

# BLG 210E Recit-2 (2021-Fall)

Ex 1 which of the matrices are invertible. Use as few calculations as possible.

a)  $\begin{bmatrix} 5 & 7 \\ -3 & -6 \end{bmatrix}$

$|A| = -9 \rightarrow$  determinant not "0" is invertible (non singular)

b)  $\begin{bmatrix} -4 & 6 \\ 6 & -9 \end{bmatrix}$

$|A| = 0$  is not invertible. Also 1.5 between columns.

c)  $\begin{bmatrix} -7 & 0 & 4 \\ 3 & 0 & -1 \\ 2 & 0 & 9 \end{bmatrix}$

zero column, linearly dependent. Not invertible (singular)

d)  $\begin{bmatrix} 0 & 3 & -5 \\ 1 & 0 & 2 \\ -4 & -9 & 7 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \\ -4 & -9 & 7 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + 4R_1} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \\ 0 & -9 & 15 \end{bmatrix}$

$\xrightarrow{R_3 \leftarrow R_3 + 3R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$  is not equivalent to the identity matrix.

e)  $\begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} \xrightarrow{\substack{R_2 \leftarrow R_2 + 3R_1 \\ R_3 \leftarrow R_3 - 2R_1}} \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & -4 & 8 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \xrightarrow{R_4 \leftarrow R_4 - \frac{1}{4}R_2} \begin{bmatrix} -1 & -3 & 0 & 1 \\ 0 & -4 & 8 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

has 4 pivot points  
also  $|A| = 12$   
(triangular matrix)

②  $A = \begin{bmatrix} 2 & -4 & 4 & -2 \\ 6 & -9 & 7 & -3 \\ -1 & -4 & 8 & 0 \end{bmatrix}$  Find LU factorization of matrix A?

$$A = \begin{bmatrix} 2 & -4 & 4 & -2 \\ 6 & -9 & 7 & -3 \\ -1 & -4 & 8 & 0 \end{bmatrix} \xrightarrow{\substack{R_2 \leftarrow R_2 - 3R_1 \\ R_3 \leftarrow R_3 + \frac{1}{2}R_1}} \begin{bmatrix} 2 & -4 & 4 & -2 \\ 0 & 3 & -5 & 3 \\ 0 & -6 & 10 & -1 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + 2R_2}$$

$$\begin{bmatrix} 2 & -4 & 4 & -2 \\ 0 & 3 & -5 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & & \\ 3 & 1 & \\ -1/2 & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1/2 & -2 & 1 \end{bmatrix}$$

$$\textcircled{3} \begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 4 & 10 & -4 & -1 \end{vmatrix} = ?$$

$$\begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 4 & 10 & -4 & -1 \end{vmatrix} \xrightarrow{R_4 \leftarrow R_4 - 2R_1} \begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 0 & 0 & 2 & 1 \end{vmatrix} \begin{array}{l} \text{make cofactor} \\ \text{expansion} \end{array}$$

make 2nd column 0

$$(-1)^{1+2} \cdot 5 \cdot \begin{vmatrix} 3 & 1 & -3 \\ -6 & -4 & 9 \\ 0 & 2 & 1 \end{vmatrix} \xrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{vmatrix} 3 & 1 & -3 \\ 0 & -2 & 3 \\ 0 & 2 & 1 \end{vmatrix} \begin{array}{l} -5 \\ \text{make first column 0} \end{array}$$

$$-5 \cdot 3 \begin{vmatrix} -2 & 3 \\ 2 & 1 \end{vmatrix} = -5 \cdot 3 (-8) = \underline{\underline{120}}$$

$$\textcircled{4} \begin{vmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{vmatrix} = ?$$

$$\begin{vmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{vmatrix} \xrightarrow{\begin{array}{l} R_2 \leftarrow R_2 - R_1 \\ R_3 \leftarrow R_3 - R_1 \end{array}} \begin{vmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix} \xrightarrow{R_3 \leftarrow R_3 - R_2}$$

$$\begin{vmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \quad \text{two rows are the same, then } \underline{\underline{|A| = 0}}$$