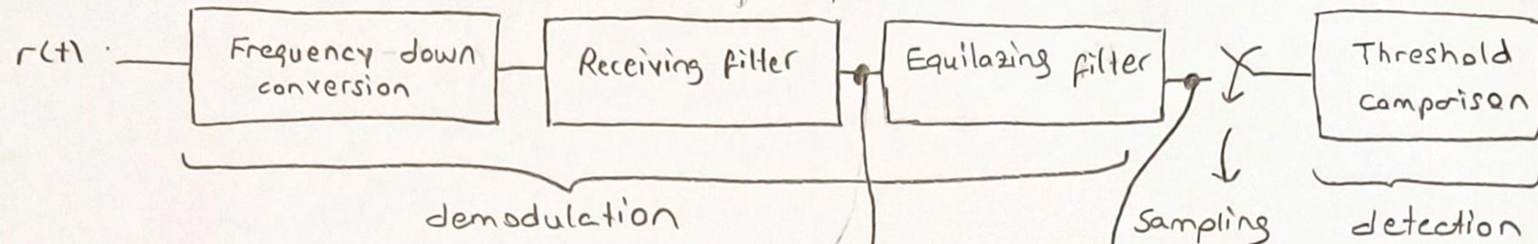
Channel Equilozitan / MMSE Equilazer



Equilozing filter : Compensation for channel induced ISI

baseband pulse

basebond pulse (possibly distorted)

Why equilozer is needed

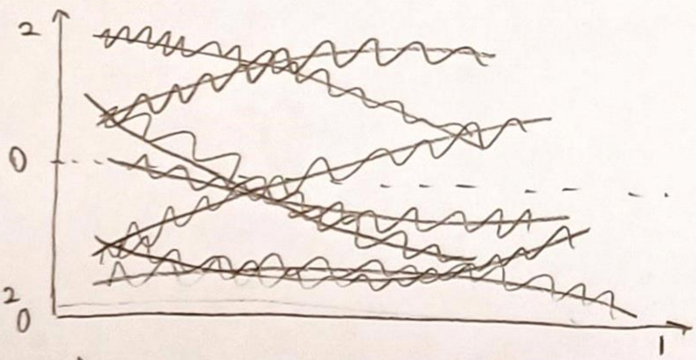
- every wireless channel is linear and time-invariant. every LTI system has certain phase and magnitude response:

$$H_c(f) = \underbrace{|H_c(f)|}_{\text{amplitude distortion}} \underbrace{e^{j\theta_c(f)}}_{\text{phase distortion}}$$

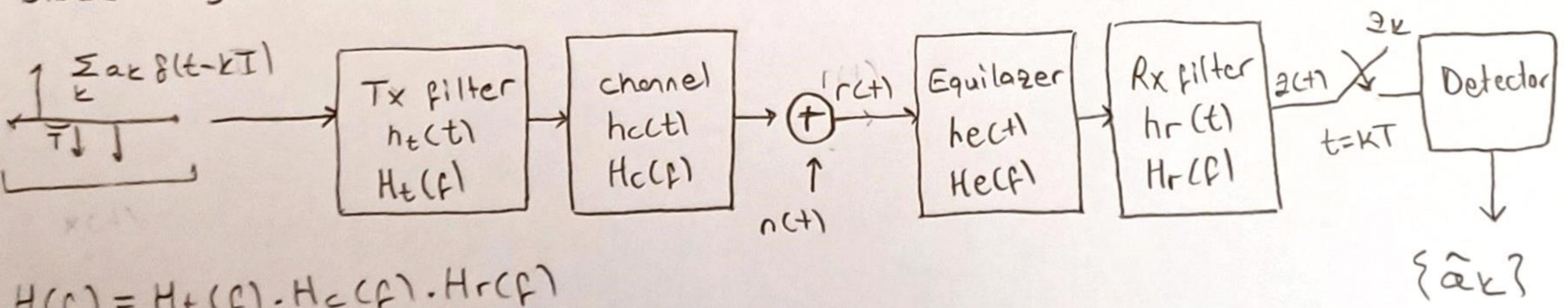
- slow fading and fast fading channels

Example : eye pattern with ISI : Binary PAM , SRRQ

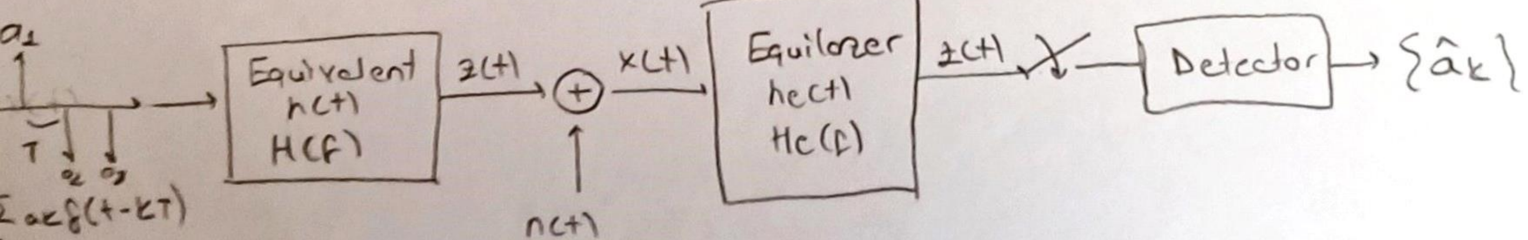
AWGN (Eb/No = 20 dB)



Block Diagram

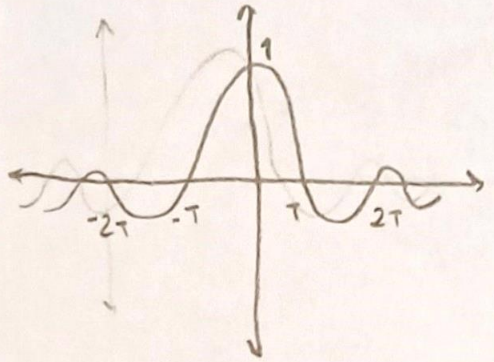


$$H(f) = H_t(f) \cdot H_c(f) \cdot H_r(f)$$



$$H(f) = \frac{H_t(f)\cdot H_r(f)}{H_{rc}(f)} \cdot \underbrace{H_c(f)\cdot H_e(f)}_{\text{cancels each other}} \qquad H_e(f) = \frac{1}{H_c(f)}$$

• if no ISI:



• but in reality it is not crossing x-axis at right sample period, so ISI introduced

• The purpose of equilizing filter is to remove this ISI.

Equalization using

- MLSE (maximum likelihood sequence estimation) (non-linear)
- Filtering (linear equalization)
  - Transversal filtering
    - zero-forcing equilizer
    - minimum mean square error equalizer (MMSE)

    } linear equalization
  - Decision feedback (non-linear)
  - Adaptive filtering

Transversal filter

$x(k) \longrightarrow \boxed{\text{Tran. filter}} \longrightarrow z(k)$

c_n

$$z(k) = c_n * x(k) = \sum_{n=-N}^{N} x(k-N)\, c_n \quad (1)$$

eqn (1)

where k is any value (running time index) $\quad -2N \leq k \leq 2N$

$$\bar{z} = \begin{bmatrix} z(-2N) \\ \vdots \\ z(0) \\ \vdots \\ z(2N) \end{bmatrix} \qquad \bar{c} = \begin{bmatrix} c_{-N} \\ \vdots \\ c_0 \\ \vdots \\ c_N \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} x(-N) & x(-N-1) & \cdots \\ x(-N+1) & x(-N) & \cdots \\ \vdots & & \\ x(N) & \cdots & x(-N) \end{bmatrix}$$

$(4N+1)\times 1 \qquad (2N+1)\times 1 \qquad (4N+1)\times(2N+1)$

eqn (1) can be written in matrix form as $\bar{z} = \bar{x}\cdot\bar{c} \longrightarrow ?$

known ↑ received preamble ↑

$\vec{c} = \vec{x}^{-1}\cdot\vec{z}$

x is not square matrix

Zero-forcing equalizer

- The filter taps are adjusted such that the equalizer output is forced to be zero at N sample points on each side:

Adjust $\{c_n\}_{n=-N}^{N} \Rightarrow z(k) = \begin{Bmatrix} 1, & k=0 \\ 0, & k=\pm 1 \ldots \pm N \end{Bmatrix}$

Mean Square Error (MSE) equalizer

- The filter taps are adjusted such that the MSE of ISI and noise power at the equalizer output is minimized.

Adjust $\{c_N\}_{n=-N}^{N} \Rightarrow \min E\left[(z(k) - x(k))^2\right]$

ZF Equalizer (main point)

- $\bar{c} = \bar{x}^{-1} \cdot \bar{z}$, adjust so that $z(k) = \begin{Bmatrix} 1, & k=0 \\ 0, & k=\pm 1 \ldots \pm N \end{Bmatrix}$ (design is in the form of δ func.)

(2N+1) is number of taps of the equalizer (design constraint)

* need to reduce $\vec{x}$ dimensions to make inverse of square matrix ($4N+1 \rightarrow 2N+1$)

* discard N rows from the top, N rows from the bottom of $\bar{x}$.

Ex:) 3-tap equalizer

$3 = 2N+1 \quad N=1$

3 coefficients c_{-1}, c_0, c_1

$\bar{z} = \begin{bmatrix} z(-1) \\ z(0) \\ z(1) \end{bmatrix}, \quad \bar{c} = \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}, \quad \bar{x} = \begin{bmatrix} x(-1) & x(-2) & x(-3) \\ x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \\ x(3) & x(2) & x(1) \end{bmatrix}$ discard

$\bar{x} = \begin{bmatrix} x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \end{bmatrix}$

given values on example

$x(-2) = 0$ • $x(0) = 0.9$ • $x(2) = 0.1$

$x(-1) = 0.2$ • $x(1) = -0.3$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.2 & 0 \\ -0.3 & 0.9 & 0.2 \\ 0.1 & -0.3 & 0.9 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = ? \Rightarrow \begin{bmatrix} -0.214 \\ 0.9631 \\ 0.3448 \end{bmatrix}$$

(coefficients for 3-tap zf equalizer. now we can use this 3-tap equalizer with this coefficients to equalize ANY kind of input.)

• $z(-3)\ z(-2)\ \underbrace{z(-1)\quad z(0)\quad z(1)}_{}\ z(2)\ z(3)$

designed for only this samples

$$\begin{bmatrix} z(-3) \\ z(-2) \\ z(-1) \\ z(0) \\ z(1) \\ z(2) \\ z(3) \end{bmatrix} = \begin{bmatrix} x(-2) & x(-3) & x(-4) \\ x(-1) & x(-2) & x(-3) \\ x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \\ x(3) & x(2) & x(1) \\ x(4) & x(3) & x(2) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

only these values are known. so other ones will be accepted as zero

$(x(-3) = x(3) = x(4) = x(-3) = 0)$

$$\bar{z} = [0, -0.0428, \underbrace{0, 1, 0}, -0.007, 0.0345]$$

• so it is obtained that 3-tap equalizer is succesfully removes the ISI from center terms, but it is failed to remove ISI from other terms. in order to success increase the size of equilozer.

$c = x^{-1} z = x^{-1}(xc + n) = c + x^{-1}n$ → enhances the level of noise

★ DRAWBACK of ZF equalizer

MMSE Equalizer

$\min E[(z(k) - x(k))^2]$ (★ minimize the error)

→ noisy

• $\bar{z} = \bar{x}\bar{c}$

$\underbrace{\bar{x}^T \cdot \bar{z}} = \underbrace{\bar{x}^T \bar{x}} \cdot \bar{c}$

$R_{xz} = R_{xx} \cdot \bar{c}$, where

R_{xz} : cross correlation between $\bar{x}$ and $\bar{z}$

R_{xx} : auto correlation

$\bar{c} = R_{xx}^{-1} \cdot R_{xz}$

$\dim(\bar{x}) = (4N+1) \times (2N+1)$, $\dim(x^T x) = (2N+1) \times (2N+1)$

so R_{xx} is always a squre matrix (not enhance the noise as ZF equalizer does, but disadvantage is you will not be able to completely zero force the output of the equalizer by making 1 and 0.) (★ TRADE OFF)

★ If you are going to completely supress the ISI, go to ZF equalizer; but if you don't want to enhance the noise as ZF does, go to MMSE equalizer and it will not enhance noise as much, but ISI will not be completely suppressed (★ DIFFERENCE)