Recitation #1 (Noise)

Q.1 The received signal $r(t) = s(t) + n(t)$ in a communication system is passed through an ideal LPF with bandwith W and unity gain. The signal component $s(t)$ has a psd

$$S_s(f) = \frac{P_0}{1+(f/B)^2}$$

, where B is the 3-dB bandwith. The noise component $n(t)$ has a psd $N_0/2$ for all frequencies. Determine and plot the SNR as a function of the ratio W/B. What is filter bandwith W that yields a maximum SNR?

S.1 The spectrum of the signal at the output of the LPF is

$$S_{s,o}(f) = S_s(f) \left| \Pi\left(\frac{f}{2W}\right)\right|^2$$

$$P_{s,o} = \int_{-\infty}^{\infty} S_{s,o}(f)\, df = \int_{-W}^{W} \frac{P_o}{1+(f/B)^2}\, df = P_o B \arctan\left(\frac{f}{B}\right)\Big|_{-W}^{W}$$

$$= 2 P_o B \arctan\left(\frac{W}{B}\right)$$

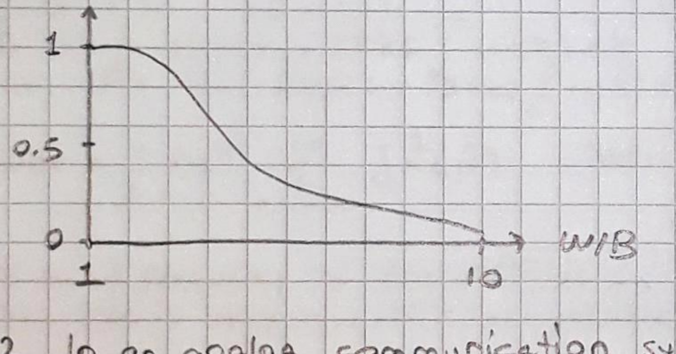
Similarly noise power at the output of LPF is,

$$P_{N,o} = \int_{-W}^{W} \frac{N_o}{2}\, df = N_o W$$

Thus the SNR,

$$SNR = \frac{2 P_o B \arctan\left(\frac{W}{B}\right)}{N_o W} = \frac{2P_o}{N_o} \frac{\arctan\left(\frac{W}{B}\right)}{\frac{W}{B}}$$

In the next figure we plot SNR as function of $\frac{W}{B}$ and for $\frac{2P_o}{N_o} = 1$



Q.2 In an analog communication system, demodulation gain is defined as the ratio of the SNR at the output of the demodulator to the SNR at the output of the noise-limiting filter at the receiver front end. Find expressions for the demodulation gain in each of the following cases:

1. DSB
2. SSB
3. Conventional AM with a modulation index of a. What is the largest possible demodulation gain in this case.
4. FM with modulation index β_f
5. PM with modulation index β_p

S.2 In the case of DSB the output of the receiver noise-limiting filter is,

$$r(t) = u(t) + n(t) = A_c m(t) \cos(2\pi f_c t + \varphi_c(t)) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$$

The power of the received signal is,

$P_s = \frac{A_c^2}{2} P_m$, whereas the power of the noise

$$P_{N,0} = \frac{1}{2} P_{N,c} + \frac{1}{2} P_{N,s} = P_N$$

the SNR at the output of the noise limity filter is

$$\left(\frac{S}{N}\right)_{0,\,lim} = \frac{A_c^2}{2} \frac{P_m}{P_N}$$

Assumming coherent demodulator the output of the demodulator is

$$y(t) = \frac{1}{2}\left[A_c\, m(t) + n_c\right]$$

The output signal power is

$P_0 = \frac{1}{4} A_c^2 P_m$ whereas the output noise power

$$P_{N,0} = \frac{1}{4} P_{N,c} = \frac{1}{4} P_N$$

$$\left(\frac{S}{N}\right)_{0,\,dem} = \frac{A_c^2 P_m}{P_N}$$

and the demodulator gain is given by

$$\text{dem. gain} = \frac{\left(\frac{S}{N}\right)_{0,\,dem}}{\left(\frac{S}{N}\right)_{0,\,lim}} = 2$$

In the case of SSB, the output of the receiver noise limiting filter is

$$r(t) = A_c\, m(t) \cos(2\pi f_c t) \pm A_c\, \tilde{m}(t) \sin(2\pi f_c t) + n(t)$$

The received signal power,

$P_s = A_c^2 P_m$, whereas the received noise power is $P_{N,0} = P_N$ at the output of the demodulator

$$y(t) = \frac{A_c}{2} m(t) + \frac{1}{2} n_c(t)$$

$P_0 = \frac{1}{4} A_c^2 P_m$ and $P_{N,0} = \frac{1}{4} P_{N,c} = \frac{1}{4} P_N$

Therefore

$$\text{dem.gain} = \frac{\left(\frac{S}{N}\right)_{o,\,dem}}{\left(\frac{S}{N}\right)_{o,\,lim}} = \frac{\dfrac{A_c^2 P_m}{P_N}}{\dfrac{A_c^2 P_m}{P_N}} = 1$$

In the case of conventional AM modulation the output of the receiver noise limiting filter is

$$r(t) = \left[A_c\left(1 + \alpha m_n(t)\right) + n_c(t)\right]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

The total predetection power in the signal is

$$P_s = \frac{A_c^2}{2}\left(1 + \alpha^2 P_{mN}\right)$$

The highest gain is achieved for $\alpha = 1$ that is %100 modulation

In this case the dem.gain is given by,

$$\text{dem.gain} = \frac{\frac{A_c^2}{2}\alpha^2 P_{mN} \,/\, P_N/4}{\frac{A_c^2}{2}\left(1+\alpha^2 P_{mN}\right) \,/\, P_N/4} = \frac{\left(\frac{S}{N}\right)_{o,\,dem}}{\left(\frac{S}{N}\right)_{o,\,lim}} = \frac{\alpha^2 P_{mN}}{1+\alpha^2 P_{mN}}$$

For the FM system the output of the receiver bandwith B_c is,

$$r(t) = A_c\cos\left(2\pi f_c t + \phi(t)\right) + n(t) = A_c\cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right) + n(t)$$

The total signal power is,

$$P_{s,\,lim} = \frac{A_c^2}{2}$$ whereas the predetection noise is,

bandwidth için carsons rule kullanld o yüzden böle

$$P_{N,\,lim} = \frac{N_o}{2}\,2B_c = N_o B_c = N_o\,2\left(\beta_f + 1\right)W$$

$$\left(\frac{S}{N}\right)_{o,\,lim} = \frac{A_c^2}{2N_o\,2\left(\beta_f+1\right)W}$$

The output post detection SNR

$$\left(\frac{S}{N}\right)_{o,\,dem} = \frac{3k_f^2 A_c^2 P_m}{2N_o W^3}$$

$$\text{dem.gain} = \frac{\left(\frac{S}{N}\right)_{o,\,dem}}{\left(\frac{S}{N}\right)_{o,\,lim}} = 6\beta_f^2\left(\beta_f+1\right)P_{mN}$$

Similarly for Pm case

$$\left(\frac{S}{N}\right)_{o,lim} = \frac{A_c^2}{2N_o 2(B_p+1)W}$$

$$\left(\frac{S}{N}\right)_{o,dem} = \frac{k_p^2 A_c^2 P_m}{2N_o W}$$

$$\text{dem.gain} = k_p^2 P_m 2(B_p+1)$$

Q.3 In a broadcasting communication system the transmitter power is 40kW, the channel attenuation is 80 dB, and the noise psd is 10^{-10} W/HZ. The messege signal has a bandwith 10^4 Hz

1. Find the predetection SNR (SNR in $r(t) = ku(t) + n(t)$).
2. Find the output SNR if the modulation is DSB.
3. Find the output SNR if the modulation is SSB.
4. Find the output SNR if the modulation is conventional AM with a modulation index of 0.85 and normalized message power of 0.2.

S.3 Since the channel attenuation is 80 dB, then,

$$10 \log\left(\frac{P_T}{P_R}\right) = 80 \quad \Rightarrow \quad P_R = 10^{-8} P_T$$

$$10^{-8} \cdot 40 \cdot 10^3 = 4 \times 10^{-4} \text{ W}$$

If the noise limiting filter has bandwith B, then the prediction noise power is

$$P_N = \int_{f_c-\frac{B}{2}}^{f_c+\frac{B}{2}} \frac{N_o}{2} df = N_o B = 2\times 10^{-10} B \text{ W}$$

In the case of DSB or conv. AM $B = 2W = 2\times 10^4$ Hz, whereas in SSB modulation $B = W = 10^4$, Thus the predetection SNR in DSB and conv. AM

$$SNR_{DSB,conv.AM} = \frac{P_R}{P_N} = \frac{4\times 10^{-4}}{2\times 10^{-10} \times 2 \times 10^4} = 10^2$$

$$SNR_{SSB} = \frac{4\times 10^{-4}}{2\times 10^{-10} \times 10^4} = 2\times 10^2$$

For the DSB, dem.gain $= 2$

$$SNR_{DSB,o} = 2\, SNR_{DSB,i} = 2\times 10^2$$

For the SSB, dem gain $= 1$

$$SNR_{SSB,o} = SNR_{SSB,i} = 2\times 10^2$$

For conv. AM with $\alpha = 0.8$ and $P_{mN} = 0.2$

$$SNR_{AM,o} = \frac{\alpha^2 P_{mN}}{1+\alpha^2 P_{mN}} \cdot SNR_{AM,i} = 0.1135 \times 2 \times 10^2$$

Q.4 Let m(t) be trasmitted using single-sideband modulation. The power spectral density of m(t) is

$$S_m(f) = \begin{cases} a \dfrac{|f|}{W} & , |f| \le W \\ 0 & , \text{else} \end{cases}$$

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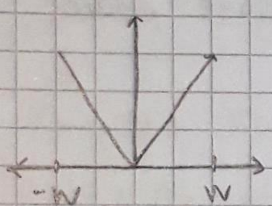
where a and W are constants. White Gaussian noise of zero mean and psd $N_0/2$ is added to the SSB mod. wave at the receiver input. Find an expression for the output SNR of the receiver.

S.4 The psd of m(t)

$S_m(f)$



$$SNR_{o,SSB} = \frac{A_c^2 P}{N_0 W} = \frac{A_c^2 aW}{N_0 W} = \frac{A_c^2 a}{N_0}$$

(from previous solutions)

The m(t) power

$$P = \int_{-\infty}^{\infty} S_m(f)\, df$$

$$= 2\int_0^W a\frac{f}{W}\, df$$

$$= aW$$

FM, PM SNR,o 242
conv AM SNR,o 222
Communication System Engineering 2nd edition

Recitation #2 (Noise)

Q.1 A DSC-SC modulated signal is transmitted over a noisy channel having a noise spectral density $N_0/2$ of 2×10^{-17} watts over hertz. The message bandwith is 4kHz and the carrier is 200 kHz. Assume the average received power of the signal is -80 dBm. Determine the post-detection SNR of the receiver.

S.2 SNR = message signal power / noise power

(10 log (watt) = dB)
(10 log (mwatt) = dBm)

$$SNR_{post}^{DSB} = \frac{A_c^2 P}{2 N_0 W}$$

$-80\text{ dBm} = 10^{-11}$ watt

average received power is $\frac{A_c^2 P}{2} = 10^{-11}$

$$SNR_{post}^{DSB} = \frac{10^{-11}}{4\times10^{-17}(4\times10^3)} = 62.5 \sim 18\text{ dB}$$

(P is average message power)

Q.2 A certain communication channel is characterized by 90-dB attenuation and additive white noise with power spectral density of $N_0/2 = 0.5 \times 10^{-14}$. The bandwith of the message signal is 1.5 MHz and its amplitude is uniformly distributed in the interval [-1, 1]. If we require that the SNR after demodulation be 30 dB, in each of the following cases find the necessary transmitted power.

1. USSB modulation
2. Conventional AM with a $\alpha = 0.5$
3. DSB-SC

$$SNR = \frac{P_R}{N_0 W} = \frac{P_R}{2 \times 0.5 \times 10^{-14} \times 1.5 \times 10^{6}} = \frac{P_R\, 10^{8}}{1.5}$$

Since the attenuation is 90 dB ; $10 \log \frac{P_T}{P_R} = 90$

$$P_R = 10^{-9} P_T$$

$$SNR = \frac{P_R\, 10^8}{1.5} = \frac{10^{-9} P_T\, 10^8}{1.5} = \frac{P_T}{15}$$

1. USSB , SNR in SSB is equivalent to DSB

$\frac{P_T}{15} = 10^3$ $\qquad P_T = 15$ kwatts

2. Conventional AM , $\alpha = 0.5$

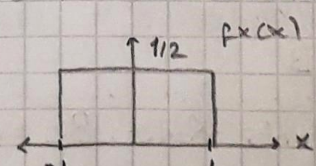
$$SNR = \eta \left(\frac{S}{N}\right) = \eta \frac{P_T}{15} \qquad \text{where } \eta = \frac{\alpha^2 P_{mN}}{1 + \alpha^2 P_{mN}}$$

$m(t) = \alpha\, m_n(t)$ where $m_n(t)$ normalized signal

$$m_n(t) = \frac{m(t)}{\max |m(t)|} \Rightarrow P_{mN}(t) = \frac{P_m}{(\max |m(t)|)^2}$$

$$s(t) = A_c [1 + \alpha\, m_n(t)] \cos(2\pi f_c t) \qquad \alpha = k_a A_m$$

$$P_m = P_{mN} = \int_{-1}^{1} \frac{1}{2} x^2\, dx = \frac{1}{3}$$



$E[m^2(x)]$ = power

$$\eta = \frac{0.25 \times (1/3)}{1 + 0.25 (1/3)} = \frac{1}{13}$$

$$SNR = \frac{1}{13} \frac{P_T}{15} = 10^3 \qquad P_T = 195 \text{ kW}$$

Q.3 A communication channel has a bandwith of 100 kHz. This channel is to be used for transmission of an analog source m(t), where |m(t)| < 1 whose bandwith is W = 4 kHz. The power content of the message signal is 0.1 W. Find the ratio of the output SNR of an Fm system that utilizes the whole bandwith, to the output SNR of a conventional Am system with a $\alpha = 0.85$? What is the ratio in dB?

S.3 $(SNR)_{fm} / (SNR)_{Am}$, where $\alpha = 0.85$

$B_c = 2(\beta + 1)W$

$100 = 2(1+\beta)4 \Rightarrow \beta = 11.5$

$$(SNR)_{fm} = \frac{3A_c^2 k_f^2 P_m}{2N_0W^3} = \frac{3A_c^2}{2}\left(\frac{\beta}{\max(m(t))}\right)^2 \frac{P_m}{N_0W}$$

$$\beta = \frac{k_f A_m}{W} \qquad (SNR)_{fm} = \frac{3A_c^2 \beta^2}{2}\frac{P_m}{N_0W}$$

$$(SNR)_{Am} = \frac{A_c^2 k_a^2 P_{mN}}{2WN_0} \qquad \alpha = k_a A_m$$

$$\frac{(SNR)_{fm}}{(SNR)_{Am}} = \frac{3\beta^2}{\alpha^2} = 27.3967 \text{ dB}$$

Q.4 The normalized $m_n(t)$ has a bandwith of 5000 Hz and power of 0.1 W, and the channel has a bandwith of 100 kHz and attenuation of 80 dB. The noise is white with power spectral density 0.5×10^{-12} and the transmitter power is 10 kW.

1. If Am with $\alpha = 0.8$ is employed what is SNR_o?
2. If Fm is employed, what is highest possible SNR_o?

S.4 $10\log\frac{P_T}{P_R} = 80 \quad \Rightarrow P_r = 10^{-8}P_T = 10^{-4}$ W

$$(SNR)_{Am} = \frac{\alpha^2 P_{mN}}{1+\alpha^2 P_{mN}}\frac{P_R}{N_0W}$$

$N_0W = 2 \times 0.5 \times 10^{-12} \times 5 \times 10^3 = 5 \times 10^{-9}$

$(SNR)_{Am} = 30.81$ dB

For Fm case Carson's Rule $B_c = 2(1+\beta)W \Rightarrow 100 \times 10^3$, $\beta = 9$

$$(SNR)_{Fm} = \frac{3A_c^2}{2}\left(\frac{\beta}{\max(m(t))}\right)^2 \frac{P_{mN}}{N_0W} = 3\beta^2 P_{mN}\frac{P_R}{N_0W} = (3.9)(0.1)\left(\frac{10^5}{5}\right)$$

$(SNR)_{Fm} = 56.87$ dB

Recitation #3 (Noise)

Q.1 m(t) has a bandwith of 10 kHz, a power of 16 W and a maximum amplitude of 6. It is desirable to transmit this message to a destination via a channel with 80 dB attenuation and additive white noise with power spectral density $S_N(f) = N_0/2 = 10^{-12}$ and achieve a SNR at the modulator output of at least 50 dB. What is required transmitter power and channel bandwith if the following modulation schemes are employed

a. DSB
b. SSB
c. Conventional AM with $\alpha = 0.8$

S.1 baseband SNR $(SNR)_b = \dfrac{P_R}{N_0 W} = \dfrac{P_R}{2 \cdot 10^{-12} \cdot 10^4} = \dfrac{10^8 P_R}{2}$

channel attenuation is 80 dB

$$10 \log \frac{P_T}{P_R} = 80 \qquad P_R = 10^{-8} P_T$$

$$\left(\frac{S}{N}\right)_b = \frac{10^8 \cdot 10^{-8} \cdot P_T}{2} = \frac{P_T}{2}$$

a) DSB

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{P_T}{2} \sim 50 \text{ dB} = 10^5$$

$P_T = 2 \times 10^5 = 200$ kW

$BW = 2W = 2 \times 10$ kHz $= 20$ kHz

b) SSB

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{P_T}{2} = 10^5, \quad P_T = 200 \text{ kW}$$

$BW = W = 10$ kHz

c) Conventional AM with $\alpha = 0.8$

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b \cdot \eta = \frac{P_T}{2} \cdot \eta$$, where η = modulation efficiency

$$\eta = \frac{\alpha^2 P_{mN}}{1 + \alpha^2 P_{mN}}$$

$$P_{mN} = \frac{P_m}{\max(m(t))^2} = \frac{P_m}{36} = \frac{16}{36} = \frac{4}{9}$$

$$\eta = \frac{(0.8)^2 (4/9)}{1 + (0.8)^2 (4/9)} = 0.22$$

$$\left(\frac{S}{N}\right)_o = 0.22 \frac{P_T}{2} = 0.11 P_T = 10^5$$

$P_T = 909$ kW

$BW = 2W = 20$ kHz

Q.2 Design an FM system that achieves an SNR at the receiver equal to 40 dB and requires the minimum amount of transmitter power. The bandwith of the channel is 120 kHz, the message bandwith 10 kHz the average-to-peak power ratio for the message, $P_{mN} = 0.5$, and the (one-sided) noise power spectral density is $N_0 = 10^{-8}$. What is required transmitter power if the signal is attenuated by 40 dB in trasmission through the channel?

S.2 Carson's Rule

$B_c = 2(\beta+1)W$

$120\ kHz = 2(\beta+1)\,10\ kHz$

$\beta = 5$

Fm is limited in BW and limited in power

using $\left(\frac{S}{N}\right)_o = 60\ \beta^2(\beta+1)P_{mN}$

SNR at the receiver $= 40\ dB = 10^4 = \left(\frac{S}{N}\right)_o$

$10^4 = 60\ \beta^2(\beta+1)\ 0.5$

$\beta = 6.6$

Since the value of β is given by the BW constraint is less than the value of β given by the power constraint

$\beta = 5$

using $\left(\frac{S}{N}\right)_o = \frac{3}{2}\beta^2\left(\frac{S}{N}\right)_b$ $\quad 10^4 = \frac{3}{2}5^2\left(\frac{S}{N}\right)_b$, $\left(\frac{S}{N}\right)_b = 266.7 = \ldots$

$= 24.26\ dB$

$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W}$ where $N_0 = 10^{-8}\ W/Hz$
$W = 10\ kHz$

$266.7 = \frac{P_R}{10^{-8}\times 10^4}$, $P_R = 0.0266 \approx -15.74\ dB$

Channel attenuation is 40 dB

$P_R(dB) = P_T(dB) - 40\ dB$

$-15.74 = P_T(dB) - 40$, $P_T = 24.26\ dB \approx 266.66\ W$

If there were no BW constraint, we could choose $\beta = 6.6$

Then, $\left(\frac{S}{N}\right)_b = 153$, $P_R \approx 0.0153$, $P_T = 153\ W$

Q.3 The message signal having a bandwith W of 4 kHz is transmitted over a noisy channel having a noise spectral density $N_0/2$ of 2×10^{-17} W/Hz using SSB modulation. If the average received power of the signal is -80 dBm, what is the post-detection SNR of the receiver? Compare the transmission bandwith of the SSB receiver to that of the DSB-SC receiver.

S.3 $SNR_{SSB,post} = \dfrac{A_c^2 P}{4 N_0 W}$

$P = E\{m^2(t)\}$ → average message power

$\dfrac{A_c^2 P}{4} = E\{s^2(t)\}$ → average received signal power

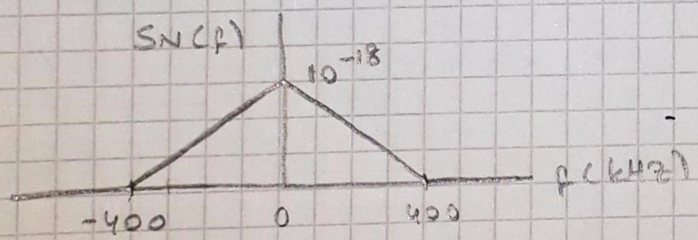
$\dfrac{A_c^2 P}{4} = -80\,dBm \Rightarrow 10^{-8}\,mW = 10^{-11}\,W$

$N_0 = 4 \times 10^{-17}$
$W = 4\,kHz$

$SNR_{SSB,post} = \dfrac{10^{-11}}{4 \times 10^{-17} \times 4000} = 62.5 \approx 18\,dB$

The transmission BW of SSB is 4kHz
The trasmission BW of DSB-SC is 8 kHz

Q.4 A DSB-SC mod signal is trasmitted over a noisy channel, with the psd of the noise as shown in figure. The message bandwith is 4kHz and the carrier frequency is 200 kHz. Assume the average received power of the signal is -80 dBm, determine the output SNR ratio of receiver.



S.3 Psd at $f_c = 200\,kHz$

$S_N(200k) = \dfrac{10^{-18}}{2} = 5 \times 10^{-19}$

$(SNR)_{DSBSC,o} = \dfrac{A_c^2 P}{2 N_0 W}$

$\dfrac{A_c^2 P}{2} = -80\,dBm = 10^{-11}\,W$

$(SNR)_{DSB,o} = \dfrac{10^{-11}}{5 \times 10^{-19} \times 4000} = 5000 = 37\,dB$

No = fc degerindeki psd degeri