Comm. 1 | EE352

Recitation (Conventional AM)

1. $m(t) = 2\cos(4000\pi t) + 5\cos(6000\pi t)$
$c(t) = 100\cos(2\pi f_c t) \quad f_c = 50\text{ kHz}$

modulated signal = $m(t)\cdot c(t) = u(t)$

$(2\cos(\underbrace{4000\pi t}_{2\text{kHz}}) + 5\cos(\underbrace{6000\pi t}_{3\text{kHz}}))\cdot(100\cos(2\pi f_c t))$

$u(t) = (200/2)\left[\cos(2\pi(52\text{ kHz})t) + \cos(2\pi(48\text{ kHz})t)\right]$
$\quad + (500/2)\left[\cos(2\pi(53\text{ kHz})t) + \cos(2\pi(47\text{ kHz})t)\right]$

$u(f) = 50\left[\delta(f-52\text{ kHz}) + \delta(f+52\text{ kHz}) + \delta(f-48\text{kHz}) + \delta(f+48\text{ kHz})\right]$
$\quad + 125\left[\delta(f-53\text{kHz}) + \delta(f+53\text{kHz}) + \delta(f-47\text{kHz}) + \delta(f+47\text{ kHz})\right]$

* Aynısının simetrisi $-50k, -52k, -53k, -48k, -47k$ da var.

$m(t) \rightarrow$ modulating signal (message)
$c(t) \rightarrow$ carrier signal
$u(t) \rightarrow$ modulated signal

DSB-SC $\rightarrow$ double side band suppress carrier

* Örnekte carrier frekansı (50kHz) baskılanmış (suppress)

2. $u(t) = A\,m(t)\cos(2\pi f_c t)$
$x_L(t) = \cos(2\pi f_c t + \theta)$ (local carrier)

$u(t) \longrightarrow \otimes \xrightarrow{y(t)} \boxed{\text{LPF}} \longrightarrow z(t)$, with $x_L(t)$ into $\otimes$

$y(t) = u(t)\cdot x_L(t) = A\,m(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \theta)$
$\quad = \frac{A}{2} m(t)\left[\cos(4\pi f_c t + \theta) + \cos(\theta)\right]$

$z(t) = \frac{A}{2} m(t)\cos\theta$ (LPF'de f_c cancel olacak)

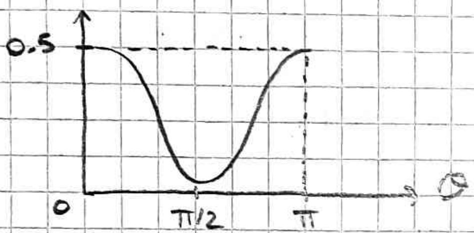
$$P_{out} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left|\frac{A}{2} m(t)\cos\theta\right|^2 dt = \frac{A^2}{4}\cos^2\theta \underbrace{\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|m(t)|^2 dt}_{P_m}$$

$$P_{out} = \frac{A^2}{4}\cos^2\theta \; P_m$$

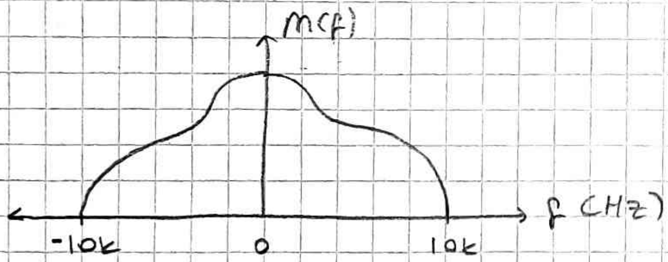
$$P_u = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|A m(t)\cos(2\pi f_c t)|^2 dt = A^2 \lim_{T\to\infty}\int_{-T/2}^{T/2} m(t)^2\cos^2(2\pi f_c t)\,dt$$

$$= \frac{A^2}{2}\left[\xrightarrow{0}\cancel{\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|m(t)|^2\cos(4\pi f_c t)\,dt} + \underbrace{\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|m(t)|^2 dt}_{P_m}\right] = \frac{A^2}{2}P_m$$

$$\frac{P_{out}}{P_u} = \frac{1}{2}\cos^2\theta$$

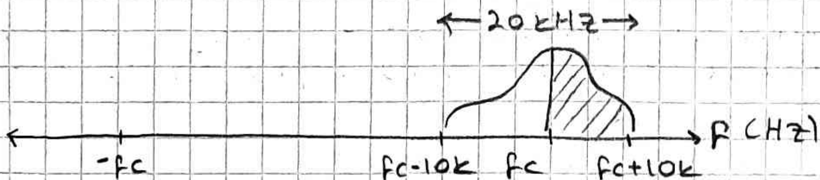


3. USSB → upper-single side band



bandwith = w = 10 kHz

i) if USSB used = w = 10 kHz



ii) if DSB is used = w = 20 kHz

iii) Conventional AM a = 0.8 => 20 kHz

4. DSB modulated

$$u(t) = m(t)\cos(2\pi f_c t + \phi) = m(t)[\cos(2\pi f_c t)\cdot\cos\phi - \sin(2\pi f_c t)\sin\phi]$$

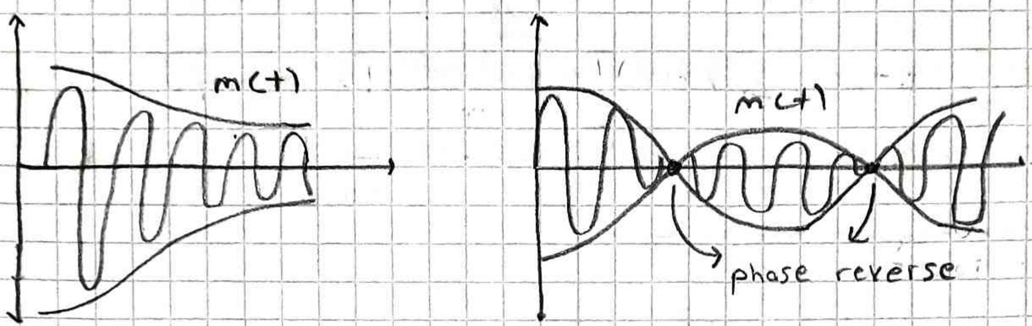
inphase component $u_c(t) = m(t)\cos\phi$

quadrature $u_s(t) = m(t)\sin\phi$

$u(t) = u_c(t)\cos(2\pi f_c t) + u_s(t)\sin(2\pi f_c t)$

Envelope

$$V_u(t) = \sqrt{u_c^2(t) + u_s^2(t)} = \sqrt{m^2(t)\,(\underbrace{\cos^2\phi + \sin^2\phi}_{1})} = |m(t)|$$



i) mesaj her zaman + ise
(envelope detector uygulanabilir)

ii) her zaman + değil

* Fourier expressions of sine, cos, triangular wave

power of s(t) ⟶ carrier power
⟶ upper side freq. response
⟶ lower side freq. response

Sınavda DSB veya SSB çıkabilir (conventional kesin)

SSB sinyali Hilbert transformation ile veya DSB + filtre ile elde edilir.

Bandwith efficient – SSB
power efficient – DSB

Recitation 2

1. For a particular case of AM using sinusoidal modulating wave, the percentage modulation is %20. Calculate the average power in (a) the carrier and (b) each side frequency, expressing your results as percentages of total transmitted power.

Conventional Am signal,

$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$

↳ modulated signal   amplitude sensitivity of modulator

where $m(t) = A_m\cos(2\pi f_m t)$

$s(t) = A_c\cos(2\pi f_c t) + k_a A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t)$

$$\cos a \cos b = \frac{\cos(a+b) + \cos(a-b)}{2}$$

$$s(t) = A_c\cos(2\pi f_c t) + \frac{k_a A_m A_c}{2}\cos(2\pi(f_c+f_m)t) + \frac{k_a A_m A_c}{2}\cos(2\pi(f_c-f_m)t)$$

$\mu = k_a A_m$ (modulation factor)
($\mu \times 100 \Rightarrow$ percentage)

percentage modulation $\Rightarrow$ 20 percent $\Rightarrow$ 0.2

$$\mathcal{F}\{\cos(2\pi A t)\} \to \frac{1}{2}\left[\delta(f-A)+\delta(f+A)\right]$$

$$s(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] + \underbrace{\frac{\overset{\mu}{k_a A_m} A_c}{4}\left[\delta(f-f_m-f_c)+\delta(f+f_m+f_c)\right]}_{\text{upper}}$$
$$\underbrace{+\frac{\mu A_c}{4}\left[\delta(f-f_c+f_m)+\delta(f+f_c-f_m)\right]}_{\text{lower}}$$

carrier power $\Rightarrow \dfrac{A_c^2}{4} + \dfrac{A_c^2}{4} = \dfrac{A_c^2}{2}$

upper side freq power $\Rightarrow \dfrac{\mu^2 A_c^2}{16} + \dfrac{\mu^2 A_c^2}{16} = \dfrac{\mu^2 A_c^2}{8}$

lower side freq power $\Rightarrow \dfrac{\mu^2 A_c^2}{16} + \dfrac{\mu^2 A_c^2}{16} = \dfrac{\mu^2 A_c^2}{18}$

a) Burası yok

b) each side frequency power as percentage of total tx power

$$\frac{\frac{\mu^2 A_c^2}{8}}{\frac{1}{2}A_c^2 + \frac{1}{4}\mu^2 A_c^2} = \frac{\mu^2}{2\mu^2+4} = \frac{0.04}{0.08+4} \cong 0.01 = \%1$$

kalanı yok

2. A message signal $m(t) = \cos(2000\pi t) + 2\cos(4000\pi t)$ modulates the carrier $c(t) = 100\cos(2\pi f_c t)$ where $f_c = 1$ MHz to produce the DSB signal $m(t)c(t)$

a) Determine the expression for the upper sideband (USB) signal.

b) Determine and sketch the spectrum of the USB signal

The modulated signal: $s(t) = c(t)\,m(t)$

$$s(t) = 100\left[\cos(2\pi 1000 t) + 2\cos(2\pi 2000 t)\right]\cos(2\pi f_c t)$$

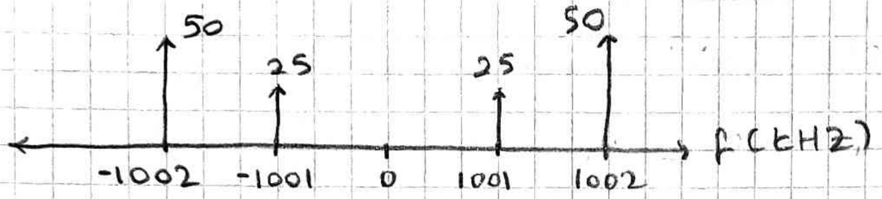
$$s(t) = 50\left[\cos(2\pi(f_c+1000)t) + \cos(2\pi(f_c-1000)t\right]$$
$$+ 100\left[\cos(2\pi(f_c+2000)t) + \cos(2\pi(f_c+1000)t\right]$$

a) Thus, the upper sideband signal is

$$s_u(t) = 50 \cos(2\pi(f_c + 1000)t) + 100 \cos(2\pi(f_c + 2000)t)$$

b) Taking the fourier transform of both sides

$$S_u(f) = 25[\delta(f - f_c - 1000) + \delta(f + f_c + 1000)]$$
$$+ 50[\delta(f - f_c - 2000) + \delta(f + f_c + 2000)]$$



$f_c = 1\,mHz = 1000\,kHz$

* lower side band freq => $\left.\begin{matrix} -f_c + f_m \\ f_c - f_m \end{matrix}\right\} |f| < f_c$

* upper side band freq => $\left.\begin{matrix} f_c + f_m \\ -f_c - f_m \end{matrix}\right\} |f| > f_c$

3. The output signal from an AM modulator is

$$u(t) = 5 \cos(1800\pi t) + 20 \cos(2000\pi t) + 5 \cos(2200\pi t)$$

a) Determine the modulating signal $m(t)$ and the carrier $c(t)$

b) Determine the modulation index

c) Determine the ratio of the power in the sidebands to the power in the carrier.

AM modulated signal

$$u(t) = 5 \cos(1800\pi t) + 20 \cos(2000\pi t) + 5 \cos(2200\pi t)$$

a) modulating signal $m(t)$, carrier $c(t)$

$$u(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

$$\cos a + \cos b = 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

$$u(t) = 10[\cos(2000\pi t) \cos(200\pi t)] + 20 \cos(2000\pi t)$$

$$u(t) = 20\left[\frac{1}{2} \cos(2000\pi t) \cos(200\pi t)\right] + 20 \cos(2000\pi t)$$

$$u(t) = 20\cos(2000\pi t)\left[1 + \frac{1}{2}\cos(200\pi t)\right]$$

$$u(t) = 20\left[1 + \frac{1}{2}\cos(200\pi t)\right]\cos(2000\pi t)$$

$\downarrow$ $\rightarrow c(t) = 20\cos(2000\pi t)$

$m(t) = \cos(200\pi t)$ $f_c = 1000\ Hz$

b) modulation index

since $-1 \leq \cos(200\pi t) \leq 1$ $\mu = \frac{1}{2}$

c) $u(f) = 10[\delta(f-1000) + \delta(f+1000)]$

$+ \frac{5}{2}[\delta(f-900) + \delta(f+900)]$ lower

$+ \frac{5}{2}[\delta(f-1100) + \delta(f+1100)]$ upper

$P_{carrier} = 100 + 100 = 200$
$P_{sidebands} = 4 \cdot \left(\frac{25}{4}\right) = 25$
$\frac{P_{sidebands}}{P_{carrier}} = \frac{25}{200} = 1/8$

4. Normalized signal $m_n(t)$ has a bandwith of 10000 Hz and its power content is 0.5 W. The carrier $A\cos(2\pi f_0 t)$ has a power content of 200 W.

a) If $m_n(t)$ modulates the carrier using SSB amplitude modulation. what will be the bandwith and the power content of the modulated signal?
b) If the modulation scheme is DSB-SC, what will be the answer to part I?
c) If the modulation scheme is AM with a modulation index of 0.6, what will be the answer to part I?

$m_n(t)$ $BW = 10\ kHz$ $P_m = 0.5\ W$
$c(t) = A\cos(2\pi f_0 t)$, $P_c = 200\ W$

a) SSB modulated signal BW, power

$S_{DSB}(t) = m(t)c(t)$ where $m(t) = A_m\cos(2\pi f_m t)$

$S_{DSB}(t) = A A_m \cos(2\pi f_m t)\cos(2\pi f_0 t)$

$$= \frac{1}{2}AA_m\cos[2\pi(f_0+f_m)t] + \frac{1}{2}AA_m\cos[2\pi(f_0-f_m)t]$$

$$S_{USSB}(t) = \frac{1}{2}AA_m\cos[2\pi(f_0+f_m)t]$$

$$\cos(a \mp b) = \cos a\cos b \mp \sin a \sin b$$

$$S_{USSB}(t) = \frac{1}{2}AA_m\cos(2\pi f_0 t)\cos(2\pi f_m t) - \frac{1}{2}AA_m\sin(2\pi f_0 t)\sin(2\pi f_m t)$$

$$S_{LSSB}(t) = \frac{1}{2} A A_m \cos(2\pi f_o t)\cos(2\pi f_m t)$$

$$+\frac{1}{2} A A_m \sin(2\pi f_o t)\sin(2\pi f_m t)$$

$$s(t) = A m_n(t)\cos(2\pi f_o t) \mp A \hat{m}_n(t)\sin(2\pi f_o t)$$

a) $s(t) = A m_n(t)\cos(2\pi f_o t) \mp A \hat{m}_n(t)\sin(2\pi f_o t)$

$$P_m = \hat{P_m} = \frac{1}{2}$$

$$P_c = \frac{A^2}{2} = 200 \qquad A^2 = 400$$

$$P_{SSB} = \frac{A^2 P_m}{2} + \frac{A^2 \hat{P_m}}{2} = A^2 P_m = 400 \frac{1}{2} = 200\,W$$

$$W_{SSB} = W_m = 10\,kHz$$

b) $S_{DSB}(t) = A m_n(t)\cos(2\pi f_o t)$

$$P_{DSB} = \frac{A^2 P_m}{2} = \frac{400\,(0.5)}{2} = 100$$

$$W_{DSB} = 2W_m = 20000\,Hz$$

c) $\mu = k_a A_m \qquad k_a = 0.6$

$$S_{C-AM}(t) = A[1 + k_a m_n(t)]\cos(2\pi f_o t)$$

$$P_m = \frac{A^2}{2} + \frac{A^2 (k_a)^2 P_m}{2} = 200 + \frac{400\,(0.36)\,0.5}{2}$$

$$P_{AM} = 236$$

$$W_{AM} = 2W_m = 20\,kHz$$