

$f(x) = \sin x$: Taylor Expansion around 0^+ .

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n \quad c=0$$

$$\begin{aligned} f(x) &= \sin x \\ f'(x) &= \cos x \\ f''(x) &= -\sin x \\ f'''(x) &= -\cos x \end{aligned}$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$f(x) = 0 + x + 0 - \frac{x^3}{3!} + 0 + \frac{x^5}{5!} + 0 - \frac{x^7}{7!} + \dots$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$f(x) = \frac{1}{1-x} \quad x \in (-1, 1)$$

$$f(x) = \sum_{n=0}^k \frac{f^{(n)}(c)}{n!} (x-c)^n + E_{n+1}$$

$$c \in (a, b) \quad \frac{f^{(n+1)}(\xi)}{(n+1)!} (x-c)^{n+1} \quad \xi \in (c, x)$$

Ex: Let's use Taylor Expansion to approximate $\sqrt{3}$.
why: $f(x) = x^2 - 3$

Consider $g(x) = \sqrt{x} = x^{1/2}$ ($\sqrt{3} = g(3)$)
Let's expand $g(x)$ around $c=4$

$$g(x) = g(4) + g'(4)(x-4) + \frac{g''(4)}{2!}(x-4)^2 + \frac{g'''(4)}{3!}(x-4)^3 + \dots$$

$$g(x) = 2 + \frac{1}{4}(x-4) + \frac{(-1/32)}{2}(x-4)^2 + E_3(x)$$

$$\begin{aligned} g'(x) &= \frac{1}{2}x^{-1/2} \\ g'(4) &= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \\ g''(x) &= -\frac{1}{4}x^{-3/2} \\ g''(4) &= -\frac{1}{4} \cdot \frac{1}{8} = -\frac{1}{32} \end{aligned}$$

$$\begin{aligned} g(3) &= 2 + \frac{1}{4}(-1) - \frac{1}{32} \cdot \frac{1}{2} \\ &= 2 - \frac{1}{4} - \frac{1}{64} = 2 - \frac{17}{64} = \frac{1.736}{1} \dots \\ \sqrt{3} &= 1.734 \dots \end{aligned}$$

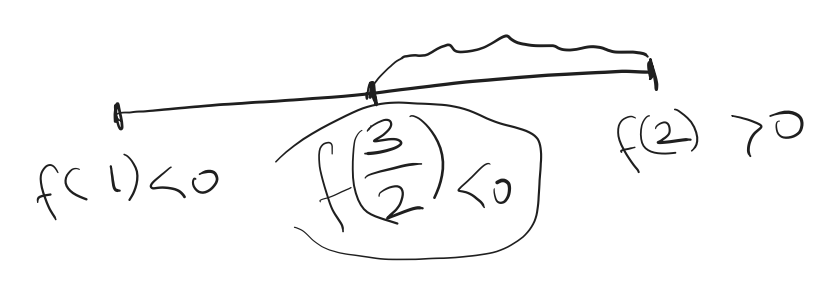
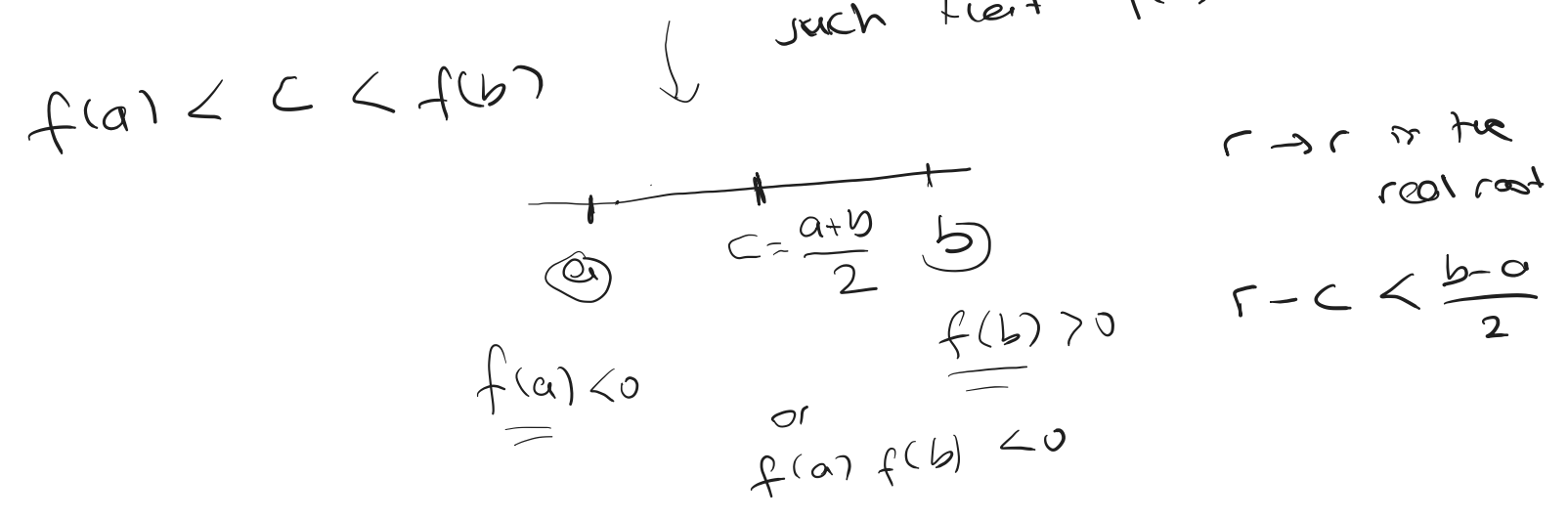
Bisection Method:

Find a root of function $f(x)$:

The Mean Value Theorem: Suppose $f(x)$ is continuous on $[a, b]$. For any y in $f(a) \leq y \leq f(b)$ there exists $c \in [a, b]$ such that $f(c) = y$.

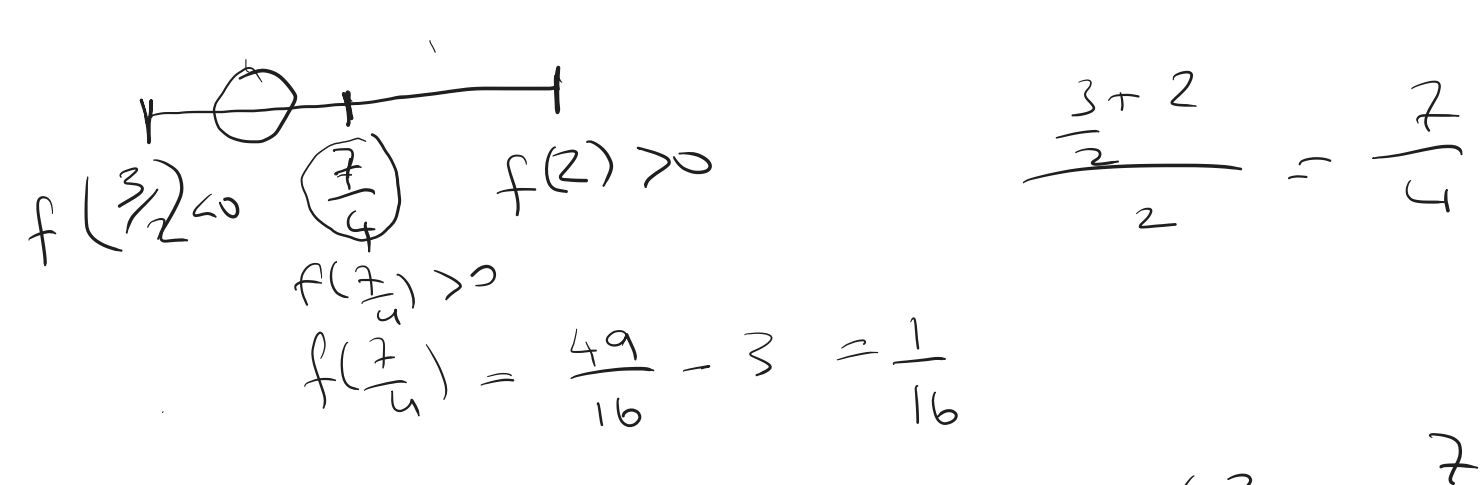
Ex: $f(x) = x^2 - 3$ we want to find a root of $f(x)$.
Find "a" and "b" so that $f(x)$ is cont on $[a, b]$
and $f(a)f(b) < 0$ or $f(a) < 0 < f(b)$

$a=1$ Satisfy what we need.
 $b=2$ $f(a) = -2$ there exist $c \in (a, b) = (1, 2)$
 $f(b) = 1$ such that $f(c) = 0$

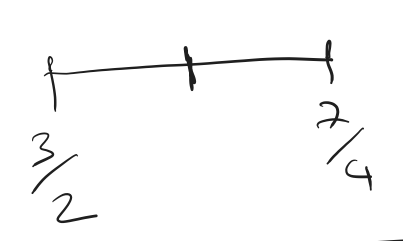


$$f\left(\frac{3}{2}\right) = \frac{9}{4} - 3 = -\frac{3}{4} < 0$$

Then I can say that the root lies in $\left(\frac{3}{2}, 2\right)$



So the root must lie in $\left(\frac{3}{2}, \frac{7}{4}\right)$



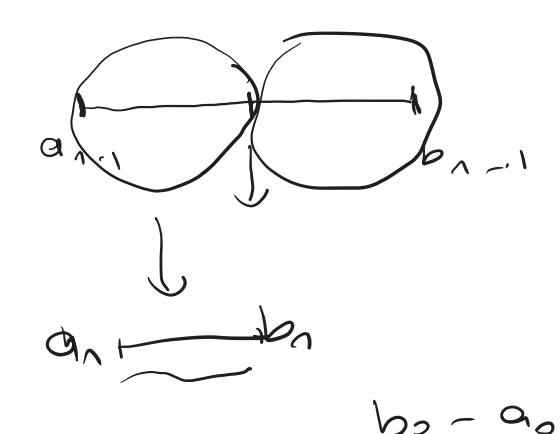
Steps of Bisection method:

- 1.) Find a, b such $f(a)f(b) < 0$
- 2.) Determine which one of $f(a)f\left(\frac{a+b}{2}\right)$ or $f(b)f\left(\frac{a+b}{2}\right)$ is negative. Then assign new a, b so that $f(a)f(b) < 0$ then proceed!

Error Computation:

$$\begin{array}{l} a_0 \\ \wedge \\ a_1 \\ \wedge \\ a_2 \\ \vdots \\ a_n \end{array} \quad \begin{array}{l} b_0 \\ \wedge \\ b_1 \\ \wedge \\ b_2 \\ \vdots \\ b_n \end{array}$$

$$\begin{aligned} b_n - a_n &= \frac{b_{n-1} - a_{n-1}}{2} \\ &= \frac{b_{n-2} - a_{n-2}}{4} = \dots = \frac{b_0 - a_0}{2^n} \end{aligned}$$



when I stop at n^{th} step

$$|b_n - a_n| = \left| \frac{b_0 - a_0}{2^n} \right|$$