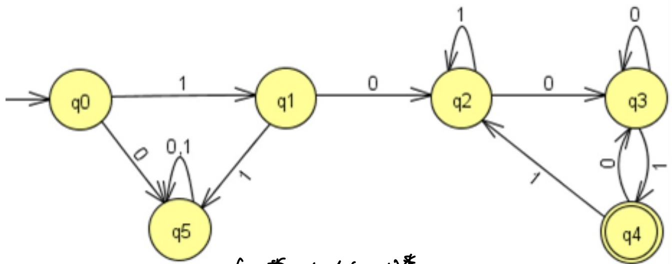
Problem 1

The state transition diagram of a Deterministic Finite Automaton (DFA) is given below.



$10(1^*0+1(01)^*$

Figure 1: Automata for Problem 1

(a) Heuristically derive the regular expression for the language recognized by this DFA.

(b) Produce a Type-3 grammar for the language recognized by this DFA.

b) $q_0 \to 1q_1$

$q_1 \to 0q_2$

$q_2 \to 1q_2 \mid 0q_3$

$q_3 \to 0q_3 \mid 1q_4 \mid 1$

$q_4 \to 1q_2 \mid 0q_3 \mid \lambda$

a)

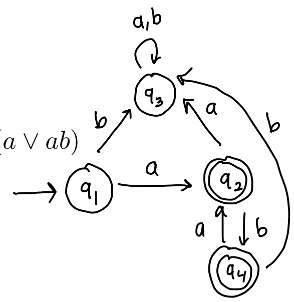
Problem 2

For the given regular expression,

$$(ab) * (a \vee ab)$$

(a) Heuristically find its automata

(b) Construct the DFA for this NFA.



Problem 3

Consider the grammar rules in BNF (Backus-Naur form) notation given below:

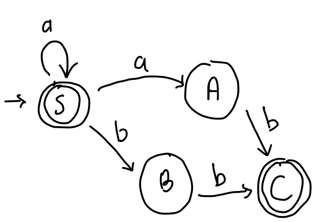
$$< S > ::= \lambda \mid a \mid a < S > \mid a < A > \mid b < B >$$
$$< A > ::= b$$
$$< B > ::= b$$

(a) Heuristically, draw the NFA diagram of the automata with this grammar.

(b) Heuristically, find it's regular expression.

(c) Construct the DFA for this NFA.

a)



b) $a^* \vee a^*(a \vee b)b$

$a^*(\lambda \vee (a \vee b)b)$

c) $q_0 = \{S\}$

$q_0, a \to \{S, A\}$ q_1

$q_0, b \to \{B\}$ q_2

$q_1, a \to \{S, A\}$ q_1

$q_1, b \to \{B, C\}$ q_3

$q_2, a \to \emptyset$

$q_2, b \to \{C\}$ q_4

$q_3, a \to \emptyset$

$q_3, b \to \{C\}$ q_4

$q_4, a \to \emptyset$

$q_4, b \to \emptyset$

$\emptyset = q_5$

Problem 4

Let $L = \{w \mid w \in \{0,1\}^* \textit{ and } w \textit{ does not contain } 011 \textit{ as a substring}\}$

(a) Draw the deterministic finite automaton (DFA) for L as a state transition diagram.

(b) Give a Type-3 grammar for this language.

(c) Write the regular expression for this language.

Problem 5

Minimize the following automata using table-filling algorithm. Show equivalent states and re-draw the minimized automata.

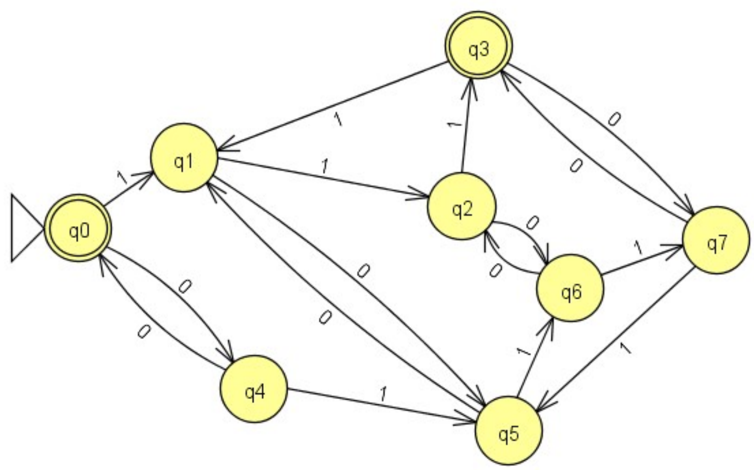


Figure 2: Automata for Problem 5

Problem 6

For the given automata,

(a) Heuristically derive the regular expression for the language recognized by the NFA whose state transition diagram is given aside.

(b) Build the equivalent DFA for this NFA.

Problem 4

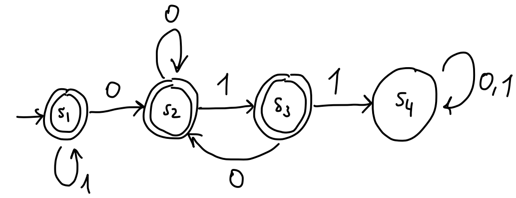
Let $L = \{w \mid w \in \{0,1\}^* \text{ and } w \text{ does not contain } 011 \text{ as a substring}\}$

(a) Draw the deterministic finite automaton (DFA) for L as a state transition diagram.

(b) Give a Type-3 grammar for this language.

(c) Write the regular expression for this language.

a)



b) $S_1 \rightarrow 1S_1 \mid 0S_2 \mid 1 \mid 0 \mid \lambda$

$S_2 \rightarrow 0S_2 \mid 1S_3 \mid 0 \mid 1 \mid \lambda$

$S_3 \rightarrow 0S_2 \mid 0 \mid \lambda$

c) $S_1 = S_1 1 \vee \lambda \qquad S_1 = 1^*$

$S_2 = S_1 0 \vee S_2 0 \vee S_3 0 \qquad S_2 = 1^* 0 + S_2 0 + S_2 10$

$S_3 = S_2 1$

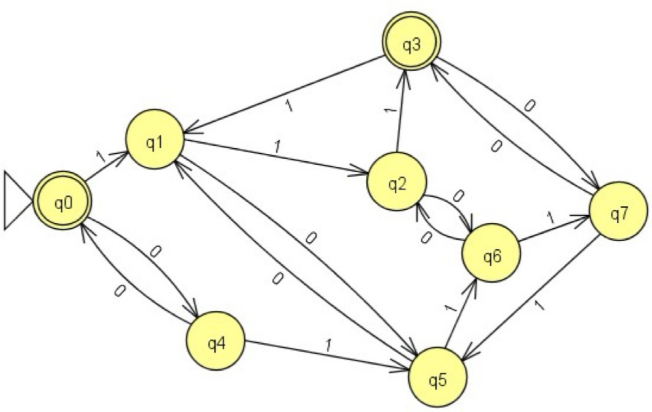
$S_2 = S_2 (10+0) + 1^* 0$

$S_2 = 1^* 0 (10+0)^*$

$\text{REGEX} = 1^* 0 (10+0)^*$

Problem 5

Minimize the following automata using table-filling algorithm. Show equivalent states and re-draw the minimized automata.



	0	1
q_0	q_4	q_1
q_1	q_5	q_2
q_2	q_6	q_3
q_3	q_7	q_1
q_4	q_0	q_5
q_5	q_1	q_6
q_6	q_2	q_7
q_7	q_3	q_5

	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7
q_0	=	X	X	(q_4,q_7)	X	X	X	X
q_1		=	X	X	X	X	X	X
q_2			=	X	X	X	X	X
q_3				=	X	X	X	X
q_4					=	X	X	(q_0,q_3)
q_5						=	X	X
q_6							=	X
q_7								=

$q_0 \sim q_3 \rightarrow (q_4, q_7)$

$q_4 \sim q_7 \rightarrow (q_0, q_3)$

	0	1	
S_0	S_3	S_1	(q_0, q_3)
S_1	S_4	S_2	q_1
S_2	S_5	S_0	q_2
S_3	S_0	S_4	(q_4, q_7)
S_4	S_1	S_5	q_5
S_5	S_2	S_4	q_6

Problem 6

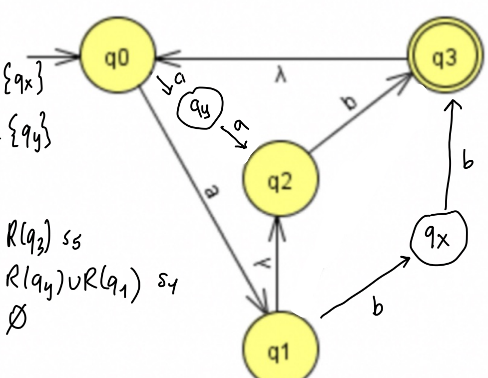
For the given automata,

(a) Heuristically derive the regular expression for the language recognized by the NFA whose state transition diagram is given aside.

(b) Build the equivalent DFA for this NFA.

(c) Produce the Type-3 grammar recognized by the DFA you found in (b).

a) $(aab + abb + ab)^+$



b) $R(q_0) = \{q_0\}$, $R(q_1) = \{q_1, q_2\}$ $R(q_x) = \{q_x\}$

$R(q_2) = \{q_2\}$, $R(q_3) = \{q_3, q_0\}$ $R(q_y) = \{q_y\}$

$S_0 = q_0$

$S_0, a \rightarrow R(q_y) \cup R(q_1) = \{q_y, q_1, q_2\}$ S_1

$S_0, b \rightarrow \emptyset$ S_2

$S_1, a \rightarrow R(q_2) = \{q_2\}$ S_3

$S_1, b \rightarrow R(q_x) \cup R(q_3) = \{q_x, q_0, q_3\}$ S_4

$S_2, a, b \rightarrow \emptyset$ S_2

$S_3, a \rightarrow \emptyset$

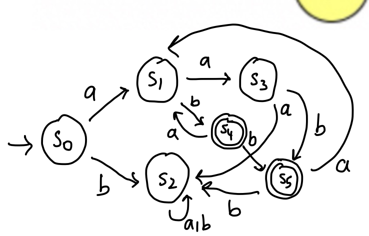
$S_3, b \rightarrow R(q_3) = \{q_0, q_3\}$ S_5

$S_4, a \rightarrow R(q_1) \cup R(q_y) = \{q_1, q_2, q_y\}$ S_1

$S_4, b \rightarrow R(q_3)$ S_5

$S_5, a \rightarrow R(q_y) \cup R(q_1)$ S_1

$S_5, b \rightarrow \emptyset$



$S_0 \rightarrow aS_1$

$S_1 \rightarrow aS_3 \mid bS_4 \mid b$

$S_3 \rightarrow bS_5 \mid b$

$S_4 \rightarrow aS_1 \mid bS_5 \mid b \mid \lambda$

$S_5 \rightarrow aS_1 \mid \lambda$

Problem 7: The Grammar of Laughter

Long before emojis and keysmashing (random atmak) people would express their amusement by typing sequences of characters resembling “haha”.
The rules for producing haha’s are:

- There should be at least 2 “ha”s.
- The first “ha” consists of exactly one h followed by one a.
- Any “ha” after the first can have a variable number of h’s and a’s depending on the intensity of the associated laughter.
- Expressions beginning with a and ending with h are not acceptable.

Some examples of acceptable haha’s are: haha, hahaaaa, hahhhhaaaaaa, hahhaaahaa...
From the first homework, regular expression of the problem was

$$hahh^*a(a + hh^*a)^*$$

Grammar of the problem was

$$
\begin{aligned}
&< S > ::= h < A > \\
&< A > ::= a < B > \\
&< B > ::= h < C > \\
&< C > ::= h < C > \ | \ a < D > \\
&< D > ::= h < C > \ | \ a < D > \ | \ a
\end{aligned}
$$

(a) Heuristically, draw the NFA diagram of the automata with this grammar.

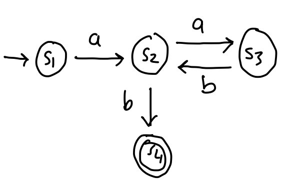
(b) Construct the DFA for this NFA.

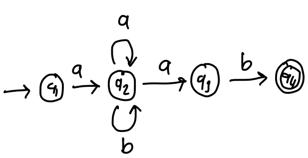
Problem 8

Using a systematic approach, check whether each of the following regular expression couples are disjoint, and if they are not, find the shortest non-empty ($\neq \lambda$) string that matches both of them.

(a) $L_1 = a(ab)^*b$ and $L_2 = a(a \vee b)^*ab$

(b) $L_1 = a(ab)^*a$ and $L_2 = a(a \vee b)^*ba$ aaba

a) L_1 :


L_2 :


$L_1 \cap L_2$:

$S = \{(s_1,q_1), (s_1,q_2), (s_1,q_3), (s_1,q_4), (s_2,q_1) \ . \ - \ - \ - \ - \ - \}$

$s_0 = \{(s_1,q_1)\}$ $F = \{(s_4,q_4)\}$

(s_1q_1), a (s_2q_2)

(s_2q_2), a (s_3,q_3)

(s_3,q_3), b, (s_2q_4)