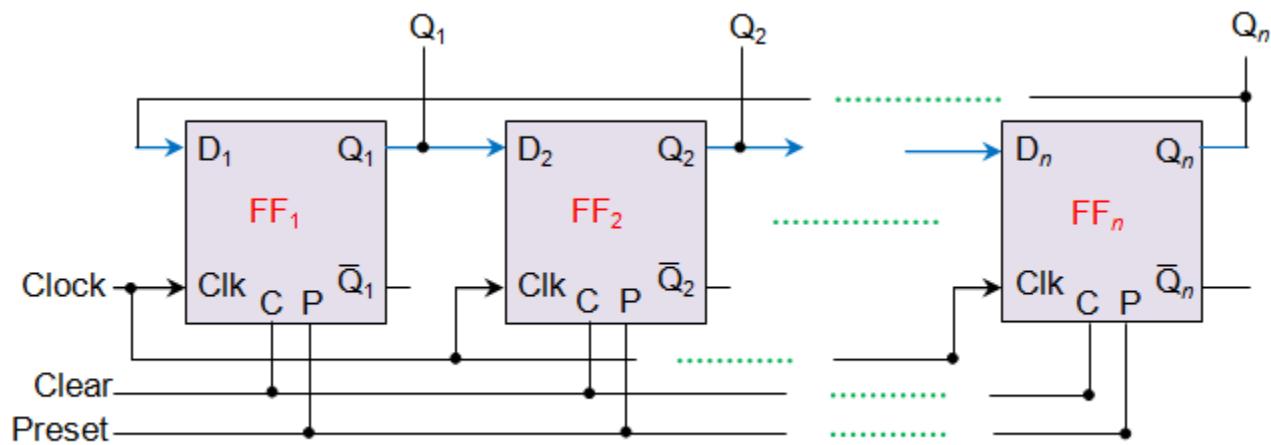


Figure 1 n -bit Ring Counter Designed Using D Flip-Flops

Clock Cycle	Q_1	Q_2	Q_3	Q_4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	1	0	0	0
6	0	1	0	0
.
.

Bit-pattern repeats for every 4 clock cycles

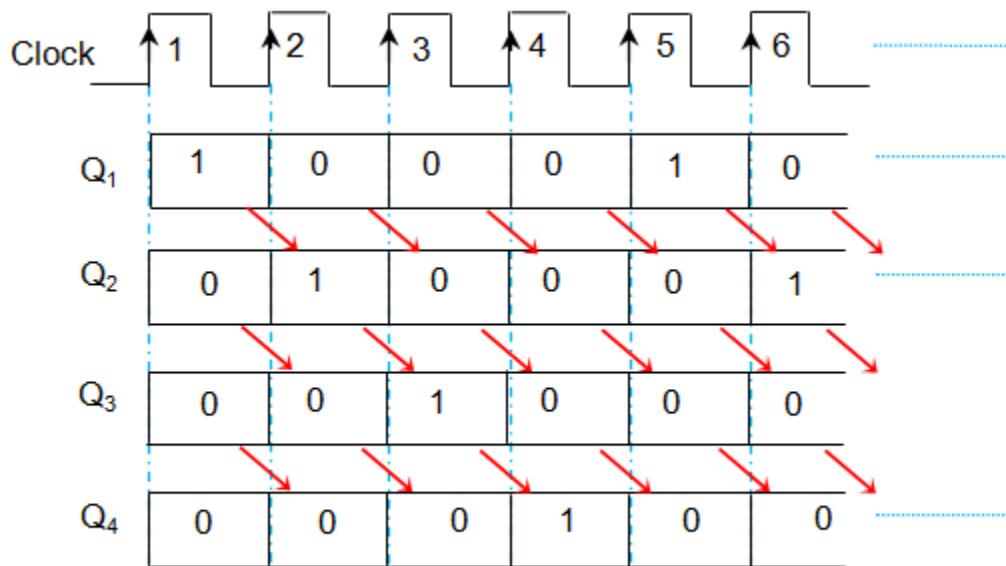


Figure 2 Output waveform of 4-bit ring counter

JOHNSON COUNTER

- A circuit which is used to count the number of times an event occurs is called a counter.
- In digital sense, these circuits comprise of bi-stable devices called flip-flops arranged in a particular fashion.
- Such a chain can be regarded to be a shift register, due to which counters can be considered as an application of shift registers.
- Either D or JK type flip-flops are preferred while designing the counter circuits. One such counter designed using D flip-flop chain is shown by Figure and is called **Johnson counter**.

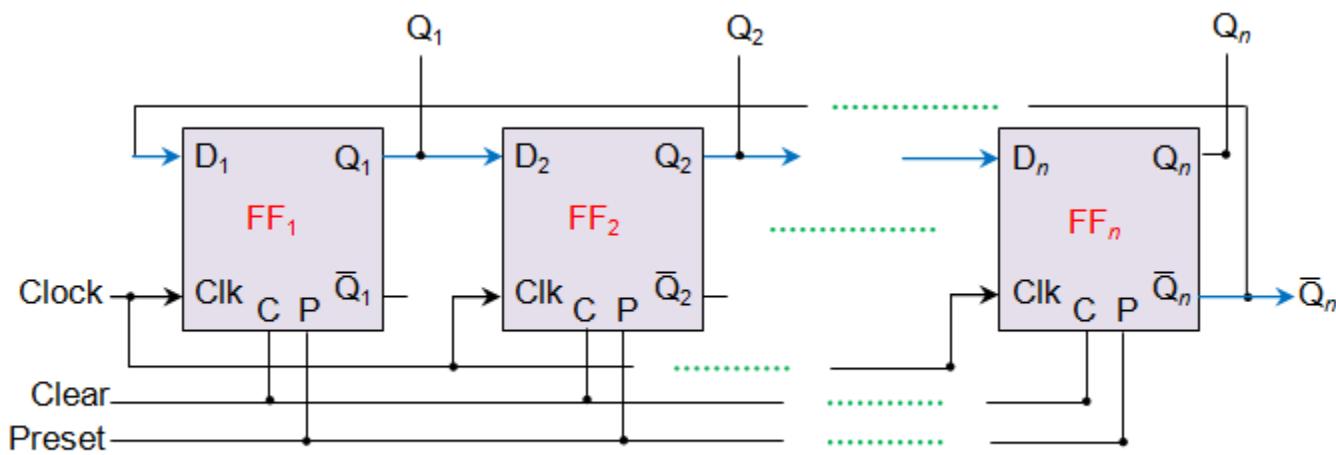


Figure 1 n -bit Johnson Counter Designed Using D Flip-Flops

Table I Truth Table of 3-bit Johnson Counter

Clock Cycle	Q_1	Q_2	Q_3	\bar{Q}_3
1	0	0	0	1
2	1	0	0	1
3	1	1	0	1
4	1	1	1	0
5	0	1	1	0
6	0	0	1	0
7	0	0	0	1
8	1	0	0	1
9	1	1	0	1
.
.
.

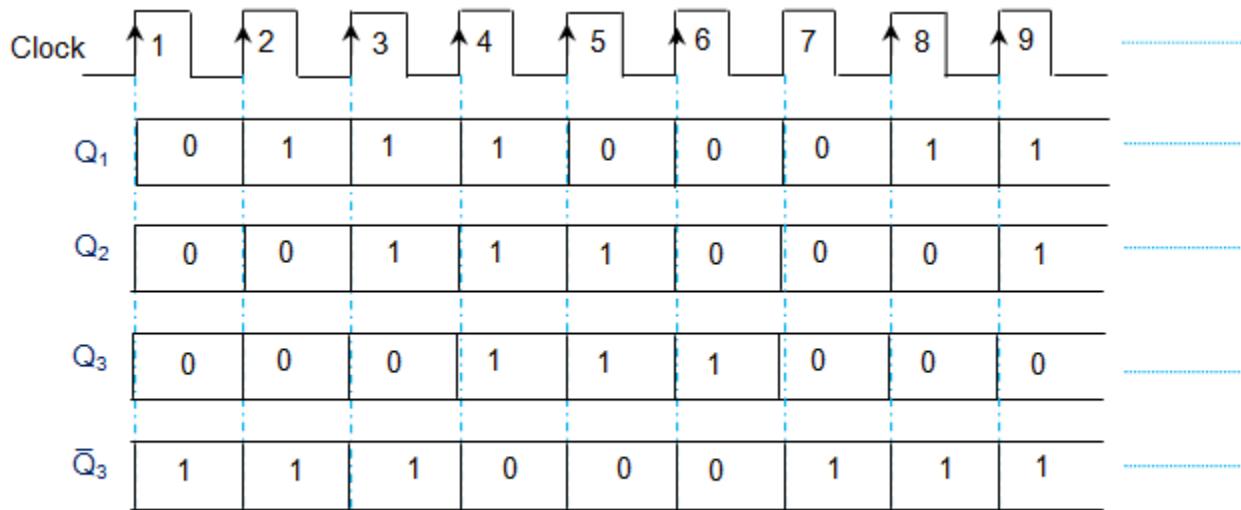
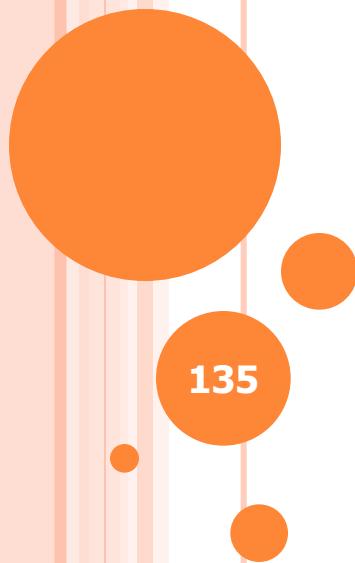


Figure 2 Output Waveform of 3-bit Johnson Counter

HDL



VERILOG HDL

Verilog HDL

VHDL → R&D
HDL ← Verilog HDL → Company

- Gate / Structural model
- Data flow model
- Behavioural model.

Gate

```

module G_M(A,B,S,X)
  input A,B,S;
  output X;
  wire S_b,w1,w2;
  begin
    not S_b(S_b,S);
    and w1(w1,A,S_b);
    and w2(w2,B,S_b);
    or X(X,w1,w2);
  end
endmodule

```

Data flow

$$X = A\bar{S} + B\bar{S}$$

```

module D_M(A,B,S,X);
  input A,B,S;
  output X;
  begin
    assign X = (A & (B & S)) | (B & S);
  end
endmodule

```

if $S = 0$

$X = A$

else

$X = B$

module B_M (A, B, S, X);

input A, B, S;

output X;

Sequential \rightarrow reg X;

memory always @ (S)

\rightarrow sensitivity list

begin

if ($S == 0$)

$X = A$

else

$X = B$

end

endmodule.

Donald D.Givone, M.Morris
Mano, Digital circuits and
design Pearson

5-5 HDL FOR SEQUENTIAL CIRCUIT

The Verilog hardware description language (HDL) is introduced in Section 3-9. The description of combinational circuits and an introduction to behavioral modeling is presented in Section 4-11.

- In this section, we continue the discussion of the behavioral modeling and present description examples of flip-flops and sequential circuits.

BEHAVIORAL MODELING

There are two kinds of behavioral statements in Verilog HDL:
initial and **always**.

```
initial
begin
    clock = 1`b0;
    repeat (30)
        #10 clock = ~clock;
end
```

```
initial
begin
    clock = 1`b0;
    #300 $finish;
end
always
    #10 clock = ~clock;
```

BEHAVIORAL MODELING

The **always** statement can be controlled by delays that wait for a certain time or by certain conditions to become true or by events to occur.

always @(event control expression)
procedural assignment statements.

always @(A **or** B **or** Reset)

always @(**posedge** clock **or** **negedge** reset)

FLIP-FLOPS AND LATCHES

HDL Example 5-1

```
//Description of D latch (See Fig. 5-6)
module D_latch (Q, D, control);
    output Q;
    input D, control;
    reg Q;
    always @(control or D)
        if (control) Q = D;      //Same as: if (control ==1)
    endmodule
```

FLIP-FLOPS AND LATCHES

HDL Example 5-2

```
//D flip-flop
module D_FF (Q, D, CLK);
    output Q;
    input D, CLK;
    reg Q;
    always @(posedge CLK)
        Q = D;
endmodule
```

FLIP-FLOPS AND LATCHES

HDL Example 5-3

```
//T flip-flop from D flip-flop and gates
module TFF (Q, T, CLK, RST);
    output Q;
    input T, CLK, RST;
    reg DT;
    assign DT = Q ^ T;
//Instantiate the D flip-flop
    DFF TF1 (Q, DT, CLK, RST);
Endmodule
```

```
// JK flip-flop from D flip-flop and gates
module JKFF (Q, J, K, CLK, RST);
    output Q;
    input J, K, CLK, RST;
    wire JK;
    assign JK = (J & ~Q) | (~K & Q);
// Instantiate D flipflop
    DFF JK1 (Q, JK, CLK, RST);
endmodule
```

FLIP-FLOPS AND LATCHES

HDL Example 5-4

```
// Functional description of JK flip-flop
module JK_FF (J, K, CLK, Q, Qnot);
    output Q, Qnot;
    input J, K, CLK;
    reg Q;
    assign Qnot = ~Q;
    always @ (posedge CLK)
        case ({J, K})
            2`b00: Q = Q;
            2`b01: Q = 1`b0;
            2`b10: Q = 1`b1;
            2`b11: Q = ~Q;
        endcase
endmodule
```

STATE DIAGRAM

HDL Example 5-5

```
//Mealy state diagram (Fig. 5-16)
module Mealy_mdl (x, y, CLK, RST);
    input x, CLK, RST;
    output y;
    reg y;
    reg [1:0] Prstate, Nxtstate;
    parameter S0 = 2`b00, S1 = 2`b01, S2 = 2`b10, S3 =
        2`b11;
    always @ (posedge CLK or negedge RST)
        if (~RST) Prstate = S0; //Initialize to state S0
        else Prstate = Nxtstate; //Clock operations

    always @ (Prstate or x)
        case (Prstate)
```

STATE DIAGRAM

```
S0: if (x) Nxtstate = S1;  
      else Nxtstate = S0;  
S1: if (x) Nxtstate = S3;  
      else Nxtstate = S0;  
S2: if (x) Nxtstate = S0;  
      else Nxtstate = S2;  
S3: if (x) Nxtstate = S2;  
      else Nxtstate = S0;  
endcase  
always @ (Prstate or x)      //Evaluate output  
    case (Prestate)  
      S0: y = 0;  
      S1: if (x) y = 1`b0; else y = 1`b1;  
      S2: if (x) y = 1`b0; else y = 1`b1;  
      S3: if (x) y = 1`b0; else y = 1`b1;  
endcase  
endmodule
```

STATE DIAGRAM

HDL Example 5-6

```
//Moore state diagram (Fig. 5-19)
module Moore_md1 (x, AB, CLK, RST);
    input x, CLK, RST;
    output [1:0] AB;
    reg [1:0] state;
    parameter S0 = 2`b00, S1 = 2`b01, S2 = 2`b10, S3 = 2`b11;
    always @ (posedge CLK or negedge RST)
        if (~RST) state = S0; //Initialize to state S0
        else
            case (state)
                S0: if (~x) state = S1; else state = S0;
                S1: if (x) state = S2; else state = S3;
                S2: if (~x) state = S3; else state = S2;
                S3: if (~x) state = S0; else state = S3;
            endcase
            assign AB = state; //Output of flip-flops
    endmodule
```

STRUCTURAL DESCRIPTION

HDL Example 5-7

```
//Structural description of sequential circuit
//See Fig. 5-20 (a)
module Tcircuit (x, y, A, B, CLK, RST);
    input x, CLK, RST;
    output y, A, B;
    wire TA, TB;
//Flip-flop input equations
    assign TB = x,
            TA = x & B;
//Output equation
    assign y = A & B;
//Instantiate T flip-flops
    T_FF BF (B, TB, CLK, RST);
    T_FF AF (A, TA, CLK, RST);
endmodule
```

STRUCTURAL DESCRIPTION

```
//T flip-flop
module T_FF (Q, T, CLK, RST);
    output Q;
    input T, CLK, RST;
    reg Q;
    always @ (posedge CLK or negedge RST)
        if (~RST) Q = 1`b0;
        else Q = Q ^ T;
endmodule
```

STRUCTURAL DESCRIPTION

```
//Stimulus for testing sequential circuit
module testTcircuit;
  reg x, CLK, RST; //inputs for circuit
  wire y, A, B; //output from circuit
  Tcircuit TC (x, y, A, B, CLK, RST); //instantiate circuit
  initial
    begin
      RST = 0;
      CLK = 0;
      #5 RST = 1;
      repeat (16)
        #5 CLK = ~CLK;
    end
  initial
    begin
      x = 0;
      #15 x = 1;
      repeat (8)
        #10 x = ~x;
    end
endmodule
```

STRUCTURAL DESCRIPTION

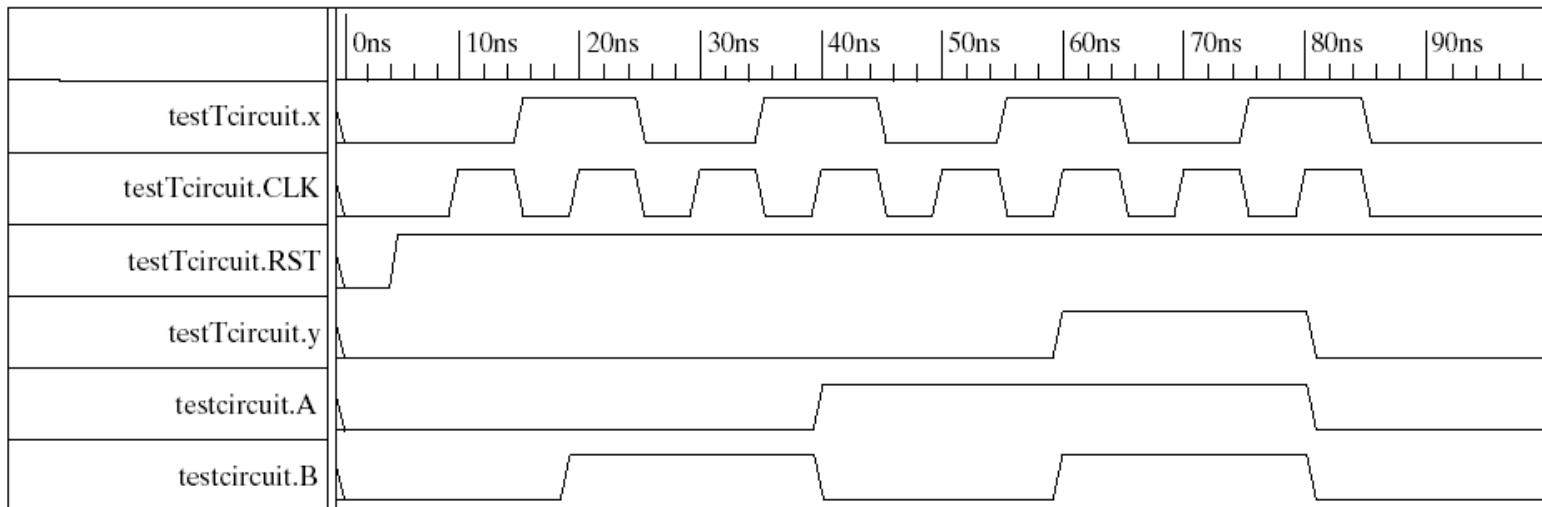


Fig. 5-21 Simulation Output of HDL Example 5-7

5-6 STATE REDUCTION AND ASSIGNMENT

- The analysis of sequential circuits starts from a circuit diagram and culminates in a state table or diagram.
- The design of a sequential circuit starts from a set of specifications and culminates discusses certain properties of sequential circuits that may be used to **reduce the number of gates and flip-flops during the design.**

STATE REDUCTION

- The reduction of the number of flip-flops in a sequential circuit is referred to as the **state-reduction** problem. **State-reduction** algorithms are concerned with procedures for reducing the number of states in a state table, while keeping the external input-output requirements unchanged.
- Since **m flip-flops produce 2^m states**, a reduction in the number of states may result in a reduction in the number of flip-flops. An unpredictable effect in reducing the number of flip-flops is that sometimes the equivalent circuit may require more combinational gates.

STATE REDUCTION

Example :

state
input
output

a	a	b	c	d	e	f	f	g	a
0	1	0	1	0	1	1	0	1	0
0	0	0	0	0	0	1	1	0	0

Initial point

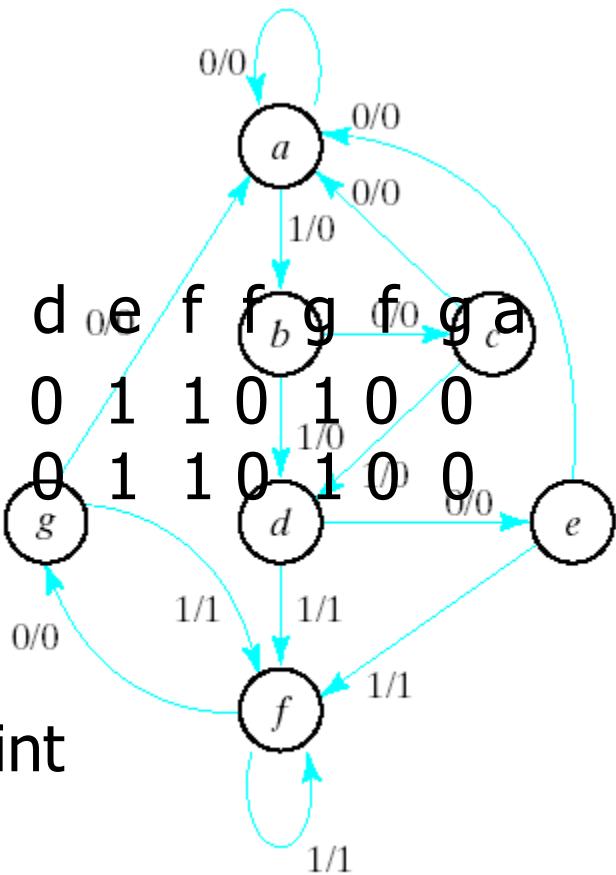


Fig. 5-22 State Diagram

STATE REDUCTION

We now proceed to reduce the number of states for this example. First, we need the **state table**; it is more convenient to apply procedures for state reduction using a table rather than a diagram. The state table of the circuit is listed in Table 5-6 and is obtained directly from the state diagram.

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	g	f	0	1
g	a	f	0	1

STATE REDUCTION

States **g** and **e** are two such states: they both go to states **a** and **f** and have outputs of **0** and **1** for $x=0$ and $x=1$, respectively. Therefore, states **g** and **e** are equivalent and one of these states can be removed. The procedure of removing a state and replacing it by its equivalent is demonstrated in Table 5-7.

The row with present **g** is removed and state **g** is replaced by state **e** each time it occurs in the next-state columns.

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
a	b		0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	e	f	0	1

STATE REDUCTION

Present state **f** now has next states **e** and **f** and **outputs 0** and **1** for **x=0** and **x=1**, respectively. The same next states and outputs appear in the row with present state **d**. Therefore, states **f** and **d** are equivalent and state **f** can be removed and replaced by **d**. The final reduced table is shown in Table 5-8.

The state diagram for the reduced table consists of only five states and is shown in Fig. 5-23.

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
a	b	c	0	0
b	c	d	0	0
c	d	d	0	0
d	e	d	0	1
e	a	d	0	1

STATE REDUCTION

state	<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>d</i>	<i>c</i>
input	0	1	0	1	0	1	1	0
output	0	0	0	0	0	1	1	0

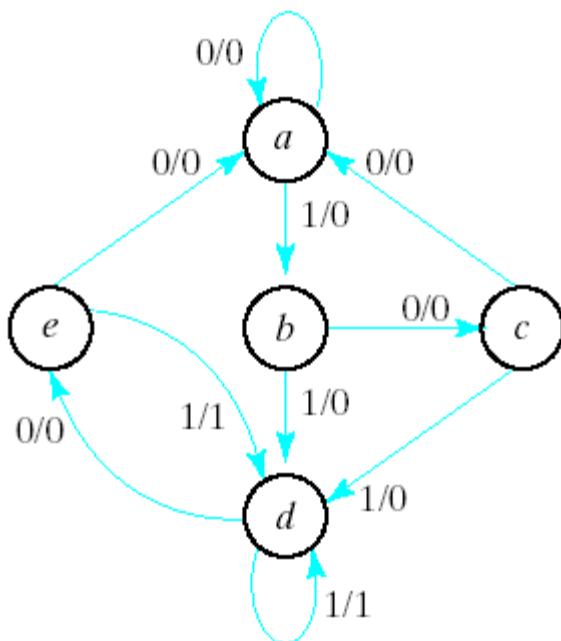


Fig. 5-23 Reduced State Diagram

STATE ASSIGNMENT

Table 5-9
Three Possible Binary State Assignments

State	Assignment 1 Binary	Assignment 2 Gray code	Assignment 3 One-hot
a	000	000	00001
b	0		
c	0		
d	0		
e	1		

Table 5-10
Reduced State Table with Binary Assignment 1

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	000	001	0	0
001	010	011	0	0
010	000	011	0	0
011	100	011	0	1
100	000	011	0	1

5-7 DESIGN PROCEDURE

The procedure for designing synchronous sequential circuits can be summarized by a list of recommended steps.

1. From the word description and specifications of the desired operation, derive a state diagram for the circuit.
2. Reduce the number of states if necessary.
3. Assign binary values to the states.
4. Obtain the binary-coded state table.
5. Choose the type of flip-flops to be used.
6. Derive the simplified flip-flop input equations and output equations.
7. Draw the logic diagram.

DESIGN PROCEDURE

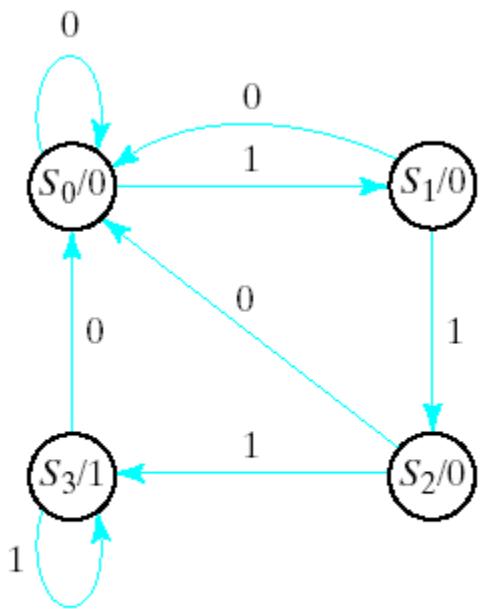


Fig. 5-24 State Diagram for Sequence Detector

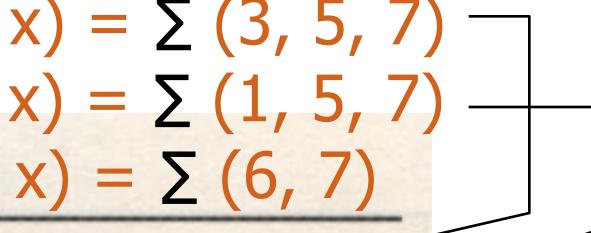
SYNTHESIS USING D FLIP-FLOPS

$$A(t + 1) = D_A(A, B, x) = \Sigma (3, 5, 7)$$

$$B(t + 1) = D_B(A, B, x) = \Sigma (1, 5, 7)$$

Table 5-11
State Table for Sequence Detector

Present State		Input	Next State		Output
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	1	1	1	1

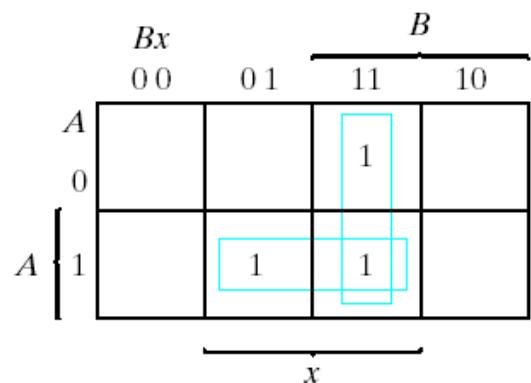


SYNTHESIS USING D FLIP-FLOPS

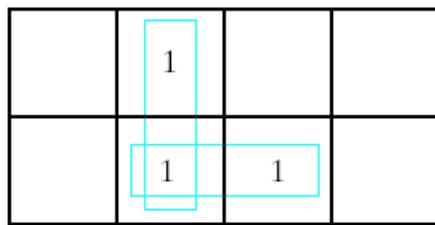
$$D_A = Ax + Bx$$

$$D_B = Ax + B'x$$

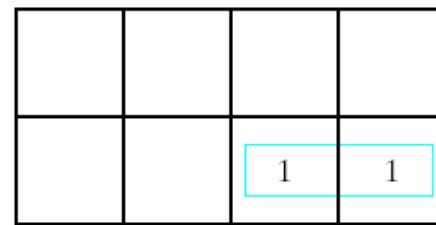
$$y = AB$$



$$D_A = Ax + Bx$$



$$D_B = Ax + B'x$$



$$y = AB$$

Fig. 5-25 Maps for Sequence Detector

SYNTHESIS USING D FLIP-FLOPS

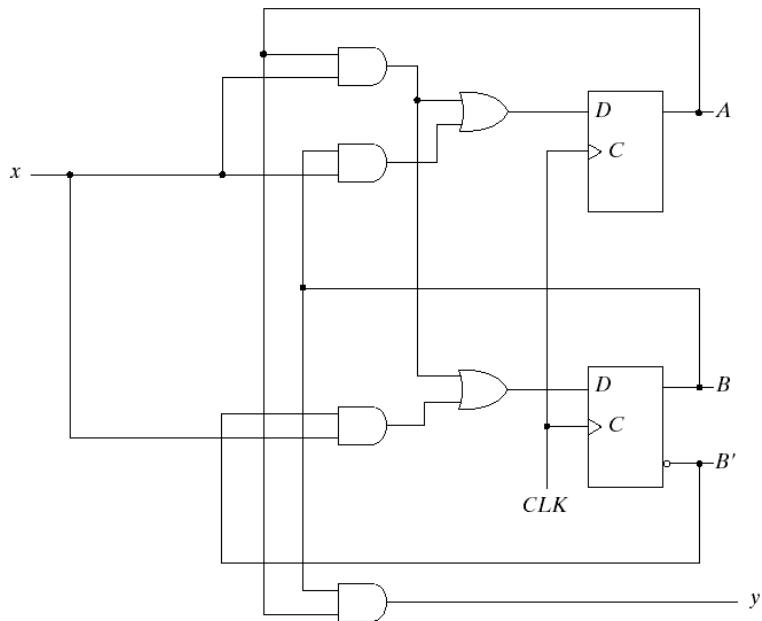


Fig. 5-26 Logic Diagram of Sequence Detector

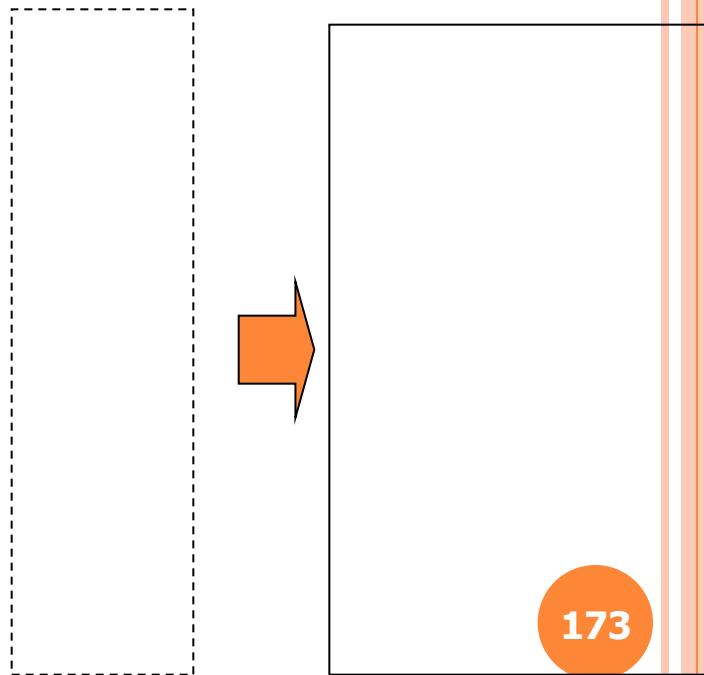
SYNTHESIS USING JK FLIP-FLOPS

Different from Table 5-11 !!

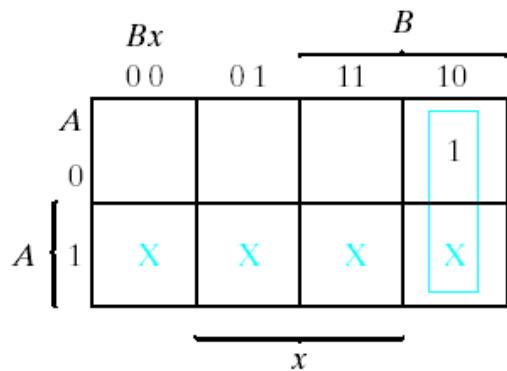
Table 5-12
Flip-Flop Excitation Tables

$Q(t)$	$Q(t + 1)$			Flop Inputs		
		J	K	J_A	J_B	K_B
0	0	0	X	0	0	X
0	1	1	X	0	1	X
1	0	X	1	0	X	0
1	1	X	0	0	1	X
		(a)JK		1	X	0
				1	X	1

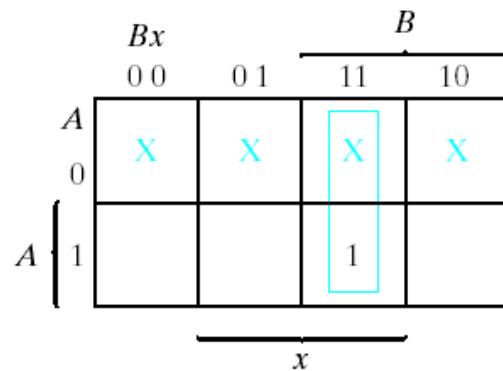
Ref. Table 5-1



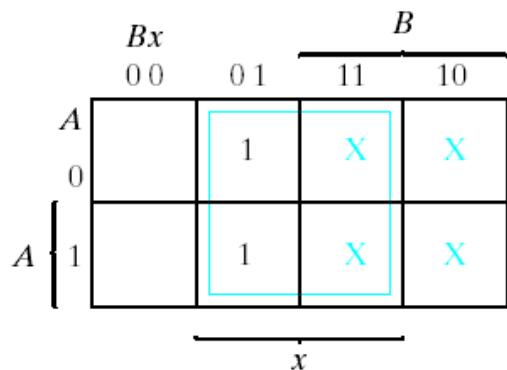
SYNTHESIS USING JK FLIP-FLOPS



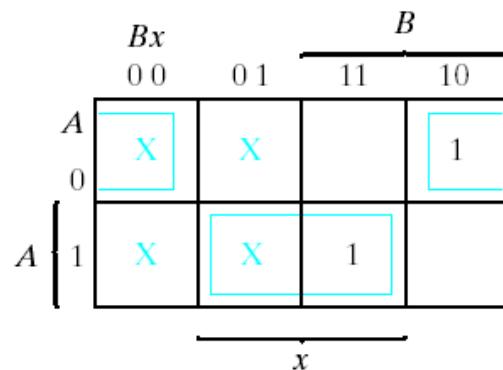
$$J_A = Bx'$$



$$K_A = Bx$$



$$J_B = x$$



$$K_B = (A \oplus x)'$$

SYNTHESIS USING JK FLIP-FLOPS

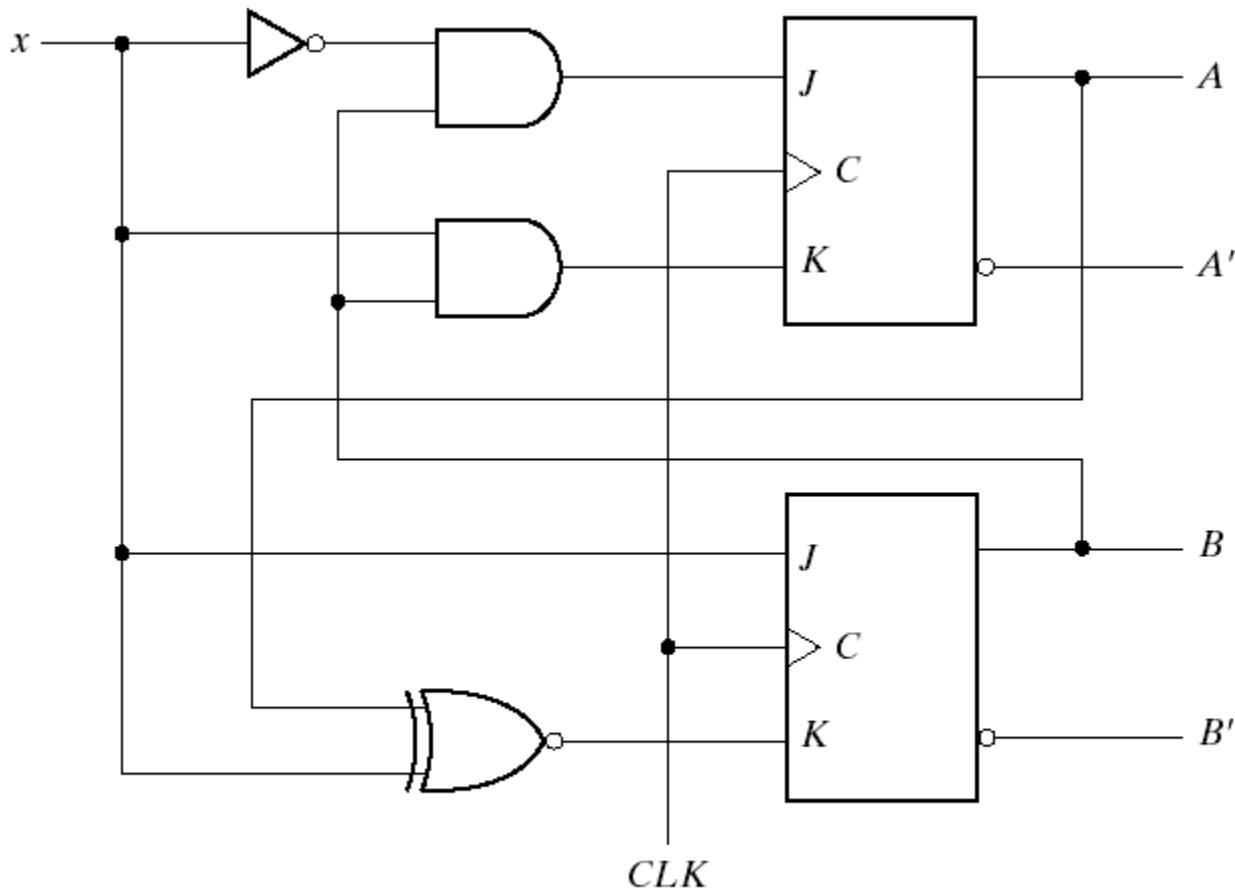


Fig. 5-28 Logic Diagram for Sequential Circuit with JK Flip-Flops

SYNTHESIS USING T FLIP-FLOPS

The synthesis using **T flip-flops** will be demonstrated by designing a binary counter. An n-bit binary counter consists of n flip-flops that can count in binary from 0 to $2^n - 1$. The state diagram of a 3-bit counter is shown in Fig. 5-29.

$Q(t)$	$Q(t + 1)$	T
0	0	0
0	1	1
1	0	1
1	1	0

Ref. Table 5-1 (b)T

SYNTHESIS USING T FLIP-FLOPS

Table 5-14
State Table for 3-Bit Counter

Present State			Next State			Flip-Flop Inputs		
A_2	A_1	A_0	A_2	A	A_0	T_{A2}	T_{A1}	T_{A0}
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	1	1
1	1	1	0	0	0	1	1	1

SYNTHESIS USING T FLIP-FLOPS

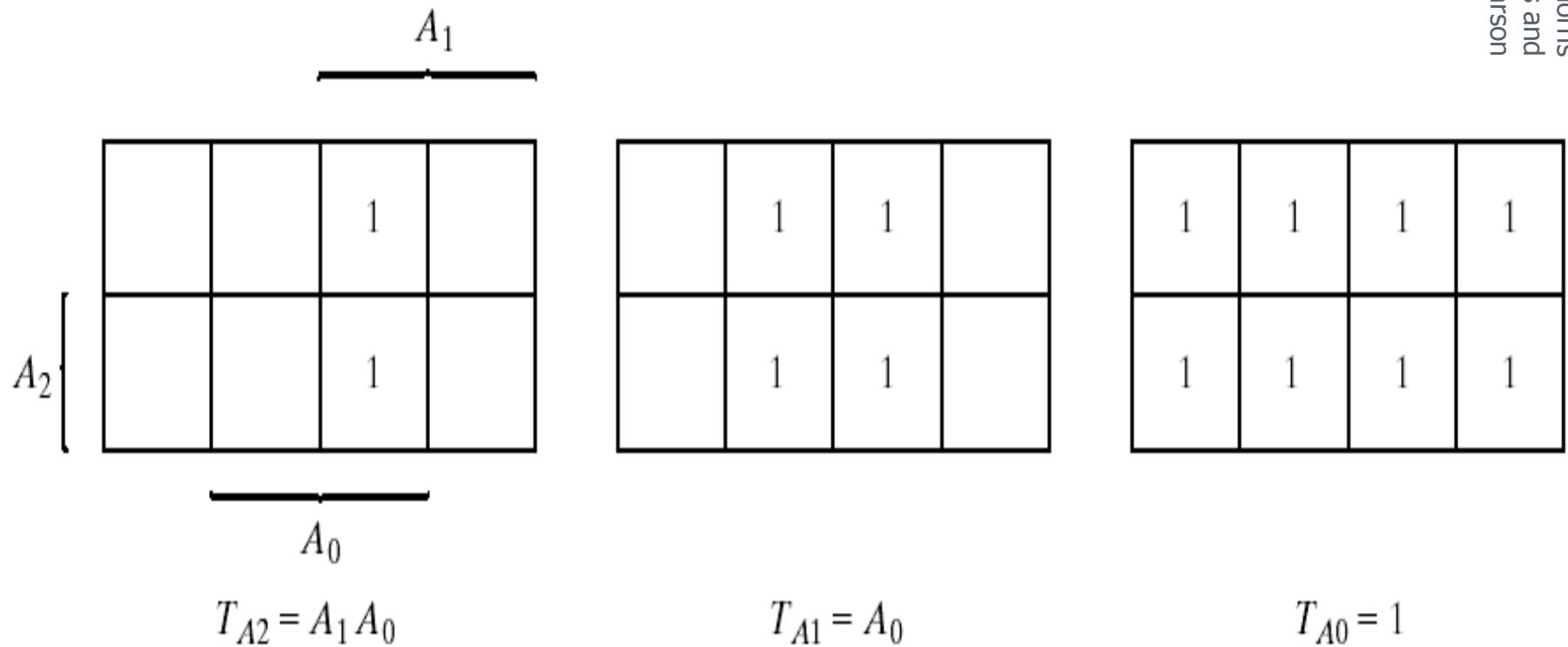


Fig. 5-30 Maps for 3-Bit Binary Counter

SYNTHESIS USING T FLIP-FLOPS

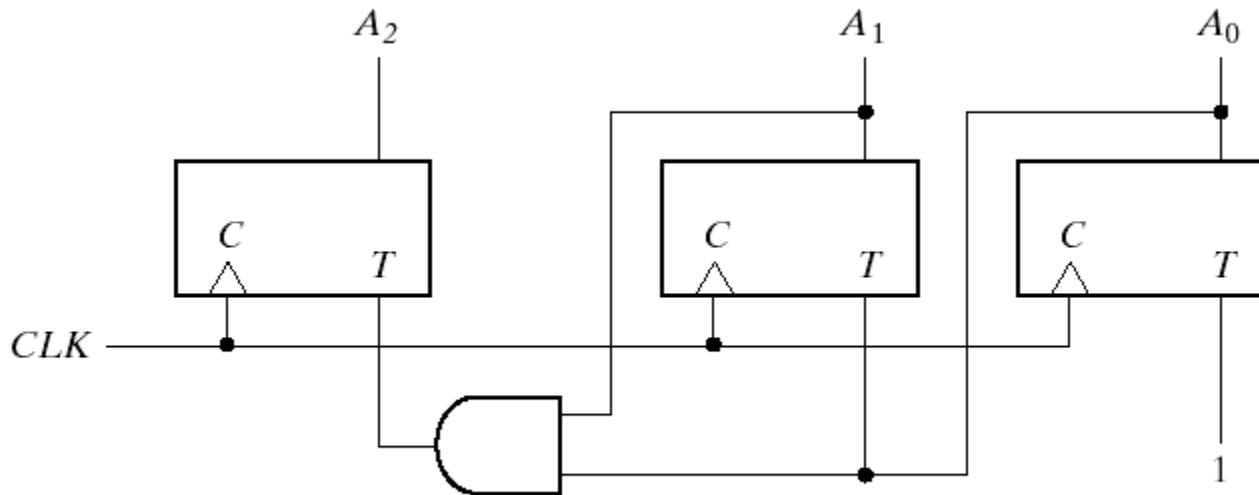


Fig. 5-31 Logic Diagram of 3-Bit Binary Counter