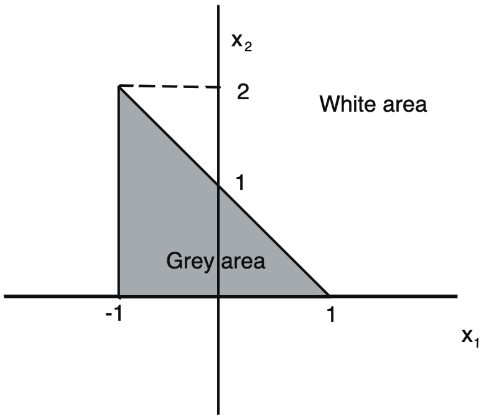
EE 401 - Neural Networks Final

18.01.2022 Tuesday
15:00-18:00

1- A planar input pattern region of class 1 and 2 is shown in the figure below. The grey area in the figure below belongs to Category 1 and the rest of the pattern space belongs to Category 2
i. Is this configuration linearly separable? Why?
ii. If yes, then construct a single layer perceptron classifier. If no, then construct a multi-layer discrete-perceptron classifier.




Category 1: White area
Category 2: Grey area

2- Prototype points are given as

$$X_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix},\ X_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ X_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ X_4 = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \text{ : Class 1}$$

$$X_5 = \begin{bmatrix} -2 \\ -1 \end{bmatrix},\ X_6 = \begin{bmatrix} -2 \\ 1 \end{bmatrix},\ X_7 = \begin{bmatrix} -3 \\ 0 \end{bmatrix},\ X_8 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \text{ :Class 2}$$

i. Determine if the classes of patterns are linearly separable.
ii. Determine the center of gravity for patterns of each class and find & draw the decision surface in pattern space.
iii. Using equations (3.9) and (3.11) design the dichotomizer for the given prototype points and determine how it would recognize the following input patterns of unknown class membership.

$$P_1 = \begin{bmatrix} 5 \\ -2 \end{bmatrix}, P_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, P_3 = \begin{bmatrix} 12 \\ -8 \end{bmatrix}, P_4 = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

3- The truncated energy function of a certain three-neuron single layer network with self-feedback is given as

$$E(\underline{\mathbf{V}}) = v_1^2 - v_2^2 + v_3^2 - 2v_1v_3 - v_2v_3 + 4v_1 + 12 \text{ where } \underline{\mathbf{V}} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Assuming high gain neurons;

i. Find the weight matrix $\underline{\underline{\mathbf{W}}} \in R^{3x3}$ and the bias current vector $\underline{\mathbf{I}} \in R^3$
ii. Find the gradient vector $\nabla E(\underline{\mathbf{V}})$
iii. Find the equilibrium point of Gradient Type Hopfield Neural Network
iv. Find the Hessian matrix $\nabla^2 E(\underline{\mathbf{V}})$
v. Decide on the definiteness of the Hessian matrix
vi. Find any minima, maxima or saddle points that the energy function may have, what are they?

4- An analog input value $x \in R$ is to be converted by a two neuron network proposed by Tank and Hopfield as an A/D converter. It is assumed the high gain neurons have continuous unipolar activation functions.

i. Write the A/D conversion error $E_c$ by defining every term in $E_c$
ii. Write the truncated energy funtion E
iii. Find the parameters of two neuron network
iv. Write the state equations and the output equations for this network ( choose $C_0 = C_1 = 1F$, $g_0 = g_1 = 1v$ and choose inverting neurons.)
v. If the analog inputs are given as $x = 0,\ x = 1,\ x = 1.5$ then find the corresponding stationary points of the network and decide on the type of the stationary points (i.e maxima, minima or saddle points ?)
vi. Decide whether the truncated energy function has unconstrained minima
vii. If this truncated energy function has no unconstrained minima then can the stationary points be reached by the network starting from some initial voltages on the capacitors?

5- The linear associator has to associate the following pairs of vectors;

$$S^{(1)} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}^t \longrightarrow f^{(1)} = [0 \quad 1 \quad 1]^t$$

$$S^{(2)} = \begin{bmatrix} \frac{1}{2} & \frac{-5}{6} & \frac{1}{6} & \frac{1}{6} \end{bmatrix}^t \longrightarrow f^{(2)} = [1 \quad 0 \quad -1]^t$$

$$S^{(3)} = \begin{bmatrix} \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{-5}{6} \end{bmatrix}^t \longrightarrow f^{(3)} = [1 \quad 0 \quad 0]^t$$

i. Verify that the vectors $S^{(i)}, i = 1,2,3$ are orthonormal,
ii. Find the memory weight matrix
iii. Veriy the association performed by the network for the stimuli $S^{(1)}$
iv. Distort $S^{(1)}$ as $S^{(1)} + \Delta^{(1)}$ by choosing $\Delta^{(1)}$ orthogonal to $S^{(1)}$ and then find the corresponding response

6- The following vectors need to be stored in a recurrent autoassociative memory;

$$S^{(1)} = [1 \quad 1 \quad 1 \quad 1 \quad 1]^t,\ S^{(2)} = [1 \quad -1 \quad -1 \quad -1 \quad 1]^t,\ S^{(3)} = [-1 \quad 1 \quad 1 \quad 1 \quad 1]^t$$

i. Use the storage algorithm to calculate the weight matrix
ii. Evaluate the energy function
iii. Apply the input vector $V^{(0)} = [1 \quad -1 \quad -1 \quad 1 \quad 1]^t$ and allow for asynchronous update sequence in retrieval algorithm in ascending node order starting at node 1 while evaluating energy values in each updating mode.
iv. Discuss the convergence of this sequence and if it exists find its limit, determine the stored memory.