CSE 321 - Introduction to Algorithms - Homework #1

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Q1)

a) $\log_2 n^2+1 \in O(n) \longrightarrow \lim_{n\to\infty} \frac{\log_2(n^2+1)}{n}$ = L'hopital = $\lim_{n\to\infty} \frac{2n}{(n^2+1).(\ln 2)} = 0$

Therefore, n increases faster than $\log(n^2+1)$. so the statement is TRUE.

b) $\sqrt{n(n+1)} \in \Omega(n) \longrightarrow \lim_{n\to\infty} \frac{(n^2+n)^{1/2}}{n}$ = L'hopital = $\lim_{n\to\infty} \frac{(2n+1).(n^2+n)^{-1/2}}{2}$

$\lim_{n\to\infty} \frac{(2n+1)}{2(n^2+n)^{1/2}} = \lim_{n\to\infty} \frac{2n}{2n+2\sqrt{n}} = 1$, Therefore $\sqrt{n(n+1)} \in \Omega(n)$. It is TRUE.

c) $n^{n-1} \in \Theta(n^n) \longrightarrow \lim_{n\to\infty} \frac{n^{n-1}}{n^n} = \lim_{n\to\infty} \frac{n^n}{n^n.n} = 0$. n^n increases faster than n^{n-1}. Therefore this statement is FALSE.

d) $O(2^n+n^3) \subset O(4^n) \longrightarrow 2^n+n^3 \in o(4^n) \longrightarrow \lim_{n\to\infty} \frac{2^n+n^3}{4^n} = 0$. Because Exponential functions grows faster than. This statement is TRUE.

e) $O(2\log_3 \sqrt[3]{n}) \subset O(3\log_2 n^2) \longrightarrow 2\log_3 \sqrt[3]{n} \in o(3\log_2 n^2) \longrightarrow \lim_{n\to\infty} \frac{2\log_3 n^{1/3}}{3\log_2 n^2}$

$\lim_{n\to\infty} \frac{2.\log_3 n}{18.\log_2 n} = \lim_{n\to\infty} \frac{2}{18} \cdot \frac{\frac{\log n}{\log 3}}{\frac{\log n}{\log 2}} = \frac{1}{9} \cdot \lim_{n\to\infty} \frac{\log 2}{\log 3}$ = constant

$2\log_3 \sqrt[3]{n} \in \Theta(3\log_2 n^2)$. Therefore this statement is FALSE.

f) $\lim_{n\to\infty} \frac{\log_2 \sqrt{n}}{(\log_2 n)^2} = \lim_{n\to\infty} \frac{\log_2 n^{1/2}}{\log_2 n.\log_2 n} = \lim_{n\to\infty} \frac{\log_2 n^{1/2}}{\log_2 2n} =$

$\lim_{n\to\infty} \frac{1}{2} \cdot \frac{\log_2 n}{\log_2 2n} =$ constant $\longrightarrow$ Therefore $\log_2 \sqrt{n} \in \Theta(\log_2 n)^2$

$\therefore$ They are of the same asymptotical order.

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Q2)

$10^n > 2^n$ because the base value is greater.

$n^3 > n^2 > \sqrt{n}$ because of exponent values of numbers.

$n^2 \log n > n^2$ because of having $\log n$.

$\lim_{n\to\infty} \frac{\sqrt{n}}{\log n} = \lim_{n\to\infty} \frac{n^{1/2}}{\log n} = \lim_{n\to\infty} \frac{1}{2} \cdot \frac{n \cdot \ln 10}{\sqrt{n}} = \infty$. Therefore $\sqrt{n} > \log n$

$\lim_{n\to\infty} \frac{2^n}{8^{\log_2 n}} = \lim_{n\to\infty} \frac{2^n}{2^{\log_8 n}} = \infty$. (Because n's growth rate > $\log_8 n$'s growth rate)

Therefore $2^n > 8^{\log_2 n}$.

The ranking of growth rates :

$\log n < \sqrt{n} < n^2 < n^2 \log n < n^3 < 8^{\log_2 n} < 2^n < 10^n$

($8^{\log_2 n} > n^3$ because exponential functions more increases than power functions.)

Q3)

a)
```
void f(int my_array[]){                               → n+1
    for (int i=0; i<sizeofArray){                     → 1
        if (my_array[i] < first_element){             → 1
            second_element = first_element;           → 1
            first_element = my_array[i];              → 1
        }
        else if ( my_array[i] < second_element){      → 1
            if (my_array[i] != first_element){        → 1
                second_element = my_array[i];         → 1
            }
        }
    }
}
```

$+$ ______

$n+7 \rightarrow O(n)$

b)
```
void f(int n){
    int count = 0;
    for (int i=2; i<n; i++){        ——→ n+1
        if (i%2 == 0){
            count++;   (?) ——→ only one times executed
        }
        else {
            i=(i-1)*i;              ——→ 1
        }
    }
}
```

i ⇒	n
2	2
3	3
7	7
43	43
1807	1807
3263443	3263443

If n is 1807 (or higher), then the loop is executed 5 times.

If n is 3263443 (or higher), then the loop is executed 6 times.

It is very fast algorithm.

Terminate $i > n$ or $n \le i$

$n \to 2, 3, \overbrace{(3*2+1)}^{7}, \overbrace{(7*6+1)}^{43}, \overbrace{(43*42+1)}^{1807} \ldots\ldots, (m*m-1+1)$

$i \to 1, 2, 3, 4, 5, \ldots\ldots\ldots, x$

$\log n \to \log 10 = 1$
$\log 100 = 10$ (Our algorithm $1807 \to 5$) (faster than $\log n$)

$\log(\log n) \to \log\overbrace{(\log 100)}^{2}$ (Our algorithm is approximately $O(\log(\log n))$.)

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Q4)

a) $\sum_{i=1}^{n} i^2 \log i$

$$\int_0^n x^2 \log x \, dx \le f(n) \le \int_1^{n+1} x^2 \log x \, dx$$

$$\frac{1}{9} x^3 \cdot (3 \log(x) - 1) \Big|_0^n \le f(n) \le \frac{1}{9} x^3 (3 \log(x) - 1) \Big|_1^{n+1}$$

$$\underbrace{\frac{1}{9} n^3 \cdot (3 \log(n) - 1)}_{n^3 \log n} \le f(n) \le \underbrace{\frac{1}{9} (n+1)^3 (3 \log(n+1) - 1) + \frac{1}{9}}_{n^3 \log n}$$

$f(n) \in \Theta(n^3 \log n)$

b) $\sum_{i=1}^{n} i^3$

$$\int_0^n x^3 dx \le f(n) \le \int_1^{n+1} x^3 dx \Rightarrow \frac{x^4}{4} \Big|_0^n \le f(n) \le \frac{x^4}{4} \Big|_1^{n+1}$$

$$\underbrace{\frac{n^4}{4}}_{n^4} \le f(n) \le \underbrace{\frac{(n+1)^4}{4} - \frac{1}{4}}_{n^4} \Rightarrow f(n) \in \Theta(n^4)$$

c) $\sum_{i=1}^{n} \frac{1}{2\sqrt{i}}$

$$\int_0^n \frac{1}{2\sqrt{x}} dx \le f(n) \le \int_1^{n+1} \frac{1}{2\sqrt{x}} dx \Rightarrow \sqrt{x} \Big|_0^n \le f(n) \le \sqrt{x} \Big|_1^{n+1}$$

$$\sqrt{n} \le f(n) \le \sqrt{n+1} - \sqrt{1} \Rightarrow f(n) \in \Theta(\sqrt{n})$$

d) $\sum_{i=1}^{n} \frac{1}{i}$

$$\int_{0}^{n} \frac{1}{x}\,dx \le f(n) \le \int_{1}^{n+1} \frac{1}{x}\,dx \Rightarrow \log(n) \le f(n) \le \log(n+1)$$

$f(n) \in \Theta(\log n)$

Q5)

```
function Linear-Search (L[1:n], x)
    for  i:=1 to n do
        if ( L[i] = x ) then
            return i
        end if
    end for
    return -1
end
```

The best case is that x is the equal to the first element of the L. Complexity of this case is O(1), and this is the best case.

$B(n) = O(1)$.

The worst case is that x is not the equal to any element of the L. In this case the loop is executed n times then the worst case occurs.

$W(n) = O(n)$.