BLG 210 - Recit 6 (23-Fall):

(1)

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

a) Find the characteristic equation of A.

b) Find the eigenvalues and eigenvectors of A.

c) Diagonalize the matrix A, if possible.

a) $A.\vec{v} = \lambda \vec{v}$

$(A - \lambda I)\vec{v} = \vec{0}$

$$\begin{vmatrix} 4-\lambda & 0 & 2 \\ 2 & 5-\lambda & 4 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0.$$ cofactor expansion along third row!

$|A| = (5-\lambda)^2 (4-\lambda) = 0$

b) For $\lambda = 5$ (algebraic Mult. = 2); $A - 5I$:

$$\left[\begin{array}{ccc|c} -1 & 0 & -2 & 0 \\ 2 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{bmatrix} -1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

x_2 and x_3 are free var.

$$\left.\begin{array}{l} x_1 = -2x_3 \\ x_2 = x_2 \\ x_3 = x_3 \end{array}\right\} \vec{x} = x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

geometric mult. = 2

(The dimension of the eigenspace corresponding to an eigenvalue)

For $\lambda = 4$ (algebraic mult. = 1); $A - 4I$:

$$\left[\begin{array}{ccc|c} 0 & 0 & 2 & 0 \\ 2 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|c} 2 & 1 & 4 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_3 \leftarrow R_3 - \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 2 & 1 & 4 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

x_2 is free var.

$$\left.\begin{array}{l} x_1 = -\dfrac{x_2}{2} \\ x_2 = x_2 \\ x_3 = 0 \end{array}\right\} \vec{x} = x_3 \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{v_3} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

c) $\lambda=5$ (alg. mult. $=2$, geometric mult. $=2$)

$\lambda=4$ (alg. mult. $=1$, geometric mult. $=1$)

$$P=[v_1\ v_2\ v_3]=\begin{bmatrix} -2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

$$D=\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

The diagonal entries of D are eigenvalues of A that correspond, respectively, to the eigenvectors in P.

$A=PDP^{-1}$