Computing various objects of an algebra from the poset of torsion classes (Demo)

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Outline

About `SageMath`

Installation

Input the poset of torsion classes

Enjoy!

About SageMath

What is SageMath?

A free open-source mathematics software system

Why SageMath?

- SageMath provides a framework for finite posets, finite lattices, simplicial complexes, and so on.
- We can construct them and compute various things, e.g. the set of join-irreducibles in the lattice.
- SageMath can check whether two objects are isomorphic.

tors_lattice

I've developed a SageMath program tors_lattice.py

1. Input the poset of torsion classes, then
2. This program can construct
    - the lattice of wide subcategories,
    - the simplicial complex $\Delta(\Lambda)$,

    etc.

Installation

Step 1: Install SageMath

- Install SageMath (ver $\geq$ 9.0) on your computer:
  Download from the official cite
  https://www.sagemath.org/index.html
  and just install it!
- From now on, we'll use SageMath Notebook

Another option:

You can use CoCalc without installing sage

Step 2: Download my program and load it

- Download my file from

  `https://haruhisa-enomoto.github.io/files/tors_lattice.py`
- Create a SageMath notebook in the same directory as you download this file.
- Execute `load("tors_lattice.py")`

Then you can use it!

Input the poset of torsion classes

Input your poset

First, input the poset of torsion classes of your algebra in `SageMath`, and name it `poset` for example.

Ways to input posets

1. For path algebras and preprojective algebras of Dynkin type, `SageMath` already has it! (later)
2. If you have a Hasse diagram, then you can input it manually.
3. You can import it from Jan Geuenich's String Applet using my other program, next slide.

String Applet to SageMath converter

String Applet can calculate the poset of torsion classes of any representation-fintie special biserial algebra.

You can import it in SageMath using my `converter`.

1. Input your algebra in String Applet,
   and show its $s\tau$-Tilting quiver.
2. Export a latex file (e.g. data.tex) in your working directory.
3. Download

   `https://haruhisa-enomoto.github.io/files/converter.py`

   in your working directory
4. Load it in your notebook by `load("converter.py")`
5. Execute `poset = Poset(SAtoSage("data.tex"))`

Enjoy!

What's next?

Now you have your `poset`. Then execute

```
tors = TorsLattice(poset)
```

Once you have done it, you can construct various things, e.g.

- `tors.wide_lattice()`
  the lattice of wide subcategories
- `tors.ice_lattice()`
  the lattice of ICE-closed subcategories
- `tors.heart_poset()`
  the poset of torsion hearts
- `tors.s_tau_tilt_complex()`
  the simplicial complex $\Delta(\Lambda)$ of τ-tilting pairs

See Manual for the list of all things you can do.

Demo

Q: Dynkin quiver, Π_Q: its preprojective algebra

algebra	tors$(-)$	wide$(-)$	$\Delta(-)$
kQ	Cambrian lattice	Non-crossing partition	Cluster cpx
Π_Q	Weyl grp with weak order	shard intersection order	(???)

All these objects are already in `SageMath`!

Let's check the above table, and guess (???)!

OEIS is useful.

Demo

Q: Dynkin quiver, Π_Q: its preprojective algebra

algebra	$\mathrm{tors}(-)$	$\Delta(-)$
kQ	Cambrian lattice	Cluster cpx (dual associahedron)
Π_Q	Weyl grp with weak order	Coxeter cpx (dual permutahedron)

Demo: lattice properties for wide Λ

There're lots of properties `SageMath` can check for lattices.

Conjecture (some have been confirmed)

If Λ is τ-tilting finite, then wide Λ is:

- ranked (graded), with its rank function given by the number of simples
- Rank-symmetric
- Relatively complemented
- (strongly) Sperner

Conjecture of ICE-closed subcategories

Sakai's Conjecture (not true...)

The number of Hasse arrows in $\mathrm{ICE}\,\Lambda$ starting at $\mathcal{C}$ is equal to the number of indecomposable Ext-projectives in $\mathcal{C}$.

True for hereditary and Nakayama algebras.

Find a counterexample!

Links

- On SageMath
  - SageMath tutorial
  - Finite Coxeter groups
  - Finite posets
  - Finite lattices and semilattices
- The lattice of torsion classes in SageMath
  - Download
  - Manual
- String Applet to SageMath converter
  - Download
  - Manual
- The SageMath notebook used in this demo
- SageMath codes in my website