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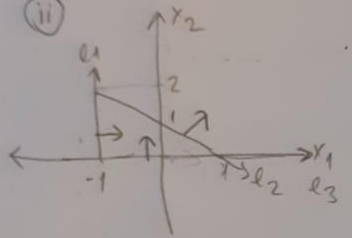
(1)

(i) No it is not linearly seperable since the shape of the areas are more complex

(ii)

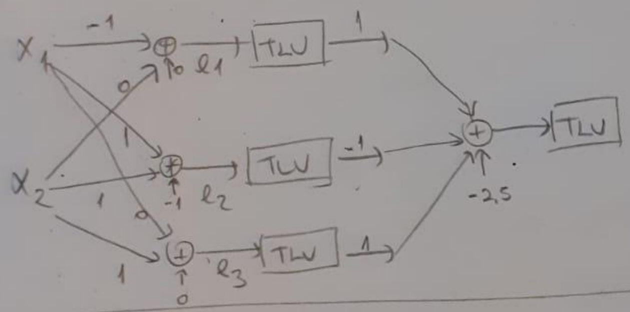


$\ell_1 \Rightarrow \quad x_1 = -1 \qquad \vec{n} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\ell_2 \Rightarrow \quad (x_2 - 2) = -(x_1 + 1)$

$x_2 + x_1 - 1 = 0 \qquad \vec{n} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\ell_3 \Rightarrow \quad x_2 = 0 \qquad \vec{n} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



Single hidden layer discrete perceptron classifier

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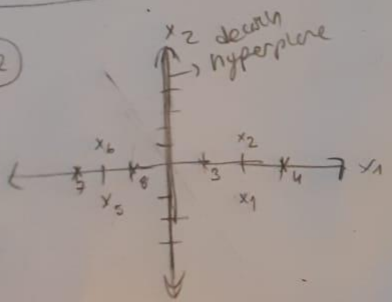
(2)



→ Yes it is linearly seperable

→ class 1 → $\dfrac{(2+2+1+3)}{4} = 2 = x_1$

$\dfrac{-1+1+0+0}{4} = 0 = x_2$

Center of gravity of class 1 is

$x_1 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Class 2 $\left(\dfrac{-2-2-3-1}{4}\right) = -2 = x_1$

$\left(\dfrac{-1+1+0+0}{4}\right) = 0 = x_2$

(1)

x_2 = Center of gravity of class 2 = $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

(2) → continue

Decision surface

is arbitrarily chosen as

$g(x) = \underline{x_1 = 0}$

(2 iii)

$(x_1 - x_2)^T x + \frac{1}{2}(\|x_2\|^2 - \|x_1\|^2) = 0$

$\begin{bmatrix} 4 \\ 0 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{2}(4 - 4) = 0$

$\boxed{x_1 = 0}$ → decision hyperplane

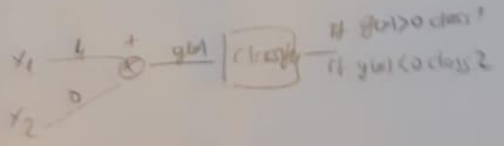
$W = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ $\quad g(x) = 4x_1 = 0$

for $P_1 = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \Rightarrow g(P_1) = 20$ $\quad g(x) > 0$ class 1

for $P_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow g(P_2) = 0$ $\quad g(x) = 0$ unknown

for $P_3 = \begin{bmatrix} 12 \\ -3 \end{bmatrix} \Rightarrow g(P_3) = 12.4$ $\quad g(x) > 0$ class 1

for $P_4 = \begin{bmatrix} -2 \\ -4 \end{bmatrix} \Rightarrow g(P_4) = -8$ $\quad g(x) < 0$ class 2



(2)

(3)

$E(\underline{v}) = v_1^2 - v_2^2 + v_3^2 - 2v_1v_3 - v_2v_3 + 4v_1 + 12$

(i) Find W and I

$$E = -\frac{1}{2} v^T W v - I v = -\frac{1}{2} \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} v - \begin{bmatrix} x \\ y \\ z \end{bmatrix}^t \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$E = -\frac{1}{2} \left(\begin{bmatrix} av_1 + v_2 d + v_3 g & bv_1 + v_2 e + v_3 h & cv_1 + fv_2 + kv_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \right) -$$

$(xv_1 + yv_2 + zv_3)$

$$E = -\frac{1}{2} \left(av_1^2 + dv_1v_2 + gv_1v_3 + bv_1v_2 + ev_2^2 + hv_3v_2 + cv_1v_3 + fv_2v_3 + kv_3^2 \right)$$

$- (xv_1 + yv_2 + zv_3)$

$a = -2$	$d = 0$	$b = 0$		$x = -1$
$e = 2$	$g + c = 4$	$g = c = 2$		$y = 0$
$k = -2$	$h + f = 2$	$h = f = 1$		$z = 0$

$$W = \begin{bmatrix} -2 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & -2 \end{bmatrix} \qquad I = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\nabla E(\bar{v}) = \begin{bmatrix} 2v_1 - 2v_3 + 4 \\ -2v_2 - v_3 \\ 2v_3 - 2v_1 - v_2 \end{bmatrix} \qquad \nabla^2 E(v) = \begin{bmatrix} 2 & 0 & -2 \\ 0 & -2 & -1 \\ -2 & -1 & 2 \end{bmatrix}$$

(3)

→ Equalibrium point of Gradient Type Hopfield NN.

$\nabla E = 0$ is the equalibrium point according to Lypnov's theorem

Equalibrium point $\quad V = \begin{bmatrix} -2V_2 - 2 \\ -V_2 \\ -2V_2 \end{bmatrix}$ found from the equations below

$\nabla E(V^*) = 0$

→ $A = W$ $\qquad$ $\det A_{33} = 2 > 0$

$\det A_{11} = -2 < 0$ $\qquad$ so it is <u>negative definite</u>

$\det A_{22} = -2 < 0$

→

$\nabla E = 0$

$2V_1 - 2V_3 + 4 = 0$

$-2V_2 - V_3 = 0$

$V_3 = -2V_2$

$2V_1 + 4V_2 + 4 = 0$

$V_1 = -2V_2 - 2$

$2V_3 - 2V_1 - V_2 = -4V_2 + 4V_2 + 4 - V_2 = 0$

$V_2 = 4 \quad V_1 = -10$

$V_3 = -8$

$\begin{bmatrix} -10 \\ 4 \\ -8 \end{bmatrix}$ → max point since negative <u>definite</u>

④

(4) Unipolar activation functions

(i) $E_C = \frac{1}{2}\left(x - \sum_{i=0}^{1} v_i 2^i\right)^2$ where x is the analog input

v_i is the

2^i is the w_{ii}

(ii) $E = -\frac{1}{2}\sum_{\substack{i=0 \\ j \neq i}}^{1}\sum_{j=0}^{1} w_{ij} v_i v_j - \sum_{i=0}^{1} i_i v_i$ is in general and for

$E = E_C + E_a = \frac{1}{2}\left(x - \sum_{i=0}^{1} v_i 2^i\right)^2 - \frac{1}{2}\sum_{i=0}^{1} 2^{2i} v_i (v_i - 1)$

$E = \frac{1}{2}x^2 + \frac{1}{2}\underbrace{\sum_{\substack{i=0 \\ i \neq j}}^{1}\sum_{j=0}^{1} 2^{i+j}}_{\text{corresponds to } W} v_i v_j + \underbrace{\sum_{i=0}^{1}\left(2^{2i-1} - 2^i x\right)}_{\text{corresponds to } I} v_i$

where $\frac{1}{2}x^2$ is additive constant

(iii) $w_{ij} = \sum_{i=0}^{1}\sum_{\substack{j=0 \\ i \neq j}}^{1} 2^{i+j}$

$i=0\ j=1$ $\quad w_{ij} = 2^1 = 2^{0+1}$ else $= 0$

$i=1\ j=0$ $\quad w_{ij} = 2^1 = 2^{1+0}$

For inverting neurons:

$W = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$

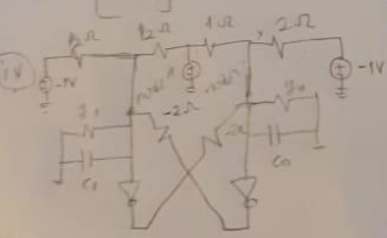
$I_i = \sum_{i=0}^{1}\left(2^{2i-1} - 2^i x\right)$

$i=0$ $\quad I_0 = \left(2^{-1} - 2^0 x\right) = \left(\frac{1}{2} - x\right)$

$i=1$ $\quad I_1 = \left(2^{2-1} - 2^1 x\right) = (2 - 2x)$

$I = \begin{bmatrix} -x + 1/2 \\ -2x + 2 \end{bmatrix}$

(iv)



KCL to node 1

$C_0 \frac{du_0}{dt} = \left(x - \frac{1}{2}\right) + (2\bar{v}_1) - v_0(2 + g_0)$

$C_1 \frac{du_1}{dt} = (2x - 2) + (-2)\bar{v}_0 - u_1(-2 + g_1)$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} du_0/dt \\ du_1/dt \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} \bar{v}_0 \\ \bar{v}_1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} v_0 \\ v_1 \end{bmatrix} + \begin{bmatrix} x - 1/2 \\ 2x - 2 \end{bmatrix}$

(5)

(v) $x=0$ $x=1$ $x=1.5$

$$E = -\frac{1}{2} v^T W v - i^t v$$

$$E = -\frac{1}{2} \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} - x \\ 2-2x \end{bmatrix}^t \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 2v_2 & 2v_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} \frac{1}{2}v_1 - xv_1 + 2v_2 - 2xv_2 \end{bmatrix}$$

$$E = -v_2 v_1 - v_1 v_2 - \frac{1}{2} v_1 - x v_1 + 2v_2 - 2x v_2$$

$$= -2v_1 v_2 - \frac{v_1}{2} + 2v_2 - x(v_1 + 2v_2)$$

$$\nabla E = \begin{bmatrix} -2v_2 - 1/2 - x \\ -2v_1 + 2 - 2x \end{bmatrix}$$

for $x=0$

$-2v_2 = 1/2$

$-2v_1 + 2 = 0$

$$\boxed{\begin{matrix} v_2 = -1/4 \\ v_1 = 1 \end{matrix}}$$

$\begin{pmatrix} 1 \\ -0.25 \end{pmatrix}$ stationary point

for $x=1$

$-2v_2 - 1/2 - 1 = 0$

$v_2 = -3/4$

$-2v_1 = 0$ $v_1 = 0$

$\begin{pmatrix} 0 \\ -3/4 \end{pmatrix} \to$ stationary point

for $x=1.5$

$v_2 = -1$

$v_1 = -1/2$

$\begin{pmatrix} -0.5 \\ -1 \end{pmatrix}$ stationary point

Types of stationary point

$\det H_{11} = 0$ not positive definite or negative definite so, they are saddle point

$\det H_{2\times2} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = -4$

(b)

(vi) It do not have unconstrant minima since it has either negative definite nor positive definite

(vii)

(5)

(i) $\Rightarrow$ $s^1 s^2 = \frac{1}{4} + \frac{1}{12} - \frac{5}{12} + \frac{1}{12} = 0$

$s^2 s^3 = \frac{1}{4} - \frac{5}{30} \frac{1}{36} - \frac{5}{36} = 0$

$s^1 s^3 = \frac{1}{4} + \frac{1}{12} + \frac{1}{12} - \frac{5}{12} = 0$

$s^1 s^{1T} = \sqrt{1}$

$s^2 s^{2T} = \sqrt{\frac{1}{4} + \frac{25}{36} + \frac{1}{36} + \frac{1}{36}} = 1$

$s^3 s^{3T} = \sqrt{\frac{1}{4} + \frac{1}{36} + \frac{1}{36} + \frac{25}{36}} = 1$

$s^i, i = 1,2,3$ are orthonormal

(ii) $W = \Sigma f^i s^i$

$$W_1' = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$W_2' = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1/2 & -5/6 & 1/6 & 1/6 \end{bmatrix} = \begin{bmatrix} 1/2 & -5/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 \\ -1/2 & +5/6 & -1/6 & -1/6 \end{bmatrix}$$

$$W_3' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1/2 & 1/6 & 1/6 & -5/6 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/6 & 1/6 & -5/6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W = W_1' + W_2' + W_3' = \begin{bmatrix} 1 & -4/6 & 2/6 & -4/6 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0 & 8/6 & 2/6 & 2/6 \end{bmatrix}$$

(iii) $f_i = W s^i$

$$f_1 = \begin{bmatrix} 1 & -4/6 & 2/6 & -4/6 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0 & 8/6 & 2/6 & 2/6 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{-4}{12} + \frac{2}{12} - \frac{4}{12} \\ \frac{1}{4} + \frac{1}{4} - \frac{1}{4} + \frac{1}{4} \\ \frac{8}{12} + \frac{2}{12} + \frac{2}{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \checkmark$$

$$f_2 = W \cdot \begin{bmatrix} 1/2 \\ -5/6 \\ 1/6 \\ 1/6 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - \frac{20}{36} + \frac{2}{36} - \frac{4}{36} \\ 1/4 - 5/12 + 1/12 + 1/12 \\ -\frac{40}{36} + \frac{2}{36} + \frac{2}{36} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \checkmark$$

(8)

$$f_3 = W \begin{bmatrix} 1/2 \\ 1/6 \\ 1/6 \\ -4/6 \end{bmatrix} = \begin{bmatrix} 1/2 - \frac{4}{36} + \frac{3}{36} + \frac{24}{36} \\ 1/4 + \frac{1}{12} + \frac{1}{12} - \frac{5}{12} \\ \frac{8}{36} + \frac{3}{36} + \frac{-10}{36} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \checkmark$$

(iv)

$$W s^1 = f^1 + \eta^1$$

$$\eta^1 = W s^1 - f^1$$

$$\eta^1 = \begin{bmatrix} 1 & -4/6 & 1/3 & -4/6 \\ 1/2 & 1/2 & 1/2 & 1/2 \\ 0 & 1/6 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4/12 \\ 3/4 \\ 10/12 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\eta^1 = \begin{bmatrix} 4/12 \\ -1/4 \\ -2/12 \end{bmatrix}$$

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(6)

$$W = \sum_i s^i s^{i\,T} - pI = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 & -1 & -1 & 1 \\ -1 & +1 & +1 & +1 & -1 \\ -1 & +1 & +1 & +1 & -1 \\ -1 & +1 & +1 & +1 & -1 \\ +1 & -1 & -1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} +1 & -1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$- 3 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 & -1 & +1 \\ -1 & 0 & 3 & 3 & 1 \\ -1 & 3 & 0 & 3 & 1 \\ -1 & 3 & 3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$E = -\frac{1}{2} v^T W v = -\frac{1}{2} \Big[-v_2 - v_3 - v_4 + v_5,\ -v_1 + 3v_3 + 3v_4 + v_5,$$
$$-v_1 + 3v_2 + 3v_4 + v_5,\ -v_1 + 3v_2 + 3v_3 + v_5$$
$$+ v_1 + v_2 + v_3 + v_4 \Big] \checkmark$$

$$E = -\frac{1}{2} \Big[-v_1v_2 - v_1v_3 - v_1v_4 + v_1v_5 - v_1v_2 + 3v_2v_3 + 3v_2v_4 + v_2v_5 - v_3v_1 + 3v_3v_2 + 3v_3v_4$$
$$+ v_3v_5 - v_4v_1 + 3v_4v_2 + 3v_4v_3 + v_4v_5 + v_1v_5 + v_2v_5 + v_3v_5 + v_4v_5 \Big]$$

(9) $$E = v_1v_2 + v_1v_3 + v_1v_4 - v_1v_5 - 3v_2v_3 - 3v_2v_4 - v_2v_5 - 3v_3v_4 - v_3v_5 - v_5v_5$$

$v^0 = [1\ -1\ -1\ 1\ 1]^t$

$v^1 = \text{sgn}(w_1 v^0) = \text{sgn}\left([0\ -1\ -1\ -1\ +1] \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}\right) = +1 \qquad v^1 = v^0$

$v^2 = \text{sgn}(w_2 v^1) = \text{sgn}\left([-1\ 0\ 3\ 3\ 1] \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}\right) = \text{sgn}(-1-3+3+1) = 0$ no change

$v^2 = v^1$

$v^3 = \text{sgn}(w_3 v^2) = \text{sgn}\left([-1\ 3\ 0\ 3\ 1] \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}\right) = \text{sgn}(-1-3+3+1) = 0$ no change

$v^3 = v_2$

$v^4 = \text{sgn}(w_4 v^3) = \text{sgn}\left([-1\ 3\ 3\ 0\ 1] \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}\right) = \text{sgn}(-1-3-3+1) = -1$

$v^4 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

$v^5 = \text{sgn}(w_5 v^4) = \text{sgn}\left([1\ 1\ 1\ 1\ 0] \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}\right) = \text{sgn}(1-1-1-1) = -1$

$v^5 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$

$v^6 = \text{sgn}(w_1 v^5) = \text{sgn}\left([0\ -1\ -1\ -1\ +1] \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}\right) = 1 \qquad v^6 = v^5$

$v^7 = \text{sgn}(w_2 v^6) = -1 \qquad v^7 = v^6$

$v^8 = \text{sgn}(w_3 v^7) = -1 \qquad v^8 = v^6$

$v^9 = \text{sgn}(w_4 v^8) = -1 \qquad v^9 = v^5$

$v^{10} = \text{sgn}(w_5 v^9) = -1 \qquad v^{10} = v^9$

fixed point at $\begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$

(iv) it converges to s_i values and their conjugates