EE401 Neural Networks
Midterm 2

1. The weight matrix W for a network with bipolar discrete binary neurons is given as

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & -1 & -1 & -3 \\ 1 & 0 & 1 & 1 & -1 \\ -1 & 1 & 0 & 3 & 1 \\ -1 & 1 & 3 & 0 & 1 \\ -3 & -1 & 1 & 1 & 0 \end{bmatrix}$$

Knowing that the thresholds and external inputs of neurons are zero, compute the energy values for all 32 bipolar binary vectors .Identify the potential attractors that may have been encoded in the system described by the specified matrix W by comparing the energy values at each of the [-1,1] cube vertices. Implement five sample asynchronous discrete-time transitions from high to low energy vertices.

2. The stationary point solution of Equations (5.28) describing the simple two-bit A/D converter can be obtained using the Newton-Raphson method by solving the nonlinear algebraic equation F(u) = 0, where

$$\mathbf{F}(\mathbf{u}) = \begin{bmatrix} -2v_1 + x - 0.5 - (g_0 - 2)u_0 \\ -2v_0 + 2x - 2 - (g_1 - 2)u_1 \end{bmatrix}$$

The iterative solution is obtained in this method as shown:

$$\begin{bmatrix} u_0^{k+1} \\ u_1^{k+1} \end{bmatrix} = \begin{bmatrix} u_0^{k} \\ u_1^{k} \end{bmatrix} - [\mathbf{J}(\mathbf{u})]^{-1} \begin{bmatrix} F_0(\mathbf{u})^k \\ F_1(\mathbf{u})^k \end{bmatrix} \text{ where Jacobian defined as } \mathbf{J} \triangleq \begin{bmatrix} \dfrac{\partial F_0}{\partial u_0} & \dfrac{\partial F_0}{\partial u_1} \\ \dfrac{\partial F_1}{\partial u_0} & \dfrac{\partial F_1}{\partial u_1} \end{bmatrix}$$

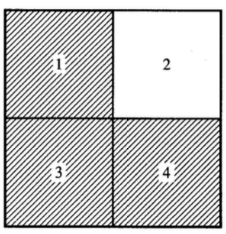
Find the closed form for the iterative solution of the equation $F(\mathbf{u}) = \mathbf{0}$ including the Jacobian matrix entries (no matrix inversion required). The activation function to be assumed is $v_i = [1 + exp(-\lambda u_i]^{-1}$ for $i = 0,\ 1$

3. Vectors $a^{(1)} = [1 \quad -1 \quad 1 \quad 1]$ and $a^{(2)} = [-1 \quad 1 \quad -1 \quad 1]$ represent the bit map shown below. They need to be associated for input vectors at HD=1 from the prototypes stored (HD stands for Hemming Distance). Design the following two autoassociators:

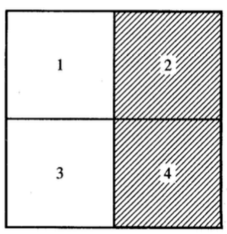
i- a linear autoassociator in the form of a linear associative memory with added thresholding element (TLU) at the output, if needed, but no feedback

ii- a recurrent autoassociative memory with asynchronous updating.

Compare the performance of both networks by evaluating responses to eight input vectors at HD = 1 from $a^{(1)}$ and $a^{(2)}$. The nodes of the memory designed in part (b) should be updated in ascending order starting at node 1.



$\mathbf{a}^{(1)}=[1 -1 1 1]'$



$\mathbf{a}^{(2)}=[-1 1 -1 1]'$

1) $E(\bar{v}) = -\frac{1}{2} \cdot v^T \cdot W \cdot v$

$$E(\bar{v}) = [v_1\ v_2\ v_3\ v_4\ v_5] \begin{bmatrix} v_2 - v_3 - v_4 - 3v_5 \\ v_1 + v_3 + v_4 - v_5 \\ -v_1 + v_2 + 3v_4 + v_5 \\ -v_1 + v_2 + 3v_3 + v_5 \\ -3v_1 - v_2 + v_3 + v_4 \end{bmatrix} = \begin{matrix} -v_1 \cdot v_2 + v_1 \cdot v_3 + v_1 \cdot v_4 + 3v_1 \cdot v_5 \\ -v_2 \cdot v_4 + v_2 \cdot v_5 - 3 \cdot v_3 \cdot v_4 - v_3 \cdot v_5 \end{matrix}$$

$-v_2 \cdot v_3 +$

$-v_4 \cdot v_5$

$\bar{v}_1 = [-1\ -1\ -1\ -1\ -1]$ $\bar{v}_2 = [-1\ -1\ -1\ -1\ 1]$ $\bar{v}_3 = [-1\ -1\ -1\ 1\ -1]$ $\bar{v}_4 = [-1\ -1\ -1\ 1\ 1]$

$\bar{v}_5 = [-1\ -1\ 1\ -1\ -1]$ $\bar{v}_6 = [-1\ -1\ 1\ -1\ 1]$ $\bar{v}_7 = [-1\ -1\ 1\ 1\ -1]$ $\bar{v}_8 = [-1\ -1\ 1\ 1\ 1]$

$\bar{v}_9 = [-1\ 1\ -1\ -1\ -1]$ $\bar{v}_{10} = [-1\ 1\ -1\ -1\ 1]$ $\bar{v}_{11} = [-1\ 1\ -1\ 1\ -1]$ $\bar{v}_{12} = [-1\ 1\ -1\ 1\ 1]$

$\bar{v}_{13} = [-1\ 1\ 1\ -1\ -1]$ $\bar{v}_{14} = [-1\ 1\ 1\ -1\ 1]$ $\bar{v}_{15} = [-1\ 1\ 1\ 1\ -1]$ $\bar{v}_{16} = [-1\ 1\ 1\ 1\ 1]$

$\bar{v}_{17} = [1\ -1\ -1\ -1\ -1]$ $\bar{v}_{18} = [1\ -1\ -1\ -1\ 1]$ $\bar{v}_{19} = [1\ -1\ -1\ 1\ -1]$ $\bar{v}_{20} = [1\ -1\ -1\ 1\ 1]$

$\bar{v}_{21} = [1\ -1\ 1\ -1\ -1]$ $\bar{v}_{22} = [1\ -1\ 1\ -1\ 1]$ $\bar{v}_{23} = [1\ -1\ 1\ 1\ -1]$ $\bar{v}_{24} = [1\ -1\ 1\ 1\ 1]$

$\bar{v}_{25} = [1\ 1\ -1\ -1\ -1]$ $\bar{v}_{26} = [1\ 1\ -1\ -1\ 1]$ $\bar{v}_{27} = [1\ 1\ -1\ 1\ -1]$ $\bar{v}_{28} = [1\ 1\ -1\ 1\ 1]$

$\bar{v}_{29} = [1\ 1\ 1\ -1\ -1]$ $\bar{v}_{30} = [1\ 1\ 1\ -1\ 1]$ $\bar{v}_{31} = [1\ 1\ 1\ 1\ -1]$ $\bar{v}_{32} = [1\ 1\ 1\ 1\ 1]$

$E(\bar{v}_1) = -2$ $E(\bar{v}_2) = -6$ $E(\bar{v}_3) = 6$ $E(\bar{v}_4) = -2$ $E(\bar{v}_5) = 6$ $E(\bar{v}_6) = -2$ $E(\bar{v}_7) = 2$ $E(\bar{v}_8) = -10$

$E(\bar{v}_9) = 2$ $E(\bar{v}_{10}) = 2$ $E(\bar{v}_{11}) = 6$ $E(\bar{v}_{12}) = 2$ $E(\bar{v}_{13}) = 6$ $E(\bar{v}_{14}) = 2$ $E(\bar{v}_{15}) = -2$ $E(\bar{v}_{16}) = -10$

$E(\bar{v}_{17}) = -10$ $E(\bar{v}_{18}) = -2$ $E(\bar{v}_{19}) = 2$ $E(\bar{v}_{20}) = 6$ $E(\bar{v}_{21}) = 2$ $E(\bar{v}_{22}) = 6$ $E(\bar{v}_{23}) = 2$ $E(\bar{v}_{24}) = 2$

$E(\bar{v}_{25}) = -10$ $E(\bar{v}_{26}) = 2$ $E(\bar{v}_{27}) = -2$ $E(\bar{v}_{28}) = 6$ $E(\bar{v}_{29}) = -2$ $E(\bar{v}_{30}) = 6$ $E(\bar{v}_{31}) = -6$ $E(\bar{v}_{32}) = -2$

Energy is minimum at $\bar{v}_8, \bar{v}_{16}, \bar{v}_{17}, \bar{v}_{25}$.

Take $\bar{v}_5 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \bar{v}_{17}$

2) $v_0 = \frac{1}{1+e^{-\lambda u_0}} \Rightarrow \frac{dv_0}{du_0} = \frac{\lambda . e^{-\lambda u_0}}{(1+e^{-\lambda . u_0})^2}$, $v_1 = \frac{1}{1+e^{-\lambda . u_1}} \Rightarrow \frac{dv_1}{du_1} = \frac{\lambda . e^{-\lambda u_1}}{(1+e^{-\lambda u_1})^2}$

$F_0 = -2v_1 + x - 0.5 - (g_0 - 2)u_0$, $F_1 = -2v_0 + 2x - 2 - (g_1 - 2)u_1$

$\frac{\partial F_0}{\partial u_0} = -(g_0 - 2)$, $\frac{\partial F_0}{\partial u_1} = \frac{-2 . \lambda . e^{-\lambda . u_1}}{(1+e^{-\lambda . u_1})^2}$, $\frac{\partial F_1}{\partial u_0} = \frac{-2 . \lambda e^{-\lambda . u_0}}{(1+e^{-\lambda u_0})^2}$, $\frac{\partial F_1}{\partial u_1} = -(g_1 - 2)$

$$\begin{bmatrix} u_0^{k+1} \\ u_1^{k+1} \end{bmatrix} = \begin{bmatrix} u_0^{k} \\ u_1^{k} \end{bmatrix} - \begin{bmatrix} 2 - g_0 & \frac{-2\lambda . e^{-\lambda u_1}}{(1+e^{-\lambda u_1})^2} \\ \frac{-2\lambda e^{-\lambda u_0}}{(1+e^{-\lambda u_0})^2} & 2 - g_1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{-2}{1+e^{-\lambda . u_1}} + x - 0.5 - (g_0 - 2) . u_0 \\ \frac{-2}{1+e^{-\lambda . u_0}} + 2x - 2 - (g_1 - 2) u_1 \end{bmatrix}$$

3-i) $S_1 = \{s_1, s_2, s_3, s_4\}$ where $s_1 = [-1\ -1\ 1\ 1]^T$, $s_2 = [1\ 1\ 1\ 1]^T$

$s_3 = [1\ -1\ -1\ 1]^T$, $s_4 = [1\ -1\ 1\ -1]^T$

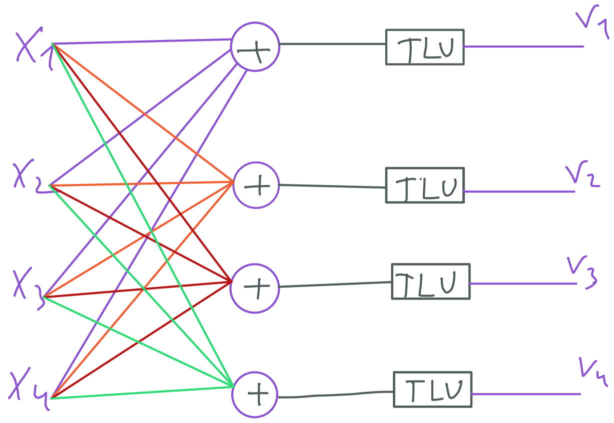
$S_2 = \{s_5, s_6, s_7, s_8\}$ where $s_5 = [1\ 1\ -1\ 1]^T$, $s_6 = [-1\ -1\ -1\ 1]^T$

$s_7 = [-1\ 1\ 1\ 1]^T$, $s_8 = [-1\ 1\ -1\ -1]^T$

$f_1 = f_2 = f_3 = f_4 = [1\ -1\ 1\ 1]^T$, $f_5 = f_6 = f_7 = f_8 = [-1\ 1\ -1\ 1]^T$

$$F = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 \end{bmatrix}$$

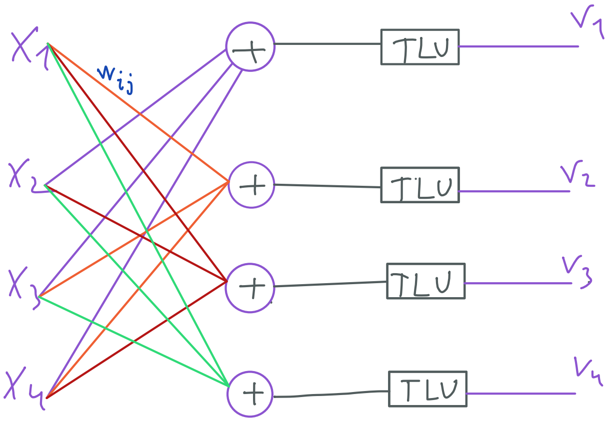
$$W = F \cdot S^T = \begin{bmatrix} 2 & -4 & 4 & 0 \\ -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$



ii)

$$S_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad S_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \qquad W = S_1 . S_1^T + S_2 . S_2^T - 2.I$$

$$W = \begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix} - 2I = \begin{bmatrix} 0 & -2 & 2 & 0 \\ -2 & 0 & -2 & 0 \\ 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



Compare for $\bar{v} = s_5 = [1\ 1\ -1\ 1]^T$

i) $\Gamma(W.s_5) = \Gamma\left(\begin{bmatrix} 2 & -4 & 4 & 0 \\ -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = a^{(2)}$

ii) Update for first row: $W_1 . s_5 = [0\ -2\ 2\ 0]\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

$= -4$ ✓

$s_5^{(2)} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

no more update

So performances are same for s_5