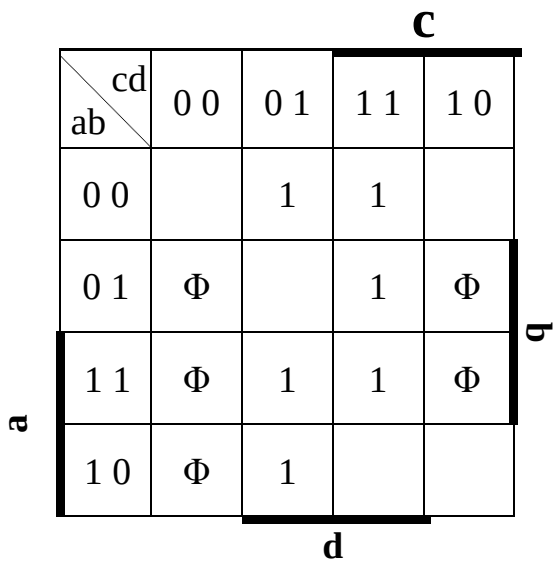
150210022 – Harun Yahya Demirpençe

1. **Question: Find the set of all prime implicants in SOP form using**
   a) **a Karnaugh map,**
   b) **the Quine-McCluskey method**

I. Solution of a

$$f(a, b, c, d) = \cup 1(1, 3, 7, 9, 13, 15) + \cup \Phi(4, 6, 8, 12, 14)$$

We assign 1 to Φ's when we are finding the set of prime implicants of a function in SOP form.



A = a′b′d cost: 8

B = b′c′d cost: 8

C = a′cd cost: 7

D = bc cost: 4

E = bd′ cost: 5

F = ab cost: 4

G = ac′ cost: 5

f (a, b, c, d) = A + B + C + D + E + F + G

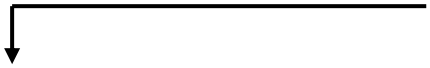
II. Solution of b

$$f(a, b, c, d) = \cup 1(1, 3, 7, 9, 13, 15) + \cup \Phi(4, 6, 8, 12, 14)$$

We assign 1 to Φ's again to find prime implicants of the function, then write the 1-generating input combinations on the truth table. Clustering the 1-generating input combinations reduce the running time of the algorithm. We compare the neighbor clusters, if a single variable changes between two combinations, then group them.

Num.	a	b	c	d
1	0	0	0	1
4	0	1	0	0
8	1	0	0	0
3	0	0	1	1
6	0	1	1	0
9	1	0	0	1
12	1	1	0	0
7	0	1	1	1
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

Num.	a	b	c	d
1,3	0	0	-	1
1,9	-	0	0	1
4,6	0	1	-	0
4,12	-	1	0	0
8,9	1	0	0	-
8,12	1	-	0	0
3,7	0	-	1	1
6,7	0	1	1	-
6,14	-	1	1	0
9,13	1	-	0	1
12,13	1	1	0	-
12,14	1	1	-	0
7,15	-	1	1	1
13,15	1	1	-	1
14,15	1	1	1	-



Num.	a	b	c	d
4,6,12,14	-	1	-	0
4,12,6,14	-	1	-	0
8,9,12,13	1	-	0	-
8,12,9,3	1	-	0	-
6,7,14,15	-	1	1	-
6,14,7,15	-	1	1	-
12,13,14,15	1	1	-	-
12,14,13,15	1	1	-	-

Since the gray ones are repetitive, it is redundant to write these again. Marked with different colors are the prime implicants of the function.

a′b′d, b′c′d, a′cd, bc, bd′, ab, ac′

2. **Question: Build the prime implicant chart considering these cost criteria: 2 units for each variable and 1 unit for each complement sign. Then, simplify it to obtain the minimal covering sum with the lowest cost. Clearly demonstrate and explain each step of the simplification process. Provide the total cost and the expression for the function with the lowest cost.**

I. Solution

There is no need to cover the unspecified value points, so we assign O to Φ's when we form the prime implicant chart of the function in SOP form. We take the symbols from the Karnaugh map.

ASSUMPTION 1

	1	3	7	9	13	15	Cost
A	X	X					8
B	X			X			8
C		X	X				7
D			X			X	4
F					X	X	4
G				X	X		5

Let's say we took the row of D because its cost is the lowest ones, then the table will look like this.

	1	3	9	13	Cost
A	X	X			8
B	X		X		8
C		X			7
F				X	4
G			X	X	5

The rows of C and F's cover just 1 point and its cost are nearly the equal to rows of A, B and G, let's see what if we eliminate them.

	1	3	9	13	Cost
A	X	X			8
B	X		X		8
G			X	X	5

The point 13 is distinguished point, so we have to take row G.

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	1	3	Cost
A	X	X	8
B	X		8

Row A covers the row B, so we take the row A.

Cost = 4 + 5 + 8 = 17

ASSUMPTION 2

	1	3	7	9	13	15	Cost
A	X	X					8
B	X			X			8
C		X	X				7
D			X			X	4
F					X	X	4
G				X	X		5

Let's say we took the row of F because its cost is the lowest ones, then the table will look like this.

	1	3	7	9	Cost
A	X	X			8
B	X			X	8
C		X	X		7
D			X		4
G				X	5

We can choose A, D, G or B, C to cover all points.

First case: A + D + F + G = 21

Second case: B + C + F = 19

We clearly see that the first assumption is the best way to represent the function with the lowest cost. The minimal covering sum with the lowest cost is f (a, b, c, d) = A + D + G => a′b′d + bc + ac′ Cost: 17