$\beta_f = \dfrac{k_f A_m}{f_m \rightarrow W} = \dfrac{\Delta f}{f_m}$ $\quad A_m = \max(m(t))$

$\beta_p = k_p A_m = \Delta\phi_{max}$

$B_T \cong 2W \quad \beta < 1$ for NB-FM

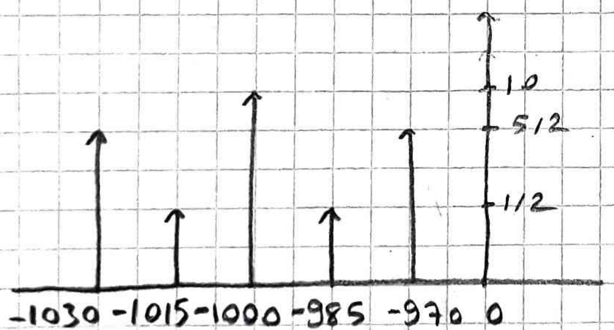
$B_T = 2(1+\beta)W$ for WB-FM (Carson's Rule at least %98 power signal)

Recitation 3

1. $u(t) = [20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)]\cos(2\pi f_c t)$
where $f_c = 100\text{ kHz}$

a) sketch spectrum of $u(t)$
b) Determine the power in each of the frequency components
c) Determine the power in the sidebands, the total power, and the ratio of the sidebands to the total power.

$U(f) = \frac{20}{2}[\delta(f-f_c)+\delta(f+f_c)] + \frac{2}{4}[\delta(f-f_c-1500)+\delta(f-f_c+1500)$
$+\delta(f+f_c-1500)+\delta(f+f_c+1500)] + \frac{10}{4}[\delta(f-f_c+3000)+\delta(f-f_c+3000)$
$+\delta(f+f_c-3000)+\delta(f+f_c+3000)]$



aynı şekil

b. $u^2(t) = 400\cos^2(2\pi f_c t) + \cos^2(2\pi(f_c-1500)t)$
$+\cos^2(2\pi(f_c+1500)t) + 25\cos^2(2\pi(f_c-3000)t) + 25\cos^2(2\pi(f_c+3000)t)$

If we integrate $u^2(t)$ from $-T/2$ to $T/2$ normalize the integral by $1/T$ and take the limit as $T \to \infty$ then all terms involving cosines thend to zero

$f_c \rightarrow P_{f_c} = \frac{400}{2} = 200$

f_c+1500, f_c-1500 $= 1/2$

f_c+3000, f_c-3000 $= 25/2$

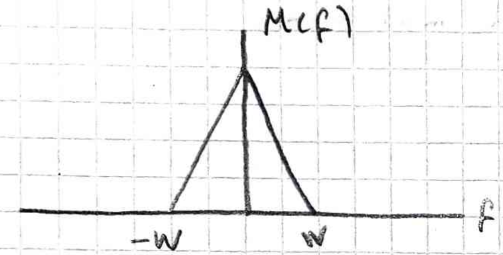
d) $u(t) = 20\cos(2\pi f_c t) + \cos(2\pi(f_c - 1500)t) + \cos(2\pi(f_c - 1500)t)$
$+ 5\cos(2\pi(f_c - 3000)t) + 5\cos(2\pi(f_c + 3000)t)$

$$P_{sidebands} = \frac{1}{2} + \frac{1}{2} + \frac{25}{2} + \frac{25}{2} = 26$$

$$P_{total} = P_{carrier} + P_{sidebands} = 200 + 26 = 226$$

$$\text{ratio} = \frac{26}{226}$$

2.



spectrum of the signal

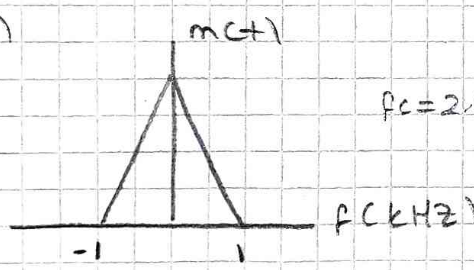
Perfect synchronization
(same carrier signals)

a) $f_c = 2.5$ kHz
b) $f_c = 0.75$ kHz

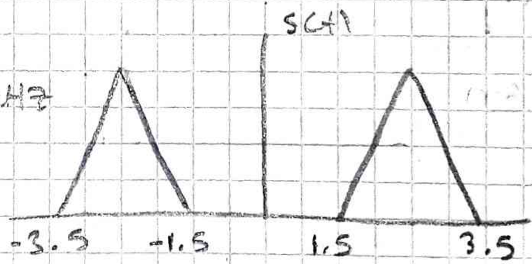
Lowest carrier frequency value for which each component of the modulated s(t) is uniquely by m(t)?

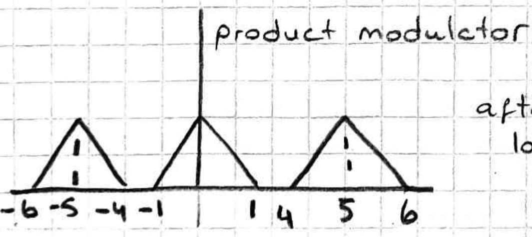
m(t) → modulator DSB-SC → coherent detector

a)

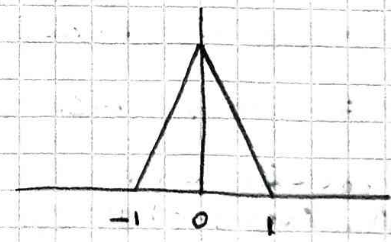


$f_c = 2.5$ kHz

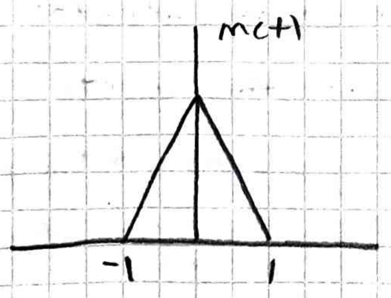


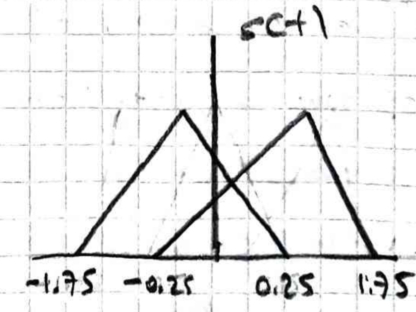


after lowpas

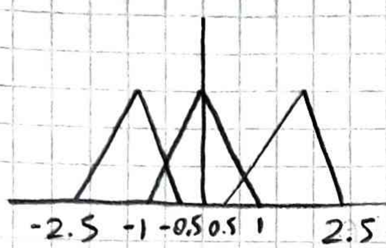


b) $f_c = 0.75$ kHz





product modulator

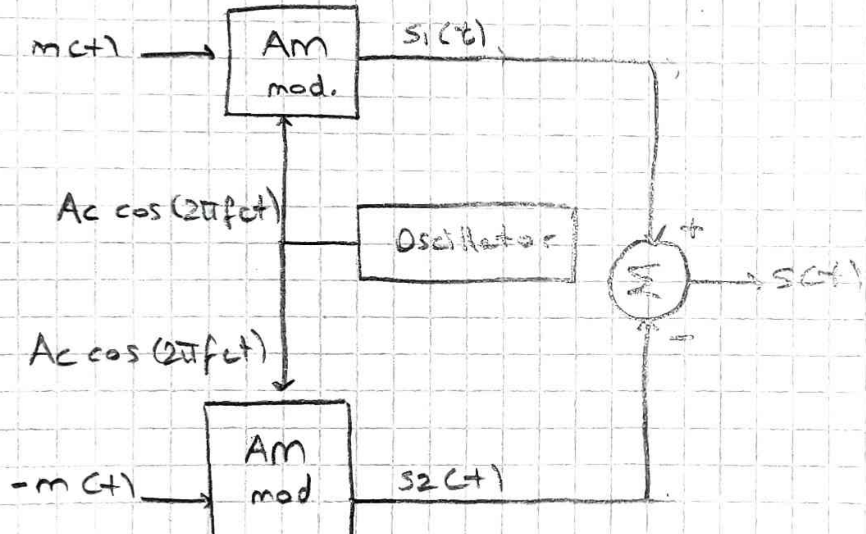


after lowpass

in order to avoid overlapping

$2f_c - W > W \qquad f_c > W$

3. Show that $s(t)$ consist of DSB-SC mod. signal



$s_1(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$

$s_2(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$

$s_2(t) - s_1(t) = 2k_a A_c m(t) \cos(2\pi f_c t)$ (DSB-SC)

4. $m(t) = A_m \cos(2\pi f_m t)$

$s(t) = 0.5 a A_m A_c \cos[2\pi(f_c + f_m)t] + 0.5 A_m A_c (1-a) [\cos 2\pi(f_c - f_m)t]$

a) Find quadrature of VSB $s(t)$

b) VSB signal plus carrier $A_c \cos(2\pi f_c t)$ to envelope detector. Determine distortion $d(t)$

c) For what value of the a distortion becomes the worst.

a) $s(t) = \frac{1}{2} a A_m A_c \cos(2\pi f_c t)\cos(2\pi f_m t) - \frac{1}{2} a A_m A_c \sin(2\pi f_c t)$
$\sin(2\pi f_m t) + \frac{1}{2}(1-a) A_m A_c \cos(2\pi f_c t)\cos(2\pi f_m t) + \frac{1}{2}(1-a) A_m A_c$
$\sin(2\pi f_c t)\sin(2\pi f_m t) = \frac{1}{2} A_m A_c \cos(2\pi f_c t)\cos(2\pi f_m t)$
$+ \frac{1}{2} A_m A_c (1-2a) \sin(2\pi f_c t) \sin(2\pi f_m t)$

$$\text{quadrature component} = -\frac{1}{2} A_c A_m (1-2a) \sin(2\pi f_m t)$$

b) adding carrier

$$s(t) = A_c\left[1 + \frac{1}{2} A_m \cos(2\pi f_m t)\right] \cos(2\pi f_c t)$$

$$+ \frac{1}{2} A_c A_m (1-2a) \sin(2\pi f_m t) \sin(2\pi f_c t)$$

Envelope

$$a(t) = A_c \sqrt{\left[1 + \frac{1}{2} A_m \cos(2\pi f_m t)\right]^2 + \left[\frac{1}{2} A_m (1-2a) \sin(2\pi f_m t)\right]^2}$$

$$= A_c\left[1 + \frac{1}{2} A_m \cos(2\pi f_m t)\right] \sqrt{1 + \left[\frac{\frac{1}{2} A_m (1-2a) \sin(2\pi f_m t)}{1 + \frac{1}{2} A_m \cos(2\pi f_m t)}\right]^2}$$

$$= A_c\left[1 + \frac{1}{2} A_m \cos(2\pi f_m t)\right] d(t)$$

$d(t) =$ köklü terim

c) $d(t)$ is greatest when $a = 0$