

Suppose you have the following mask

1 2 3

4 5 6

7 8 9

Perform convolution process on the following image matrix when stride =2

2 4 6

8 10 12

14 16 18

### Step 1: Understanding Convolution

The **convolution operation** slides the mask over the image matrix and calculates the **dot product** between the overlapping elements.

**Given:**

**Image Matrix:**

$$\begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

**Mask (Kernel):**

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

**Stride = 2**, meaning we slide the mask **two pixels at a time** instead of one.

### Step 2: Extracting Submatrices

Since the image matrix is **3×3**, applying a **3×3 mask** with a stride of **2** results in the following valid positions:

1. The mask aligns with the **top-left** region:

$$\begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

Performing element-wise multiplication and summing:

$$(1 \times 2) + (2 \times 4) + (3 \times 6) + (4 \times 8) + (5 \times 10) + (6 \times 12) + (7 \times 14) + (8 \times 16) + (9 \times 18) \\ = 2 + 8 + 18 + 32 + 50 + 72 + 98 + 128 + 162 = 570$$

### Step 3: Final Output

Since the stride is **2**, the mask moves completely out of bounds after the first step, meaning we only compute **one convolution result**.

Thus, the **output matrix** consists of:

[570]

Suppose you have the following image matrix

38	66	65
14	35	64
12	15	42

Apply the Prewitt operator and find out the direction and magnitude of the matrix.

Applying the **Prewitt operator** to an image matrix involves calculating the gradient in the **x-direction** and **y-direction** using the Prewitt kernels. This helps determine the **edge strength (magnitude)** and **edge direction**.

**Given Matrix:**

$$\begin{bmatrix} 38 & 66 & 65 \\ 14 & 35 & 64 \\ 12 & 15 & 42 \end{bmatrix}$$

### Prewitt Kernels:

For **horizontal gradient** ( $G_x$ ):

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

For **vertical gradient** ( $G_y$ ):

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

### Step 1: Compute $G_x$

Each pixel's horizontal gradient is computed as:

$$G_x(i, j) = (-1 \cdot A_{i-1, j-1}) + (0 \cdot A_{i-1, j}) + (1 \cdot A_{i-1, j+1}) + (-1 \cdot A_{i, j-1}) + (0 \cdot A_{i, j}) + (1 \cdot A_{i, j+1})$$

Computing for the center pixel ( $A_{1,1} = 35$ ):

$$G_x(1, 1) = (-1 \cdot 38) + (0 \cdot 66) + (1 \cdot 65) + (-1 \cdot 14) + (0 \cdot 35) + (1 \cdot 64) + (-1 \cdot 12) + (0 \cdot 15)$$

$$G_x(1, 1) = (-38 + 65 - 14 + 64 - 12 + 42) = 107$$

### Step 2: Compute $G_y$

Each pixel's vertical gradient is computed as:

$$G_y(i, j) = (-1 \cdot A_{i-1, j-1}) + (-1 \cdot A_{i-1, j}) + (-1 \cdot A_{i-1, j+1}) + (0 \cdot A_{i, j-1}) + (0 \cdot A_{i, j}) + (0 \cdot A_{i, j+1}) + (1 \cdot A_{i+1, j-1}) + (1 \cdot A_{i+1, j}) + (1 \cdot A_{i+1, j+1})$$

Computing for the center pixel ( $A_{1,1} = 35$ ):

$$G_y(1, 1) = (-1 \cdot 38) + (-1 \cdot 66) + (-1 \cdot 65) + (0 \cdot 14) + (0 \cdot 35) + (0 \cdot 64) + (1 \cdot 12) + (1 \cdot 15) + (1 \cdot 15)$$

$$G_y(1, 1) = (-38 - 66 - 65 + 12 + 15 + 42) = -100$$

### Step 3: Compute Gradient Magnitude

$$M = \sqrt{G_x^2 + G_y^2}$$

$$M = \sqrt{(107)^2 + (-100)^2}$$

$$M = \sqrt{11449 + 10000} = \sqrt{21449} \approx 146.43$$

### Step 4: Compute Direction

$$\theta = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{-100}{107}\right) = \tan^{-1}(-0.9346)$$

$$\theta \approx -43.86^\circ$$

### Final Result:

For the **center pixel** ( $A_{1,1} = 35$ ), the edge strength (magnitude) is **146.43**, and the edge direction is **-43.86°**.