EE451 Comm. II

1. Review of digital repr. of analog signals
2. Baseband Trans. (binary and m-ary) and optimum receivers
3. Matched filter and correalator and optimum detection
4. Error prob. for binary transmission
5. ISI and RC spectrum
6. Equalization
7. Recitation
8. Midterm (22 Kasım)

- 30 % midterm, % 30 lab (project presentation + 5 lab), % 40 final
- Simon Haykin kitap
- Comm. Systems Proakis

9. Bandpass transmission
10. Bandpass M-ary transmission and QAM.
11. Entrophy and source coding
12. Channel capacity and channel coding
13. Recitation
14. Project presentations

1. Analog signals ⇄ analog transmission schemes (AM, FM, PM)

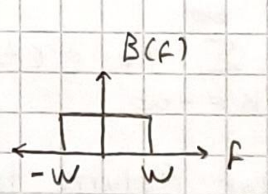
⇓

Analog to digital conversion (A\D)

⇓

digital modulation schemes

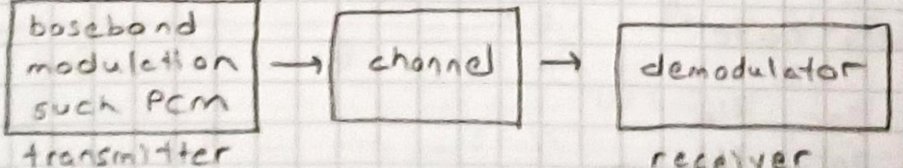
- First step is that sampling the analog source.
- $B_m = W$ (the spectrum of the positive side)
- at discrete time, $T_s = \frac{1}{F_s} = \frac{1}{2W}$ (Nyquist Rate $= 2W$)



- analog pulse amplitude mod.
- analog pulse width mod.
- analog pulse positioning mod.

- Second step is Quantization
- For each sample, you will have k bits. k bits for $M = 2^k$ levels
- Digital is less sensitive than the analog
- Multiplexing techniques

2. Base band Transmission (no carrier frequency)
- pulse (PCM)

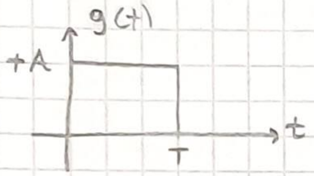


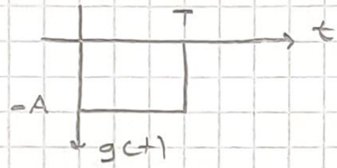
- Non-return to zero (NRZ) code.
  - Transmitted bits $b_k \in \{0, 1\}$
  - symbol period T

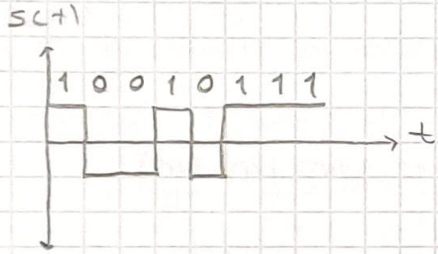
$[kT, (k+1)T]$ where $\frac{T_s}{k} = T$

after line codes
$s_k(t)$

$+A$ for bit 1
$-A$ for bit 0





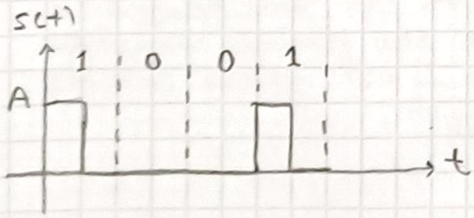


$$s(t) = \sum_{k=-\infty}^{\infty} a_k \, g(t - kT)$$

- Unipolar Return to Zero Code (On/Off keying)

$d_k = 1$ A $T/2$
$d_k = 0$ 0

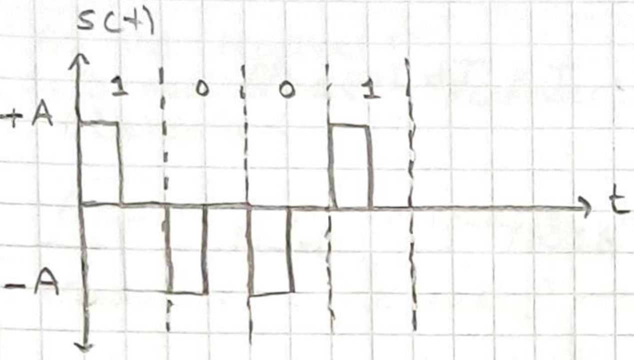
$$g(t) = \begin{cases} A & \text{if } t \in \{0, T/2\} \\ 0 & \text{if } t \in \{T/2, T\} \end{cases}$$



- Bipolar Return to Zero Code

If $d_k = 1$, A for $T/2$
, 0 for $T/2$

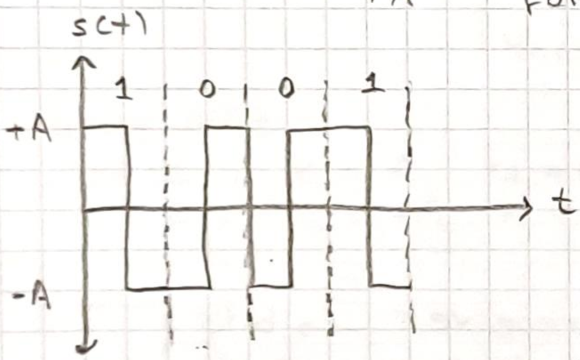
If $d_k = 0$, $-A$ for $T/2$
0 for $T/2$



Manchester Code

If $d_k = 1$ +A for T/2
               -A for T/2

If $d_k = 0$ -A for T/2
               +A for T/2



page 364, figure 7.25

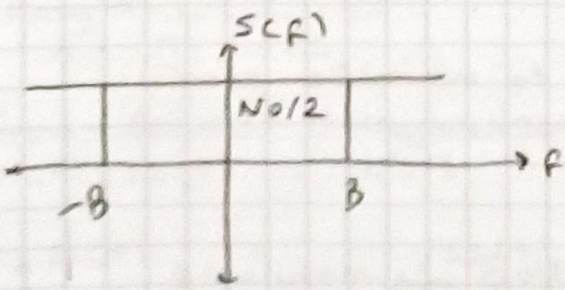
$\xrightarrow[\text{Pcm}]{s(t)}$ channel $\longrightarrow$ demodulator

↳ AWGN channel + band limited channel

$s(t) \longrightarrow \oplus \longrightarrow s(t) + n(t)$

$\uparrow$

$n(t)$



B: required bandwith

$P_N = N_0 B$

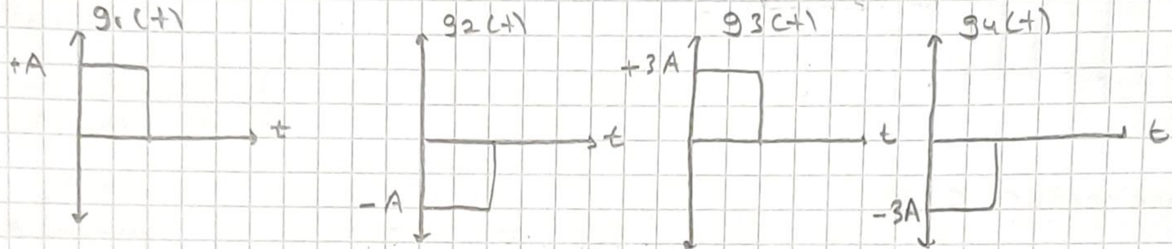
$W(0, \sigma^2)$

$\sigma^2 = N_0 B$ $N_0 = -174$ dBm/Hz

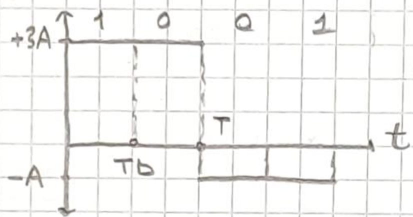
• z-transform for filter design (hardware - embedded)

m-ary NRZ code
Instead of trasmitting binary NRZ, $T = T_b \log_2 M$

$A = \{\pm 1, \pm 3\}$ if $m = 4$



00 → $g_1(t)$
01 → $g_2(t)$
10 → $g_3(t)$
11 → $g_4(t)$



Review of 1st week

Transmitter → channel → receiver → bits

data (analog) — sampling — quantization — line coding (NRZ) → $s(t)$

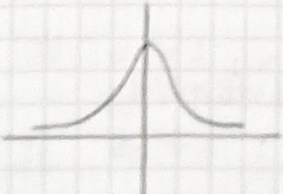
mapping
- binary (each symbol carries only one bit)
- m-ary (each symbol carries $\log_2 m$ bits.)

channel:
- bandlimited channel
- AWGN

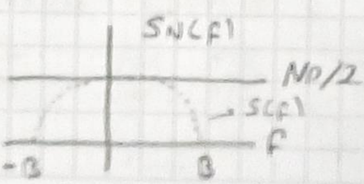
$s(t) \rightarrow \oplus \rightarrow$ demodulator (for obtaining bits back)
$\uparrow$
$n(t)$

• design optimum receiver under AWGN to minimize error.

$r(t) = s(t) + n(t)$



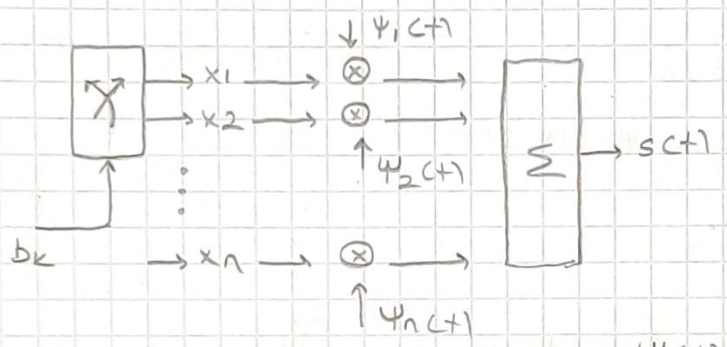
$N(0, \sigma^2)$ $\quad \sigma^2 = N_0 B \rightarrow$ modulated signal bandwidth



At the receiver:
1. Demodulation
2. Detection
} Receiver Design

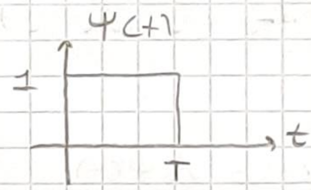
- Assume that the receiver is coherent (frequency, the symbol timing is known)

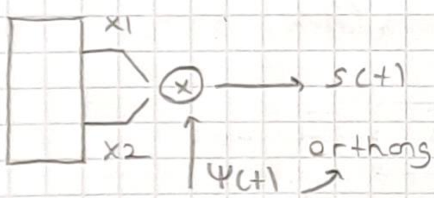
Modulator (line coding)



- binary NRZ:

bit 1, $x_1 = A$ $x_2 = 0$
bit 0, $x_2 = -A$ $x_1 = 0$





orthongonal basis

- $\int_0^T \psi_i(t) \cdot \psi_j(t)\, dt = 0 \quad i \neq j$ (birbirlerine dik bazlar)

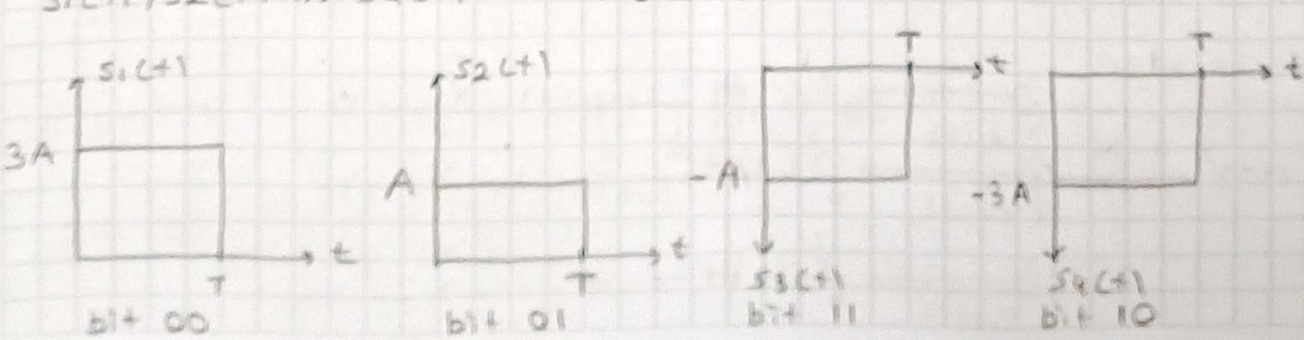
- $s(t) = \sum_{i=1}^{N} x_i \psi_i(t)$

- $E_i = \int_0^T s_i^2(t)\, dt$

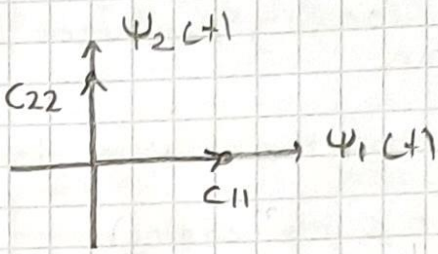
- $E_{avg} = \frac{1}{m} \sum_{m=1}^{m} E_i$

Gram-Schmidt Orthagonalization Procedure

$s_1(t), s_2(t) \ldots s_4(t)$ $\quad 0 \leq t < T$



$\psi_1(t) = \dfrac{s_1(t)}{\sqrt{E_1}}$ → the first waveform is constructed by using $s_1(t)$



$c_{21} = \int_{-\infty}^{\infty} s_2(t)\, \psi_1(t)\, dt$

$\psi_2'(t) = s_2(t) - c_{21}\, \psi_1(t)$

↓

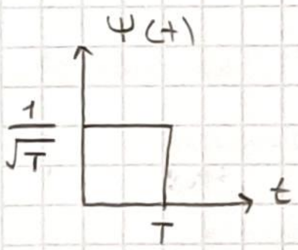
sıfır çıkmazsa c_{31} ve c_{32}

$\psi_3'(t) = s_3(t) - c_{31}\, \psi_1(t) - c_{32}\, \psi_2(t)$

- Normalization

$$\psi_2(t) = \frac{\psi_2'(t)}{\sqrt{E_{\psi_2(t)}}}$$

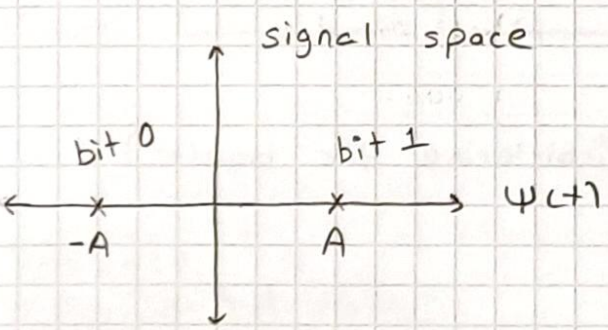
Binary NRZ



$s_1(t) = A\,\psi(t)$

$s_2(t) = -A\,\psi(t)$

$$E_s = \frac{A^2 + A^2}{2} = A^2$$



Optimum Receiver for AWGN at the receiver input:

$r(t) = s_i(t) + n(t) \quad 0 \le t < T$

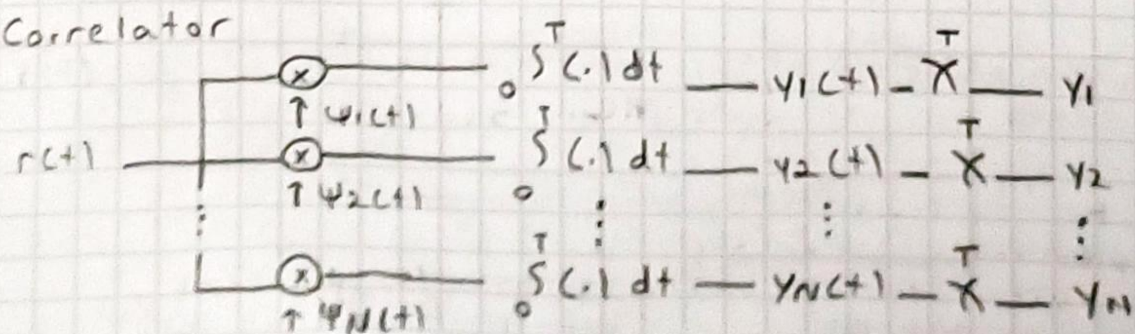
$s_i(t)$, ith transmitted symbol among M possible symbols

First phase : demodulation (optimum)

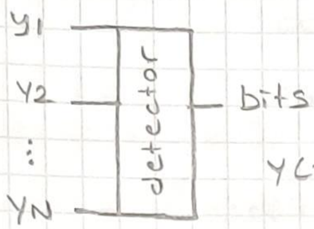
- correalator based demod.
- matched filter based demod.

=> to maximize signal to noise ratio (SNR)

Correlator



Second phase : Detection (optimum)



$$y_j = y(jT) = \int_0^T r(t)\,\psi_j(t)\,dt$$

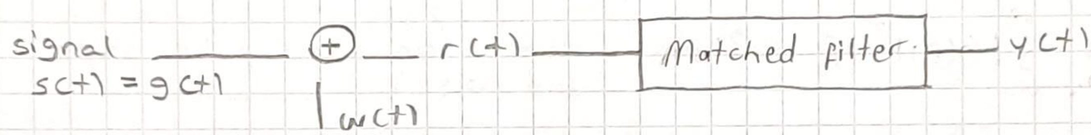
$$y(t) = \int_0^T s_i(t)\,\psi_j(t)\,dt + \int_0^T n(t)\,\psi_j(t)\,dt$$

after sempled at $t = jT$

$$y_j = s_{ij} + n_j \qquad j = 1, \dots, N$$

$$E[n_j] = 0$$

$$\sigma^2 = N_0 B$$



$$y(t) = g_o(t) + n(t)$$

signal to noise (SNR) power $\quad \eta = \dfrac{|g_o(T)^2|}{E[n^2(t)]}$ ⇒ instantenous power in the output

⇓

average noise power

- Design a filter to maximize SNR at the output.

frequency response $H(f)$ at matched filter
pulse response $G(f)$

$$g_o(t) = \int_{-\infty}^{\infty} G(f) \cdot H(f) \exp(j2\pi f t)\,df$$

impulse response of the filter $h(t)$

Filter output is sampled at $t = T$

$$|g_o(T)| = \left| \int_{-\infty}^{\infty} H(f)\,G(f) \exp(j2\pi f T)\,df \right|$$

$$S_N(f) = \frac{N_0}{2} |H(f)|^2$$

$$E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)\,df = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\,df$$

$$\eta = \frac{\left| \int_{-\infty}^{\infty} G(f)\,H(f) \exp(j2\pi f T)\,df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\,df}$$

- problem is to find $H(f)$ for a given $G(f)$ to maximize η.

Schwarz Inequality

$$\int_{-\infty}^{\infty} |\phi_1(x)|^2 dx < \infty \quad , \quad \int_{-\infty}^{\infty} |\phi_2(x)|^2 dx < \infty$$

then we can write

$$\left| \int_{-\infty}^{\infty} \phi_1(x)\,\phi_2(x)\,dx \right|^2 \leq \int_{-\infty}^{\infty} |\phi_1(x)|^2 dx \cdot \int_{-\infty}^{\infty} |\phi_2(x)|^2 dx$$

$\phi_1(x) = H(f)$
$\phi_2(x) = G(f) \cdot \exp(j2\pi f T)$

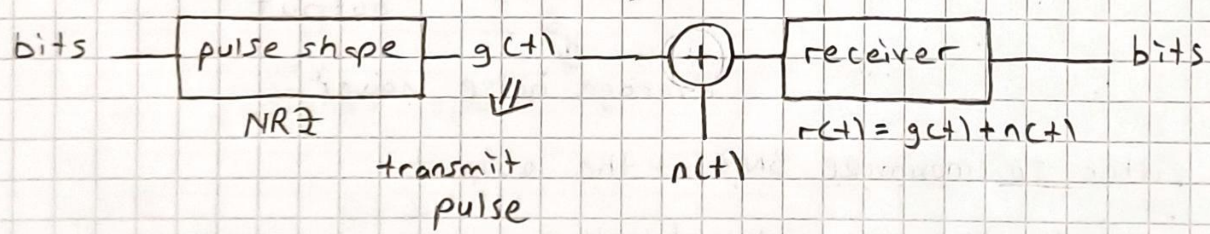
$$\left| \int_{-\infty}^{\infty} H(f)\,G(f)\,\exp(j2\pi f T)\,df \right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |G(f)|^2 df$$

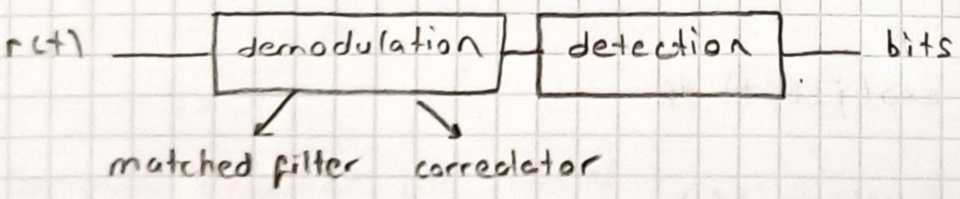
$\eta = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df = \eta_{max}$ (depends on signal pulse shape and noise)

$\phi_1(x) = k\,\phi_2^*(x)$ (schwarz inequality is valid if and only if this)

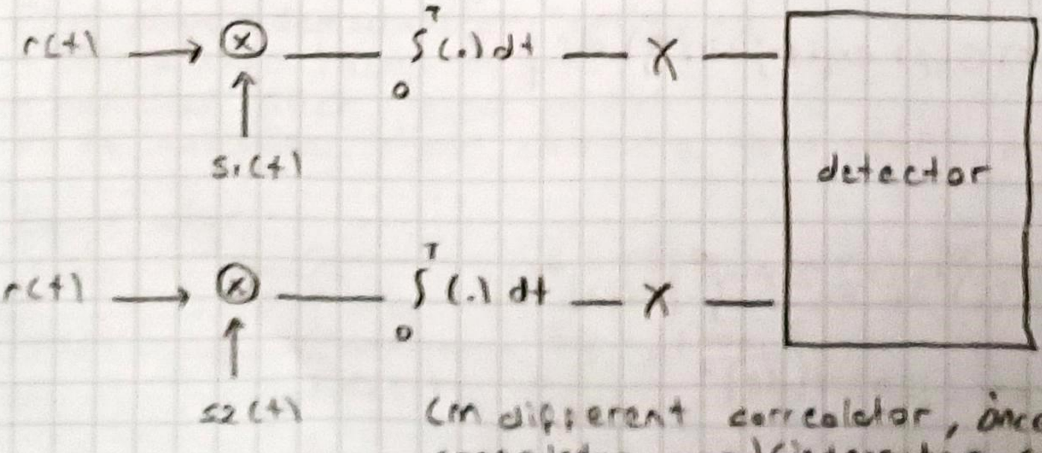
$H(f) = k\,G^*(f)\exp(-j2\pi f T)$

SUMMARY





different kind of correlator



(m different correlator, öncekinde $N=1$ correlator var |(integratör sayısı gibi bi şey

$r(t) = g(t) + n(t)$ —— [matched filter] —— γ —— [detector] —— bits

$h(t), H(f)$

If $\phi_1(x) = k\,\phi_2^*(x)$ (reference to schwarz inequality)

- for real signals, $G(f) = G^*(f)$

$$h_{opt}(t) = k\int_{-\infty}^{\infty} G^*(f)\exp(-j2\pi fT)\exp(j2\pi ft)\,df$$

$h_{opt}(t) = k\,g(T-t)$ => scale with k, time reversed and delayed by T

match to input ($g(t)$) => matched filter

$$H_{opt}(f) = k\,G^*(f)\exp(-j2\pi fT)$$

$G(f)$ —— [$H(f)$] —— $G_o(f) = G(f)\cdot H_{opt}(f) = k|G(f)|^2\exp(-j2\pi fT)$

M.F.

for 2max,

$$g_o(t) = k\underbrace{\int_{-\infty}^{\infty}|G(f)|^2\,df}$$ => energy of pulse

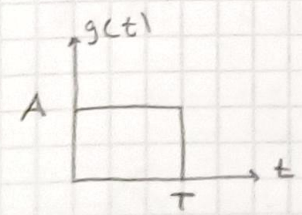
Rayleigh Energy Theorem

$$g_o(t) = k\cdot E$$

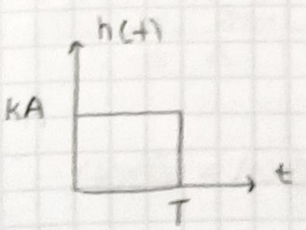
$$E\{n^2(t)\} = \frac{k^2 N_0}{2}\int_{-\infty}^{\infty}|G(f)|^2\,df = \frac{k^2 N_0 E}{2}$$

$$2max = \frac{(kE)^2}{\frac{k^2 N_0 E}{2}} = \frac{2E}{N_0}$$

Example: Matched Filter for Rectangular Pulse

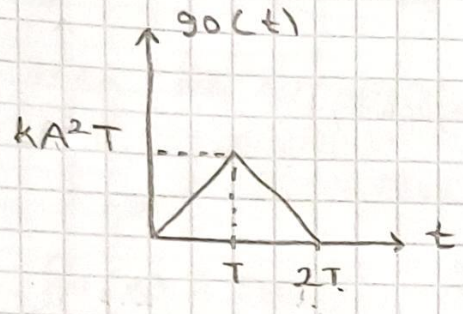


$E = A^2T$

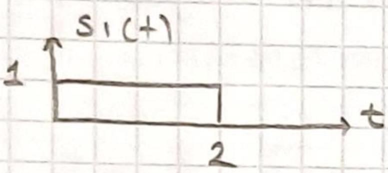


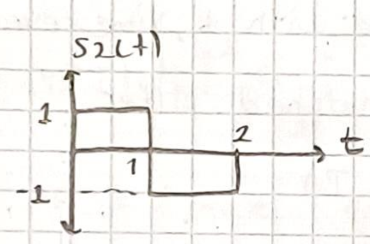
$h(t) = k\,g(T-t)$

$g(t)$ — $\boxed{h(t)}$ — $g_o(t) = g(t) * h(t)$



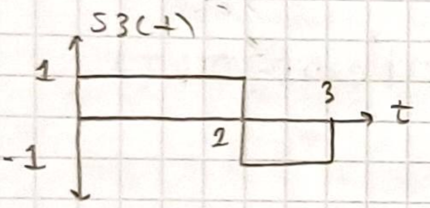
Example:

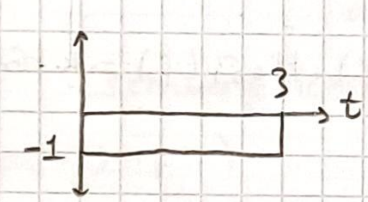




construct signal space diagram

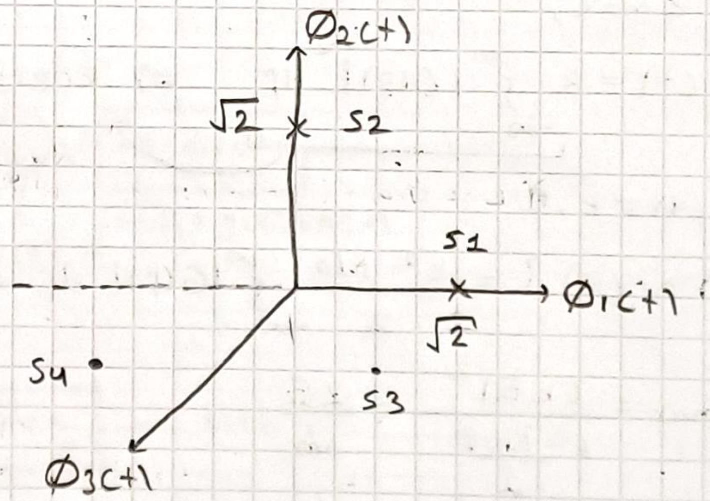
$M = 4 \quad N = ?$

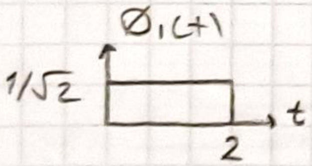




$\mathcal{E}_1 = 2 \qquad \phi_1(t) = \dfrac{s_1(t)}{\sqrt{\mathcal{E}_1}}$

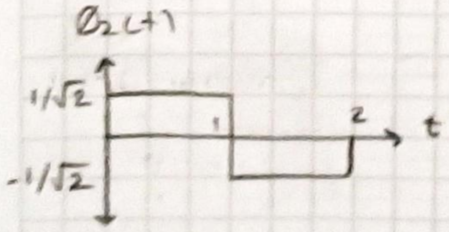
$c_{21} = \int_0^2 s_2(t)\,\phi_1(t)\,dt = 0$

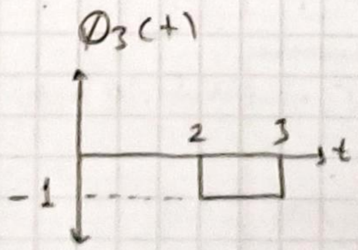




$\mathcal{E}_2 = 2$

$\phi_2(t) = \dfrac{s_2(t)}{\sqrt{\mathcal{E}_2}}$





$c_{31} = \sqrt{2}$

$c_{32} = 0$

$\gamma_3(t) = s_3(t) - c_{31}\,\phi_1(t) = \begin{cases} -1 & 2 \le t \le 3 \\ 0 & \text{else} \end{cases} \qquad \gamma_3(t) = \phi_3(t)$

$c_{41} = -\sqrt{2}$ $\quad 0 \le t \le 2$

$c_{42} = 0$

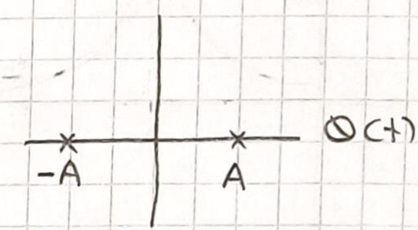
$c_{43} = 1$ $\quad 2 \le t \le 3$

4. Probability of Error due to Noise

• use polar NRZ signaling

bit 1 → +A
bit 0 → −A } (baseband) pulse amplitude mod.

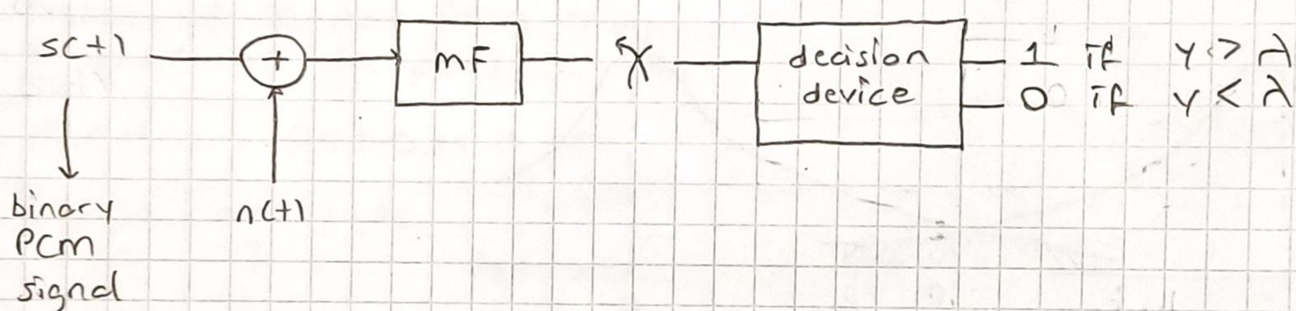
signal space



$x(t) = \begin{cases} +A + w(t) & \text{if bit 1 is sent} \\ -A + w(t) & \text{if bit 0 is sent} \end{cases}$

⇓
received signal

• for a given noisy x(t), the receiver make a decision in each signaling interval to decide whether the transmitted bit is 1 or 0.



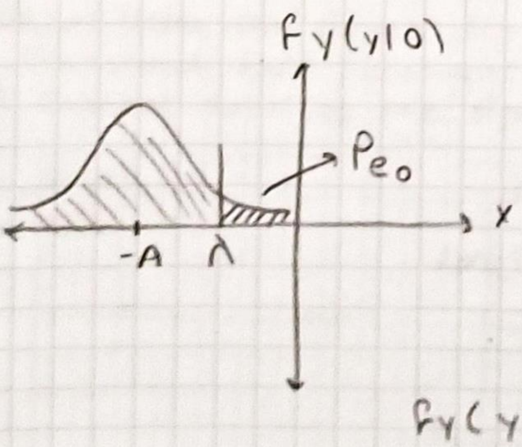
• 1101
1111 $P_b = 1/4$ (error probability)

There are two possibilities for errors:
1. Symbol 1 is chosen when 0 was transmitted
2. else

$x(t) = -A + w(t)$ $\quad$ bit 0 $\quad 0 \le t \le T_b$

$y(t) = -A + \frac{1}{T_b}\int_0^{T_b} w(t)\,dt$ ⇒ filtered noise (random variable)

$\sigma_y^2 = ?$ (y is gaussian dist. with mean −A, variance σ_y^2)



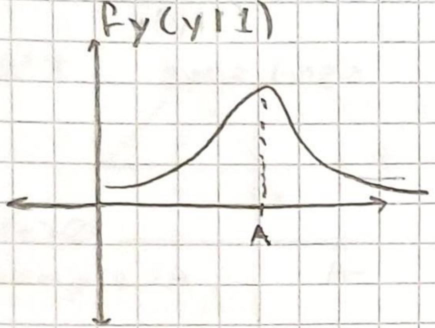
$\sigma_y^2 = E\{(y+A)^2\} = \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} \underbrace{E(w(t)w(v))}_{R_w(t,u)}\,dt\,dv$

white noise autocorrelation function
$= \frac{N_0}{2}\delta(t-u)$

$f_Y(y|0) = \frac{1}{\sqrt{\pi N_0/T_b}} \exp\left(-\frac{(y+A)^2}{N_0/T_b}\right)$

$$\sigma_y^2 = \frac{N_o}{2T_b}$$

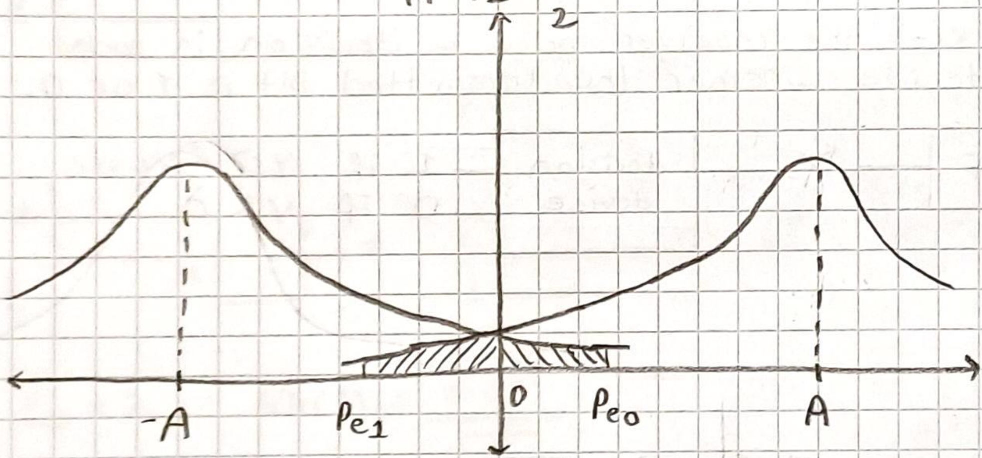
$$P_{e_0} = P(y > \lambda) = \int_{-\lambda}^{\infty} f_Y(y|0)\,dy$$ (error prob. of bit 0)



* Bununla ilgili recitation sorusu ver. Sınavda da çıkabilir

$$\omega\left(A, \frac{N_o}{2T_b}\right)$$

$$P_1 = P_2 = \frac{1}{2}$$



$$P_e = \frac{1}{2}P_{e_0} + \frac{1}{2}P_{e_1} \Rightarrow P_{e_0} = P_{e_1}$$

$$P_{e_0} = \int_{-\infty}^{\infty} f_Y(y|0)\,dy$$

$$P_1 \neq P_2 \qquad \lambda \neq 0$$

$$P_{e_0} = \frac{1}{\sqrt{\pi N_o/T_b}} \int_0^{\infty} \exp\left(\frac{-(y+A)^2}{N_o/T_b}\right) dy$$

$$z = \frac{y+A}{\sqrt{N_o/2T_b}}$$

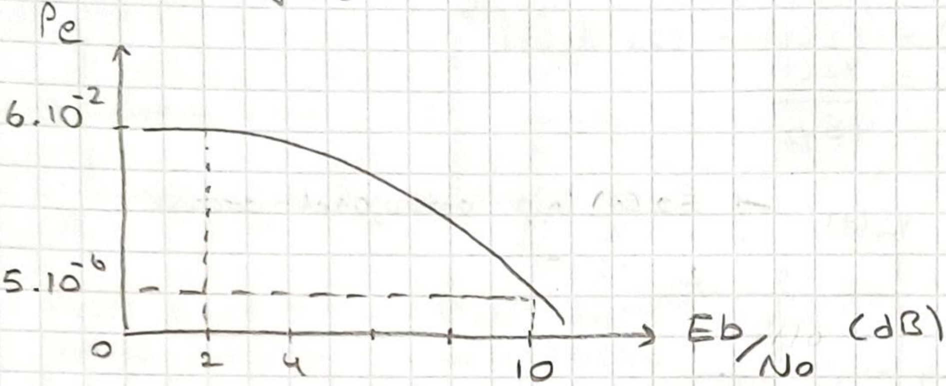
$$P_{e_0} = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2E_b/N_o}}^{\infty} \exp\left(\frac{-z^2}{2}\right) dz$$

Q-function:

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} \exp\left(\frac{-z^2}{2}\right) dz$$

$E_b = A^2 T_b$ => energy of transmitted signal

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_o}}\right) \quad \text{fig 8.7}$$

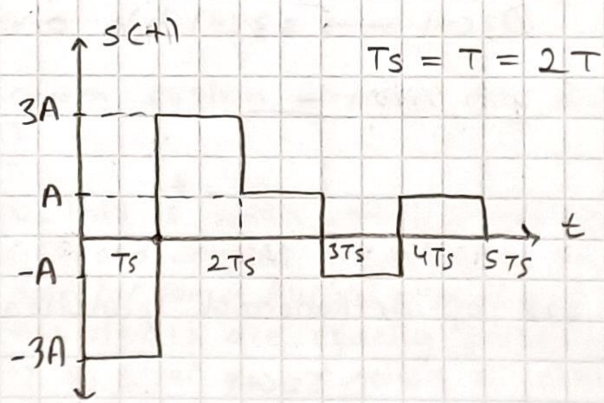


- Theoretical Result
- Monte Carlo Result
  10e6 bits

Baseband M-Ary PAM Transmission
=> produces M possible amplitude levels

$m = 4$, 0010110111

Digit	amplitude
00	-3
01	-1
11	+1
10	+3



$T_s = T = 2T_b$

$$R_s = \frac{1}{T_s}$$

$R_b = kR_s$ for a given BW

$B_T = \frac{1}{T_s}$ (resources : BW, power)