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Question 1:

a)

i) “+112” is a signed (positive) integer, so its binary representation starts with 0.

1. 112 / 2 = 56 => (remainder 0) **(LSB)**

56 / 2 = 28 => (remainder 0)

28 / 2 = 14 => (remainder 0)

14 / 2 = 7 => (remainder 0)

7 / 2 = 3 => (remainder 1)

3 / 2 = 1 => (remainder 1)

1 / 2 = 0 => (remainder 1) **(MSB)**

2. Binary representation: $\mathbf{(+112)_{10} = (0111\ 0000)_2}$

ii) “-41” is a signed (negative) integer, so its binary representation starts with 1.

1. Unsigned magnitude of -41 => 41.

2. 41 / 2 = 20 => (remainder 1) **(LSB)**

20 / 2 = 10 => (remainder 0)

10 / 2 = 5 => (remainder 0)

5 / 2 = 2 => (remainder 1)

2 / 2 = 1 => (remainder 0)

1 / 2 = 1 => (remainder 1) **(MSB)**

3. Binary representation of 41: $\mathbf{(41)_{10} = (0010\ 1001)_2}$

4. 2’s complement of 41: $\mathbf{(1101\ 0110)_2 + 1 = (1101\ 0111)_2}$

5. Then: $\mathbf{(-41)_{10} = (1101\ 0111)_2}$

b)

i) The expression "(-112) – (-41)" is equivalent to "(-112) + (+41)."

1. $(112)_{10}$ = (0111 0000) Binary representation of 112

$(-112)_{10} = (1000\ 1111)_2 + 1$ 1's complement of 112

$(-112)_{10} = (1001\ 0000)_2$ 2's complement of 112

and

$(-41)_{10} = (1101\ 0111)_2$ Binary representation of -41

$(+41)_{10} = (0010\ 1000)_2 + 1$ 1's complement of -41

$(+41)_{10} = (0010\ 1001)_2$ 2's complement of -41

2. (-112) + (+41) = $(1001\ 0000)_2 + (0010\ 1001)_2$

= **$(0\ 1011\ 1001)_2$**

3. It is a subtraction operation between two signed integers, so the carry bit (9^{th}) is ignored.

4. The result: **$(1011\ 1001)_2$**

5. There are neither carry nor borrow in this subtraction since carry and borrow are the terms for unsigned integers, and overflow is not a concern in this operation as subtracting negative value from another negative value cannot result in overflow.

ii) As this is the addition operation, the expression result can be calculated directly.

1. The result of the (-112) + (-41):

$(-112)_{10} = (1001\ 0000)_2$

$(-41)_{10} = (1101\ 0111)_2$

+______________________

$(1\ 0110\ 0111)_2$

2. The carry bit (9^{th}) is ignored since the addition operation between signed integers.

3. The result: **$(0110\ 0111)_2$**

4. While carry and borrow typically related to unsigned integers, in this addition, they are not relevant because both terms do not apply. However, there is indeed an overflow in this case. This is because when adding two negative integers, the result is positive, and this situation meets the condition for an overflow in an addition expression involving negatives (neg. + neg. = pos.).

iii)

1. $(41)_{10} = (0010\ 1001)_2$

$(112)_{10} = (0111\ 0000)_2$

2. Two's complement of $(112)_{10}$:

$(-112)_{10} = (1001\ 0000)_2$

3. The expression:

$(0010\ 1001)_2$

$(1001\ 0000)_2$

+______________

$(0\ 1011\ 1001)_2$

4. The carry bit (9^{th}) is 0. (No carry)

5. There is no carry but there is a borrow due to the 9^{th} bit being 0. The result is invalid since when a borrow occurs it indicates that the first number is smaller than the second one, making it impossible to represent the result using unsigned integers.

Question 2:

a) E (X, Y, Z) = X'Y + YZ'+ YZ + XY'Z'

1. YZ' + YZ = Y (Z'+ Z) = Y . 1 = Y (Distributivity, Inverse, Identity)

E (X, Y, Z) = X'Y + Y + XY'Z'

2. X'Y + Y = Y (X' + 1) = Y . 1 = Y (Distributivity, Annihilator, Identity)

E (X, Y, Z) = Y + XY'Z'

3. Y + XY'Z' = Y + XY'Z' + XZ' (Consensus Theorem with respect to Y)

XY'Z' + XZ' = XZ' (Y' + 1) = XZ' . 1 = XZ' (Distributivity, Annihilator, Identity)

E (X, Y, Z) = **Y + XZ'** (The Final Result)

b) E (T, W, X, Y, Z) = TW + X'YZ' + TW'Z + W'XY + T'WX + W'XY' + TW'Z'

1. TW'Z + TW'Z' = TW' (Z + Z') = TW' . 1 = TW' (Distributivity, Inverse, Identity)

E (T, W, X, Y, Z) = TW + X'YZ' + W'XY + T'WX + W'XY' + TW'

2. W'XY + W'XY' = W'X (Y + Y') = W'X . 1 = W'X (Distributivity, Inverse, Identity)

E (T, W, X, Y, Z) = TW + X'YZ' + T'WX + TW' + W'X

3. TW + TW' = T (W + W') = T . 1 = T (Distributivity, Inverse, Identity)

E (T, W, X, Y, Z) = X'YZ' + T'WX + W'X + T

4. T'WX + W'X = T'WX + W'X + T'X (Consensus Theorem with respect to W)

T'WX + T'X = T'X (W + 1) = T'X . 1 = T'X (Distributivity, Annihilator, Identity)

E (T, W, X, Y, Z) = X'YZ' + T'X + W'X + T

5. T'X + T = T'X + T + X (Consensus Theorem with respect to T)

T'X + X = (T' + 1) X = 1 . X = X (Distributivity, Annihilator, Identity)

E (T, W, X, Y, Z) = X'YZ' + W'X + T + X

6. W'X + X = (W' + 1) X = 1 . X = X (Distributivity, Annihilator, Identity)

E (T, W, X, Y, Z) = X'YZ' + T + X

7. X'YZ' + X = X'YZ' + X + YZ' (Consensus Theorem with respect to X)

X'YZ' + YZ' = YZ' (X' + 1) = YZ' . 1 = YZ' (Distributivity, Annihilator, Identity)

E (T, W, X, Y, Z) = **T + X + YZ'** (The Final Result)