M-Ary Data Transmission Systems

- Binary Transmission Systems : $s_0(t)$, $s_1(t)$ for bit 0 and 1. T_s (symbol duration), T_b (bit duration). $T_s = kT_b$ and $k = \log_2 M = 1$ when $M = 2$. For a given symbol duration T_s, we carry k bits. $R_b = 1/T_b$ (bit rate), $R_s = 1/kT_b$ and $R_s = R_b/k$ so $R_b = kR_s$ (for a given R_s) for M-ary schemes.
- $B_T = 1/T_s$ (ideal case). For a given BW, the data rate is increased by a factor of k in M-ary transmission schemes for M-PSK and M-QAM. It is not valid for M-FSK.
- $\mathcal{E}_{avg} = \frac{1}{M} \sum_{i=1}^{M} \mathcal{E}_m \rightarrow$ energy per symbol
- $\mathcal{E}_{b,avg} = \frac{\mathcal{E}_{avg}}{k}$

QPSK data trasmission systems

$s_0(t) = \sqrt{2} A_c \cos(2\pi f_c t + \pi/4)$ for pair 11

$s_1(t) = \sqrt{2} A_c \cos(2\pi f_c t + 3\pi/4)$ for pair 01

$s_2(t) = \sqrt{2} A_c \cos(2\pi f_c t + 5\pi/4)$ for pair 00

$s_3(t) = \sqrt{2} A_c \cos(2\pi f_c t + 7\pi/4)$ for pair 10

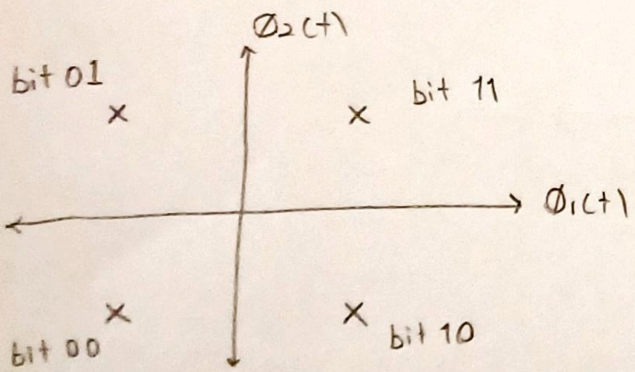
$s(t) = \pm A_c \cos(2\pi f_c t) \pm A_c \sin(2\pi f_c t)$

Orthonormal Basis

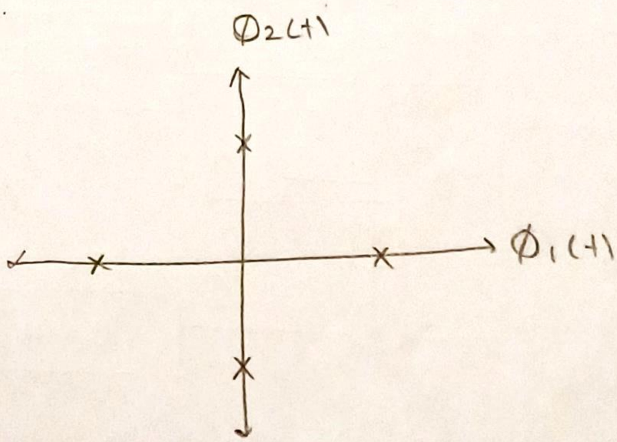
$\phi_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}}\, g_T(t) \cos(2\pi f_c t)$

$\phi_2(t) = -\sqrt{\frac{2}{\mathcal{E}_g}}\, g_T(t) \sin(2\pi f_c t)$

$g_T(t)$ terimi yukarda da var ama rectangular fonksiyon olduğu için yazmadık



or

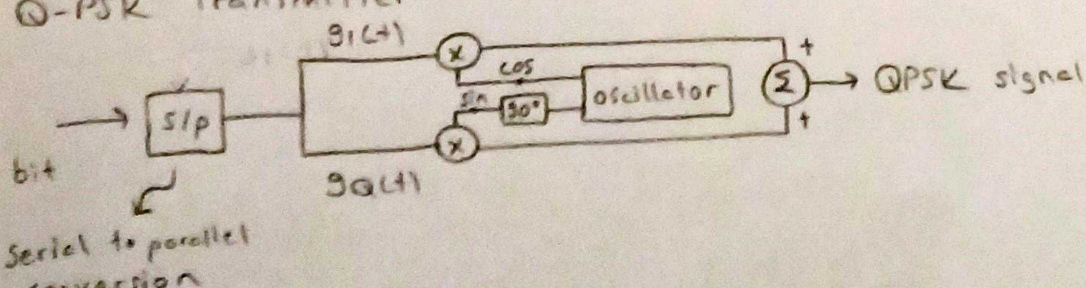


$\mathcal{E}_{s,avg} = \mathcal{E}_{s,avg}$

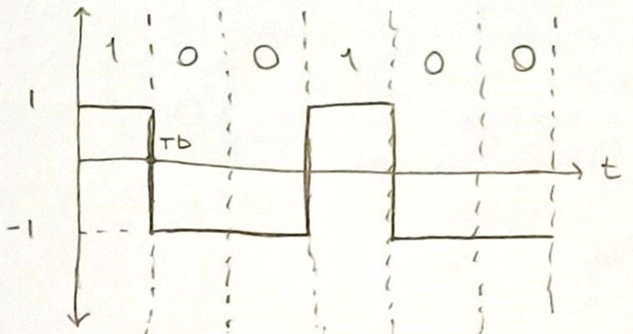
$s(t) = s_{m,1}(t)\,\phi_1(t) + s_{m,2}(t)\,\phi_2(t)$

- Soldaki tercih ediliyor

Q-PSK Transmitter

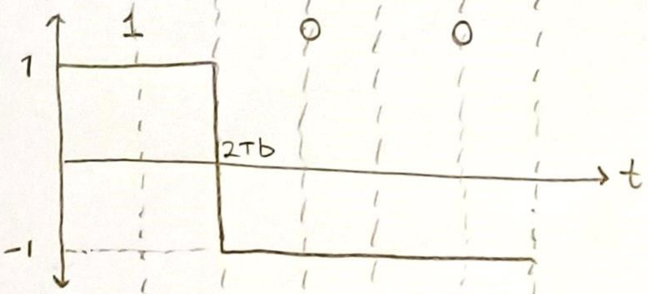


Binary data



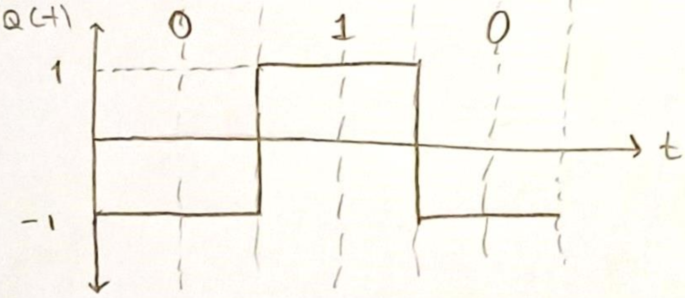
$T_b = T$

$g_I(t)$



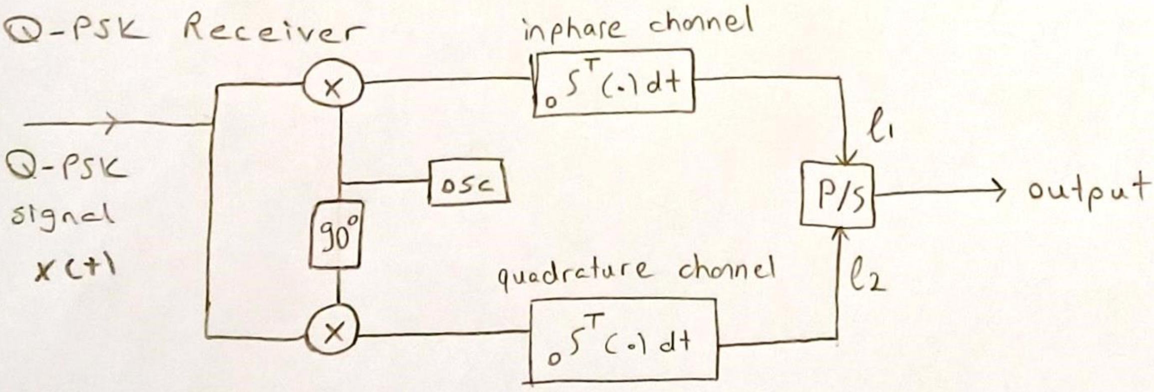
$2T_b = T$

$g_Q(t)$



- Bit rate is decreased. Symbol duration is increased.

Q-PSK Receiver



$$x(t) = s(t) + w(t)\ (\text{AWGN}) = \pm A_c\, g_T(t) \cos(2\pi f_c t) \pm A_c\, g_T(t) \sin(2\pi f_c t)$$

$$0 \le t \le T$$

$x(t)$ ⇓ received signal

$s(t)$ ⇓ QPSK signal

$$l_1 = \pm \frac{1}{2} A_c T + \int_0^T w(t) \cos(2\pi f_c t)\, dt$$

$$l_2 = \pm \frac{1}{2} A_c T + \int_0^T w(t) \sin(2\pi f_c t)\, dt$$

$E\{L_1\} = ?$

$E\{L_2\} = ?$

$\operatorname{var}\{L_1\} = ?$

$\operatorname{var}\{L_2\} = ?$

$E\{L_1\} = \pm A_c \frac{T}{2}$, $E\{L_2\} = A_c \frac{T}{2}$

$$\mathrm{var}(L_1) = E\left\{\left(\int_0^T w(t)\cos(2\pi f_c t)\,dt\right)^2\right\} = E\left\{\left(\int_0^T\int_0^T w(t)w(u)\cos(2\pi f_c t)\cos(2\pi f_c t)\,dt\,du\right)\right\}$$

$$= \frac{N_0}{2}\int_0^T \cos^2(2\pi f_c t)\,dt = \frac{N_0 T}{4}$$

$\mathrm{var}(L_1) = \frac{N_0 T}{4}$, $\mathrm{ver}(L_1) = \frac{N_0 T}{4}$

$$P_{e1} = P_{e2} = Q\left[\sqrt{\frac{A_c^2 T}{N_0}}\right] = Q\left(\sqrt{\frac{\mathcal{E}}{N_0}}\right) = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$

$\mathcal{E} = A_c^2 T = 2\mathcal{E}_b$

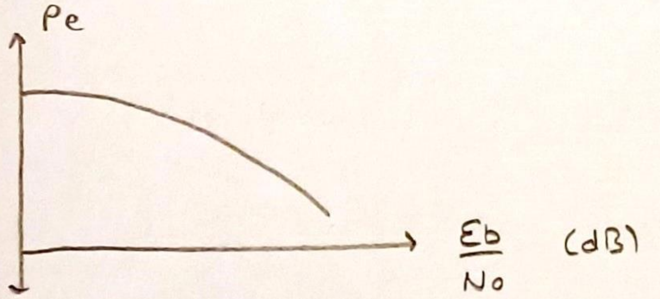
$\Downarrow$

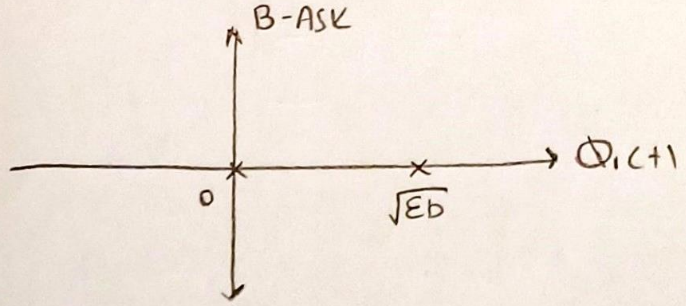
symbol energy

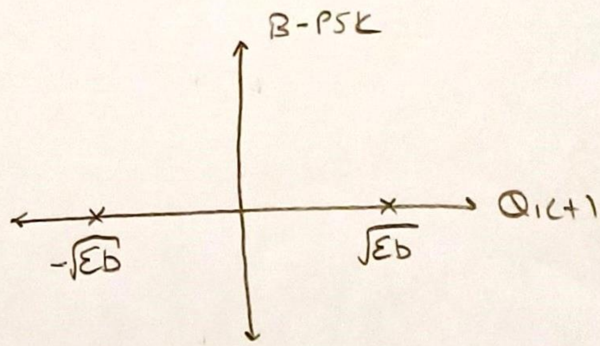
$$P_e = \frac{1}{2}P_{e1} + \frac{1}{2}P_{e2} = Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right]$$

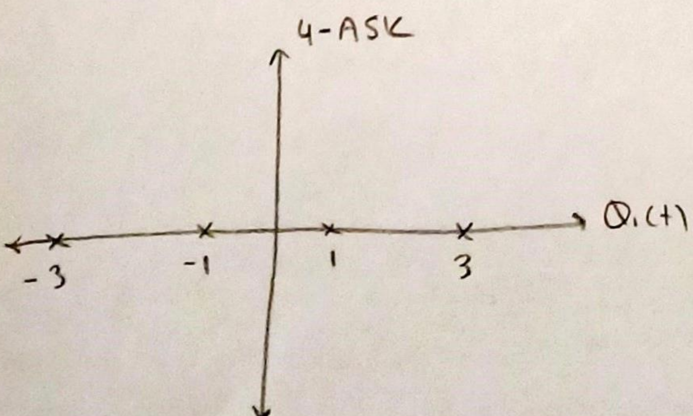
$\Downarrow$

average prob. error









$s_m = A_m$

$$s_m(t) = A_m\sqrt{\frac{\mathcal{E}_g}{2}}\,Q(t)$$

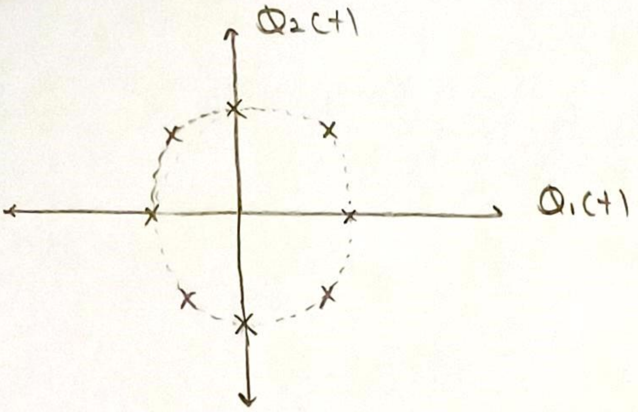
$A_m = \{\pm 1, \pm 3, \pm 5, \ldots, \pm(m-1)\}$

$\mathcal{E}_m = A_m^2$

$s_m(t) = A_m\, g_T(t)\cos(2\pi f_c t)$

M-ASK $\quad Q_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}}\, g_T(t)\cos(2\pi f_c t)$

M-PSK (M=8)



$$s_m(t) = Re\{g(t)\, e^{-j2\pi \frac{(m-1)}{M}}\, e^{j2\pi f_c t}\}$$

$$s_m(t) = g(t)\cos\left(2\pi f_c t + \frac{2\pi}{M}(m-1)\right)$$

$\Downarrow$

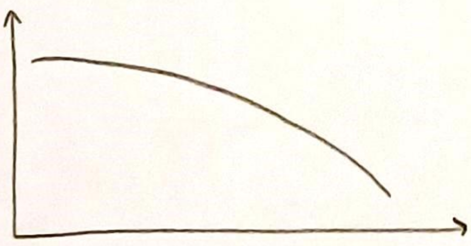
M-PSK

$$\mathcal{E}_m = \mathcal{E}$$

$$\phi_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}}\, g(t)\cos(2\pi f_c t)$$

$$\phi_2(t) = -\sqrt{\frac{2}{\mathcal{E}_g}}\, g(t)\sin(2\pi f_c t)$$

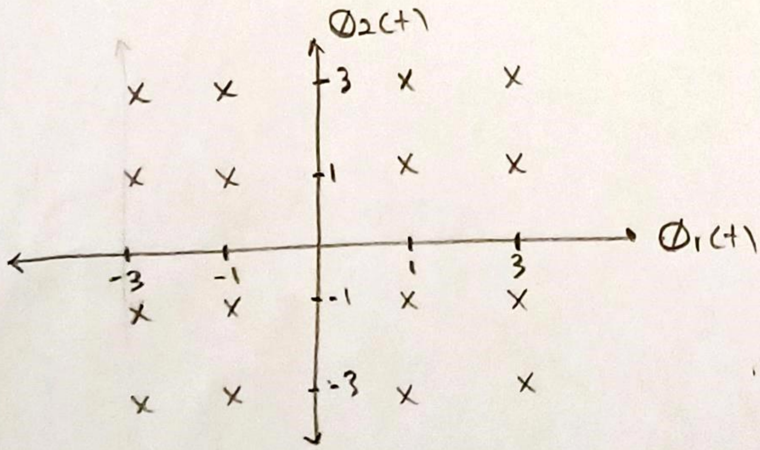
- If we wanto decrease the error, we need to increase the energy ($\mathcal{E} = A_m^2$), also distance between symbols



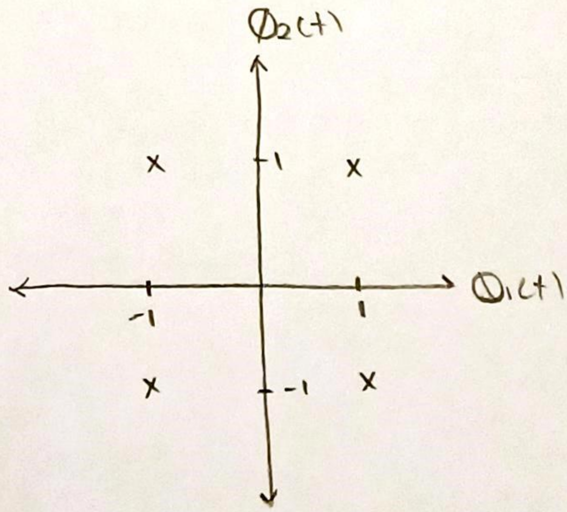
M-QAM

$$s_m(t) = Re\{(A_{m,i} + jA_{m,q})\, g(t)\, e^{j2\pi f_c t}\}$$

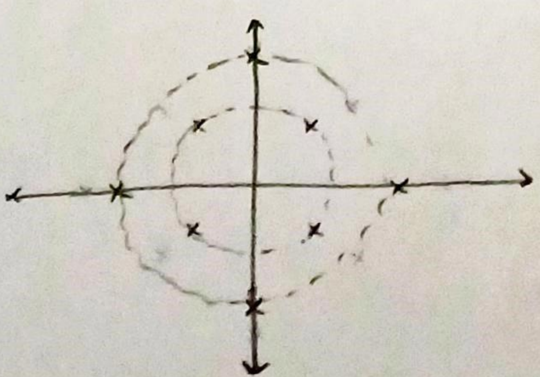
$$= A_{m,i}\, g(t)\cos(2\pi f_c t) - A_{m,q}\, g(t)\sin(2\pi f_c t)$$



M=16



M=4



M = 8

minimum distance in the constellations

$d_{min} \uparrow$ $P_e \downarrow$

Comparing 16-QAM and 8-QAM

$\mathcal{E}_{s,16QAM} = \mathcal{E}_x = 1$ same energy levels and compare the Pe

$\mathcal{E}_{s,8QAM} = \mathcal{E}_y = 1$

M-FSK

$s_m(t) = A_c \cos(2\pi f_m t)$ $m = 1, 2, \ldots, M$ (M different orthonormal basis)

$\Delta f = f_{m+1} - f_m$

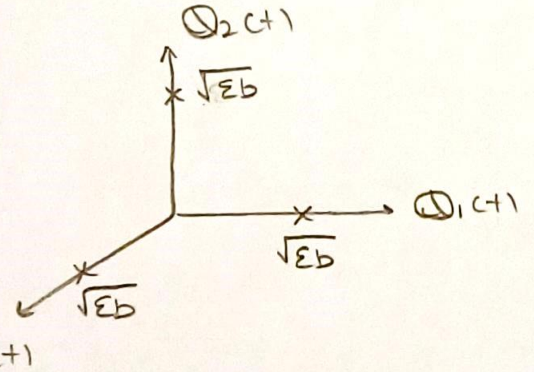
$\Delta f = \frac{1}{2T}$ (minimum seperation to have orthogonal frequencies)

$M = 3$

$s_1(t) = A_c \cos(2\pi f_1 t)$ $f_1 = f_c$

$s_2(t) = A_c \cos(2\pi f_2 t)$ $f_2 = f_c + \Delta f$

$s_3(t) = A_c \cos(2\pi f_3 t)$ $f_3 = f_c + 2\Delta f$



$s_1 = (\sqrt{\mathcal{E}_b}, 0, 0)$

$s_2 = (0, \sqrt{\mathcal{E}_b}, 0)$

$s_3 = (0, 0, \sqrt{\mathcal{E}_b})$

$M \uparrow$ $B_T \uparrow$

- Uyduda SNR düşük M-FSK tercih ediliyor, ama cellularda bant genişliği dar o yüzden M-QAM daha iyi bir alternatif

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Recitation #3

Binary PAM → rectangular pulse
→ duration Tb, amplitude ±A

Q7.10 (Proakis) → to transmit at $R_b = 10^5$ bps
→ AWGN ($N_0 = 10^{-2}$ W/Hz)

a) A required = ? that $P_e = 10^{-6}$

$$P_e = Q\left[\sqrt{\frac{2E_b}{N_0}}\right] = 10^{-6} \quad , \quad \sqrt{\frac{2E_b}{N_0}} = 4.75$$

$T_s = T_b$ (since Binary PAM) $M = 2$ $k = 1$

$$\sqrt{\frac{2A^2}{10^5 10^{-2}}} = 4.75 \qquad A_{req} = 106.21$$

Q4.39 Proakis (Digital comm)

Same as previous question

$\longrightarrow N_0 = 2 \cdot 10^{-10}$

$\longrightarrow R_b = \{10^4, 10^5, 10^6\}$ $A_{req} = ?$ $P_e = 10^{-6}$

$$\mathcal{E}_b = \frac{A^2 T_S}{2}$$

$T_S = T_b$ (since binary)

$$\sqrt{\frac{2A^2}{2 \cdot R_b \cdot N_0}} = \sqrt{\frac{A^2}{10^4 \cdot 2 \cdot 10^{-10}}} = 4.75$$

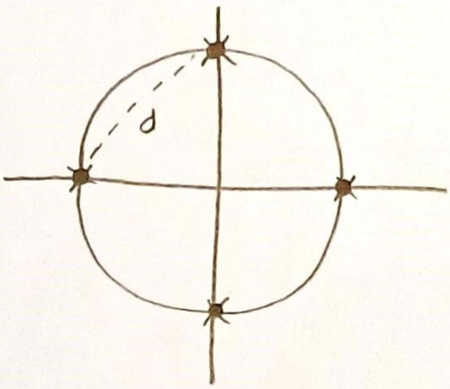
$A_1 = 6.7 \times 10^{-3}$

$A_2 = 2.1 \times 10^{-2}$

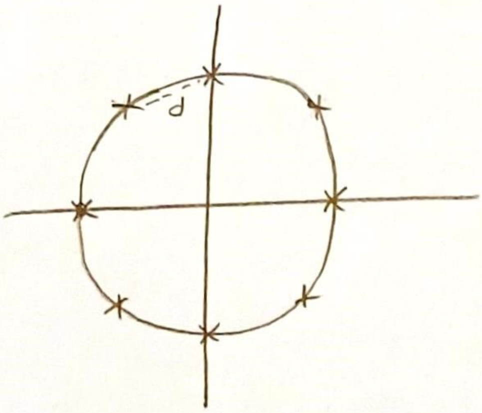
$A_3 = 6.7 \times 10^{-2}$

Q7.42 Proakis (comm systems)

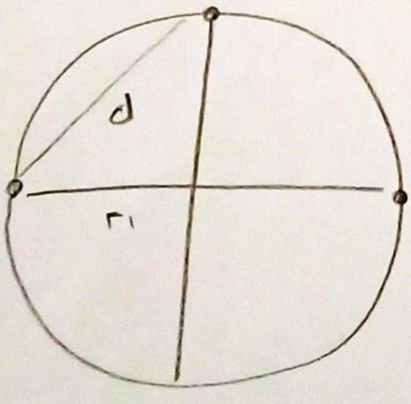
4-PSK

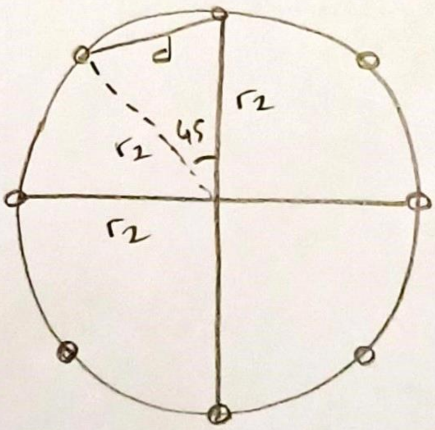


8-PSK



- $P_{e,4\text{-PSK}} = P_{e,8\text{-PSK}}$
- assume that error occurs only in selecting adjacent points
- what is the additional transmitted energy repuired to equilaze $P_{e,4\text{-PSK}}$ and $P_{e,8\text{-PSK}}$?





$$r_1^2 + r_1^2 = d^2$$

$$r_1 = \frac{d}{\sqrt{2}}$$

$$\mathcal{E} = r_1^2$$

$$\mathcal{E}_{avg} = \frac{r_1^2 \cdot 4}{4} = r_1^2 = \frac{d^2}{2}$$

$$r_2^2 + r_2^2 - 2 r_2^2 \cos(45) = d^2$$

$$r_2 = \frac{d}{\sqrt{2-\sqrt{2}}}$$

$$\mathcal{E} = \frac{d^2}{2-\sqrt{2}}$$

$$\mathcal{E}_{diff} = 10\log_{10}\left(\frac{d^2}{2-\sqrt{2}}\right) - 10\log_{10}\left(\frac{d^2}{2}\right)$$

$$= 10\log_{10}\left(\frac{2}{2-\sqrt{2}}\right) = 5.33 \text{ dB}$$

Q.7.34 Proakis
(Comm systems)

→ Desired rate of transmission = R bit/s
→ Determine the required transmission bandwidth for
- B-FSK
- 8-PSK
- QPSK
- BPSK
- 16 FSK

$r = \frac{R}{W}$ (bandwidth efficiency) $r = \frac{2\log_2 M}{N}$

	M	N	W
BFSK	2	2	R
8-PSK	8	2	R/3
QPSK	4	2	R/2
BPSK	2	2	R
16-FSK	16	16	2R

- 8-PSK is the most bandwidth efficient

$r_{8\text{-PSK}} > r_{QPSK} > r_{BFSK} = r_{BPSK} > r_{16\text{-FSK}}$

Information and Forward Error Correction
Information Sources
- video
- image
- audio

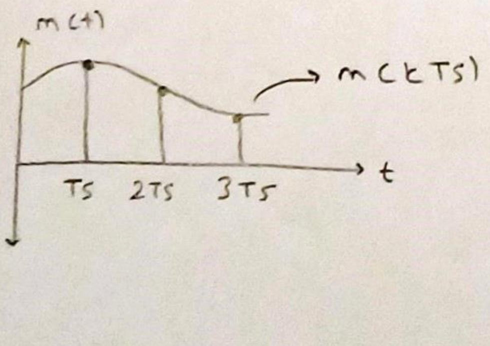
) analog and digital information sources

* sampling at a rate of Nyquist
* quantization (bits per sample)
* source encoding
* compressing the data

- use less bits for information sources.

$x(t) \rightarrow x(kTs) \rightarrow x_p(kTs)$

Discrete memoryless Source



- xi's are generated independently with some random distribution:
- $\{P_i\}_{i=1}^{N}$ (independent identically distributed)
- DMS generates iid.r.v.
- If the source is binary, $\mathcal{A} = \{0,1\}$.
  - binary source $P_1 = p = P\{x_1 = 1\}$
    $P_0 = 1 - p$
- BSS (binary symmetric source), if $p = 0.5$

measure of information

- If there is an uncertainity on time event, this contains less information than the event has in doubt.
- Self-information is a measure of the information content
- The amount of self information contained in a probabilistic event depends only on the probability of event.
- The smaller its probobility, the larger self information associated with receiving information related to this event.

$$I(p) = -\log_a(p) = \frac{1}{\log_a(p)} \quad , \text{ if } a=2 \text{ it is bits. } \left(\log_2\left(\frac{1}{p}\right)\right)$$

Entropy

- The information content of the source is a weighted average of the self information of all source outputs.

$x_0, \dots, x_{N-1} \Rightarrow$ source outputs

$\downarrow$

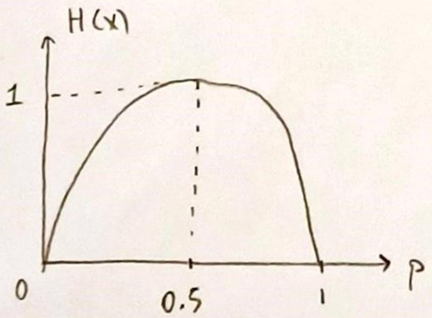
$p_0, \dots, p_{N-1}$

$$H(x) = \sum_{i=0}^{N-1} p_i \, I(p_i) = \sum_{i=0}^{N-1} p_i \log_2\left(\frac{1}{p_i}\right)$$

If binary source

- $p_0 = p \quad , \quad p_1 = 1-p$

$$H_b(x) = -\sum_{i=0}^{1} p_i \log_2(p_i) = -p\log_2 p - (1-p)\log_2(1-p)$$



- Entropy is the meausure of the uncertainity about the source

Source-Coding Theorem

- Fundamental limit on the rate at which the output of information source can be compressed without causing a large error.
- A source with entropy H can be encoded with arbitrarily small errors at a rate of R (bits / source output) as long as $R > H$. If $R < H$ causes large error.

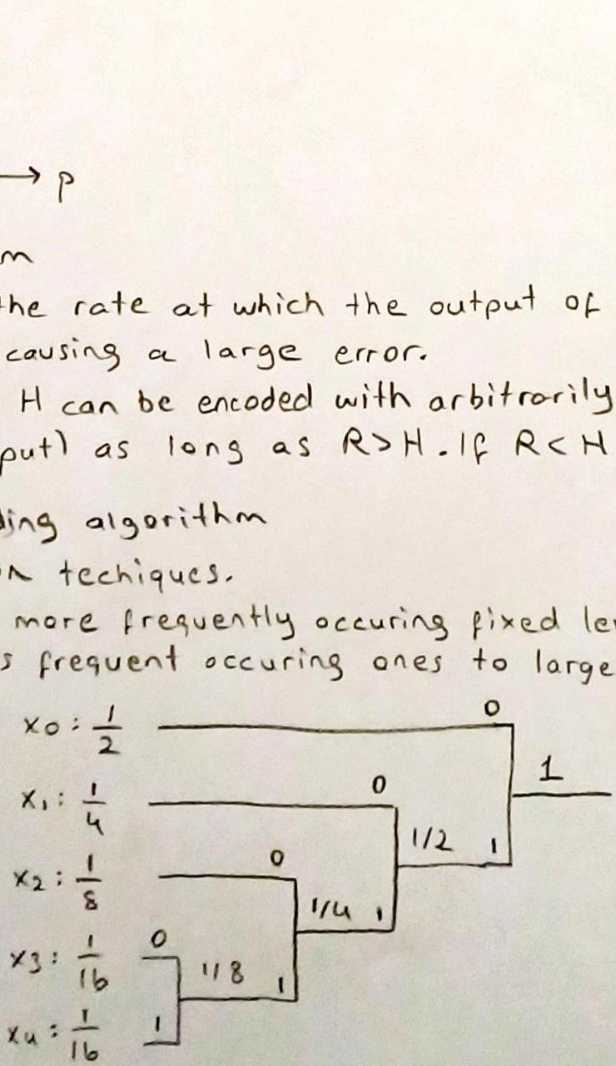
Huffman source-encoding algorithm

- lossless data compression techiques.
- the idea is to map the more frequently occuring fixed length sequences to shorter binary sequences and the less frequent occuring ones to larger binary sequences.

Ex: x_0, x_1, x_2, x_3, x_4

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

$1/2 \quad 1/4 \quad 1/8 \quad 1/16 \quad 1/16$



$x_0 \rightarrow 0$

$x_1 \rightarrow 10$

$x_2 \rightarrow 110$

$x_3 \rightarrow 1110$

$x_4 \rightarrow 1111$

average number of bits / source output

$$\bar{L} = \frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 3 + \frac{1}{16}\cdot 4 + \frac{1}{16}\cdot 4 = \frac{15}{8}$$

$\bar{L} = R$ here, so $\bar{L} > H$ it can be encoded

Ex:)

$x_0 \rightarrow 0.4$

$x_1 \rightarrow 0.2$

$x_2 \rightarrow 0.2$

$x_3 \rightarrow 0.1$

$x_4 \rightarrow 0.1$

$$H(x) = 0.4 \log_2\left(\frac{1}{0.4}\right) + 0.2 \log_2\left(\frac{1}{0.2}\right) + \ldots = 2.12193$$

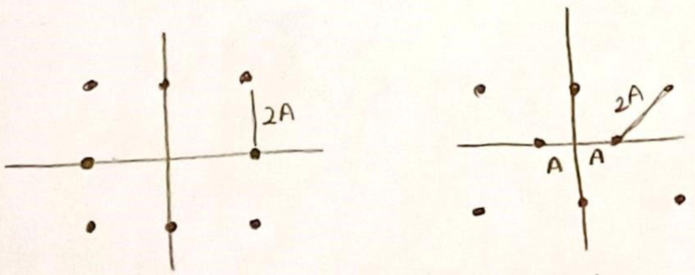
Figure 10.5

şu sıkındırık şemayı yaparken yüksek olasılıklıyı en üste çekmek lazım, sort etmek lazım her toplanden önce.

Recitation

Q1. 7.43 Proakis Comm System

A1.



(a) (b)

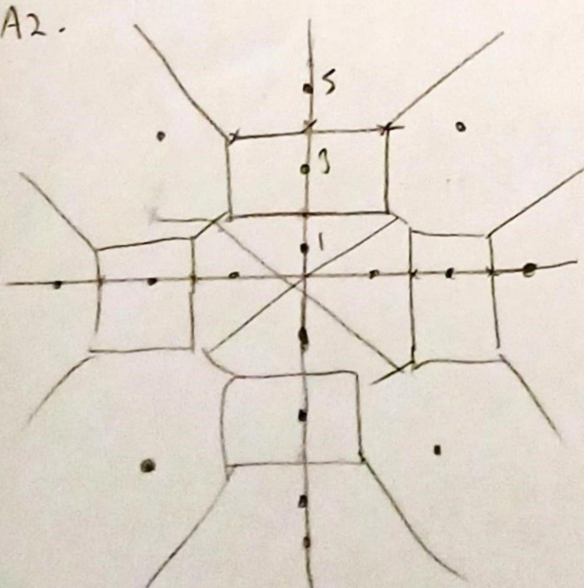
$$P_{avg} = \frac{1}{8}\left[4(2A)^2 + 4(2\sqrt{2}A)^2\right] = 6A^2 \quad (a)$$

$$P_e \approx \frac{1}{d_{min}}$$

$$P_{avg} = \frac{1}{8}\left[4\cdot(\sqrt{7}A)^2 + 2\cdot(\sqrt{3}A)^2 + 2\cdot A^2\right] = \frac{9}{2}A^2 \quad (b) \quad \text{(power efficient)}$$

Q2. 7.44 Proakis Comm System

A2.



Q3. 7.46 Proakis Comm Systems (2nd ed.)

A3.

$$A^2 = r^2 + r^2 - 2r^2 \cos(45)$$

$$r = \frac{A}{\sqrt{2-\sqrt{2}}}$$

PSK

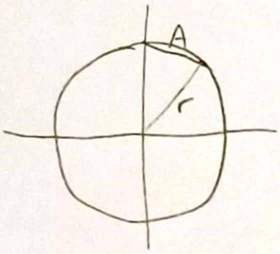
phase değişiyor

PAM

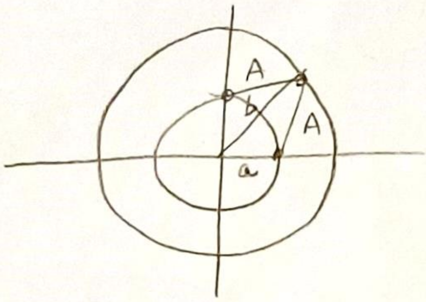
amplitude değişiyor

QAM

hem phase hem amplitude değişiyor



(a)



(b)

$$a = \frac{A}{\sqrt{2}}$$

$$b = \frac{a\sqrt{2}}{2} + \frac{A\sqrt{3}}{2} = \left(\frac{\sqrt{3}+1}{2}\right)A$$

$$P_{avg} = \frac{1}{8}(8r^2) = \frac{A^2}{2-\sqrt{2}} \quad (a)$$

$$P_{avg} = \frac{1}{8}\left[4a^2 + 4b^2\right] = \frac{1}{2}\left[\frac{A^2}{2} + \left(\frac{\sqrt{3}+1}{2}\right)^2 A^2\right]$$

$$\frac{P_{8-PSK}}{P_{8-QAM}} = 1.44 \approx 1.57 \text{ dB}$$ QAM more power efficient

Q4. Diğer proakis kitabı

a) $k = \frac{4800}{2400} = 2$ (4-QAM)

$$P_m = 1 - \left(1 - 2\left(1 - \frac{1}{\sqrt{m}}\right) Q\left(\sqrt{\frac{3kEb}{(m-1)No}}\right)\right)^2$$

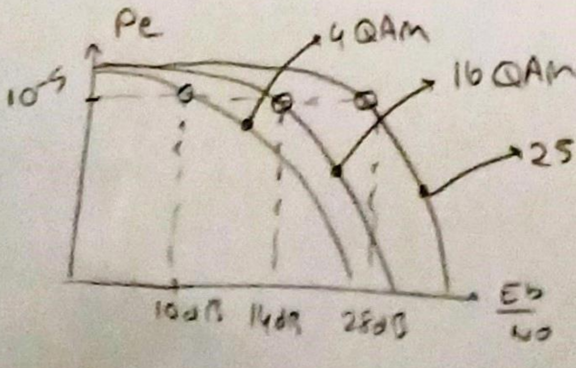
$k = 2$ $\quad$ $m = 4$

$P_m = 10^{-5}$

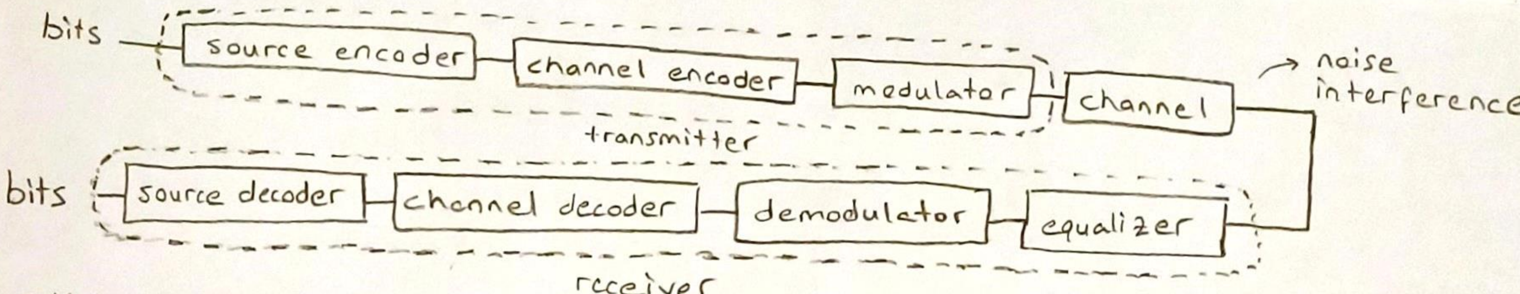
$$Q\left(\sqrt{\frac{2Eb}{No}}\right) = 5 \times 10^{-6} \quad \approx\Rightarrow \quad 9.89 \text{ dB} = \frac{Eb}{No}$$

b) $\frac{Eb}{No} = 14.04$ dB (16-QAM)

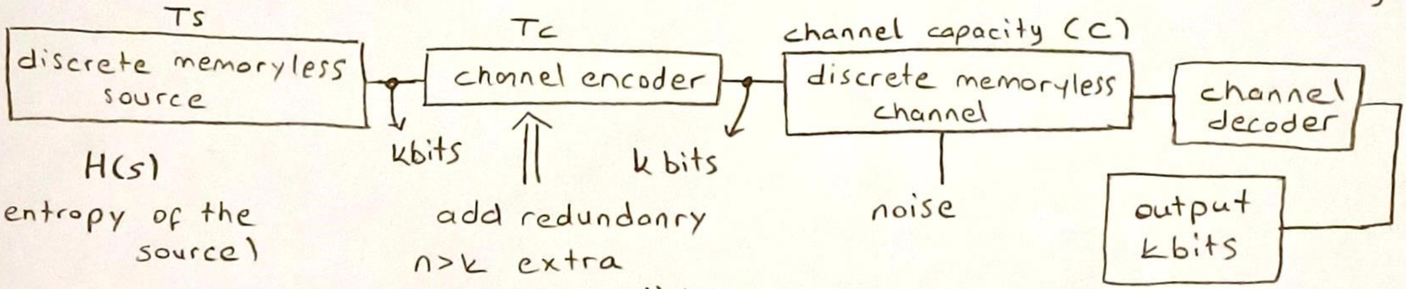
c) $\frac{Eb}{No} = 28.19$ dB (256-QAM)



...Coding Theorem



- Voice application, 10^{-3} or less error probability is expected
- Data application, 10^{-6} or less error is expected. (using channel coding schemes)



H(s)
(entropy of the source)

add redundanry
$n>k$ extra
$n-k$ (redundant) bits

$r=\frac{k}{n}$ code rate

- average rate of the source : $H(S)/Ts$
- the maximum rate of information of transmission over the channel : $\frac{C}{Tc}$

$$\frac{H(S)}{Ts} \le \frac{C}{Tc}$$

- DMS with alphabet S having entropy H(S) and produce symbols every Ts seconds. DMS with C every TS.

Application of the chanel coding theorem to binary symmetric chanels

DMS $\rightarrow$ 0 s, 1 s } equally likely every Ts

$H(S)=1$ bit per source output. Information rate : $1/Ts$ bps.

- Then, this source output is applied to the channel encoder with code rate $r=\frac{k}{n}$
- Then, channel encoder generates a symbol every Tc
- Chanel capacity is C , $1/Tc$ (anlamadım), C/Tc bps (anlamadım)

information rate

$$\frac{1}{Ts} \le \frac{C}{Tc}$$

channel capacity per unit time

$$\frac{Tc}{Ts} = r \text{ (code rate)}, \text{ so } r \le C$$

Repetition Code
repeat n times the bit
k=1 1/3 RC
bit is 1 111
bit is 0 000

input → DMC → output

$p = 10^{-2}$

$C = 1 - H(x)$

$C = (1-p)\log_2(1-p) + p\log_2 p + 1$

$C = 0.9192$, so $r \leq 0.9192$ $\quad r = \frac{k}{n}$

- the error probability is not very high.

r	P_e
1/3	3.10^{-4}
1/5	10^{-6}
1/9	10^{-8}

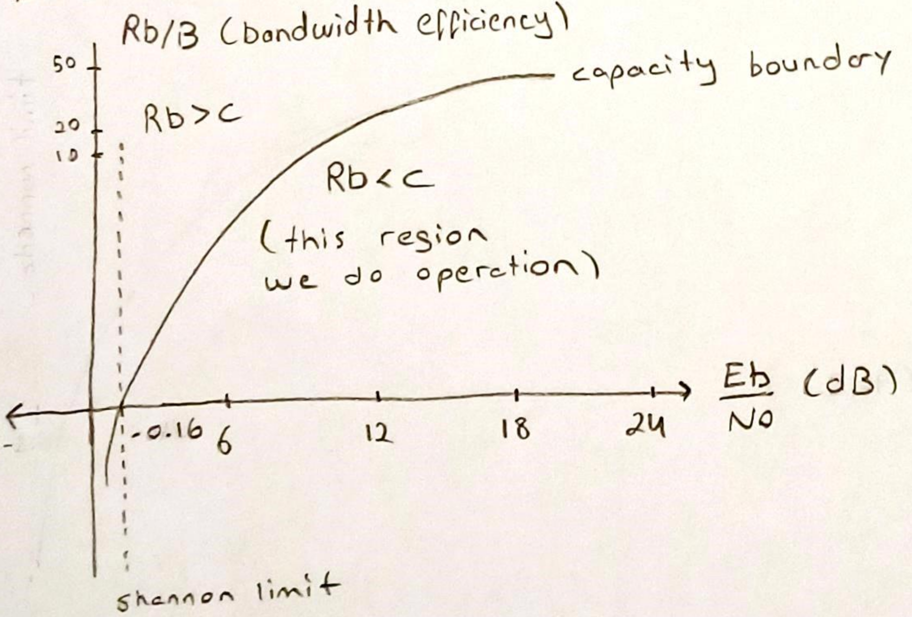
The capacity of Gaussian channel

- Shannon capacity $C = \underbrace{B}_{\text{channel BW}} \log_2\left(1 + \frac{\overbrace{P}^{\text{received power}}}{N_0 B}\right)$

$w(0, \sigma^2)$ $\quad \sigma^2 = N_0 B$

- Gaussian, White noise psd is $N_0/2$
- spectral efficiency : C/B (bps/Hz)

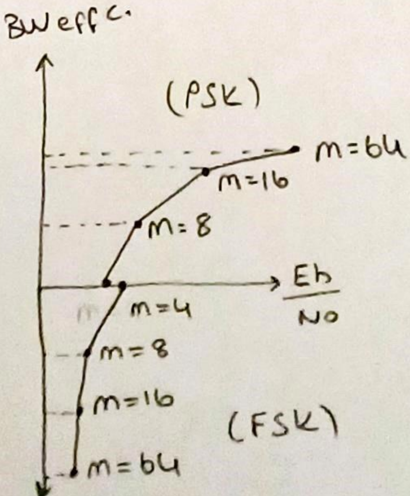
Bandwidth Efficiency Diagram



(Fig. 10.15)

m-Ary PSK and m-Ary FSK

- For PSK : $\frac{R_b}{B} = \frac{\log_2 M}{2}$
- For FSK : $\frac{R_b}{B} = \frac{2\log_2 M}{M}$



Recitation

Q1. A source has an alphabet $\{a_1, .., a_6\}$ with the corresponding probabilities $\{0.1, 0.2, 0.3, 0.05, 0.15, 0.2\}$ find the entropy of this source. Compose this entropy of a uniformly distributed source with the same alphabet.

$H(x) = -\sum p_i \log_2 p_i$

$H(x) = 2.4087$ bits/symbol

unifom olsaydı $p_i = 1/6$ hepsi için

$H(x) = -6 \cdot \frac{1}{6} \log_2 (1/6) = -\log_2 (1/6) = 2.585$ bits/symbol

Q2. A source alphabet $\{a_1, a_2, a_3, a_4, a_5, a_6\}$ with corresponding probabilities $\{0.1, 0.2, 0.3, 0.4\}$

a) Entropy

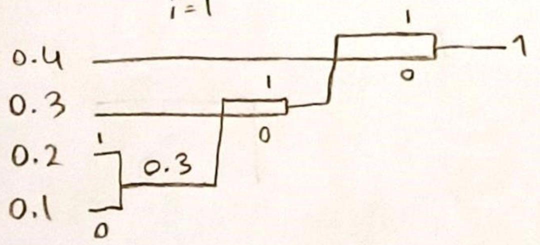
b) What is the minimum required average code word length to represent this source for error free reconstruction?

c) Huffman code, compare average length of code with entropy?

S2.

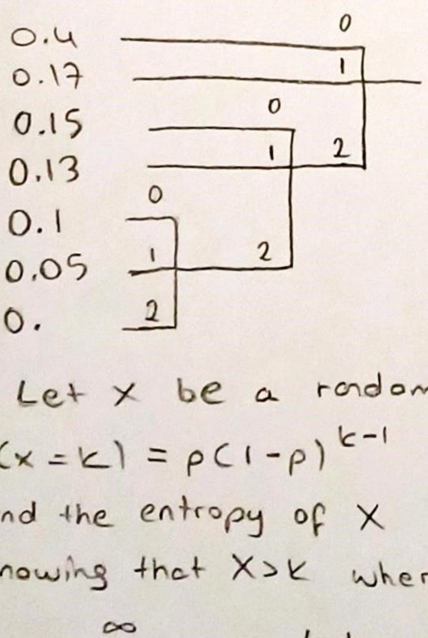
a) $H(x) = -\sum_{i=1}^{4} p_i \log_2 p_i = 1.8464$ bits / output symbol ((b) has same result)

c)



0.4	$\to$	0	1×0.4
0.3	$\to$	10	2×0.3
0.2	$\to$	111	3×0.2
0.1	$\to$	110	$+ \; 3 \times 0.1$
			1.9

Q3. $\{0.4, 0.17, 0.15, 0.13, 0.1, 0.05\}$ ternary Huffman (1 0 2)



0.4	0
0.17	1
0.15	20
0.13	21
0.1	220
0.05	221
0.	222

Q4. Let X be a random variable

$P(x = k) = p(1-p)^{k-1} \qquad k = 1, 2, \ldots$

1) find the entropy of X

2) Knowing that $X > k$ where k is a positive integer, what is $H(x)$?

S4.

$H(x) = -\sum_{k=1}^{\infty} p(1-p)^{k-1} \log_2 (p(1-p)^{k-1})$

$-p\log_2(p) \sum_{k=1}^{\infty} (1-p)^{k-1} - p\log_2(1-p) \sum_{k=1}^{\infty} (k-1)(1-p)^{k-1}$

k yerine k+1 ilk kısım için $\sum_{r=0}^{\infty} \alpha^r = \frac{1}{1-\alpha}$

$$-p\log_2(p)\frac{1}{1-(1-p)}$$

ikinci kısım için $\sum_{r=0}^{\infty} r\alpha^r = \frac{\alpha}{(1-\alpha)^2}$

$$-p\log_2(1-p)\frac{(1-p)}{(1-(1-p))^2}$$

$$= -\log_2(p) - \frac{1-p}{p}\log_2(1-p)$$

$k > K$

2) $P(x=k \mid x>K) = \frac{P(x=k, x>K)}{P(x>K)}$

$$= \frac{p(1-p)^{k-1}}{P(x>K)}$$

$$P(x>K) = \sum_{k=K+1}^{\infty} p(1-p)^{k-1}$$

$$p\Big(\underbrace{\sum_{k=1}^{\infty}(1-p)^{k-1}} - \underbrace{\sum_{k=1}^{K}(1-p)^{k-1}}\Big)$$

$$p\Big(\frac{1}{1-(1-p)} \qquad \sum_{r=0}^{k}\alpha^r = \frac{1-\alpha^{k-1}}{1-\alpha}$$

$$p\left(\frac{1}{1-(1-p)} - \frac{1-(1-p)^K}{1-(1-p)}\right) = \frac{p(1-p)^{k-1}}{(1-p)^K}$$

$k > K$
$k = K + \ell$
assumption

$$\frac{p(1-p)^{K+\ell-1}}{(1-p)^K} = p(1-p)^{\ell-1}$$

Q1. Determine the average energy of a set of MPAM signals of the form

$s_m(t) = s_m \varphi_o(t) \quad m = 1,2, \ldots m \quad 0 \le t \le T$

$s_m = \sqrt{E_g}\, A_m \quad m = 1,2 \ldots m$

$A_m = (2m - 1 - M)\dfrac{d}{2}$

$$\mathcal{E}_a = \frac{1}{M}\sum_{m=1}^{M} s_m^2 = \frac{d^2}{4M}\mathcal{E}_g \sum_{m=1}^{M}(2m-1-M)^2 =$$

$$\frac{d^2}{4M}\mathcal{E}_g \sum_{m=1}^{M}\left[4m^2 + (M+1)^2 - 4m(M+1)\right] = \frac{d^2}{4M}\mathcal{E}_g\left(M(M+1)^2 + 4\sum_{m=1}^{M} m^2 - 4(M+1)\sum_{m=}\right.$$

$$\frac{d^2}{4M}\mathcal{E}_g\left(M(M+1)^2 - \frac{4M(M+1)^2}{2} + \frac{4M(M+1)(2M+1)}{6}\right)$$

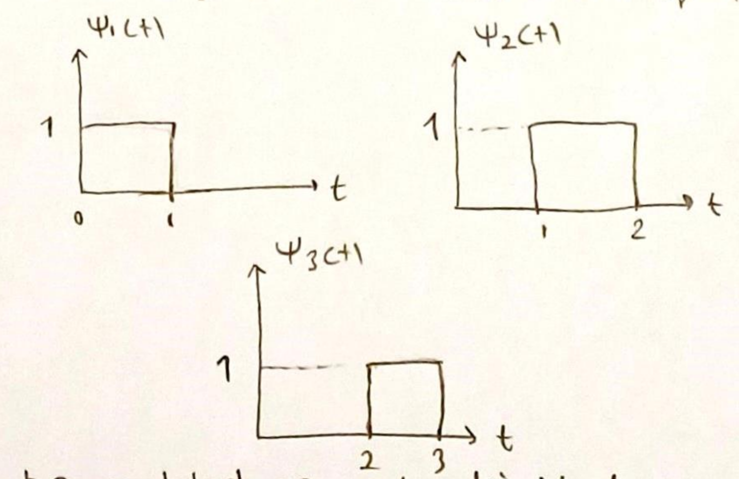
$$= \frac{M^2-1}{3}\,\frac{d^2}{4}\,\mathcal{E}_g$$

Q2. Consider three waveforms $\psi_n(t)$ shown in the figure

1) Show that they are orthogonal

2) Express the waveform $x(t)$ as a weighted linear combination of $\psi_n(t)$

$$x(t) = \begin{cases} -1 & 0 \le t \le 1 \\ 1 & 1 \le t \le 3 \\ -1 & 3 \le t \le 4 \end{cases}$$



Q3. An information source can be modeled as a bandlimited process with a bandwidth at 6000 Hz. This process is sampled at a rate higher than Nyquist rate to provide a guard band at 2000 Hz. It is observed that the resulting samples take waves in a set {-4, -3, -1, 2, 4, 7} with probs {0.2 0.1 0.15 0.05 0.3 0.2}. What is the H(x) in terms of bit/sec?

$f_s = 2000 + 12000 = 14\text{kHz}$ symbol/sec

entropy = bits/symbol

$H(x) = -0.2\log_2 0.2 \ldots = 2.4087$ bits/symbol $\times$ 14k symbol/sec

$= 33721.8$ bits/sec

Q4. Let $Y = g(x)$ where g donetes a deterministic function. Show that in general $H(Y) \leq H(x)$. When does equclity holds

$$H(x,y) = H(x, g(x)) = H(x) + H(g(x)|x) = H(Y) + H(x|g(x))$$

x verildiği zaman g(x)'te kalan belirsizlik demek. x verilince g(x)'te belirsizlik kalmıyor. 0 oluyor.

$$H(x) = H(Y) + H(x|g(x))$$

$H(x) \geq H(Y)$ sağlandı

Q5. Binary PAM is used to transmit information over an unequalized linear filter channel, when $a=1$ is transmitted, the noise free output of the demod is

$$x_m = \begin{cases} 0.3 & m=1 \\ 0.9 & m=0 \\ 0.3 & m=-1 \\ 0 & \text{else} \end{cases}$$

a) Design a zero forcing linear equalizer so that output

$$q_m = \begin{cases} 1 & m=0 \\ 0 & m\neq 0 \end{cases}$$

K değeri en yüksek gecikmeli coeff'e göre belirleniyor.

$$q_m = \sum_{n=1}^{1} c_n h_{m-n}$$

$$\begin{pmatrix} 0.9 & 0.3 & 0 \\ 0.3 & 0.9 & 0.3 \\ 0 & 0.3 & 0.9 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} -0.47 \\ 1.4286 \\ -0.4762 \end{pmatrix}$$

$c = h^{-1} q$

Q.7 $M=4$ PAM modulation is used for transmitting at a bit rate of 9600 bits/sec on a channel having a frequency response

$$C(f) = \frac{1}{1 + j\frac{f}{400}} \qquad |f| \le 2400$$

Determine optimum tx, rx filters. frequency response characteristics

4-PAM

$K = 2$

$$T = \frac{k}{9600} = \frac{1}{4800} \qquad R_{max} = \frac{1}{2T} \Rightarrow W = 2400$$

$$X(f) = T\,\Pi\left(\frac{f}{2W}\right)$$

$$|G_T(f)| = |G_R(f)| = \left[1 + \left(\frac{f}{2400}\right)^2\right]^{1/4}$$

Q8. Consider a digital communication system wit QAM over a voice band telephone channel at rate 2400 sym/sec. The AWGN noise.

1) Determine the $\mathcal{E}_g/N_0$ requires to achieve an error probability at 10^{-5} at 4800 bps.

2) 9600 bps

3) 19200 bps

4) conclusions

1) $K = \frac{4800}{R} = \frac{4800}{2400} = 2$

4 QAM

$$P_M = 1 - \left(1 - 2\left(1 - \frac{1}{\sqrt{M}}\right)\right) Q\left(\sqrt{\frac{3k\mathcal{E}_b}{(M-1)N_0}}\right)$$

$P_M = 10^{-5}$ $\quad k = 2$ $\quad M = 4$

$$Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) = 5\times10^{-6} \Rightarrow \frac{\mathcal{E}_b}{N_0} = 9.76$$

2) $P_M = 1-\left(1-2\left(1-\frac{1}{\sqrt{M}}\right)\right)Q\left(\sqrt{\frac{3k\mathcal{E}_0}{(M-1)N_0}}\right)$ MQAM kare constellation

$$Q\left(\sqrt{\frac{1}{5}\frac{\mathcal{E}_b}{N_0}}\right) = \frac{1}{3} \cdot 10^{-5} \Rightarrow \frac{\mathcal{E}_b}{N_0} = 2.536$$

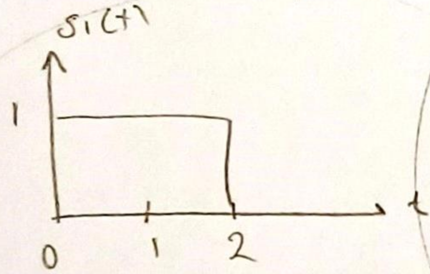
3) 256 QAM

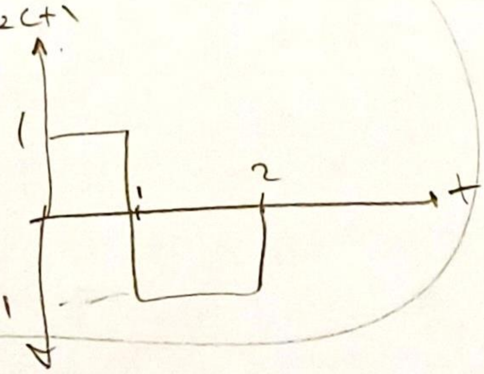
Midterm Questions

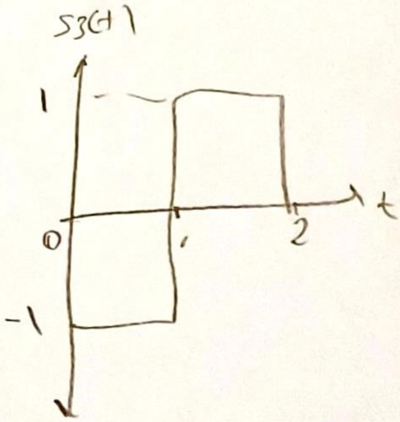
Q1) a) Find a set of orthonormal basis functions to represent the four signals

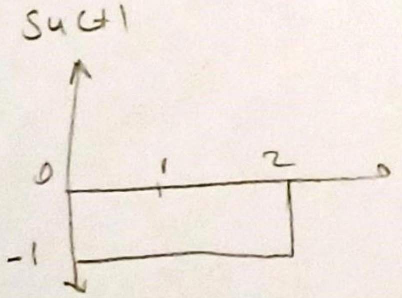
b) Express each of the signals using the basis functions found in (a)

c) Design optimum receiver for these signals assume AWGN

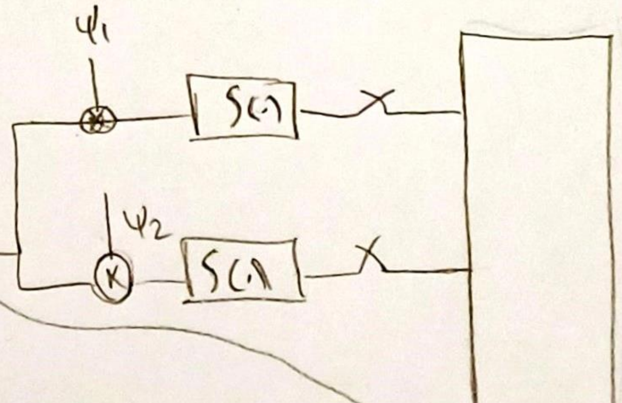








c) $r(t)$



a)

$\psi_1 = \frac{s_1}{\sqrt{2}}$ $\quad \psi_2 = \frac{s_2}{\sqrt{2}}$

b)

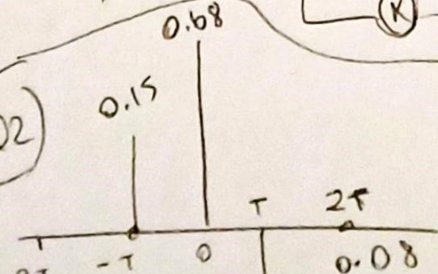
$s_1(t) = \sqrt{2}\,\psi_1$

$s_2(t) = \sqrt{2}\,\psi_2$

$s_3(t) = -\sqrt{2}\,\psi_2$

$s_4(t) = -\sqrt{2}\,\psi_1$

Q2)



$k = 2 \quad 2k+1 = 5$

$$\begin{bmatrix} c_0 & c_{-1} & c_{-2} & c_{-3} & c_{-4} \\ \vdots & & & & \vdots \\ c_4 & & \cdots & & c_0 \end{bmatrix}$$