$$\text{quadrature component} = -\frac{1}{2} A_c A_m (1-2a) \sin(2\pi f_m t)$$

b) adding carrier

$$s(t) = A_c \left[1 + \frac{1}{2} A_m \cos(2\pi f_m t)\right] \cos(2\pi f_c t)$$

$$+ \frac{1}{2} A_c A_m (1-2a) \sin(2\pi f_m t) \sin(2\pi f_c t)$$

Envelope

$$a(t) = A_c \sqrt{\left[1+\frac{1}{2} A_m \cos(2\pi f_m t)\right]^2 + \left[\frac{1}{2} A_m (1-2a) \sin(2\pi f_m t)\right]^2}$$

$$= A_c \left[1+\frac{1}{2} A_m \cos(2\pi f_m t)\right] \sqrt{1 + \left[\frac{\frac{1}{2} A_m (1-2a) \sin(2\pi f_m t)}{1+\frac{1}{2} A_m \cos(2\pi f_m t)}\right]^2}$$

$$= A_c \left[1+\frac{1}{2} A_m \cos(2\pi f_m t)\right] d(t)$$

$d(t) =$ köklü terim

c) $d(t)$ is greatest when $a = 0$

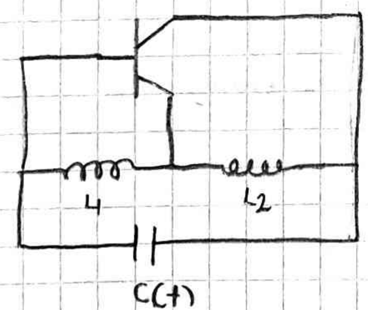
Lecture

* Midterm konuları AM ve FM

FM signals generation

- In direct generating from message signal, the freq response of signal

FM Modulator



hardware oscillator

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C(t)}}$$

$$c(t) = C_0(t) \Delta C \cos(2\pi f_m(t))$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)[C_0(t) \Delta C \cos(2\pi f_m(t))]}}$$

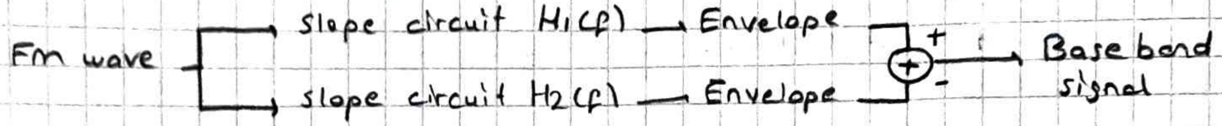
$$f_0 = \frac{1}{2\pi\sqrt{L C_0}}$$

$$f_i = f_0 \left[1 + \frac{\Delta C}{C_0} \cos 2\pi f_0 t\right]^{-1/2}$$

$$\sqrt{1+x} \approx \sqrt{1+\frac{x}{2}}$$

$$f_i = f_0 \left[1 - \frac{\Delta C}{C_0} \cos 2\pi f_0 t\right]$$

Frequency Discriminator for the FM Demodulator



Carson Rule for B_T of slope circuits

$$H_1(f) = \begin{cases} j2\pi a\left(f - f_c + \frac{B_T}{2}\right), & f_c - \frac{B_T}{2} \le f \le f_c + \frac{B_T}{2} \\ j2\pi a\left(f + f_c - \frac{B_T}{2}\right), & -f_c - \frac{B_T}{2} \le f \le -f_c + \frac{B_T}{2} \\ 0, & \text{else} \end{cases}$$

$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$ FM signal

$\tilde{s}(t) = A_c \exp\left(j2\pi k_f \int_0^t m(\tau)\,d\tau\right)$ complex envelope of $s(t)$

$\tilde{H}_1(f)$ is the low-pass representation of the signal $s(t)$

$$\tilde{H}_1(f) = \begin{cases} j4\pi a\left(f + \frac{B_T}{2}\right), & -\frac{B_T}{2} \le f \le \frac{B_T}{2} \\ 0 & \text{else} \end{cases}$$

$\tilde{H}_1(f - f_c) = 2H_1(f) \quad f > 0$

$s_{fm}(t) \longrightarrow$ frequency discriminator $\longrightarrow m(t)$

$\tilde{S}_1(f) = \frac{1}{2}\tilde{H}_1(f)\tilde{S}(f)$ (as a result of upper branch) (low-pass rep.)

$S_1(f) = H_1(f)\, S(f)$

$$\tilde{S}_1(f) = \begin{cases} j2\pi a\left(f + \frac{B_T}{2}\right)\tilde{S}(f), & -\frac{B_T}{2} \le f \le \frac{B_T}{2} \\ 0, & \text{else} \end{cases}$$

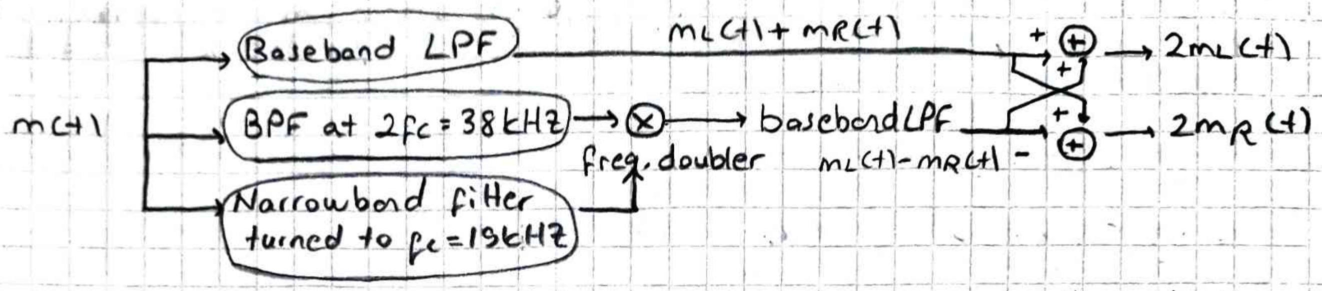
$\tilde{s}_1(t) = a\left[\frac{d\tilde{s}(t)}{dt} + j\pi B_T \tilde{s}(t)\right]$ (4.52 - 4.53 eqns)

- Both amplitude and frequency of the carrier wave is changing with the envelope detector

$$\left|\frac{2k_f m(t)}{B_T}\right| < 1$$

FM Stereo Multiplexing - Transmitter

$$m(t) = (m_L(t) + m_R(t)) + (m_L(t) - m_R(t)) \cdot \underbrace{(\cos 4\pi f_c t)}_{\substack{\text{DSB-SC} \\ \text{product modulation}}} + \underbrace{K \cos 2\pi f_c t}_{\text{pilot signal}}$$



We have two different signal is trasmitted at the same time, that's why it is called multiplexing

Phase Noise

- It is introduced by oscillator.

$$s(t).c(t) = A_c \cos(2\pi f_c t + \phi(t)).\cos(2\pi(f_c - f_B)t + \phi_n(t))$$

$c(t)$ ↳ oscillator; $(f_c - f_B)$: freq. shift; $\phi_n(t)$: phase shift

$$\sim A_c \cos(2\pi f_b + \phi(t) - \phi_n(t))$$

Recitation 4

1. $u(t) = 100\cos(2\pi f_c t + 4\sin 2000\pi t)$ angle mod. signal

$f_c = 10\ MHz$

1. Determine average transmitted power
2. Determine peak-phase deviation
3. Determine peak-frequency deviation
4. Is this an FM or a PM signal?

1. Since an angle mod. signal is essentially a sinusoidal signal with constant amplitude we have

$$P = \frac{A_c^2}{2} \Rightarrow \frac{100^2}{2} = 5000$$

The same result is obtained if we use the expansion

$$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos(2\pi(f_c + nf_m)t)$$

along with the identity

$$J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta) = 1$$

2. The max phase deviction

$$\Delta\phi_{max} = \max|4\sin(2000\pi t)| = 4$$

3. The instantenous frequency

$$f_i = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt} = f_c + \frac{4}{2\pi}\cos(2000\pi t)2000\pi = f_c + 4000\cos(2000\pi t)$$

$$\Delta f_{max} = \max|f_i - f_c| = 4000$$

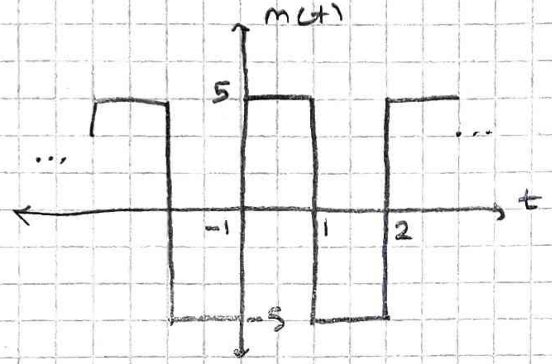
4. The angle modulated signal can be interpreted both as a PM and FM signal. It is a PM signal with phase deviation constant $k_p = 4$ and message signal $m(t) = \sin(2000\pi t)$ and it is an FM signal with frequency deviation constant $k_f = 4000$ and message signal $m(t) = \cos(2000\pi t)$

2. An FM signal given as

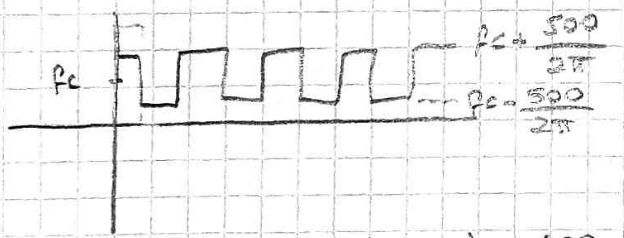
$$u(t) = 100 \cos\left[2\pi f_c t + 100 \int_{-\infty}^{t} m(\tau)\, d\tau\right]$$

1. Sketch $f_i(t)$
2. Determine Δf_{max}

1.

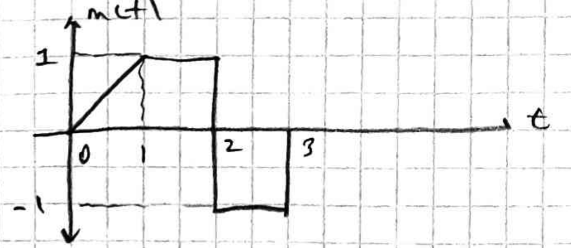


1. $f_i(t) = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt} = f_c + \frac{1}{2\pi} 100\, m(t)$



2. $\Delta f_{max} = k_f \max|m(t)| = \frac{100}{2\pi} 5 = \frac{250}{\pi}$

3. signal is used once freq mod carrier and once to phase mod carrier



1. Find relation between k_f and k_p such that the max phase of the mod signal are both equal.

2. $k_p = k_f = 1$ $\quad f_{i\,max}$?

1. $\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau = 2\pi k_f \int_{0}^{t} m(\tau)\,d\tau$

$$= \begin{cases} 2\pi k_f \int_0^t \tau\,d\tau = \pi k_f t^2 & , \quad 0 \le t \le 1 \\ \pi k_f + 2\pi k_f \int_1^t d\tau = \pi k_f + 2\pi k_f (t-1) & , \quad 1 \le t \le 2 \\ \pi k_f + 2\pi k_f - 2\pi k_f \int_2^t d\tau = 3\pi k_f - 2\pi k_f (t-2) & , \quad 2 \le t \le 3 \\ \pi k_f & , \quad 3 \le t \end{cases}$$

2. $k_p = 1$ $\quad$ f_i for PM mod signal,

$$f_i(t) = f_c + \frac{1}{2\pi} k_p \frac{d\,\phi(t)}{dt}$$

for $m(t)$ max value of $\frac{d}{dt}\phi(t)$ is obtained for $t \in [0, 1)$

so, $\max(f_i(t)) = f_c + \frac{1}{2\pi}$

f_i for FM mod signal

$f_i(t) = f_c + k_f m(t)$, $\max(f_i(t)) = f_c + k_f = f_c + 1$

4. $u(t) = 100\cos(2\pi f_c t + 4\sin(2\pi f_m t))$, and $f_c = 10\,mHz$
$f_m = 1000\ Hz$

1. Assumming this is FM, determine modulation index and BW_T
2. Repeat part 1 if fm is doubled.
3. Assumming this is PM, determine modulation index and BW_T
4. Repeat part 3 if fm is doubled.

1. $u(t) = 100\cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{\infty} \alpha \cos(2\pi f_m \tau)\,d\tau$

$= 100\cos(2\pi f_c t + \frac{k_f \alpha}{f_m} \sin(2\pi f_m t)$

mod index $\beta_f = \frac{k_f \alpha}{f_m} = \frac{\Delta f}{f_m} = 4$

$\beta_{fm} = 2(\beta_f + 1) f_m = 10\ kHz$

2. If we double the frequency then,

$u(t) = 100\cos(2\pi f_c t + 4\sin(4\pi f_m t))$

$\beta_f = 2$ $\quad$ $\beta_{fm} = 2(\beta_f + 1) 2 f_m = 12\ kHz$

3. If PM

$\beta_p = \Delta\phi_{max} = max|4\sin(2\pi f_m t)| = 4$

$B_{pm} = 2(\beta_p + 1) f_m = 10kHz$

4. If f_m is doubled $\beta_p = \Delta\phi_{max}$ still

$B_{pm} = 2(\beta_p + 1) 2f_m = 20\ kHz$