BLG 210E
Engineering maths
Recit 1. week 2
09.10.2023

1) Find values of "h" which make the following systems of eqns. consistent.

a) $\left[\begin{array}{cc|c} 1 & h & 4 \\ 3 & 6 & 8 \end{array}\right] \xrightarrow{R_2 \leftarrow -3R_1 + R_2} \left[\begin{array}{cc|c} 1 & h & 4 \\ 0 & -3h+6 & -4 \end{array}\right]$

if $-3h+6=0$, the system becomes inconsistent.

$-3h+6=0 \Rightarrow h=2.$

$h \neq 2$ makes the system consistent

$\{\forall h \in \mathbb{R},\ h \neq 2\}$

b) $\left[\begin{array}{cc|c} 2 & -3 & h \\ -6 & 9 & 5 \end{array}\right] \xrightarrow{R_2 \leftarrow 3R_1 + R_2} \left[\begin{array}{cc|c} 2 & -3 & h \\ 0 & 0 & 3h+5 \end{array}\right]$

if $3h+5=0$, the system becomes consistent.

$3h+5=0 \Rightarrow h=-5/3$

2) Solve the following linear sys. of eqns with row reduction method.

$$x_1 - 3x_2 + 4x_3 = -4$$
$$3x_1 - 7x_2 + 7x_3 = -8$$
$$-4x_1 + 6x_2 - x_3 = 7$$

$\longrightarrow$ Augmented matrix becomes: $A = \left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{array}\right]$

$R_2 \leftarrow -3R_1 + R_2$
$R_3 \leftarrow 4R_1 + R_3$

$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -6 & 15 & -9 \end{array}\right]$

$R_3 \leftarrow 3R_2 + R_3$
$R_2 \leftarrow \frac{1}{2} R_2$

$\rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 1 & -5/2 & 2 \\ 0 & 0 & 0 & 3 \end{array}\right]$

The last row implies that $0=3$. This system of eqns is inconsistent. Hence, there is no solution.

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3) Solve the following linear sys. of eqns using a) row reduction, b) LU factorization, c) matrix inversion methods.

$$\left.\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}\right\}\quad Ax=b \quad A=\begin{bmatrix} 1 & 0 & -3 \\ 2 & 2 & 9 \\ 0 & 1 & 5 \end{bmatrix},\; b=\begin{bmatrix} 8 \\ 7 \\ -2 \end{bmatrix}$$

a) Row reduction.

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right] \xrightarrow{R_2 \leftarrow -2R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right] \xrightarrow{R_3 \leftarrow -\frac{1}{2}R_2 + R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 0 & -5/2 & 5/2 \end{array}\right] \xrightarrow[R_3 \leftarrow -\frac{2}{5}R_3]{R_2 \leftarrow \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 15/2 & -9/2 \\ 0 & 0 & 1 & -1 \end{array}\right] \xrightarrow[R_1 \leftarrow 3R_3 + R_1]{R_2 \leftarrow -\frac{15}{2}R_3 + R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \Rightarrow x = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$$

b) LU Factorization.

$$L=\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1/2 & 1 \end{bmatrix},\quad U=\begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 15 \\ 0 & 0 & -5/2 \end{bmatrix}$$

$Ax=b$, $A=LU \Rightarrow LUx=b$

$\Rightarrow L(Ux)=b \Rightarrow Ly=b.$

$$\begin{bmatrix} 1 & 0 & -3 \\ 2 & 2 & 9 \\ 0 & 1 & 5 \end{bmatrix} \xrightarrow{R_2 \leftarrow \boxed{-2}R_1 + R_2} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 15 \\ 0 & 1 & 5 \end{bmatrix} \xrightarrow[R_3 \leftarrow \boxed{\frac{-1}{2}}R_2 + R_3]{R_3 \leftarrow \boxed{0}R_1 + R_3} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 15 \\ 0 & 0 & -\frac{5}{2} \end{bmatrix} = U$$

$$Ly=b \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -2 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 2 & 1 & 0 & 7 \\ 0 & 1/2 & 1 & -2 \end{array}\right] \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -9 \\ 0 & 1/2 & 1 & -2 \end{array}\right] \xrightarrow{R_3 \leftarrow -\frac{1}{2}R_2 + R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5/2 \end{array}\right] \Rightarrow y = \begin{bmatrix} 8 \\ -9 \\ 5/2 \end{bmatrix}$$

$Ux = y$,

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 15 \\ 0 & 0 & -5/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -9 \\ 5/2 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 0 & -5/2 & 15/2 \end{array}\right] \xrightarrow[R_2 \leftarrow \frac{1}{2} R_2]{R_3 \leftarrow -\frac{2}{5} R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 15/2 & -9/2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$R_2 \leftarrow -\frac{2}{15} R_3 + R_2$

$R_1 \leftarrow 3R_3 + R_1$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \Rightarrow x = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$$

c) Matrix Inversion.

$$Ax = b \Rightarrow A^{-1}Ax = A^{-1}b \Rightarrow Ix = A^{-1}b \Rightarrow x = A^{-1}b$$

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 2 & 9 \\ 0 & 1 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} 8 \\ 7 \\ -2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 2 & 2 & 9 & 0 & 1 & 0 \\ 0 & 1 & 5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftarrow -2R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 0 & 2 & 15 & -2 & 1 & 0 \\ 0 & 1 & 5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 \leftarrow -1/2 R_2 + R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 0 & 2 & 15 & -2 & 1 & 0 \\ 0 & 0 & -5/2 & +1 & -1/2 & 1 \end{array}\right] \xrightarrow[R_2 \leftarrow 1/2 R_2]{R_3 \leftarrow -2/5 R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 0 & 1 & 15/2 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & -2/5 & 1/5 & -2/5 \end{array}\right] \xrightarrow[R_2 \leftarrow -15/2 R_3 + R_2]{R_1 \leftarrow 3R_3 + R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/5 & 3/5 & -6/5 \\ 0 & 1 & 0 & 2 & -1 & 3 \\ 0 & 0 & 1 & -2/5 & 1/5 & -2/5 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} -1/5 & 3/5 & -6/5 \\ 2 & -1 & 3 \\ -2/5 & 1/5 & -2/5 \end{bmatrix}$$

$$x = A^{-1}b = \begin{bmatrix} -1/5 & 3/5 & -6/5 \\ 2 & -1 & 3 \\ -2/5 & 1/5 & -2/5 \end{bmatrix} \begin{bmatrix} 8 \\ 7 \\ -2 \end{bmatrix} = \begin{bmatrix} -8/5 + 21/5 + 12/5 = 5 \\ 16 - 7 - 6 = 3 \\ -16/5 + 7/5 + 4/5 = -1 \end{bmatrix}$$

4) Determine if b is a linear combination of a_1, a_2, a_3.

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

$$x_1 a_1 + x_2 a_2 + x_3 a_3 = b$$

→ Create an augmented matrix.

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \xrightarrow{R_2 \leftarrow 2R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{array}\right] \xrightarrow{R_3 \leftarrow -2R_2 + R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

→ The system is consistent and it has a solution. Hence, a_1, a_2, and a_3 are a linear combination of the vector b.

5) Determine if b is a linear combination of the vectors formed from the columns of matrix A.

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{array}\right] \xrightarrow{R_3 \leftarrow 2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

The last row creates an inconsistency. Therefore, this system doesn't have a solution. Hence, the columns of A ~~are~~ not a linear combination of b.