



# *Autonomous Gyroscopic 2-Wheel Differential Robot*

Haruka Kido, James Vrtis, Luke Anderson

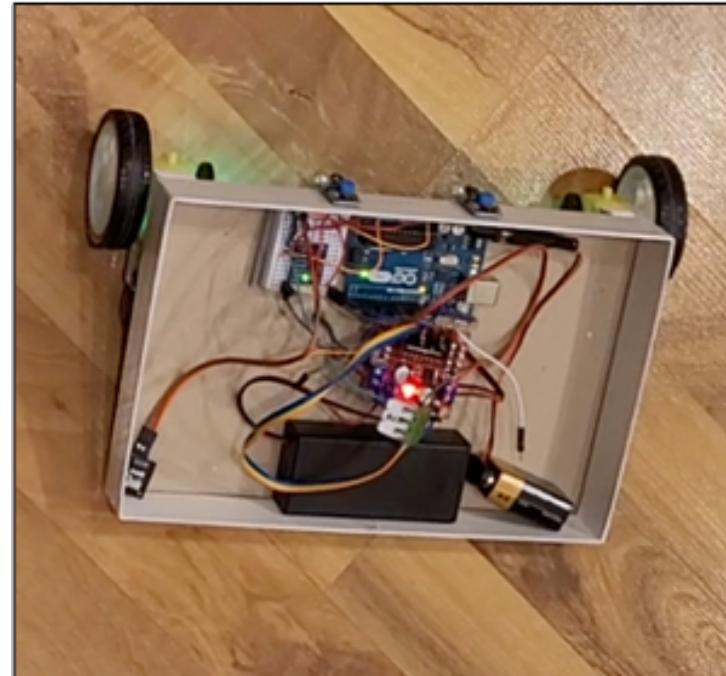
Professor Tarek Elderini  
**EE428: Robotics Fundamentals**  
University of North Dakota

# Outline

1. Introduction
2. Mathematical Model
3. Simulation Work
4. Experimental Work
5. Discussion/Future Works
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# Introduction/Background

This presentation demonstrates the implementation of an autonomous gyroscopic 2-wheel differential robot, including a forward and inverse kinematics simulation in MatLAB, a test hardware robot and programmed demonstration of a simple move forward and spin left motion, and a final configuration of a complete square path based on programming kinematics, gyroscopic speed responses, and remote-control functionality.





# Mathematical Model

The forward kinematics of a differential drive robot, the following variables are needed:  $L$  (length between wheels to center),  $r$  (diameter of wheels),  $\mathbf{x}$ ,  $\mathbf{y}$ ,  $\theta$  (pose matrix), &  $\varphi_1$  /  $\varphi_2$  (speed of the right /left wheels) [8]. (The inverse kinematics are explained through the simulation)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \varphi_1, \varphi_2) \quad (\text{overall speed in global reference frame})$$

$$x_{r1}(\text{wheel translation speed in reference position}) = \frac{r\varphi_1}{2} \quad (1)$$

$$x_{r2}(\text{wheel translation speed in reference position}) = \frac{r\varphi_2}{2} \quad (2)$$

$$\omega_1(\text{right wheel rotation velocity}) = \frac{r\varphi_1}{2*l} \quad (3)$$

$$\omega_2(\text{left wheel rotoation velocity}) = -\frac{r\varphi_2}{2*l} \quad (4)$$

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \theta \end{bmatrix} = \begin{bmatrix} \frac{r_R}{2} * \cos\theta & \frac{r_L}{2} * \cos\theta \\ \frac{r_R}{2} * \sin\theta & \frac{r_L}{2} * \sin\theta \\ \frac{r_R}{2L} & -\frac{r_R}{2L} \end{bmatrix} * \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \quad (5)$$

# Simulation Work

The  $\dot{\eta}$  is the desired velocity for the center of the robot; containing  $x$ ,  $y$ , and  $\Theta$ , which respectively are the distance and angular displacement of the robot. The  $u$  and  $r$  variables are the forward and angular velocities, respectively [1]. The variable  $\rho$  in Equation 7 calculate the distance between the desired pose and current pose.

Equation 8 & 9 calculate the angular displacement of the robot. Equation 10 calculates the angular displacement **error** between  $\eta$  the existing pose, once the margin of error is below a specified tolerance. Equation 11 is the **Jacobian matrix** vector by angle  $\theta$ .

$$\dot{\eta} = \begin{bmatrix} x_d \\ y_d \\ \theta_d \end{bmatrix} \quad (6)$$

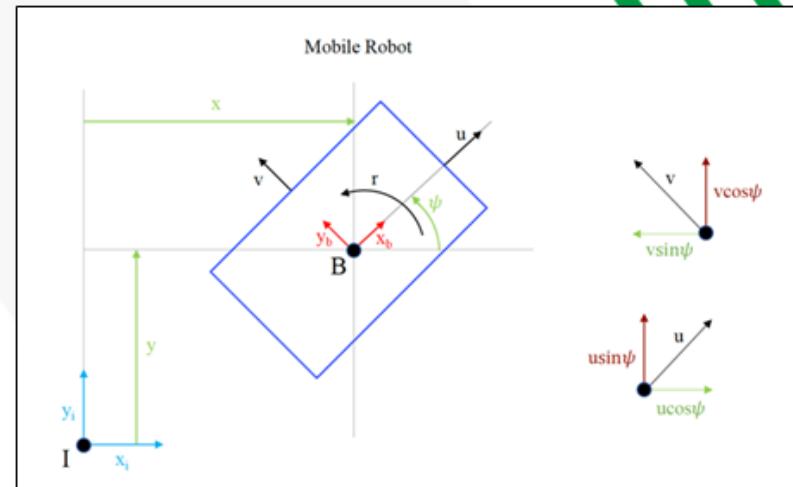
$$\rho = \sqrt{(\dot{\eta}(x) - \eta(x, t))^2 + (\dot{\eta}(y) - \eta(y, t))^2} \quad (7)$$

$$\theta_d = \text{atan}\left(\frac{y_d - y}{x_d - x}\right)(t) \quad (8)$$

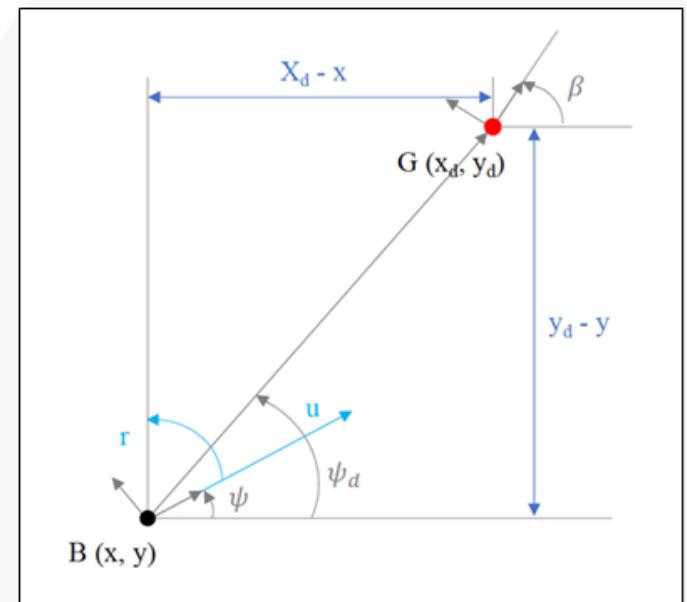
$$e_{error} = \begin{bmatrix} x_d \\ y_d \\ \theta_d(t) \end{bmatrix} \quad (9)$$

$$e_{radius-error} = \dot{\eta} - \eta(t) \quad (10)$$

$$\text{Jacobian Matrix} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$



Mobile robot reference frame [1]



Mobile robot line of site reference frame [1]

# Simulation Work Cont.

Found in Equation 12.b is where the matrix controller adjusts the forward and angular velocities, and can be tested upon the robot platform parameters in Equation 12.a [1].

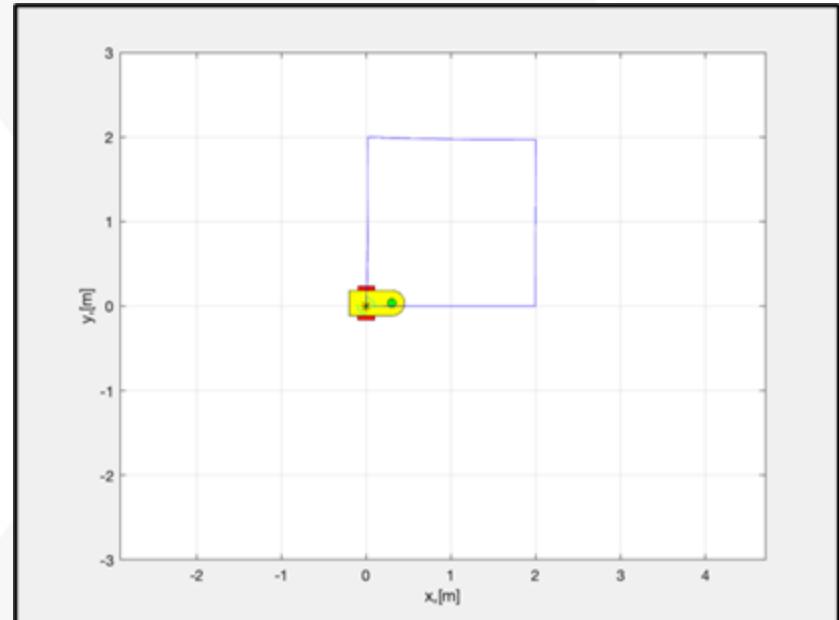
## Kinematics Velocity Controller

$$\begin{bmatrix} u(t) \\ 0 \\ r(t) \end{bmatrix} = Z = J^{-1} * \begin{pmatrix} K_{vel-x} & 0 & 0 \\ 0 & K_{vel-y} & 0 \\ 0 & 0 & K_{vel-\theta} \end{pmatrix} * \begin{bmatrix} x_d - x(t) \\ y_d - y(t) \\ \theta_d - \theta(t) \end{bmatrix} \quad (12.a)$$

$$\begin{bmatrix} u_z(t) \\ 0 \\ r_z(t) \end{bmatrix} = \left( \begin{bmatrix} a/2 & a/2 \\ 0 & 0 \\ \frac{a}{2*L} & -\frac{a}{2*L} \end{bmatrix} * \begin{bmatrix} u(t) \\ 0 \\ r(t) \end{bmatrix} \right) \quad (12.b)$$

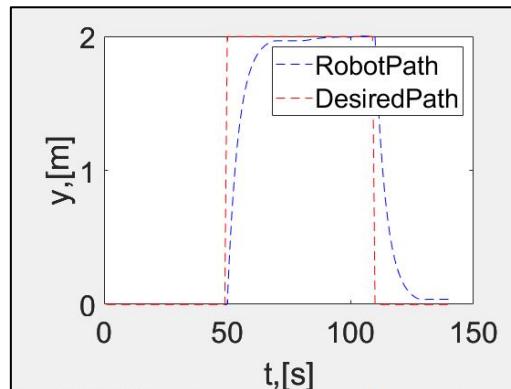
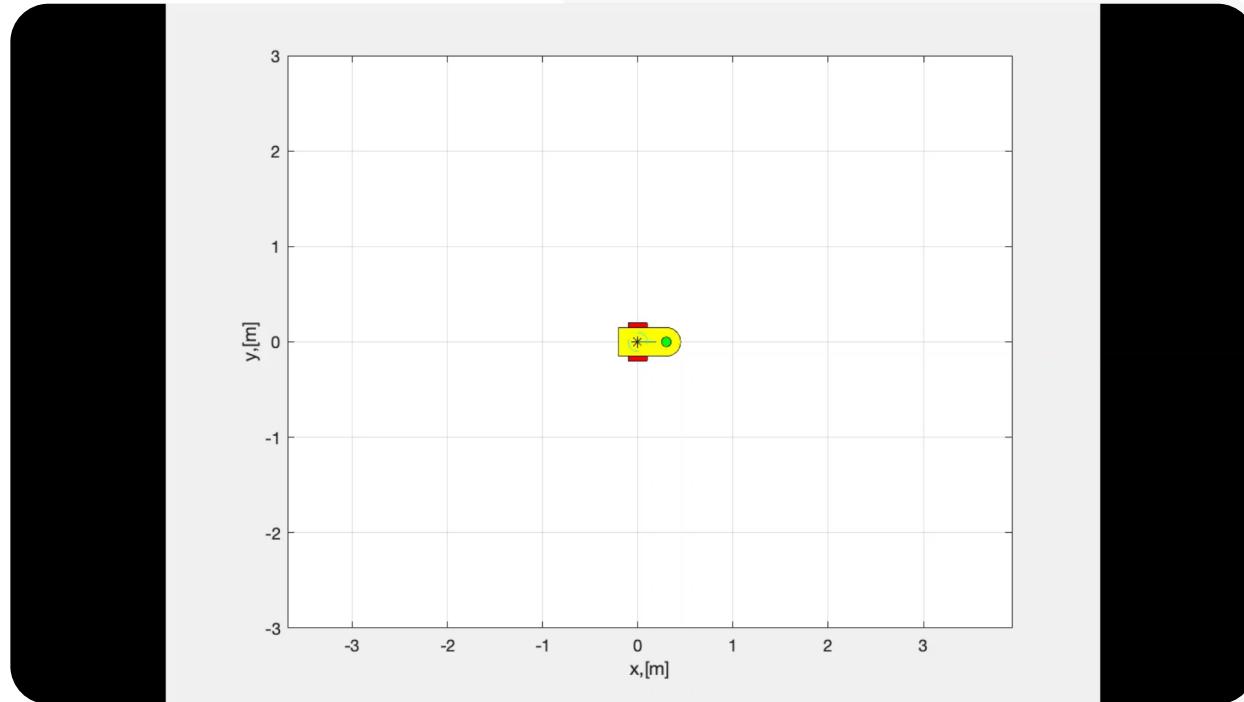
## Position Controller

$$\eta(:, t+1) = \eta(t) + \int_t^{\infty} error(t) dt = 0 \quad (13)$$

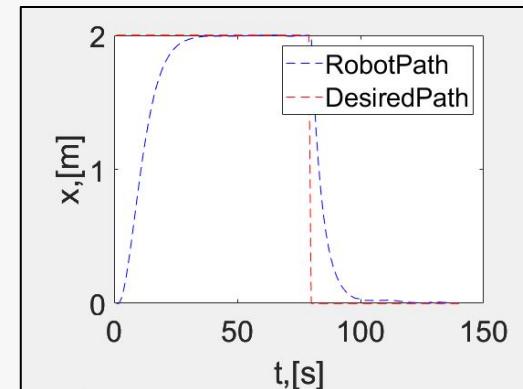


MatLAB Simulation

# Simulation Work Cont.



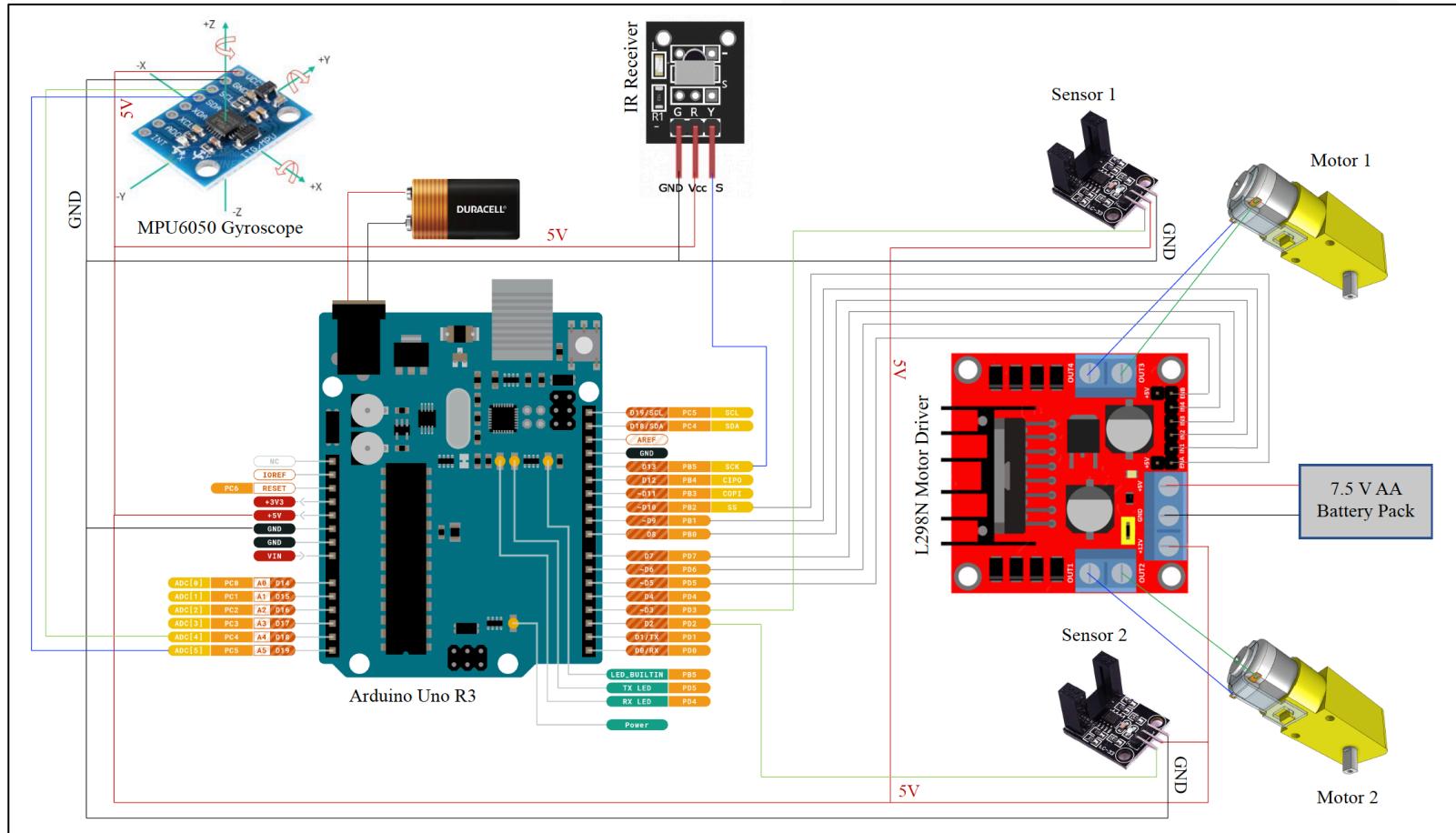
Graphical model of y-axis robot path versus time



Graphical model of x-axis robot path versus time

# Experimental Work:

## Circuit Diagram

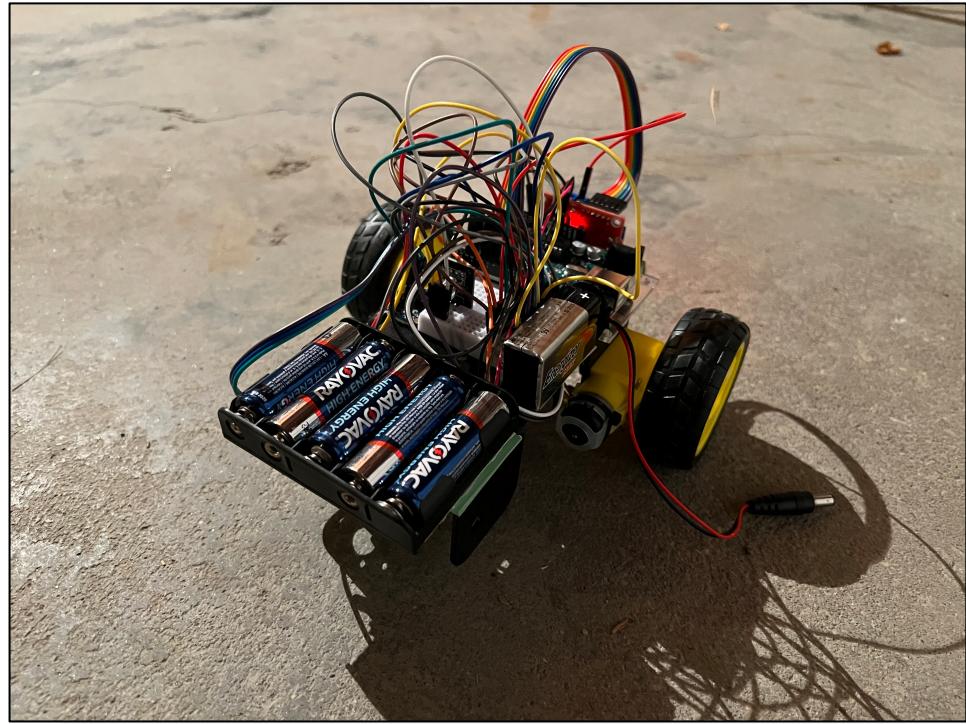


Autonomous Gyroscopic 2-Wheel Differential Drive Hardware Circuit

# Experimental Work: Preliminary Configuration and Testing

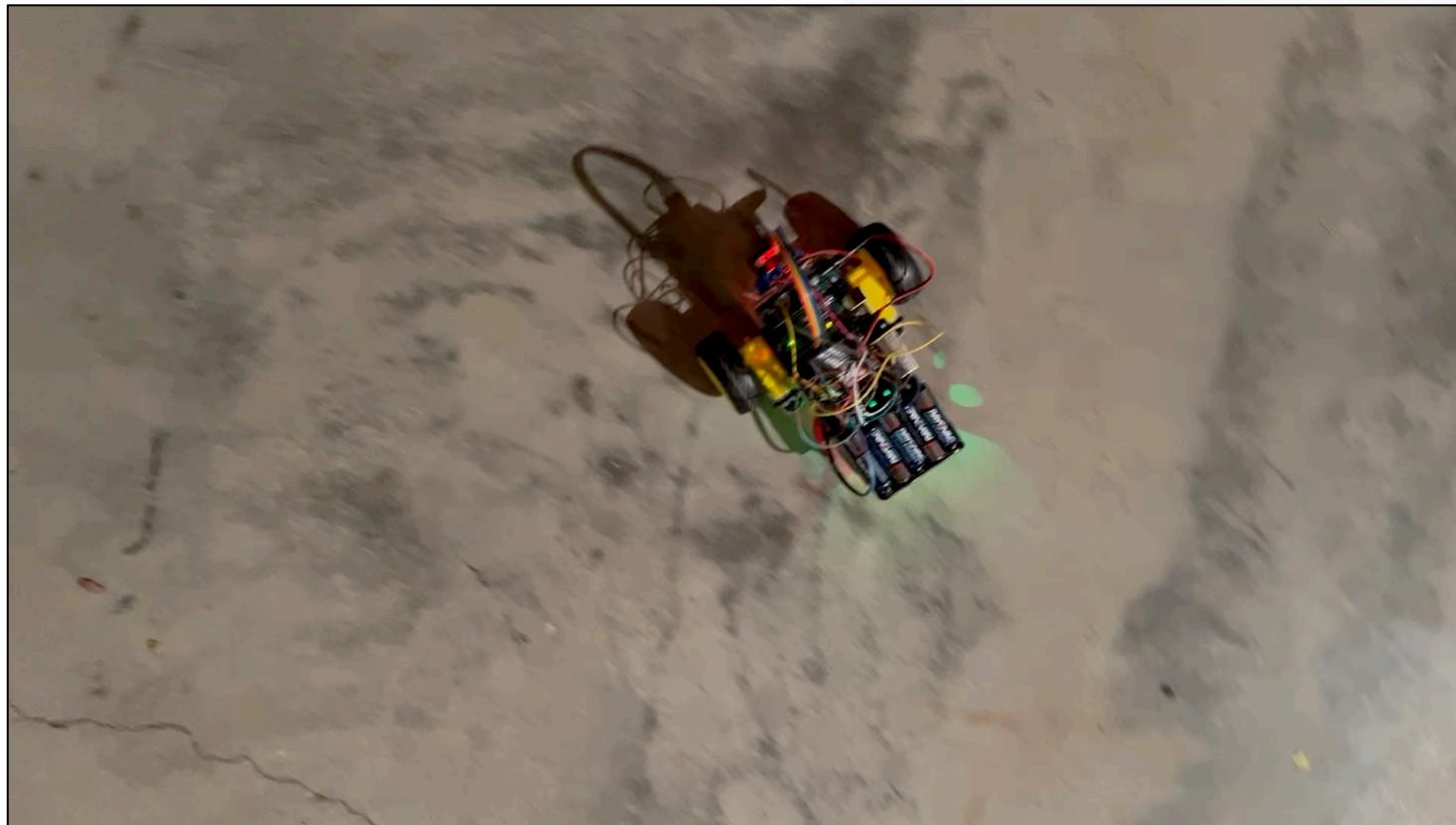


Soldering electrical leads to motor



Preliminary Test Differential Robot with front castor wheel

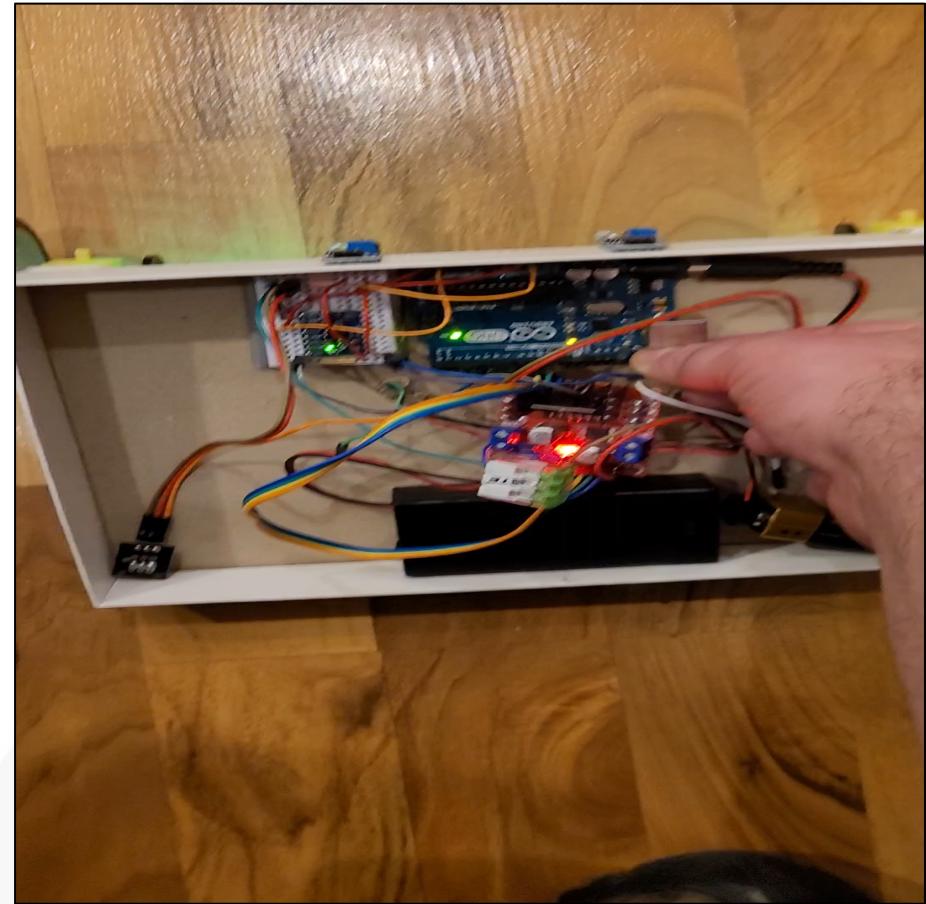
# Experimental Work: Preliminary Configuration and Testing



Preliminary Differential Test Robot in Autonomous Action

# Experimental Work

- Testing to see if we could reproduce the square simulation.
- Using the MPU6050 gyro, we could control the turning angle.
- Using the encoders, we could control the distance travel from the wheels.



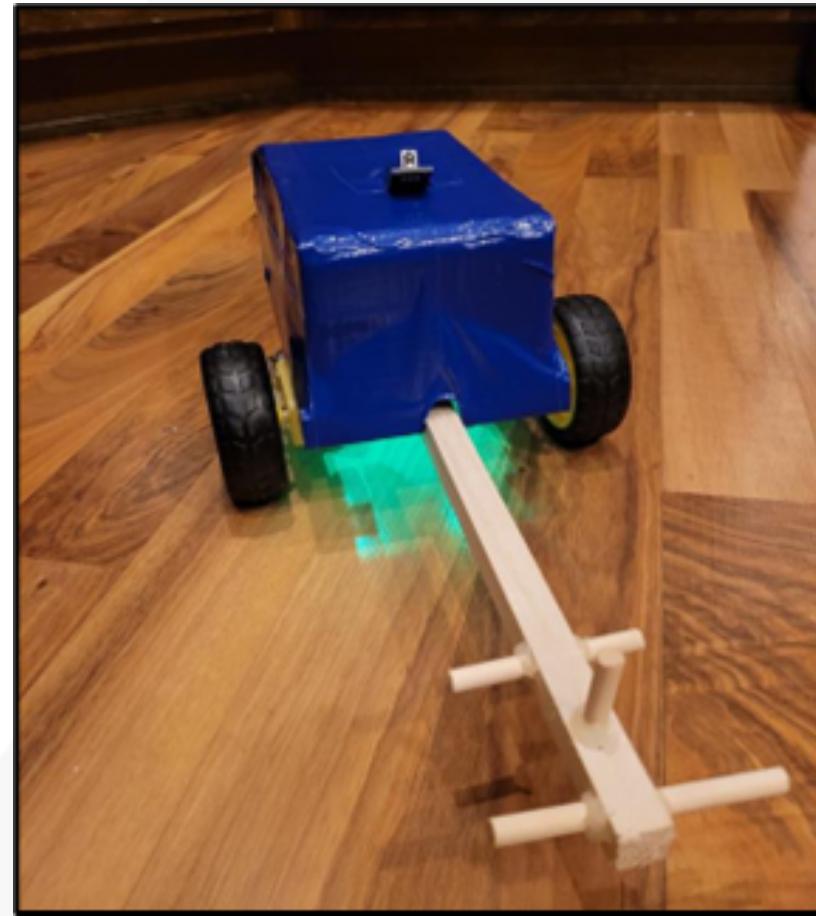
Experimental Robot with Gyro and Encoders: Square Path

# Experimental Work Cont.

Combined the two robots to produce the battle bot.

General Specs:

- 2x lithium-ion batteries primary power
- 9V battery for the Arduino
- IR remote control
- Swinging spike arm for weapon



Experimental Battle Bot



# Discussion/Future Works

## Explore Error Sources:

- Simulations:
  - Error with the line-of-sight method used to perform the autonomous motion: would occur when  $\dot{\eta}$  contained a value that was previously used in a preceding desired destination
  - Corrected when the robot aligns itself in the negative  $\dot{\theta}$  orientation
  - May be caused by the mathematical line of site method to find the robot's path from the starting location to the endpoint location
- Test Drives:
  - Initial failure possibly due to electrical conductivity insufficiencies, power consumption issues, or motor driver inefficiencies, did not function to continue the motion drive in any direction after the first implementation

## Explore Areas of Advancement:

- Adjusting the velocity and computational time for a loop needed for the robot to move from the starting to the ending point for the simulation
- Adding machine vision for image-processing in an autonomous robot
- Physical limitations and design changes of hardware parts (power supplies, board and sensors)
- Higher-level software adaptations and libraries for more complex manipulator systems

# References

- [1] A. Thondiyath and S. Mohan, "Wheeled Mobile Robots," Indian Institute of Technology, Palakkad.
- [2] Siciliano, Bruno, Sciavicco, Lorenzo, Villani, Luigi, Oriolo, Giuseppe, "Robotics Modelling, Planning and Control," Springer, 2009, ISBN: 978-1-84628-642-1
- [3] Warren, John-David, Adams, Josh, Molle, Harald, "Arduino Robotics," Technology in Action, 2011
- [4] Edwards, Stephen A., "Assembly Instructions for a Motor Robot Car Kit 2WD, L298N Motor driver, HC-SC04 Ultrasonic module, Arduino," July 2018
- [5] B. Siciliano, L. Sciavicco, L. Villani and G. Oriolo, "Robotics Modelling, Planning and Control," Springer, 2009.
- [6] "Arduino UNO R3," Arduino.cc, <https://docs.arduino.cc/hardware/uno-rev3>
- [7] Muir, Patrick F., Neuman Charles P., "Kinematic Modeling of Wheeled Mobile Robots," Department of Electrical and Computer Engineering, The Robotics Institute, Carnegie-Mellon University, 1986.
- [8] Siegwart, Roland., Nourbakhsh Illah R. "Introduction to Autonomous Mobile Robots" Massachusetts Institute of Technology, 2004.



# Thank You! Questions?