LECTURE 1
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LECTURE 8
SINAV KONULARI
- Causality, stability (in terms of input-output signals, or h[n], or H(z), or H(jw)), ROC?
- Basit bir sinyal üzerinden Autocorrelation hesaplanması.
- Basit iki sinyal üzerinden Crosscorrelation hesaplanması
- Phase türleri? Zero, Linear, Nonlinear. Özellikleri?
- FIR, IIR filtreler. Avantaj dezavantajları? Phase özellikleri?
- Convolution.
- LCCDEs.
- Fourier Transform türleri ve aralarındaki farklar (ayrıklık, süreklilik, periyodiklik, aperiyodiklik: in time and in frequency domains)
- What are stationary and nonstationary signals? Differences?
- How Fourier Transform (FT) works on signals?
- What are the disadvantages/shortcomings of FT to work on nonstationary signals?
- How Short Time Fourier Transform (STFT) works on signals? Also talk about resoloution (time,frequency) issues for STFT.
- What is Heisenberg principle?
- What is Wavelet Transform (WT)? Briefly explain. How it overcomes the preset resolution problem?
- What is wavelet?
- What is the relationship between scale and frequency, and translation and time?
- What are the conditions for wavelet functions to satisfy?
- Fourier Transform, Short Time Fourier Transform ve Wavelet transform tarifleri ve karşılaştırmaları?
- Tanımlar: Deterministic signals? Random signals? Random (Stochastic) processes? Realization? Random variable?
- Random signals konusunda: random variable, realization, stochastic process nedir? Lecture 5'dekinden farklı bir örnekle açıklayınız.
- What are random signal statistical properties? Tanımlar: Mean (Expected) value? Mean square value? Variance? Autocorrelation sequence? Autocovariance? Difference between Autocorrelation and Autocovariance? Crosscorrelation? Crosscovariance? Stationarity/Stationary? Wide-Sense-Stationary (WSS) process? Special cases fro WSS processes? Ergodicity? Ergodic processes? Formül ezberlemeyin. Neyin ne olduğunu bilin.
- Why can't directly we use FT to represent random signals in frequency (transform) domain? Power spectrum (Wiener – Khintchine theorem)? Power Density Spectrum or Power Spectral Density? (PSD)?
- What is special for (uniform) white noise?
- Since random signals change from one realization to the next, what can we say about their spectral properties when they are filtered? What is the relationship between the autocorrelation functions of filter input and output?
- What are the ways to estimate PSD?
- What is periodogram method? How it works? What are the differences among using different

type of windows for periodogram method?
Yukaridaki bilgileri direk veya dolayli kullanacaginiz sorular.Formülleri veririm.

1. <u>Causality, stability (in terms of input-output signals, or h[n], or H(z), or H(jw)), ROC?</u>

1. **Causality:**
   - **Definition:** A system is causal if the output at any given time depends only on the current and past values of the input. In other words, the output does not depend on future values of the input.
   - **Implication:** Causal systems are physically realizable, as they do not require knowledge of future inputs to produce an output.
2. **Stability:**
   - **In Terms of Input-Output Signals:**
     - A system is stable if, for any bounded input, the output remains bounded.
     - Mathematically, if $|x[n]| < M$ for all n implies $|y[n]| < N$ for all n, where M and N are finite.
   - **In Terms of Transfer Function (H(z) or H(jw)):**
     - For a continuous-time system, stability is determined by the poles of the transfer function lying in the left-half plane.
     - For a discrete-time system, stability is determined by the poles of the transfer function lying inside the unit circle in the z-plane.
   - **Implication:** Stability ensures that the system does not produce unbounded or oscillatory responses for bounded inputs.
3. **Region of Convergence (ROC):**
   - **Definition:** The ROC is the set of values in the complex plane for which the Z-transform converges, ensuring that the Z-transform is well-defined.
   - **In Terms of H(z):**
     - For a discrete-time system with a transfer function $H(z)$, the ROC is the region in the z-plane for which the Z-transform converges.
   - **Implication:** The ROC is crucial for the convergence of the system representation in the Z-domain.

In summary, causality ensures that the system's output depends only on past and present inputs, stability guarantees that the system response remains bounded for bounded inputs, and the ROC is a region in the complex plane where the Z-transform converges, influencing the validity of the system representation in the Z-domain. These concepts are fundamental in the analysis and design of linear time-invariant systems.

2. <u>Basit bir sinyal üzerinden Autocorrelation hesaplanması.</u>
   Lecture 05 Sayfa 8
3. <u>Basit iki sinyal üzerinden Crosscorrelation hesaplanması</u>
4. ***Lecture 05 Sayfa 12***

5. <u>Phase türleri? Zero, Linear, Nonlinear. Özellikleri?</u>

Zero-Phase:

In a zero-phase system or filter, all frequencies experience the same phase shift, and this phase shift is constant across the entire frequency spectrum.

The term "zero-phase" indicates that there is zero phase delay for all frequencies, meaning the output signal is aligned in time with the input signal.

Achieving zero-phase typically implies a symmetric impulse response. In other words, the filter introduces no phase distortion.

Linear-Phase:

A system or filter is said to have a linear phase if the phase shift is directly proportional to frequency.

In a linear-phase system, the phase response can be expressed as a linear function of frequency, such as $\varphi(\omega) = a\omega + b$, where φ is the phase shift, ω is the angular frequency, and a and b are constants.

The key characteristic of linear phase is that the group delay, which represents the delay experienced by each frequency component of the signal, is constant for all frequencies. This implies that the filter introduces a constant time delay across the entire frequency spectrum.

Non-Linear Phase:

In contrast to zero-phase and linear-phase, non-linear phase systems or filters exhibit a phase response that is not proportional to frequency.

Non-linear phase introduces varying amounts of delay for different frequencies, leading to distortion in the temporal alignment of different components of the signal.

Non-linear phase can be undesirable in certain applications, especially in audio signal processing, where it may cause phase distortion and affect the fidelity of the reproduced signal.

Properties:

Zero-Phase Properties:

Zero-phase systems are useful in applications where maintaining the temporal alignment of different frequency components is critical.

They are often employed in applications like audio processing to preserve the integrity of the signal.

Linear-Phase Properties:

Linear-phase systems are desirable because they introduce a constant group delay, ensuring that all frequencies are delayed by the same amount.

This property is beneficial in applications such as audio equalization and filtering, where maintaining the relative timing of different frequencies is important.

Non-Linear Phase Properties:

Non-linear phase systems may be more computationally efficient in some cases, but they can introduce distortion in the temporal alignment of signals.

They are generally avoided in applications where maintaining phase relationships is critical, such as in audio and communication systems.

6. FIR, IIR filtreler. Avantaj dezavantajları? Phase özellikleri?

Lecture 02 Sayfa 24'ten itibaren (FIR, IIR Filters)

Lecture 03 Sayfa 42'den itibaren (Advantages and Disadvantages)

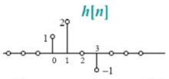
- If the impulse response $h[n]$ of a system is of **finite length**, that system is referred to as a **finite impulse response (FIR) system**

$$h[n] = 0 \text{ for } n < N_1 \text{ and } n > N_2,\ N_1 < N_2$$

- The output of such a system can then be computed as a finite convolution sum

$$y[n] = \sum_{k=N_1}^{N_2} h[k]x[n-k]$$

- E.g., $h[n]$ =[**1** 2 0 -1] is a **FIR system (filter)**



- **FIR systems** are also called **nonrecursive systems** (for reasons that will later become obvious), where the output can be computed from the current and past input values only – without requiring the values of previous outputs

- If the impulse response is of infinite length, then the system is referred to as an **infinite impulse response (IIR) system**. These systems cannot be characterized by the convolution sum due to infinite sum
- Instead, they are typically characterized by **Linear Constant Coefficient Difference Equations**, as we will see later

- Recall accumulator and note that it can have an alternate – and more compact - representation that makes the current output a function of previous inputs and outputs

$$y[n] = \sum_{l=-\infty}^{\infty} x[l] \quad \Rightarrow \quad y[n] = y[n-1] + x[n]$$

- The **impulse response of this system** (which is of infinite length), cannot be represented with a finite convolution sum. Note that, since the current output depends on the previous outputs, this is also called a **recursive system**

FIR Filters (Advantages and Disadvantages):

Advantages:

- Linear Phase: FIR filters can be designed to have a linear phase response, which means all frequencies are delayed by the same amount. This is beneficial for applications where phase distortion needs to be minimized.
- Stability: FIR filters are inherently stable for all types of filter designs.
- Flexibility: FIR filters provide more flexibility in designing the frequency response, and it is easier to achieve a specific desired response.
- No Feedback: FIR filters do not use feedback, making them simpler to implement in certain applications.

Disadvantages:

- Higher Order: FIR filters often require higher order (more coefficients) to achieve the same performance as an IIR filter. This can lead to increased computational complexity.
- Delay: Due to the linear phase characteristic, FIR filters introduce a delay, which can be a disadvantage in real-time applications.

IIR Filters (Advantages and Disadvantages):

Advantages:

- Lower Order: IIR filters often achieve similar performance as FIR filters with lower order, leading to reduced computational complexity.
- Efficiency: IIR filters are generally more computationally efficient than FIR filters for the same level of performance.
- Real-Time Processing: IIR filters introduce less delay compared to FIR filters, making them more suitable for real-time applications.

Disadvantages:

- Phase Distortion: IIR filters may introduce phase distortion, which means different frequencies experience different delays. This can be a concern in applications where preserving the phase relationship is crucial.

- Stability: Care must be taken in designing IIR filters to ensure stability. Some IIR filter designs may be prone to instability, especially with high Q-factor or sharp transitions in the frequency response.
- Limited Flexibility: Achieving a specific frequency response may be more challenging with IIR filters compared to FIR filters.

In summary, the choice between FIR and IIR filters depends on the specific requirements of the application. FIR filters are often preferred when linear phase and precise control over the frequency response are crucial, while IIR filters may be more suitable for applications where computational efficiency and lower order are priorities. The selection also depends on the trade-offs between phase distortion, filter order, and stability.

7. Convolution.
8. LCCDEs.

Lecture 02 Sayfa 33

9. Fourier Transform türleri ve aralarındaki farklar (ayrıklık, süreklilik, periyodiklik, aperiyodiklik: in time and in frequency domains)

Lecture 02 Sayfa 35

- Frequency representation of a signal is typically obtained in one of **Fourier transforms** or **z-transform**:
- **Fourier Transforms**:
  - **Fourier series** – for periodic continuous time signals
  - **Continuous Time Fourier Transform (CTFT)** – for aperiodic continuous time signals $x(t) \overset{\Im}{\leftrightarrow} X(\Omega)$
  - **Discrete Time Fourier Transform (DTFT)** – for aperiodic discrete time signals (frequency domain is still continuous however) $x[n] \overset{\Im}{\leftrightarrow} X(\omega)$
  - **Discrete Fourier Transform (DFT)** – DTFT sampled in the frequency domain $x[n] \overset{\Im}{\leftrightarrow} X[k]$
  - **Fast Fourier Transform (FFT)** – Same as DFT, except calculated very efficiently

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Biomedical Sig. Proc Lecture # 2 **The Frequency Domain**

- **z-transform**:
  - A generalized version of the DTFT. The de-facto transform used in representing discrete signals and systems in frequency domain. Also used in designing filters $x[n] \overset{\Im}{\leftrightarrow} X(z)$

10. What are stationary and nonstationary signals? Differences?

Lecture 04 Sayfa 8

Lecture 05 Sayfa 13

Stationary Signal:

- Statistical properties (mean, variance, etc.) don't change over time.
- Autocorrelation function is time-invariant.
- Examples: Constant-frequency sine wave.

Non-Stationary Signal:

- Statistical properties change over time.
- Autocorrelation function may vary with time.
- Examples: Biological signals, Stock prices, signals with trends or seasonality.

11. How Fourier Transform (FT) works on signals?

Lecture 04 Sayfa 3

12. What are the disadvantages/shortcomings of FT to work on nonstationary signals?

The Fourier Transform (FT) is a powerful tool for analyzing signals in the frequency domain, but it has some disadvantages when dealing with nonstationary signals, i.e., signals whose characteristics change over time. Here are the main shortcomings:

Assumes Stationarity:

Disadvantage: FT assumes that the signal is stationary over its entire duration. This assumption is not valid for signals that exhibit variations or changes in frequency content over time.

Impact: When applied to nonstationary signals, FT may provide a misleading representation, as it does not capture the time-varying nature of the signal.

Lack of Time Localization:

Disadvantage: FT does not provide information about when specific frequency components are present in the signal. It gives a global frequency representation but lacks time localization.

Impact: In the case of nonstationary signals, where frequency content changes over time, FT fails to reveal the temporal evolution of these changes.

Fixed Resolution:

Disadvantage: FT provides a fixed frequency resolution for the entire signal. It cannot adapt to changes in the signal's frequency content, especially when the signal contains both low and high-frequency components.

Impact: Fine details in the time or frequency domain may be overlooked, making it challenging to capture rapid changes or transients in nonstationary signals.

Uncertainty Principle:

Disadvantage: The uncertainty principle states that there is a fundamental trade-off between time and frequency resolution in the FT. It is not possible to simultaneously achieve high precision in both domains.

Impact: In nonstationary signals, where time-frequency variations are significant, the uncertainty principle limits the ability to precisely localize both time and frequency information.

Sensitivity to Signal Duration:

Disadvantage: FT results can be sensitive to the duration of the analyzed signal. Padding the signal with zeros or using windowing techniques can affect the frequency resolution and introduce artifacts.

Impact: This sensitivity makes FT less robust when dealing with signals of varying lengths, common in nonstationary scenarios.

To overcome these limitations for nonstationary signals, alternative techniques such as the Short-Time Fourier Transform (STFT) or the Wavelet Transform are often employed. These methods offer time-frequency analysis, providing a more suitable representation for signals with time-varying characteristics.

13. <u>How Short Time Fourier Transform (STFT) works on signals? Also talk about resoloution (time,frequency) issues for STFT.</u>

The Short-Time Fourier Transform (STFT) is a time-frequency analysis technique used to analyze signals in both the time and frequency domains. It overcomes the limitation of the Fourier Transform (FT) by providing a time-varying representation of the signal's frequency content. Here's how the STFT works and its resolution-related considerations:

Short-Time Fourier Transform (STFT) Process:

Windowing:

The STFT involves dividing the signal into short, overlapping segments or windows.

A window function (e.g., Hamming, Hanning) is applied to each segment, emphasizing the signal within that interval and reducing spectral leakage.

Fourier Transform:

The Fourier Transform is applied to each windowed segment independently.

This produces a set of spectra, one for each time segment, revealing the frequency content at different moments.

Time Evolution:

By choosing appropriate window lengths and overlaps, the STFT captures how the signal's frequency content evolves over time.

Time-Frequency Representation:

The result is a 2D representation, typically displayed as a spectrogram, with time on one axis, frequency on the other, and the color/intensity indicating the magnitude of the signal's frequency components.

Resolution Issues for STFT:

Time Resolution:

Short Windows: Short windows provide better time resolution, capturing rapid changes in the signal.

Trade-Off: However, shorter windows result in poorer frequency resolution.

Frequency Resolution:

Long Windows: Longer windows improve frequency resolution, allowing for more accurate estimation of the frequency components.

Trade-Off: Longer windows result in poorer time resolution and may smooth out rapid changes.

Window Function Selection:

The choice of window function affects the trade-off between time and frequency resolution.

Some window functions offer better frequency resolution at the expense of wider main lobes in the time domain and vice versa.

Overlap:

Overlapping windows mitigate the trade-off between time and frequency resolution by providing redundancy in the time domain.

Higher overlap improves time resolution but increases computational complexity.

Time-Frequency Trade-Off:

The time-frequency trade-off in STFT is a fundamental limitation known as the uncertainty principle.

It states that there is a fundamental limitation on how well time and frequency can be simultaneously localized.

Summary:

Advantages: STFT provides a time-varying representation, capturing changes in the frequency content of a signal.

Trade-Off: There is an inherent trade-off between time and frequency resolution, and the parameters (window length, overlap) must be carefully chosen based on the characteristics of the signal.

Applications: STFT is widely used in audio signal processing, speech analysis, and other applications where the time-varying nature of signals is critical.

14. What is Heisenberg principle?

Lecture 04 Sayfa 22

15. What is Wavelet Transform (WT)? Briefly explain. How it overcomes the preset resolution problem?

Lecture 04 Sayfa 23

Wavelet Transform (WT) is a mathematical tool that analyzes signals by breaking them down into localized components called wavelets. It overcomes the preset resolution problem by offering adaptive resolution, allowing it to capture details at different scales within a signal. This flexibility is useful for analyzing signals with varying frequency components and transient features.

16. What is wavelet?

A wavelet is a mathematical function that is characterized by being localized in both time and frequency domains. Unlike traditional sine and cosine functions used in Fourier analysis, which are global and extend infinitely, wavelets are compact and have a finite duration. This localization property makes wavelets well-suited for analyzing signals that have localized features or transients.

The basic idea of a wavelet is to oscillate for a short duration and be modulated in frequency. The term "wavelet" is derived from "wave" and "diminutive" (small), indicating its ability to capture both high and low-frequency information in a signal while being focused in time.

Mathematically, a wavelet function ψ(t) is defined by a dilation and translation of a mother wavelet function $\psi_0(t)$:

- ψa,b(t)=a1ψ(at−b)

Here, a represents the scale or dilation factor, b represents the translation or shift, and ψ(t) is the mother wavelet function.

Wavelets are used in various applications, such as signal processing, image compression, and data analysis. In signal processing, the Wavelet Transform decomposes a signal into different scales and provides a localized representation of signal features, making it a powerful tool for analyzing signals with both high and low-frequency components.

17. What is the relationship between scale and frequency, and translation and time?

Lecture 04 Sayfa 34

In signal processing, the terms "scale" and "frequency," as well as "translation" and "time," are related concepts that are often associated with the analysis and processing of signals, especially in the context of Fourier analysis and wavelet analysis.

Scale and Frequency:

Frequency: In the context of signal processing, frequency refers to the rate at which a signal oscillates or repeats over time. It is often measured in hertz (Hz). For example, a signal with a frequency of 10 Hz completes 10 cycles per second.

Scale: Scale is related to the concept of frequency through the idea of scaling a signal. In wavelet analysis, signals can be decomposed into different scales. Scaling a signal involves changing the duration or size of the waveform without changing its shape. Larger scales correspond to lower frequencies, and smaller scales correspond to higher frequencies. This relationship is particularly evident in wavelet transforms, where a signal

can be decomposed into various scales to analyze its frequency content at different resolutions.
Translation and Time:
Time: In signal processing, time represents the independent variable along which a signal is measured or observed. It is a fundamental dimension in the analysis of signals.
Translation: Translation, in the context of signal processing, refers to shifting a signal along the time axis. When you translate a signal, you are essentially changing the time at which it starts or ends. Translating a signal in time does not alter its frequency content; it simply changes the temporal location of the signal.
In summary, the relationship between scale and frequency is tied to the concept of scaling in wavelet analysis, where changing the scale corresponds to analyzing different frequency components of a signal. On the other hand, translation and time are related by the shifting of a signal along the time axis without altering its frequency characteristics. These concepts are crucial in the analysis and processing of signals in various applications, including image processing, audio analysis, and communication systems.

18. What are the conditions for wavelet functions to satisfy?

In signal processing, wavelet functions must meet specific conditions to be effective:

1. **Admissibility Condition:** Wavelet functions must balance time and frequency localization, ensuring finite energy for effective representation:
   $\int_{-\infty}^{\infty} |\psi(t)|^2 \, dt < \infty$
2. **Normalization Condition:** Wavelet functions should be normalized to preserve unit energy:
   $\int_{-\infty}^{\infty} |\psi(t)|^2 \, dt = 1$
3. **Orthogonality (Optional):** Optionally, wavelets may form an orthonormal basis:
   $\langle \psi_a, \psi_b \rangle = \delta_{ab}$
4. **Compact Support Condition:** Wavelets with compact support (nonzero over a finite interval) are preferred for capturing localized features.
5. **Smoothness Condition:** Wavelet functions need to be smooth for a balance between time and frequency localization, crucial in tasks like denoising.
6. **Dilation Equation:** The wavelet function should satisfy a dilation equation for scaling:
   $\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$

These conditions ensure wavelet functions are suitable for signal processing. Different wavelet families (Daubechies, Morlet, Haar) satisfy these conditions to varying degrees, chosen based on specific application needs.

19. <u>Fourier Transform, Short Time Fourier Transform ve Wavelet transform tarifleri ve karşılaştırmaları?</u>

Fourier Transform, Short-Time Fourier Transform (STFT), and Wavelet Transform are all signal processing techniques used for analyzing and representing signals in the time and frequency domains. Here's a brief description of each, along with some comparisons:

Fourier Transform:

Description: The Fourier Transform is a mathematical technique that transforms a signal from the time domain to the frequency domain. It represents a signal as a sum of sinusoidal functions with different frequencies.

Advantages:

- Provides a global view of the frequency content of the entire signal.
- Well-suited for stationary signals.

Limitations:

- Assumes that the signal is stationary over its entire duration.
- Lacks time information.

Short-Time Fourier Transform (STFT):

Description: The STFT is an extension of the Fourier Transform that analyzes the frequency content of a signal over short, overlapping time intervals. It uses a window function to obtain a time-varying spectrum.

Advantages:

- Provides a time-varying representation of the frequency content.
- Suitable for non-stationary signals.

Limitations:

- Limited time and frequency resolution trade-off due to the use of fixed-size windows.
- Limited ability to capture fine details in time and frequency.

Wavelet Transform:

Description: The Wavelet Transform is a signal processing technique that uses wavelet functions to represent signals in both time and frequency domains simultaneously. It provides a multi-resolution analysis, capturing high and low-frequency components with different resolutions.

Advantages:

- Localized in both time and frequency, allowing for better representation of non-stationary signals.
- Provides a time-frequency representation with variable resolution.

Limitations:

- The choice of wavelet and parameters can significantly affect the results.
- More computationally intensive compared to the Fourier Transform and STF

Comparisons:

Time-Frequency Localization:

- Fourier Transform has global frequency information but lacks time localization.
- STFT provides time-varying frequency information but with limited time and frequency resolution.
- Wavelet Transform offers excellent time and frequency localization simultaneously.

Resolution:

- Fourier Transform and STFT have fixed resolutions in both time and frequency.
- Wavelet Transform provides variable resolution, allowing for better adaptation to signal characteristics.

Computational Complexity:

- Fourier Transform is computationally efficient.
- STFT is more computationally demanding due to overlapping windows.
- Wavelet Transform is generally more computationally intensive, especially for continuous wavelet transforms.

Applications:

- Fourier Transform is suitable for analyzing stationary signals.
- STFT is useful for analyzing non-stationary signals with some time localization.
- Wavelet Transform is well-suited for analyzing signals with both time and frequency-varying characteristics, making it versatile in applications like signal denoising, compression, and feature extraction.

20. <u>Tanımlar: Deterministic signals? Random signals? Random (Stochastic) processes? Realization? Random variable?</u>

1. Deterministic signals:

• A deterministic signal is one whose values in the future can be predicted exactly with absolute confidence if enough information about its past is available
• Encountered commonly in textbook examples but less frequently in the real world
• Impulses, steps, and exponential functions are deterministic
• In fact, any signal which can be expressed exactly in closed mathematical form as a function of time or other variable is deterministic

2. Stochastic (Random) signals:

• Stochastic signals are signals for which it is impossible to predict an exact future value even if one knows its entire past history
• Also the name “random signal” is often used for this signals
• Some random signals are completely unpredictable (uncorrelated), some others can be predicted with greater (but not absolute) confidence
• Random signals are abundant in physical processes
• Example: Noise generated by electronic components in instrumentation is a common type of random signal that is present in much biomedical data

21. Random signals konusunda: random variable, realization, stochastic process nedir? Lecture 5'dekinden farklı bir örnekle açıklayınız.

1. **Random Variable in Signal Processing:**
   In signal processing, a random variable could represent a parameter associated with a signal. For example, consider the amplitude of a signal received by a sensor in a noisy environment. The amplitude at any given moment could be considered a random variable because it varies due to noise and other unpredictable factors.
2. **Realization in Signal Processing:**
   In the context of signal processing, a realization could be a specific instance or sample of a signal. For instance, if you are measuring the voltage of a signal over time, a realization would be a specific voltage-time sequence obtained from an actual measurement. This specific sequence is one possible outcome among many that the signal could take.
3. **Stochastic Process in Signal Processing:**
   A stochastic process in signal processing refers to a collection of signals that evolve over time in a probabilistic manner. For instance, consider the signal generated by a communication channel in the presence of random interference. The received signal at any given moment is a random variable, and the entire sequence of received signals over time constitutes a stochastic process. Stochastic processes are particularly relevant in modeling and analyzing signals in dynamic and uncertain environments.

Different Example in Signal Processing:
Consider a communication channel that transmits digital data, and the received signal is affected by random noise:

- **Random Variable in Signal Processing:** The amplitude of the noise added to the transmitted signal can be considered a random variable. Let's denote it as $N(t)$, where t is time.
- **Realization in Signal Processing:** If you observe the actual noise amplitude at a specific instant, say $N(2\ \text{seconds}) = 0.1\ \text{volts}$, this specific value is a realization of the random variable $N(t)$ at that particular time.
- **Stochastic Process in Signal Processing:** The entire sequence of noise amplitudes over time, i.e., $N(t)$ for all t, forms a stochastic process. It represents the random variations in the noise affecting the signal during the entire transmission.

22. What are random signal statistical properties? Tanımlar: Mean (Expected) value? Mean square value? Variance? Autocorrelation sequence? Autocovariance? Difference between Autocorrelation and Autocovariance? Crosscorrelation? Crosscovariance? Stationarity/Stationary? Wide-Sense-Stationary (WSS) process? Special cases fro WSS processes? Ergodicity? Ergodic processes? Formül ezberlemeyin. Neyin ne olduğunu bilin.

1. **Mean (Expected) Value:**
   - **Definition:** The mean or expected value of a random signal is a measure of its central tendency. It represents the average value the signal is expected to have.
   - **Notation:** If X is a random signal, its mean is denoted by $E[X]$ or μ.
2. **Mean Square Value:**
   - **Definition:** The mean square value is the expected value of the square of the signal. It provides a measure of the signal's energy.
   - **Notation:** For a random signal X, the mean square value is denoted as $E[X^2]$.
3. **Variance:**
   - **Definition:** Variance measures the spread or dispersion of a random signal. It is the expected value of the squared deviation from the mean.
   - **Notation:** If X is a random signal, its variance is denoted by $Var(X)$.
4. **Autocorrelation Sequence:**
   - **Definition:** Autocorrelation sequence describes how a signal correlates with a delayed version of itself at different time instants.
   - **Notation:** Autocorrelation of a signal X at lag k is denoted as $R_X(k)$.
5. **Autocovariance:**
   - **Definition:** Autocovariance is similar to autocorrelation but without normalizing by the signal's variance. It is the covariance between a signal and its delayed version.
   - **Notation:** Autocovariance of a signal X at lag k is denoted as $C_X(k)$.
6. **Difference between Autocorrelation and Autocovariance:**
   - Autocorrelation is the normalized version of autocovariance. Autocorrelation is obtained by dividing the autocovariance by the product of the standard deviations of the signal at the two lags involved.
7. **Crosscorrelation:**
   - **Definition:** Crosscorrelation measures the similarity between two different signals at different time instants.
   - **Notation:** Crosscorrelation between signals X and Y at lag k is denoted as $R_{XY}(k)$.

8. **Crosscovariance:**
   - **Definition:** Similar to crosscorrelation but without normalization by the standard deviations of the signals.
   - **Notation:** Crosscovariance between signals X and Y at lag k is denoted as $C_{XY}(k)$.
9. **Stationarity/Stationary:**
   - **Definition:** A signal is stationary if its statistical properties do not change with time. This includes mean, variance, and autocorrelation.
10. **Wide-Sense-Stationary (WSS) Process:**
    - **Definition:** A process is wide-sense stationary if its mean and variance are constant, and its autocorrelation function depends only on the time difference.
11. **Special Cases for WSS Processes:**
    - For a strict-sense stationary process, the second-order moments (variance and autocovariance) are constant.
    - For a wide-sense stationary uncorrelated scattering (WSSUS) process, the mean is constant, the variance is constant, and the autocorrelation function is zero for all lags except when the lag is zero.
12. **Ergodicity:**
    - **Definition:** An ergodic process is one in which the statistical properties estimated from a single realization converge to the ensemble averages as the observation time becomes infinite.
13. **Ergodic Processes:**
    - Ergodic processes are those for which time averages are equal to ensemble averages over an infinite time duration.

These concepts are fundamental in the analysis of random signals and stochastic processes, particularly in fields such as communication systems, signal processing, and statistical signal processing.

23. Why can't directly we use FT to represent random signals in frequency (transform) domain? Power spectrum (Wiener – Khintchine theorem)? Power Density Spectrum or Power Spectral Density? (PSD)?

The direct application of the Fourier Transform (FT) to represent random signals in the frequency domain is not suitable for several reasons, primarily because random signals are not deterministic and may not have a well-defined Fourier Transform due to their stochastic nature. Here are the key reasons:

1. **Non-Deterministic Nature of Random Signals:**
   - Random signals are inherently non-deterministic, and their values are not precisely predictable. Unlike deterministic signals with well-defined amplitudes at each time instant, the concept of a fixed frequency spectrum for a random signal doesn't hold.
2. **Statistical Properties:**
   - Random signals are characterized by statistical properties such as mean, variance, and correlation. The Fourier Transform does not capture these statistical properties.
3. **Infinite Signal Duration:**
   - Random signals often have infinite duration, and their frequency content can be spread over an infinite range. Applying the Fourier Transform directly to an infinite-duration signal might not converge or might not provide meaningful information.

To analyze random signals in the frequency domain, statistical tools are employed. The Power Spectral Density (PSD) is one such tool, and it is closely related to the Wiener–Khintchine theorem:

- **Wiener–Khintchine Theorem:**
  - This theorem establishes a relationship between the autocorrelation function of a random signal and its Power Spectral Density (PSD). It states that the Fourier Transform of the autocorrelation function of a wide-sense stationary process is equal to the PSD of the process.
- **Power Spectral Density (PSD):**
  - PSD is a statistical measure that describes how the power of a signal is distributed across different frequencies. It is essentially the Fourier Transform of the autocorrelation function of a random signal. The PSD provides a way to understand the frequency content of a random signal in a statistical sense.

So, instead of directly using the Fourier Transform for random signals, the PSD provides a more appropriate tool to analyze the frequency characteristics of stochastic processes, considering their statistical properties. The PSD provides a measure of how the power of a random signal is distributed across different frequencies, and it is a fundamental concept in the analysis of random signals and systems.

24. What is special for (uniform) white noise?

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25. Since random signals change from one realization to the next, what can we say about their spectral properties when they are filtered? What is the relationship between the autocorrelation functions of filter input and output?

When random signals are filtered, their spectral properties can be influenced by the characteristics of the filter. Filtering a random signal essentially involves modifying its frequency content based on the frequency response of the filter. The relationship between the autocorrelation functions of the filter input and output provides insights into how the filtering process affects the temporal characteristics of the signal.

Spectral Properties after Filtering:

Frequency Modification: The spectral properties of a random signal can be altered when it is passed through a filter. The filter may attenuate or amplify certain frequency components, and this is reflected in the power spectral density (PSD) of the filtered signal.

Frequency Response: The frequency response of the filter, which describes how the filter behaves at different frequencies, plays a crucial role. Filters can be designed to pass certain frequencies (passband), attenuate others (stopband), or have specific characteristics such as low-pass, high-pass, bandpass, or bandstop.

Effect on Randomness: While the filtering process modifies the spectral content, it's important to note that filtering does not introduce new randomness. If the input signal is truly random, the filtered output will also be random, but with a modified spectral distribution.

Autocorrelation Functions:

Autocorrelation Input vs. Output: The autocorrelation function of a signal describes the correlation between different time instants within the signal. The relationship between the autocorrelation functions of the filter input and output is given by Wiener-Khinchin theorem.

Wiener-Khinchin Theorem: The power spectral density (PSD) of a signal is the Fourier transform of its autocorrelation function. Mathematically, if $Rx(\tau)$is the autocorrelation function of the input signal and $Ry(\tau)$is the autocorrelation function of the output signal after filtering, then the relationship is given by:

$Ry(\tau)=Rx(\tau)*h(\tau)$,where $**$ denotes convolution, and $h(\tau)$ is the impulse response of the filter.

Correlation and Filtering: The autocorrelation function of the filtered signal is related to the autocorrelation function of the input signal through the convolution with the impulse response of the filter. This convolution reflects how the filter modifies the temporal correlations within the signal.

In summary, filtering random signals can modify their spectral properties based on the characteristics of the filter. The relationship between the autocorrelation functions of the filter input and output is given by the Wiener-Khinchin theorem, linking the temporal correlation structures before and after filtering through convolution with the filter's impulse response.

26. What are the ways to estimate PSD?

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27. <u>What is periodogram method? How it works? What are the differences among using different type of windows for periodogram method?</u>

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The periodogram is a method used in signal processing and spectral analysis to estimate the power spectral density (PSD) of a signal. The power spectral density describes how the power of a signal is distributed across different frequencies. The periodogram is a commonly used tool for understanding the frequency content of a signal.

Here's how the periodogram method works:

Discrete Fourier Transform (DFT): The periodogram is based on the Discrete Fourier Transform (DFT), which is a mathematical technique used to analyze the frequency content of a discrete signal. The DFT transforms a sequence of equally spaced samples of a function into a sequence of complex numbers representing the amplitudes and phases of sinusoidal functions.

Power Spectral Density (PSD): The periodogram estimates the PSD of a signal by computing the squared magnitude of the DFT. The PSD gives information about the distribution of power across different frequencies in the signal.

Windowing: When applying the periodogram to a finite-length signal, a common practice is to multiply the signal by a window function before computing the DFT. This is called windowing. Windowing helps mitigate issues related to spectral leakage and finite-length effects.

Different Types of Windows: The choice of window function can impact the performance of the periodogram. Common window functions include rectangular, Hamming, Hanning, Blackman, and more. Each window has its own characteristics, and the choice depends on the specific requirements of the analysis.

Rectangular Window: This is the simplest window, where the signal is multiplied by a rectangular function. While it is easy to compute, it can result in high spectral leakage.

Hamming Window: The Hamming window reduces spectral leakage compared to the rectangular window but has a wider main lobe.

Hanning Window: Similar to the Hamming window but with a different shape, the Hanning window also reduces spectral leakage.

Blackman Window: The Blackman window has a more complicated shape and provides better frequency resolution at the expense of a wider main lobe.

The choice of window depends on the specific characteristics of the signal and the trade-offs between frequency resolution and spectral leakage.

In summary, the periodogram is a method for estimating the power spectral density of a signal using the Discrete Fourier Transform. The choice of window function in the periodogram method affects the trade-off between frequency resolution and spectral leakage, and different windows may be suitable for different types of signals and analysis goals.

Extra Informations

- **The action potential** – mother of all biological signals
- **The electroneurogram (ENG)** – propagation of nerve action potential
- **The electromyogram (EMG)** – electrical activity of the muscle cells
- **The electrocardiogram (ECG)** – electrical activity of the heart / cardiac cells
- **The electroencephalogram (EEG)** – electrical activity of the brain
- **The electrogastogram (EGG)** – electrical activity of the stomach
- **The phonocardiogram (PCG)** – audio recording of the heart's mechanical activity
- **The carotid pulse (CP)** – pressure of the carotid artery
- **The electoretinogram (ERG)** – electrical activity of the retinal cells
- **The electrooculogram (EOG)** – electrical activity of the eye muscles