15021022 – Harun Yahya Demirpençe

Questions:

1) Write the expression for the complement of function f in second canonical form. z' = f (a, b, c, d)' =?

	a	b	c	d	z	$\bar{z}$
0	0	0	0	0	1	0
1	0	0	0	1	1	0
2	0	0	1	0	0	1
3	0	0	1	1	0	1
4	0	1	0	0	1	0
5	0	1	0	1	1	0
6	0	1	1	0	1	0
7	0	1	1	1	1	0
8	1	0	0	0	0	1
9	1	0	0	1	0	1
10	1	0	1	0	1	0
11	1	0	1	1	1	0
12	1	1	0	0	0	1
13	1	1	0	1	0	1
14	1	1	1	0	1	0
15	1	1	1	1	1	0

a) We can obtain the 2nd canonical form of a function by considering the false ('0') rows of the complement of the function results. To obtain the individual maxterms, we substitute variables for zeros and complements of variables for ones in the truth table. (0, 1, 4, 5, 6, 7, 10, 11, 14, 15)

f' (a, b, c, d) = (a + b + c + d) (a + b + c + d') (a + b' + c + d)
(a + b' + c + d') (a + b' + c' + d) (a + b' + c' + d') (a' + b + c' + d)
(a' + b + c' + d') (a' + b' + c' + d) (a' + b' + c' + d')

2) Find the expression for the function f (a, b, c, d) in first canonical form. To do this, apply theorems of Boolean algebra to the expression for the complement of function *f* in the second canonical form that you found in answer to Question 1 above. Show all steps and write the name of the theorems you use. z = f (a, b, c, d) =?

b) Applying De Morgan's theorem to the 2nd canonical form of the f' (a, b, c, d) will give us the 1st canonical form of f (a, b, c, d). With De Morgan's rule the (+) operation turns to (.) operation, (.) operation turns to (+), and we take the complement of the variables.

f'' (a, b, c, d) = f (a, b, c, d)
f (a, b, c, d) = a'b'c'd' + a'b'c'd + a'bc'd' + a'bc'd + a'bcd' + a'bcd + ab'cd' + ab'cd + abcd' + abcd

3) Minimize the expression for f (a, b, c, d) in the first canonical form that you found above in Question 2 using axioms and theorems of Boolean algebra. Show all steps in your minimization, and write the name of the axiom/theorem/property you use on the right-hand side of the expression at each step.

c) **f (a, b, c, d) =** a'b'c'd'+a'b'c'd+a'bc'd'+a'bc'd+a'bcd'+a'bcd+ab'cd'+ab'cd+abcd'+abcd =>
a'b'c'd' + a'b'c'd (consensus respect to d) =>
a'b'c'd' + a'b'c'd + a'b'c' (absorption) => a'b'c' =>
a'b'c' + a'bc'd' + a'bc'd + a'bcd' + a'bcd + ab'cd' + ab'cd + abcd' + abcd =>
a'bc'd' + a'bc'd (consensus respect to d) =>
a'bc'd' + a'bc'd + a'bc' (absorption) => a'bc' =>
a'b'c' + a'bc' + a'bcd' + a'bcd + ab'cd' + ab'cd + abcd' + abcd =>
a'bcd' + a'bcd (consensus respect to d) =>
a'bcd' + a'bcd + a'bc (absorption) => a'bc =>

a'b'c' + a'bc' + a'bc + ab'cd' + ab'cd + abcd' + abcd =>
ab'cd' + ab'cd (consensus respect to d) =>
ab'cd' + ab'cd + ab'c (absorption) => ab'c =>
a'b'c' + a'bc' + a'bc + ab'c + abcd' + abcd =>
abcd' + abcd (consensus respect to d) =>
abcd' + abcd + abc (absorption) => abc =>
a'b'c' + a'bc' + a'bc + ab'c + abc =>
a'b'c' + a'bc' (consensus respect to b) =>
a'b'c' + a'bc' + a'c' (absorption) => a'c' =>
a'c' + a'bc + ab'c + abc =>
ab'c + abc (consensus respect to b) =>
ab'c + abc + ac (absorption) => ac =>
a'c' + a'bc + ac =>
a'c' + a'bc (consensus respect to c) =>
a'c' + a'bc + a'b (absorption) => a'c' + a'b =>
a'c' + a'b + ac

Minimize expression of f (a, b, c, d) is “a'c' + a'b + ac”.

4) Draw the circuit for the minimized expression you found in Question 3 above using 2-input NAND gates only.

d)