

## **EE451 PROJECT MIDPHASE REPORT**

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### **COMPARISON OF ZERO FORCING AND MMSE EQUALIZERS IN DIFFERENT AWGN CHANNELS**

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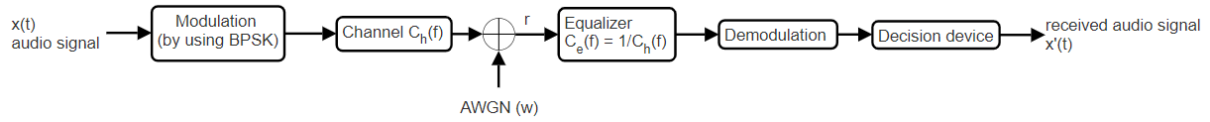
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#### **1. Abstract**

This project aims to conduct a comprehensive comparison of the performance of Zero Forcing and MMSE (Minimum Mean Square Error) equalizers in Band Limited AWGN (Additive White Gaussian Noise) channels. Zero Forcing leverages the direct relationships between transmitted and received signals, whereas MMSE optimizes received signals based on the Minimum Mean Square Error principle. The project will employ simulations to investigate performance differences between both equalizer types and compare the results. The primary objective is to determine which equalizer type demonstrates superior performance in a Band Limited AWGN channel.

#### **2. Introduction**

In communication systems, equalization plays a pivotal role in suppressing channel-induced interference and ensuring reliable data transmission. This project delves into the comparison of two common equalization techniques: Zero Forcing and MMSE, and their effect in suppressing intersymbol interference (ISI) in Band Limited AWGN channels.



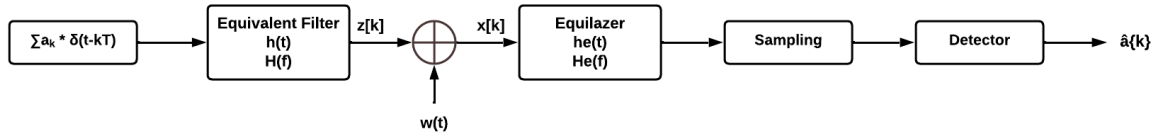
**Figure 1:** General block diagram of the process of this project

## 2.1 Equalization Overview

Equalization is a critical component of wireless communication systems that serves to compensate for channel-induced intersymbol interference (ISI). Every channel can be characterized as linear and time-invariant, with specific phase and magnitude responses. These responses influence the quality of signal transmission.

### Transversal Filtering

Transversal filtering is a common approach to equalization, involving the use of a filter with adjustable taps. The filter operates on the incoming signal to minimize ISI and noise.



The equivalent filter is equal to below:

- $H(f) = H_t(f) \cdot H_c(f) \cdot H_r(f)$ , where  $H_t(f)$  is TX filter,  $H_c(f)$  is channel and  $H_r(f)$  is Rx filter responses.

### Zero Forcing Equalizer

Zero Forcing equalization is achieved by adjusting the filter taps to ensure that the equalizer output is forced to zero at  $N$  sample points on each side of the signal. This approach aims to entirely suppress ISI but can result in an increase in noise.

- adjust  $\{c_n\}$  ( $n = -N$  to  $N$ ), so that  $z[k] = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k = \pm 1, \pm 2, \dots, \pm N \end{cases}$

### MMSE Equalizer

The MMSE Equalizer seeks to minimize the mean square error (MSE) of ISI and noise power at the equalizer output. Unlike Zero Forcing, MMSE Equalization does not entirely eliminate ISI but provides a trade-off by not enhancing noise as much.

- adjust  $\{c_n\}$  ( $n = -N$  to  $N$ ), so that  $\min(E\{|z[k] - x[k]|^2\})$

This project represents an opportunity to gain deeper insights into the practical aspects of Zero Forcing and MMSE equalizers in real-world communication scenarios. The results of this study will contribute to our understanding of equalization techniques in communication systems.

### 3. Communication System Design

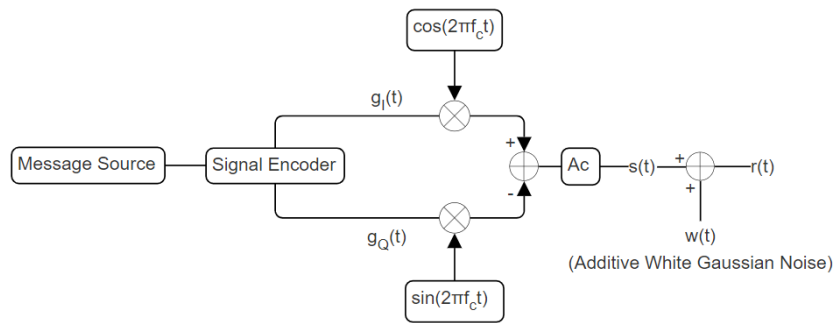
#### 3.1 Modulation

##### Basic Concepts of Digital Modulation

Digital modulation<sup>[1]</sup> is a process used in telecommunications to transfer a digital bit stream over an analog communication channel. This method involves modifying the properties of a carrier wave - such as amplitude, frequency, or phase - to encode digital information. The primary aim of digital modulation is to translate digital data into a form suitable for reliable transmission through a physical medium, typically radio waves, cables, or optical fibers.

##### Overview of Different Modulation Types: ASK, FSK, and PSK

Digital modulation encompasses various techniques, each with its unique characteristics and applications.



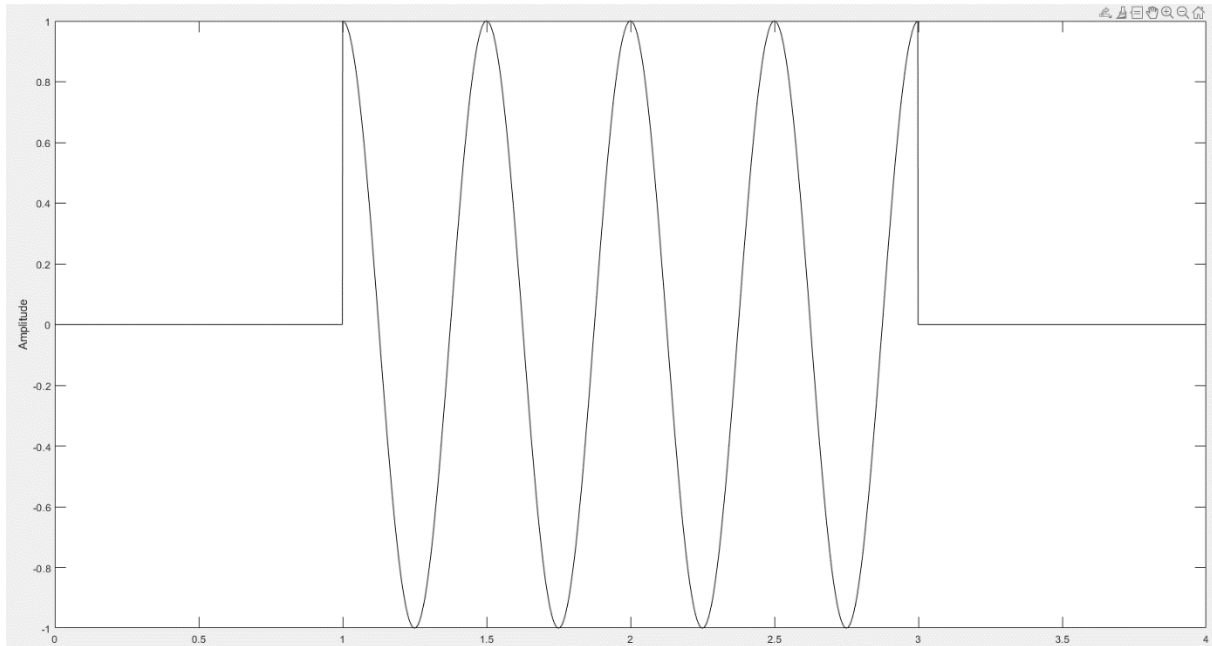
**Figure 2:** General Representation of Modulation

In the figure above, structure of the modulation part in bandpass transmission is shown. Considering binary transmission, three different methods are mentioned by handling an example of binary symbols  $s_0(t)$  and  $s_1(t)$  below:

- Bit 0,  $s_0(t)$  , where  $0 \leq t \leq T_b$  and also  $0 \leq t \leq T_s$ , where  $T_b = T_s$
- Bit 1,  $s_1(t)$

In the cases below  $A_c = 1$  and  $f_c = 2000$  Hz assumptions are made for obtaining plot figures.

**Amplitude Shift Keying (ASK):** Amplitude Shift Keying (ASK) is a modulation technique where the amplitude of the carrier wave is varied in direct proportion to the digital data being transmitted. This method encodes binary data: a high amplitude of the carrier wave represents a binary '1', while a low amplitude represents a binary '0'. ASK is straightforward and cost-effective, making it suitable for applications where complexity and cost need to be minimized, such as in RFID systems and infrared remote controls. However, its major drawback is its susceptibility to noise and interference, as amplitude variations can be easily distorted, leading to errors in the received signal. This limitation makes ASK less ideal for noisy environments or long-distance transmission.

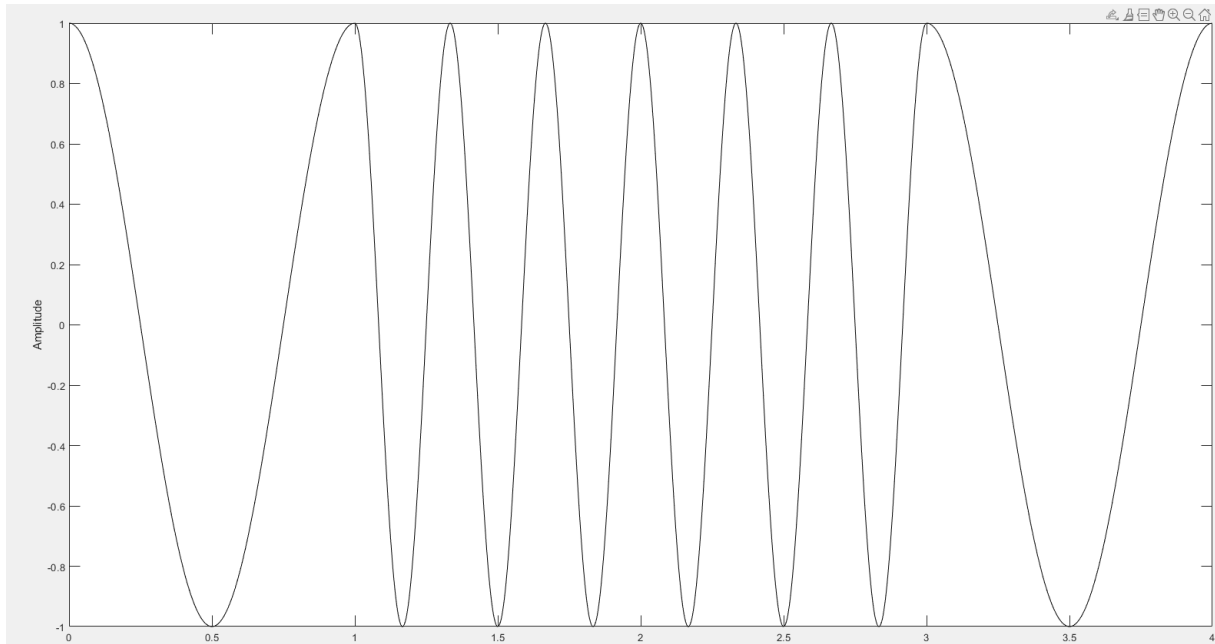


**Figure 3: On-Off Keying with B-ASK<sup>[3]</sup>**

Orthonormal basis functions for the case can be calculated as below and used for signal constellation in further steps:

- $\phi_1(t) = (2/T_b)^{1/2} \cos(2\pi f_c t)$

**Frequency Shift Keying (FSK):** Frequency Shift Keying (FSK) is a digital modulation technique in which the frequency of the carrier wave is varied to represent the digital data. Unlike ASK, FSK does not alter the amplitude but instead shifts between different frequencies to denote binary '1s' and '0s'. This feature makes FSK more resistant to signal amplitude noise, thus more reliable in noisy environments compared to ASK. FSK is widely used in applications like traditional radio systems, caller ID systems, and modem technology. Its resilience to noise and ability to operate effectively over various transmission media make it a versatile choice. However, it generally requires a larger bandwidth than ASK, which can be a limiting factor in bandwidth-constrained systems.



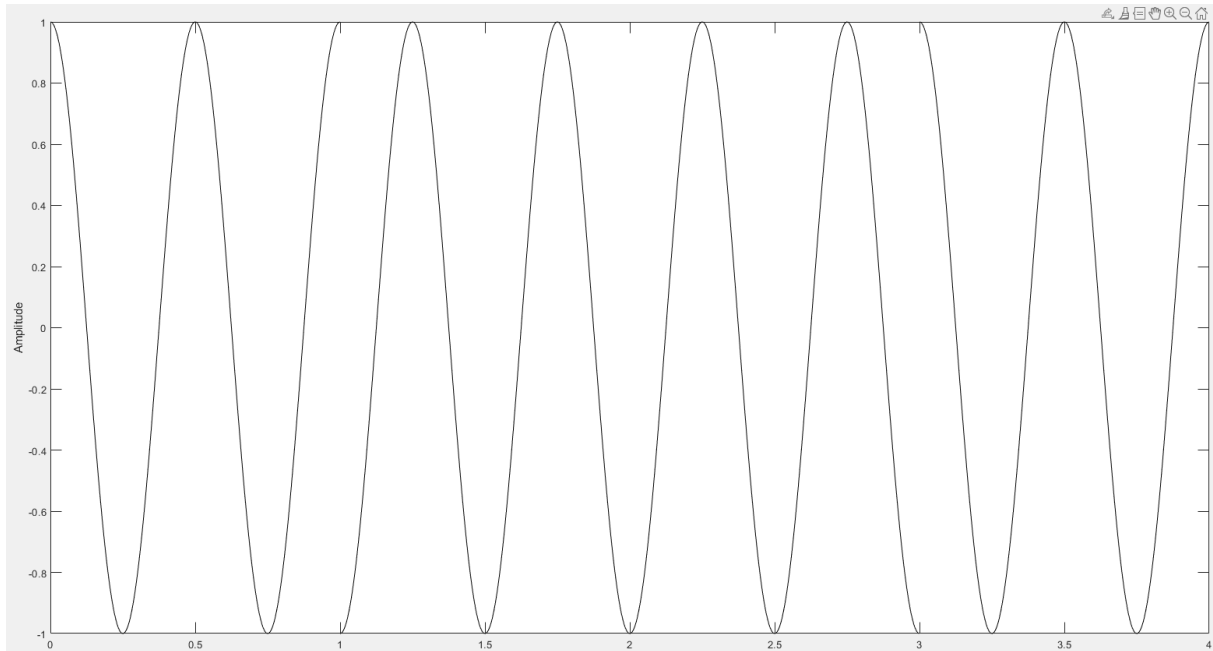
**Figure 4:** Signal Representation by using B-FSK<sup>[3]</sup>

- $\phi_0(t) = (2/T_b)^{1/2} \cos(2\pi f_0 t)$
- $\phi_1(t) = (2/T_b)^{1/2} \cos(2\pi f_1 t)$

There are two basis functions.

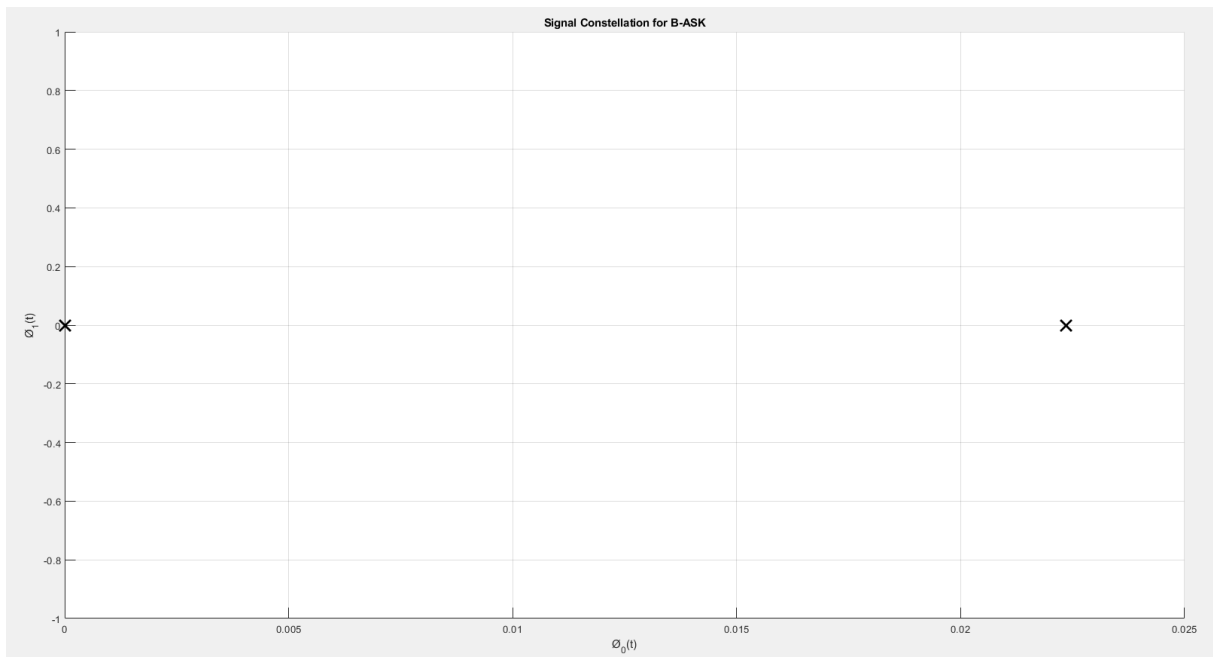
**Phase Shift Keying (PSK):** Phase Shift Keying (PSK) is a form of digital modulation that involves changing the phase of the carrier wave to convey information. The phase of the carrier signal is shifted in relation to the baseline or unmodulated signal to represent the digital data. PSK is more bandwidth-efficient than both ASK and FSK and offers a higher noise immunity, making it well-suited for high-data-rate applications in wireless communication. The simplest form of PSK is Binary PSK, where only two phase shifts are used, each representing a binary digit. PSK is widely used in technologies like Wi-Fi, RFID, and Bluetooth, where maintaining signal integrity in the presence of noise is crucial. Its main challenge is in the receiver design, which must accurately detect phase changes, a task that becomes more complex in environments with signal reflection and fading.

- $\phi_1(t) = (2/T_b)^{1/2} \cos(2\pi f_c t)$

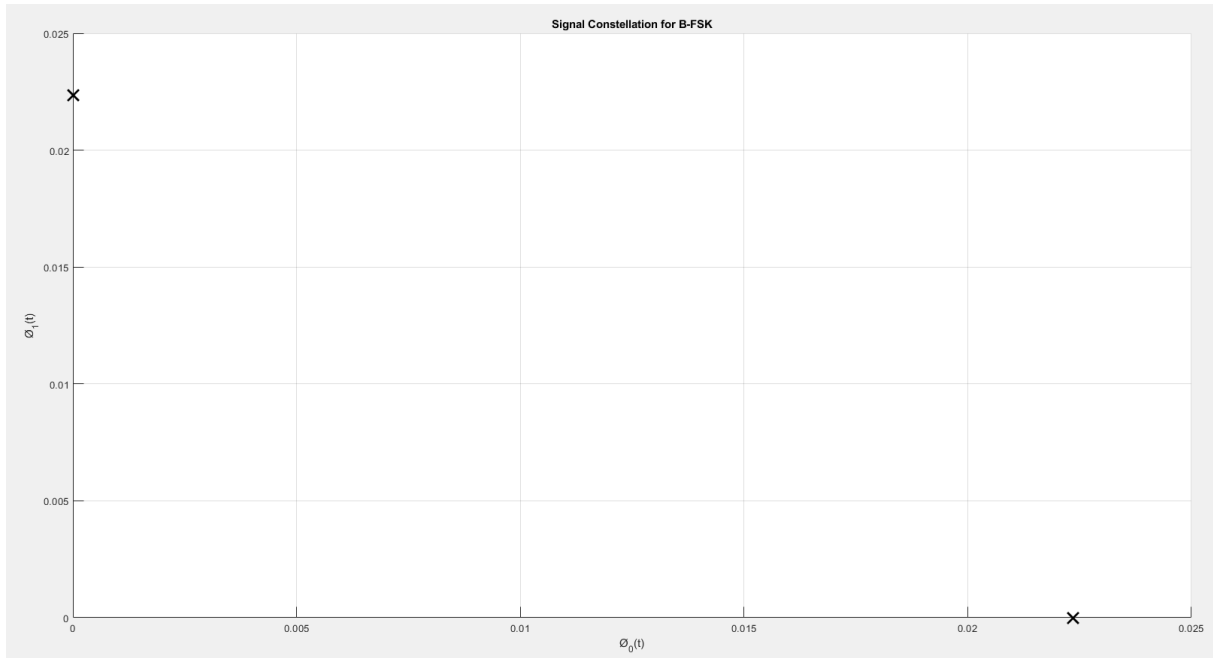


**Figure 5:** Signal Representation by using B-PSK<sup>[3]</sup>

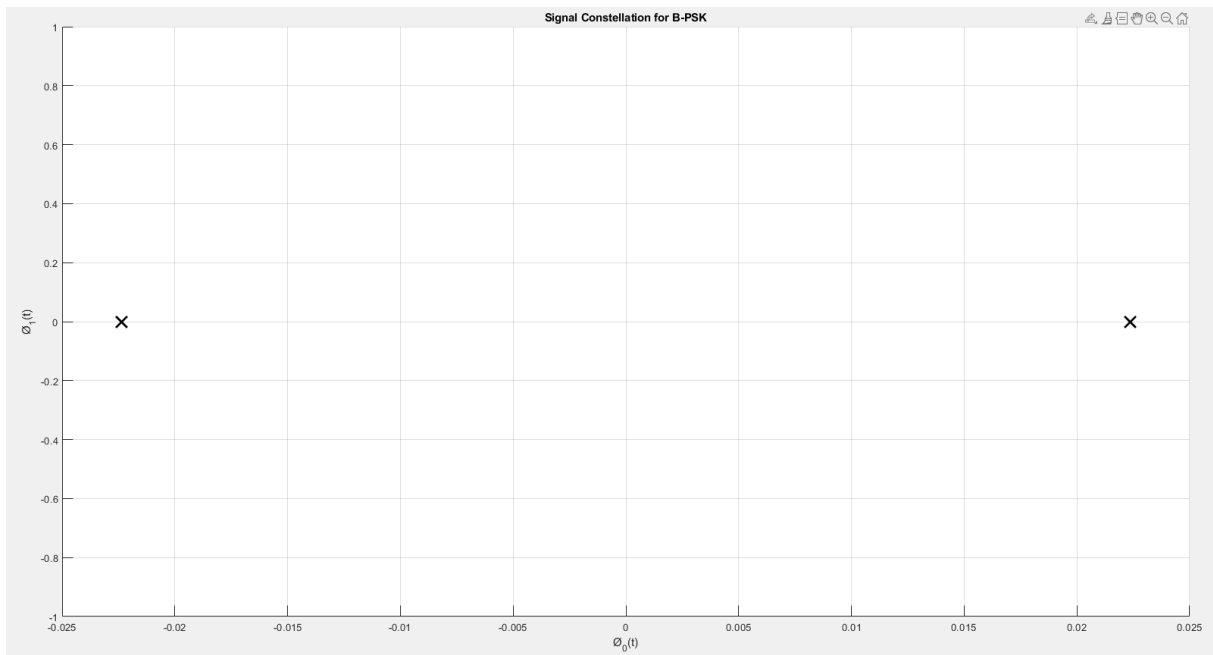
Obtaining the signal constellation graphs from the calculated orthonormal basis functions for each method, it is observed that B-PSK is the best method since the probability of error is minimum in that case, assuming variance of the noise components are same.



**Figure 6:** Signal Constellation for B-ASK<sup>[4]</sup>



**Figure 7:** Signal Constellation for B-FSK<sup>[5]</sup>



**Figure 8:** Signal Constellation for B-PSK<sup>[6]</sup>

- $\sqrt{E_b} < \sqrt{2E_b} < 2\sqrt{E_b}$

Since the maximum distance of between energies of two bits  $s_0(t)$  and  $s_1(t)$  in signal constellation is  $2\sqrt{E_b}$  which is obtained with B-PSK, the minimum error will be occur. Probability of error can be calculated by using Q functions.

### 3.2 Channel

In communication systems<sup>[2]</sup>, a channel serves as the medium for transmitting information from a sender to a receiver. This medium varies depending on the type of communication system. For instance, in wired communication systems like traditional telephone networks or Ethernet connections, the channel consists of physical wires or cables that carry electrical signals. In wireless communication systems such as mobile networks or Wi-Fi, the channel is the air or space through which radio waves or electromagnetic signals travel. Optical communication systems use fiber optic cables, where the channel is the optical fiber through which data is transmitted using light signals. In satellite communication, the channel involves transmitting signals between ground stations and satellites or between different satellites. Lastly, in underwater communication systems like those used for submarine communication, the channel is represented by submerged cables.

The channel is a crucial concept in communication theory, and the characteristics of the channel can have a significant impact on the quality and reliability of the communication. Factors such as noise, interference, and bandwidth limitations can affect how well information is transmitted through the channel. Engineers and researchers study these factors to design communication systems that can effectively overcome challenges and deliver reliable and efficient communication.

Before delving into the other channel types that play a pivotal role in the field of communication systems, it's essential to understand the dynamic nature of signal propagation. In the world of wireless and wired communications, signals don't always travel smoothly from sender to receiver. Instead, they encounter a myriad of challenges along the way, from reflections off buildings in urban settings to the interference caused by multiple signal paths. These challenges give rise to a fascinating array of channel models, each designed to capture the unique characteristics of these real-world scenarios. Let's explore these channel types, from Rayleigh and Rician fading channels to frequency-selective and Nakagami fading channels, shedding light on the intricacies of signal transmission in diverse environments:

- **Rayleigh Fading Channel:** The Rayleigh Fading Channel is characterized by the introduction of random amplitude variations to the transmitted signal, effectively simulating the impact of multipath propagation in wireless communication. A prime example of this occurs in mobile communication within urban environments, where signals can bounce off buildings, resulting in fluctuations in signal strength.
- **Rician Fading Channel:** The Rician fading channel exhibits characteristics which is similar to the Rayleigh fading channel, featuring random multipath fading, but distinguished by the presence of a dominant line-of-sight signal component, making it suitable for modeling scenarios involving a robust direct signal amidst scattered signals. An illustrative example of its applicability lies in wireless communication within open environments with occasional obstructions, where a distinct line of sight to the transmitter coexists with multipath propagation.
- **Frequency-Selective Fading Channel:** The Frequency-Selective Fading Channel exhibits varying levels of attenuation and phase shifts at different frequencies, making it a suitable model for scenarios where the channel response varies with signal frequency. An example of its applicability is in high-frequency radio communication



in urban areas characterized by varying building densities, causing different frequency components to experience distinct fading levels.

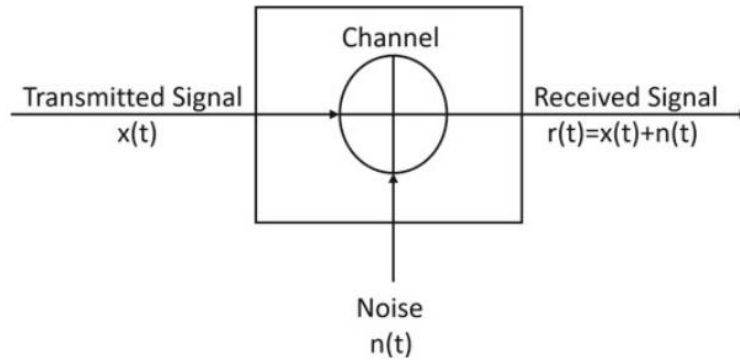
- **Nakagami Fading Channel:** The Nakagami Fading Channel is characterized by a fading envelope following a Nakagami probability distribution, which can manifest varying fading severity. This channel is particularly relevant in applications like underwater acoustic communication, where the signal experiences different levels of attenuation due to changing water conditions.
- **Gilbert-Elliott Channel:** The Gilbert-Elliott Channel model incorporates both "good" and "bad" states, representing periods of reliable and unreliable transmission, making it a valuable tool for modeling communication in mobile networks. It reflects scenarios where channel conditions may switch between good and bad states due to factors such as shadowing or signal blockage.
- **Multipath Fading Channel:** The Multipath Fading Channel simulates the effects of signals arriving at the receiver through multiple paths, leading to constructive and destructive interference. This channel model is commonly used for indoor wireless communication, where signals may reflect off various surfaces like walls and floors, resulting in multipath propagation.
- **Slow Fading Channel:** The Slow Fading Channel experiences variations in signal strength over relatively extended periods, often attributed to factors like vehicle movement or gradual changes in the environment. This channel model finds application in communication within vehicular networks, where channel conditions change gradually as vehicles move. These channel models collectively aid researchers and engineers in the analysis and design of communication systems that can operate effectively in realistic and challenging conditions.

As a result of examining these channel types, and in addition, as a result of the task given in the project, it is decided to use Additive White Gaussian Noise Channel and observe the response of the system in different signal-to-noise ratios.

### **The AWGN (Additive White Gaussian Noise) Channel**

This channel introduces random additive noise to the transmitted signal, simulating the effects of various environmental interferences in communication systems. For instance, in digital communication systems, the AWGN channel can represent the cumulative impact of thermal noise, electronic interference, and other unpredictable factors. For instance, in wireless communication, AWGN may model the collective impact of atmospheric noise, electronic device interference, and other random disturbances that affect the clarity of the received signal. Here are the key effects of the AWGN channel on the received signal:

**Additive Noise:** The AWGN channel adds random noise to the transmitted signal during its journey from the sender to the receiver. This noise is independent of the original signal and is considered to be unrelated to the communication process.



**Figure 9:** Noise addition part

**Signal Degradation:** The noise introduced by the AWGN channel can degrade the quality of the received signal. As the noise is added to the signal, it can make it more challenging for the receiver to accurately detect and interpret the original information.

**Impact on Signal-to-Noise Ratio (SNR):** The quality of the received signal is often measured by the Signal-to-Noise Ratio (SNR), which is the ratio of the signal power to the noise power. In the presence of AWGN, the SNR decreases, making it more difficult for the receiver to distinguish the signal from the noise.

**Bit Errors:** The presence of noise in the received signal can lead to errors in the detection of individual bits. In digital communication systems, this may result in bit errors, where the received bits do not match the transmitted bits.

**Reduced Communication Performance:** The random nature of the Gaussian noise in the AWGN channel means that the received signal experiences fluctuations that can impact communication performance. This is especially critical in scenarios where a high level of accuracy is required, such as in data transmission or digital audio/video communication.

**Uncertainty:** The randomness of the noise introduces uncertainty into the received signal. This uncertainty poses a challenge for the receiver to reliably reconstruct the original information sent by the transmitter.

Despite these challenges, engineers use various techniques, such as error-correcting codes, modulation schemes, and signal processing algorithms, to mitigate the effects of AWGN and improve the reliability of communication systems in the presence of noise.

### 3.3 Equalization

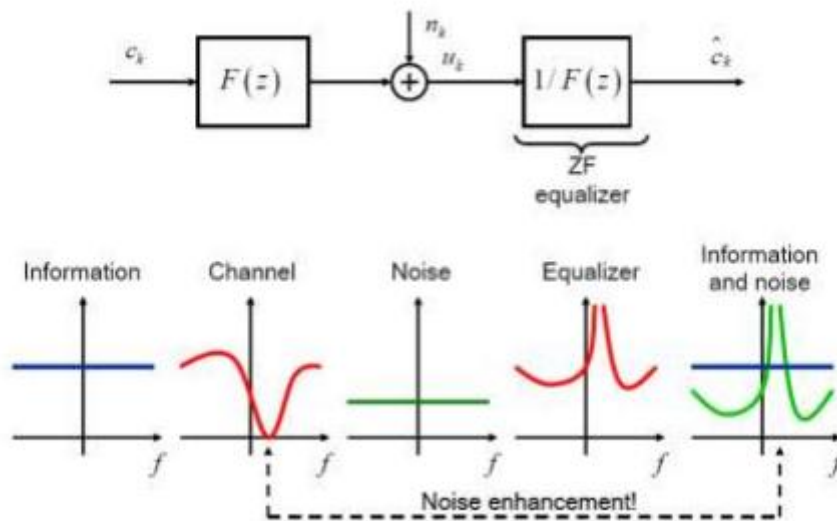
#### Zero Forcing Equalizer

Zero Forcing Equalizer refers to a form of linear equalization algorithm used in communication systems which applies the inverse of the frequency response of the channel. The Zero-Forcing Equalizer applies the inverse of the channel frequency response to the received signal, to restore the signal after the channel<sup>[7]</sup>. The name Zero Forcing corresponds to bringing down the intersymbol interference (ISI) to zero in a noise free case. This will be useful when ISI is significant compared to noise. For a channel with frequency response the zero forcing equalizer is constructed by:

$$C(f) = \frac{1}{F(f)}$$

Thus the combination of channel and equalizer gives a flat frequency response and linear phase. In reality, zero-forcing equalization does not work in some applications, for the following reasons:

Even though the channel impulse response has finite length, the impulse response of the equalizer needs to be infinitely long - At some frequencies the received signal may be weak. To compensate, the magnitude of the zero-forcing filter ("gain") grows very large. As a consequence, any noise added after the channel gets boosted by a large factor and destroys the overall signal-to-noise ratio. Furthermore, the channel may have zeroes in its frequency response that cannot be inverted at all.



**Figure 10:** Zero Forcing Equalizer

The noise enhancement problem is defined at the beginning, in the equalization overview section.

### MMSE Equalizer

To minimize the intersymbol interference and additive noise effects, the equalizer coefficients can be optimized using the minimum mean squared error (MMSE) criterion. When the SNR has elevated values the MMSE equalizer works as Zero Forcing does, but when the SNR has lower values, the fact that MMSE equalizer takes into account the noise and signal variance, makes to not amplify the noise as Zero Forcing does.

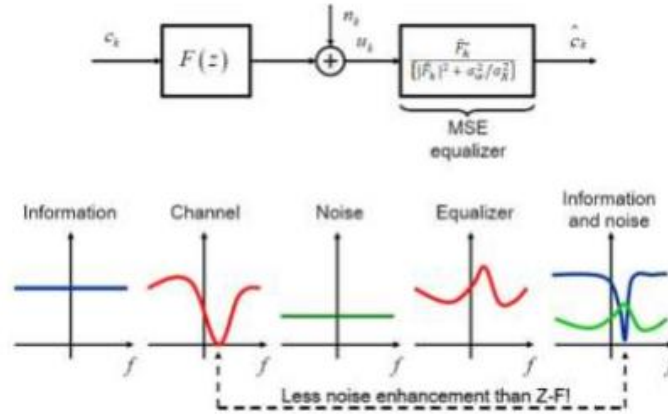


Figure 11: MMSE Equalizer

As it can be seen in the Figure 11, when the Signal to Noise Ratio (SNR) has high values, the MMSE equalizer works as the Zero Forcing does, but for the rest of values that SNR can take, the MMSE equalizer works better in terms of distortion.

That is got for the term  $\sigma_w^2 / \sigma_x^2$ , because when a deep null appears in the frequency response of the channel, this equalizer does not amplifies the received signal just multiplying for the inverse of the channel, it takes into account the SNR in order to not amplify so much the noise term.

This is the big difference about the two equalizers and it can be clearly appreciated in the deep nulls of the channel. While Zero Forcing equalizer increases significantly the noise term in order to recover the original signal, MMSE does not do.

### 3.4 Demodulation

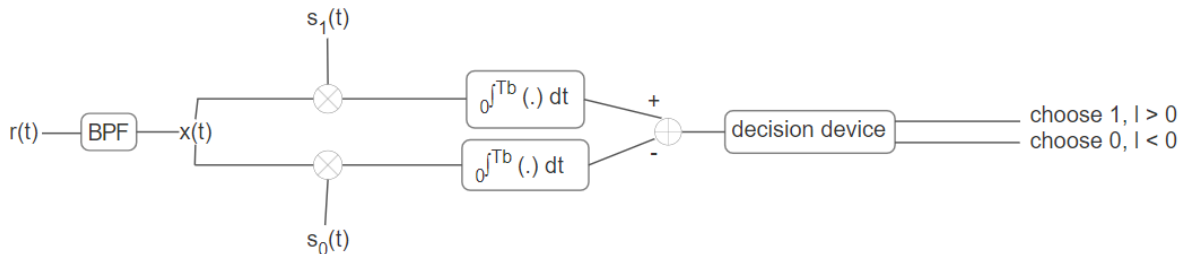
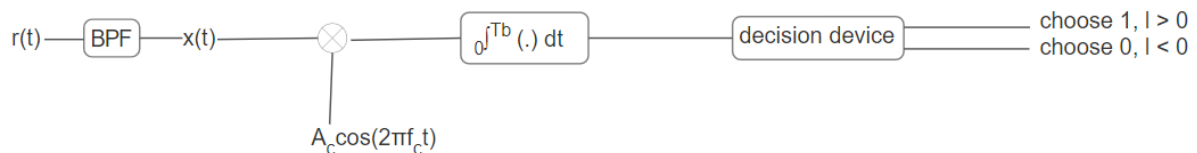


Figure 12: Demodulation Block Diagram

Considering two path correlation receiver a general case the demodulation of the B-ASK signal is described above. But for designing optimum receiver, coherent demodulation of the BPSK signal is performed at the receiver. The received signal is multiplied by a carrier with the same frequency and phase as at the transmitter, which is required for coherent demodulation.

Phase synchronization is typically achieved using the Phase Locked Loop (PLL) in the receiver. However, perfect phase synchronization is assumed here, and PLL implementation is not undertaken. The block diagram of the BPSK modulator is depicted in the figure above. After multiplication with the carrier (orthonormal basis function), the signal is integrated and sampled over symbol time 'T'. Thresholding is then applied to determine whether a '1' is transmitted or a '0' is transmitted.

Coherent receiver of B-PSK has only one branch for correlation since there is only one basis function.



**Figure 13:** Coherent Detector of B-PSK

The demodulation of B-PSK signals involves several key steps. Initially, the demodulator detects the phase of the incoming signal and compares it against a reference phase. This comparison is critical to determine the binary value represented by each phase. Accurate demodulation requires precise synchronization between the transmitter and receiver to ensure that the phase comparison correctly interprets the data. Fluctuations in phase, known as phase noise, or any synchronization mismatches can lead to errors in interpreting the binary data. Despite these challenges, B-PSK remains a preferred choice due to its relative resistance to noise and efficient use of bandwidth compared to other modulation schemes like Amplitude Shift Keying (ASK).

## References

- [1] *Digital Communications*, John G. Proakis, 4th ed. New York: McGraw Hill, 2001.
- [2] *Fundamentals of Communication Systems*, John G. Proakis and Masoud Salehi, 2nd edition, 2005.
- [7] Jon Mark and Weihua Zhuang (2003). "Ch.4", *Wireless Communications and Networking*. Prentice Hall. p. 139.

## Appendices

- [3] MATLAB code for signal representations

```
clc;clear;close all;
tb = 1/1000;
ta= tb/1000;
t = 0:ta:(4*tb)-ta;
s=cos(2*pi*2000*t);
s(1:1000)=0;
s(3000:4000)=0;
plot(t,s,"black")
xlabel("Tb")
ylabel("Amplitude");
%%
clc;clear;close all;
```

## EE451 – Communication Systems II

### P3 - Comparison of Zero Forcing and MMSE Equalizers in Different AWGN Channels

```
tb = 1/1000;
ta= tb/1000;
t = 0:ta:(4*tb)-ta;
s1=cos(2*pi*3000*t(1001:3000));
s0_1=cos(2*pi*1000*t(1:1000));
s0_2=cos(2*pi*1000*t(3001:4000));
figure(1);
plot(t(1:1000),s0_1,"black")
hold on
plot(t(1001:3000),s1,"black")
hold on
plot(t(3001:4000),s0_2,"black")
xlabel("Tb")
ylabel("Amplitude");
%%
clc;clear;close all;
tb = 1/1000;
ta= tb/1000;
t = 0:ta:(4*tb)-ta;
s0_1= cos(2*pi*2000*t(1:1000));
s1= cos(2*pi*2000*t(1001:3000)+pi);
s0_2= cos(2*pi*2000*t(3001:4000));
figure(1);
plot(t(1:1000),s0_1,"black")
hold on
plot(t(1001:3000),s1,"black")
hold on
plot(t(3001:4000),s0_2,"black")
xlabel("Tb")
ylabel("Amplitude");
```

#### [4] MATLAB code for signal constellation with B-ASK

```
clc;
clear all;
%%
Fs = 1000;
Eb= (1/sqrt(2))*sqrt(1/Fs);
sz = 250;
figure(1)
scatter(0,0,sz,'x',"black");
hold on
scatter(Eb,0,sz,'x',"black");
grid on
xlabel("ϕ0(t)");
ylabel("ϕ1(t)");
title("Signal Constellation for B-FSK");
%%
```

#### [5] MATLAB code for signal constellation with B-FSK

```
clc;
clear all;
%%
```

## EE451 – Communication Systems II

### P3 - Comparison of Zero Forcing and MMSE Equalizers in Different AWGN Channels

```
Fs = 1000;
Eb= (1/sqrt(2))*sqrt(1/Fs);
fi2 = 0:0001:0.5;
fi1 = 0:0001:0.5;
sz = 250;
figure(1)
scatter(Eb,0,sz,'x','black');
hold on
scatter(0,Eb,sz,'x','black');
grid on
xlabel("\phi_0(t)");
ylabel("\phi_1(t)");
title("Signal Constellation for B-FSK");
%%
```

[6] MATLAB code for signal constellation with B-PSK

```
clc;
clear all;
%%
Fs = 1000;
Eb= (1/sqrt(2))*sqrt(1/Fs);
fi2 = 0:0001:0.5;
fi1 = 0:0001:0.5;
sz = 250;
figure(1)
scatter(-Eb,0,sz,'x','black');
hold on
scatter(Eb,0,sz,'x','black');
grid on
xlabel("\phi_0(t)");
ylabel("\phi_1(t)");
title("Signal Constellation for B-PSK");
%%
```