BLG 210E Recit_4 (2023-FALL)

(1)

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \text{ and } y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$$

for what value(s) of h will y be in the subspace of $\mathbb{R}^3$ spanned by v_1, v_2, v_3?

$$c_1 \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} + c_2 \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix} + c_3 \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$$

consistent

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ -2 & -7 & 0 & h \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 + 2R_1]{R_2 \leftarrow R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 3 & -6 & h-8 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 - 3R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & h-5 \end{array}\right] \qquad h-5=0 \qquad \underline{h=5}$$

consistent

(2) Find a spanning set for the null space of the matrix.

$$A = \left[\begin{array}{ccccc|c} -3 & 6 & -1 & 1 & -7 & 0 \\ 1 & -2 & 2 & 3 & -1 & 0 \\ 2 & -4 & 5 & 8 & -4 & 0 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccccc|c} 1 & -2 & 2 & 3 & -1 & 0 \\ -3 & 6 & -1 & 1 & -7 & 0 \\ 2 & -4 & 5 & 8 & -4 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 \leftarrow R_3 - 2R_1]{R_2 \leftarrow R_2 + 3R_1} \left[\begin{array}{ccccc|c} 1 & -2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 5 & 10 & -10 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}\right] \xrightarrow{R_2 \leftarrow \frac{1}{5}R_2} \left[\begin{array}{ccccc|c} 1 & -2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \left[\begin{array}{ccccc|c} 1 & -2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \leftarrow R_1 - 2R_2} \left[\begin{array}{ccccc|c} 1 & -2 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

if you want continue but RREF

x_2, x_3 and x_5 are free variables:

$$S = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\left.\begin{aligned} x_1 &= 2x_2 + x_4 - 3x_5 \\ x_3 &= -2x_4 + 2x_5 \end{aligned}\right\} \quad \vec{x} = \begin{bmatrix} 2x_2 + x_4 - 3x_5 \\ x_2 \\ -2x_4 + 2x_5 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

(3)

$$A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$$

a) Determine if u is in Nul A. Could u be in Col A?

b) Determine if v is in Col A. Could v be in Nul A?

a) $\begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ u is not in Nul A

u has 4 entries (not 3) (in R^4), since Col A is a subspace of R^3.

b) $\left[\begin{array}{cccc|c} 2 & 4 & -2 & 1 & 3 \\ -2 & -5 & 7 & 3 & -1 \\ 3 & 7 & -8 & 6 & 3 \end{array}\right] \xrightarrow[R_3 \leftarrow R_3 - \frac{3}{2}R_1]{R_2 \leftarrow R_2 + R_1} \left[\begin{array}{cccc|c} 2 & 4 & -2 & 1 & 3 \\ 0 & -1 & 5 & 4 & 2 \\ 0 & 1 & -5 & 9/2 & -3/2 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 + R_2}$

should be consistent

$$\left[\begin{array}{cccc|c} 2 & 4 & -2 & 1 & 3 \\ 0 & -1 & 5 & 4 & 2 \\ 0 & 0 & 0 & 17/2 & 1/2 \end{array}\right]$$

consistent, so v in Col A.

Nul A is a subspace of R^4 so v could not possibly be in Nul A.