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A Systems Approach to Channel Equalization

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***Abstract*— In this paper we present a control theoretic framework for channel equalization. Channel equalization methods are used to mitigate the effects of inter-symbol interference (ISI). Traditional methods, maximize the signal to noise ratio (SNR) in an attempt to convert a bandlimited ISI channel into a memoryless AWGN channel, which is then followed by symbol detection. Nevertheless, for the purpose of reliable symbol detection both problems — SNR maximization and AWGN channel conversion — are not reflective of the error probability and lead typically to suboptimal solutions. Our viewpoint in this paper is to directly characterize the overall probability of symbol error by means of a Chernoff type bound for a given channel/receiver combination. The main idea behind our technique is to exploit the randomness of transmitted symbols to average out ISI rather than invert the channel dynamics. The problem reduces to choosing a receiver that minimizes the exponent in the Chernoff bound. This problem is shown to reduce to a mixed ℓ_1/ℓ_∞ problem whose solution can be completely characterized. We comment on how the solution methodology can have implications for a fundamental understanding of the tradeoff between channel uncertainty and bit error probability, a situation commonly encountered in wireless communications.**

Keywords: Equalization, large deviations, ℓ_1/ℓ_∞ optimality, worst case, discrete data reconstruction.

I. Introduction

In this paper we present a control theoretic framework for channel equalization. Channel equalization methods [7] are used to mitigate the effects of inter-symbol interference (ISI) that result from the frequency and time selectivity characteristics of practical channels. Frequency selectivity in wireless channels [7], [12] results from multi-path effects — when transmitted signals are reflected and delayed and scaled copies are received at the receiver. The changing environmental mediums, such as the ones frequently encountered in mobile settings, lead to time variation. In addition, wireless channels pose additional challenges due to multi-user interference (MAI). As data rates become more demanding with respect to the channel bandwidth, channel dispersion and the attendant ISI become a critical performance limiting issue. In many situations (but not all), diversity and space-time coding methods — which in effect spread the symbol across time, space or frequency — can in principle be used to mitigate ISI and MAI [6], [5]. However, the practical bandwidth limits coupled with the number of users and the difficulty of maintaining synchronization results in performance degradation [6]. Therefore, equalization methods are critical for overcoming performance degradation.

Maximum likelihood sequence detection [7] is optimal purely from a probability of error view point. The idea here is to estimate from the set of all possible sequences a sequence that has maximum likelihood. The main difficulty is that the ML technique has a computational complexity that grows with channel time dispersion. If the size of the symbol alphabet is p and the number of symbols contributing to ISI is T, the optimal Viterbi algorithm has a complexity of p^{T+1} for each new received symbol. Therefore, the ML technique is rarely used as a stand alone technique in most practical systems.

Most equalization methods [7], [12], [6], [5], [3], [10] employ sub-optimal methods, which attempt to convert a channel with ISI into one that is memoryless (or close to memoryless). Once a memoryless the channel is obtained, the ML technique yields the so called symbol-by-symbol detection scheme which is in effect a likelihood ratio threshold test [8]. The question now arises as to how to perform the main step of inversion of a channel with ISI. Linear methods can be broadly classified into those that: (a) minimize the peak distortion; or (b) minimize the mean square distortion. The former results in the so called zero forcing equalizer and amounts to exactly inverting the channel, while the latter is based on finding a real valued estimate for the symbol that minimizes the mean squared error. The latter technique can be appropriately viewed as maximizing the SNR among all linear techniques. A common nonlinear technique is the decision feedback equalizer which can be viewed as minimizing the mean squared error conditioned on correct decoding of the previous symbols. In general, all of these techniques can ultimately viewed as methods that maximize the SNR or equivalently as attempts to approximately invert the channel to obtain a memoryless channel. Nevertheless, SNR maximization is sub-optimal and not truly reflective of the actual error probability.

In this paper we formulate a new direction for channel equalization based on a combination of well-known ideas

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from probability and control theory. Some of the results have appeared in [11]. Our approach exploits the independent identically distributed (i.i.d.) nature of the input sequence. The point is that when a linear equalization method is employed the ISI together with noise can be viewed as a weighted sum of i.i.d. random variables. It is well known from large-deviation theory [9] that such sums of random variables rarely deviate from their mean, i.e., the probability that a large deviation of a sum of i.i.d. random variables occurs goes to zero exponentially fast. This hinges on the fact that in this sum, different i.i.d. random variables tend to cancel the contribution from each other. Consequently, in the context of equalization our goal is to maximize these cancellations by appropriately choosing the equalization filter coefficients such that ISI and noise contribution do not exceed the noise immunity set by the symbol-by-symbol detector. We will show that by using standard well known results in probability theory, it is possible to bound the error contribution in terms of a ratio between ℓ_1 and ℓ_∞ norms. Next, our task reduces to maximizing this ratio while constraining the ℓ_1 norm. This problem can be re-formulated as a convex mixed ℓ_1, ℓ_∞ problem and the solution to the problem is then presented.

The organization of the paper is as follows. In Section 2 we present the basic setup. In Section 3 we derive a modified version of zero forcing equalizers that accounts for noise enhancement in addition to peak distortion. In Section 4 we present a new formulation of the channel equalization problem based on large-deviation bounds for error probability. We formulate the mixed ℓ_1/ℓ_∞ problem and present the corresponding solution.

II. Basic Problem Definition

The model for the channel between a particular transmitter and receiver pair in a general communication system consists of two components: a linear time-varying filter that captures the effects of frequency and time selectivity of the channel, and an additive noise term representing noise and possibly interference from other users [7], [12]. We represent the channel dynamics in terms of an equivalent discrete-time baseband model. Although, sometimes such models appear over-simplistic of the physical systems they represent, they are generally adequate for capturing the key features of such systems and the associated channels.

The basic problem we are concerned with is depicted in Figure 1 where $s(\cdot)$ is a i.i.d. binary signal—corresponding to a binary PAM constellation—to be transmitted with $s(k) \in \{-1,1\}$ for all $k = 0,1,\ldots$; n is noise which accounts for both co-channel interference and receiver noise. Co-channel interference results from interference due to multiple users in the communication medium and can be modeled as the output of a linearly filtered binary signal. On the other hand, receiver noise is usually modeled as additive white Gaussian noise (AWGN) with mean zero and variance σ^2. The system $H = \{h_0, h_1, \ldots\}$ is a discrete-time LTI system[1] that represents the channel dynamics which are assumed known *a priori* for now. Our task is to design a receiver structure Q so that its output $\hat{s}(k)$ has the same sign as $s(k)$ at all times (so that a threshold device can correctly determine $s(k)$ based on $\hat{s}(k)$). The basic problem and our solution methodology can be extended in many directions which are not explicitly discussed here. These directions include:

- Colored noise: The receiver noise can be modified to include filtered Gaussian noise processes.
- LTV extensions: The LTI channel dynamics, H, can be modified to a linear time-varying setup. The methods presented in this paper can be readily modified to account for time variation.
- MIMO equalization: Multiple users can be accounted for in the channel dynamics and, as in multi-user detection, we can simultaneously attempt to decode messages from multiple users and thus realize larger performance gains.

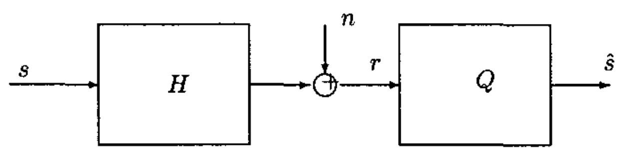

Fig. 1. Basic set-up

III. A Modification of the Linear Zero Forcing Equalizer

In this section we derive a modification of the well-known zero forcing equalizer (ZFE) [7], [6]. The zero forcing equalizer is a linear filter that can be thought of as optimizing the peak ISI distortion. This strategy amounts to forcing ISI to zero and is optimal in the absence of noise. However, it is well-known that ZFE leads to poor performance in the presence of noise. This is because the filter $Q(z) = 1/H(z)$ will typically have a large gain for certain ranges of frequencies, leading to significant enhancement of noise. To modify the scheme, we proceed by considering the equalization in Figure 1 with Q being an LTI filter to be designed. clearly,

$$\tilde{s}(k) = QHs(k) + Qn(k) = v(k) + z(k) + g_0 s(k)\,, \quad (1)$$

where $v = Qn$, $z = \widetilde{G}s$, $\widetilde{G} = G - g_0$, $G = QH$. Without loss of generality we assume that $g_0 \geq 0$.

Next, we calculate the probability of symbol error P_e at time instant k as

$$\begin{aligned} P_e &= \frac{1}{2}\text{Prob }\{\tilde{s}(k) > 0 | s(k) = -1\} + \\ &\quad + \frac{1}{2}\text{Prob }\{\tilde{s}(k) < 0 | s(k) = 1\} \\ &= \text{Prob }\{\tilde{s}(k) > 0 | s(k) = -1\}\,, \end{aligned}$$

[1]The setup can be extended to cover the linear time-varying case as well. Here, we deal with LTI channels for the sake of exposition.

where the last equality follows from symmetry. Now,

$$\begin{aligned} P_e &= \text{Prob }\{\tilde{s}(k) > 0 | \, s(k) = -1\} & (2) \\ &= \text{Prob }\{v(k) + z(k) - g_0 > 0\} & (3) \\ &\leq \text{Prob }\{v(k) + \max|z(k)| - g_0 > 0\} & (4) \\ &= \text{Prob }\left\{v(k) > g_0 - \|\widetilde{G}\|_1\right\} = \bar{P}_e \,. & (5) \end{aligned}$$

Since n is Gaussian, $v(k)$ is a zero mean Gaussian random variable with variance $\|Q\|_2^2\sigma^2$ and Stirling's approximation provides $\bar{P}_e = \text{ErrFun}\left(\frac{g_0 - \|\widetilde{G}\|_1}{\|Q\|_2\sigma}\right)$ where $\text{ErrFun}(x)$ satisfies

$$\begin{aligned} \text{ErrFun}(x) &= \frac{1}{2\pi}\int_x^\infty \exp(-x^2/2) \\ &\leq \frac{1}{2\pi}\exp\left[-x^2 - \log(x)\o 2\right] \,. \end{aligned}$$

On the other hand, if $v(k)$ is a sum of bounded random variables (which is the case when MAI is the significant form of noise) we then appeal to the Hoeffding inequality [4]

$$\text{Prob }\{\sum_{i=1}^n Y_i \geq \alpha\} \leq \exp\left[-2\alpha^2 \o \sum_{i=1}^n (b_i - a_i)^2)\right] \,,$$

where $Y_i \in [a_i, b_i]$ respectively. In both cases, we see that the upper bound $\bar{P}_e$ is minimized if and only if

$$r := \frac{|g_0| - \|\widetilde{G}\|_1}{\|Q\|_2} \qquad (6)$$

is maximized. Now, if we let $Q = q_0(1 + \lambda\widetilde{Q})$, r can be written as

$$r = \frac{|h_0| - \|(1 + \lambda\widetilde{Q})H - h_0\|_1}{\|1 + \lambda\widetilde{Q}\|_2}$$

or, if $H = h_0 + \lambda\widetilde{H}$,

$$r = \frac{|h_0| - \|\widetilde{H} + \widetilde{Q}H\|_1}{\|1 + \lambda\widetilde{Q}\|_2}.$$

To maximize r, we consider the related problem

$$\begin{aligned} \inf_Q \|1 + \lambda\widetilde{Q}\|_2 &:= \mu_\gamma \\ \text{s.t. } \|\widetilde{H} + \widetilde{Q}H\|_1 &\leq \gamma \end{aligned} \qquad (7)$$

where $\gamma \geq 0$ is a parameter.

Performing the optimization in (7) is a mixed H_2/ℓ_1 problem which can be solved using standard methods [2]. By varying γ one can obtain the solution that maximizes r. Indeed, if $\bar{\gamma}$ is the maximum γ for which the ℓ_1 constraint is active, then $\mu_\gamma = 1$ for all $\gamma \geq \bar{\gamma}$ and the optimal $\widetilde{Q} = 0$. Also, it follows that $\bar{\gamma} = \|\widetilde{H}\|_1$. The minimum $\underline{\gamma}$ for which a solution to (7) exists corresponds to the optimal ℓ_1 problem

$$\underline{\gamma} := \inf_Q \|\widetilde{H} + \widetilde{Q}H\|_1$$

It can be also be shown that μ_γ is a continuous non-increasing function of γ and consequently the function

$$f(\gamma) := \frac{|h_0| - \gamma}{\mu_\gamma}$$

is a continuous function of γ. Therefore, the maximum $r_{\max}$ is obtained by the one parameter search

$$r_{\max} = \max_{\underline{\gamma} \leq \gamma \leq \bar{\gamma}} f(\gamma).$$

Moreover, the maximizing γ_0 can be obtained from the above search and consequently the optimal $\widetilde{Q}_0$, which can be shown [2] to be unique.

In summary, the method presented in this section modifies the existing ZFE structure by seeking to minimize the peak distortion, while at the same time constraining noise enhancement. This can be observed in Eq. 6 where the denominator constraints the enhancement of noise, while the numerator accounts for the peak distortion.

IV. Probability of Error Bounds Based on Large Deviations

In the previous section we proposed a modification of existing ZFE techniques by formulating a mixed optimization problem. Nevertheless, the methods can be potentially conservative. One of the reasons for the conservatism stems from the fact that the peak distortion is rarely achieved in practice, especially when the channel suffers from significant channel dispersion. This can be seen as follows. Suppose there are 20 symbols contributing to ISI then the probability that the peak distortion is ever achieved has a probability of $2^{-20} \approx 10^{-6}$. This is because all the input symbols have to exactly align with the channel transfer function. A variant of this situation is described in the following example and will motivate our objective.

Example 1: Consider a channel with $h = [1, 0.1, 0.1, \ldots, 0.1]$ with 10 symbols contributing to ISI. Further, the receiver noise is known to be AWGN Gaussian noise and the corresponding $SNR = 10dB$ for the binary PAM signaling system. We wish to compare the performance of several linear equalization schemes.

First observe that the peak distortion is equal to 1 and the peak distortion bound would lead us to conclude that the probability of error is 1/2 without any equalization (which is the maximum possible). Now applying the linear ZFE results, it turns out that the probability of error is equivalent to a memoryless channel with $SNR = 3dB$ ($P_e \equiv 0.2$ which is quite large). Minimum mean square equalization provides marginal improvement as the noise is relatively insignificant compared to ISI. The method presented in the previous section also performs only marginally better than ZFE.

Nevertheless, it turns out that the smallest probability of error is obtained when no equalization is performed on the received signal. Indeed, the actual probability of error based on large-deviation inequality turns out to be approximately 10^{-3}. This amounts to a channel with $SNR = 10dB$, the signal to noise ratio we originally started with.

The above example illustrates that "actual" contribution of ISI can be negligible and yet equalization techniques can degrade the performance by attempting to convert the channel into a memoryless channel. Therefore, our idea is to exploit the randomness of the input to average out the noise rather than inverting the channel (which could lead to enhancing noise as in the example). To do this we refer back to Eq. (2). To simplify our analysis we denote by $\Phi_1 = \widetilde{G}$, $\Phi_2 = Q$, $\Phi = (\Phi_1 \ \Phi_2)$ and $u(k) = [s(k) \ n(k)]^T$. From Eq. (2) it follows that,

$$\begin{aligned} P_e &= \text{Prob } \{\tilde{s}(k) > 0 | \, s(k) = -1\} \\ &= \text{Prob } \{v(k) + z(k) - g_0 > 0\} \\ &= \text{Prob } \left\{ [\Phi_1 \ \Phi_2] \begin{bmatrix} s \\ n \end{bmatrix} (k) > g_0 \right\} \\ &= \text{Prob } \{\Phi u(k) > g_0\} \ . \end{aligned}$$

Notice that s and n are independent random variables and $s(\cdot)$ and $n(\cdot)$ are i.i.d. random processes. Based on these observations we can derive a large deviation type bound as stated below.

Theorem 1: For the setup described above the ISI contribution to the error can be characterized as:

$$Prob\{\Phi_1 s(k) \geq t\} \leq C_0 \exp\left(-t\frac{\|\Phi_1\|_1}{\|\Phi_1\|_\infty}\psi\left(\frac{t}{\|\Phi\|_1}\right)\right)$$

where, $k \in Z^+$, $t > 0$ and $\psi(\cdot)$ is a monotonically increasing function.

Proof: The main steps of the proof follows along the lines of the proof of Hoeffding inequality. ∎

We note that $(\Phi_2 n)(k) \equiv \mathcal{N}(0, \|\Phi_2\|_2^2)$ and the probability of error will be given by the convolution of the upper bound above with a gaussian. It turns out that the problem involves a mixed optimization problem that maximizes the ratio $\|\Phi_1\|_1/\|\Phi_1\|_\infty$ while constraining the two-norm of Φ_2. Alternatively, for multiple-access channels, where noise plays a less significant role this ratio fundamentally characterizes the probability of error. We now go back to our earlier example and notice how the ratio is close to 10 while $\|\Phi\|_1 \leq 2$. For a binary random variable this yields a P_e of 10^{-3} verifying the fact that $Q = I$ is a good equalizer.

It can also be shown that if the ratio $\frac{\|\Phi\|_1}{\|\Phi\|_\infty}$ is large relative to Φ_1 the upper bound is essentially tight. Notice now for our example, how the ratio is close to 10 while $\|\Phi\|_1 \leq 2$. For a binary random variable this yields a P_e of 10^{-3} verifying the fact that $Q = I$ is a good equalizer.

A. Optimal Equalization Filter

In this section we present techniques for deriving equalizers that minimize the bounds obtained in Theorem 1. Based on the previous analysis, the bound $\bar{P}_e$ on P_e depends on the ratio

$$r = \frac{\|\Phi\|_1}{\|\Phi\|_\infty}.$$

To minimize $\bar{P}_e$ one has to determine what is

$$\sup_{s.t. \|\Phi\|_1 = \rho} \frac{\|\Phi\|_1}{\|\Phi\|_\infty}$$

or equivalently,

$$\inf_{s.t. \|\Phi\|_1 = \rho} \frac{\|\Phi\|_\infty}{\|\Phi\|_1} =: \nu_\rho$$

as a function of the parameter $\rho \geq 0$.

This leads us to investigate the following relevant problem

$$\inf_{s.t. \|\Phi\|_1 \leq \rho} \|\Phi\|_\infty \ =: \ \mu_\rho \ .$$

Let $\bar{\rho}$ be the (minimal) ℓ_1 norm of the unconstrained problem $\inf_{Q \in \ell_1} \|\Phi\|_\infty$. Then, for $\rho > \bar{\rho}$ the constraint is inactive. For $\rho \leq \bar{\rho}$ the ℓ_1-constraint is active and thus

$$\nu_\rho = \inf_{s.t. \|\Phi\|_1 \leq \rho} \frac{\|\Phi\|_\infty}{\|\Phi\|_1} = \inf_{s.t. \|\Phi\|_1 \leq \rho} \frac{\|\Phi\|_\infty}{\|\Phi\|_1} = \frac{\mu_\rho}{\rho},$$

where $\rho \leq \bar{\rho}$. For $\rho > \bar{\rho}$ one can construct Φ with $\|\Phi\|_1 = \rho$ and $\|\Phi\|_\infty = \mu_{\bar{\rho}}$ and thus

$$\nu_\rho = \frac{\mu_{\bar{\rho}}}{\rho}, \quad \rho > \bar{\rho} \ . \tag{8}$$

Indeed if $Q_{\bar{\rho}}$ is an optimal solution for $\nu_{\bar{\rho}}$, then one can use $Q = Q_{\bar{\rho}} + \lambda^n Q_s$ with n sufficiently large to obtain a Φ with $\|\Phi\|_\infty = \nu_{\bar{\rho}}$ and $\|\Phi\|_1 = \rho$ for any $\rho > \bar{\rho}$. This can be done by appropriately choosing a "slack" Q_s as $Q_s = \alpha\frac{1}{1-\beta\lambda}$ with $|\beta| = 1 - \epsilon$ with $\epsilon > 0$ arbitrarily small, and $\alpha > 0$ sufficiently small. Now, Φ is of the form $\Phi = T_1 - QT_3$ with $T_1, Q, T_3 \in \ell_1$ and hence

$$\Phi = T_1 - Q_{\bar{\rho}}T_3 - \lambda^n Q_s T_3 \ .$$

Since only finite terms in $T_1 - Q_{\bar{\rho}}T_3$ matter for $\|\Phi\|_\infty$, say $n-1$, the remaining terms have a gap. Specifically, if $\Phi_{\bar{\rho}} = T_1 - Q_{\bar{\rho}}T_3$ with $\Phi_{\bar{\rho}} = \sum_{i=0}^{\infty} \Phi_{\bar{\rho}}(i)\lambda^i$ then $|\Phi_{\bar{\rho}}|_\infty < \|\Phi\|_\infty$ for $i \geq n$. As the term $\lambda^n Q_s T_3$ affects Φ only after $n-1$ terms the parameters α, β can be selected so that $\|\Phi\|_\infty = \|\Phi_{\bar{\rho}}\|_\infty$ and, at the same time, $\|\Phi\|_1 = \rho$ for any $\rho > \bar{\rho}$ arbitrarily large.

The other term in the bound for $\bar{P}_e$ depends on $\frac{\|\Phi\|_1}{|q_0 h_0|}$. Thus, by a two-parameter search, namely ρ, q_0, one can completely characterize the minimum possible $\bar{P}_e$ along with the minimizing ρ, q_0, and hence Q.

V. Dealing with Channel Uncertainty

In this section we provide brief comments on how the methods developed in this paper can be modified in a straightforward way to incorporate robustness to channel uncertainty. This issue gains significance in highly time and frequency selective channels. Such channels arise naturally in air to ground communications [12]. The point is that since the coherence time of the channel is significant relative to channel dispersion, uncertainty in the channel estimates is unavoidable. The schematic of channel model and equalization for such situations is shown in Figure 2. Our main goal is to understand the fundamental tradeoff between channel uncertainty and probability of symbol error. This will eventually lead to fundamental understanding of fundamental capacity limits for time-varying channels.

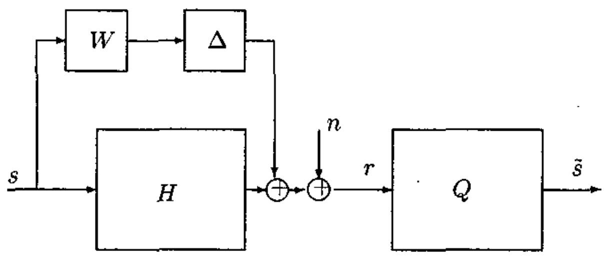

Fig. 2. A fading channel model with linear equalization

The uncertainty here is given in terms of an additive weighted block ΔW, where Δ is assumed to be an unknown perturbation, possibly time-varying and even nonlinear, that has a bounded ℓ_∞ to ℓ_∞ norm $\|\Delta\|_{\infty-\infty} \leq 1$. The weight W is a known stable LTI dynamical system that may reflect magnitude normalization and partial information on the magnitude of the uncertainty over different frequencies (i.e., it "shapes" the uncertainty block).

As an example, consider the actual channel H_a as $H_a = H + E$ where $H = \{h_0, h_1, \ldots\}$ is the nominal LTI channel and E represents time-varying perturbations on the parameters of H leading to a response

$$(H_a s)(k) = \sum_{i=0}^{k}(h_{k-i} + \epsilon_{ki})s(i)$$

with the perturbation ϵ_{ki} bounded as $|\epsilon_{ki}| \leq \epsilon_i$ for all $k = 0, 1, 2, \ldots$, but otherwise arbitrary. If $\sum_{i=0}^{\infty} \epsilon_i = \epsilon$, then this amounts to modeling E as $E = \Delta W$ with $W = \epsilon$ and $\|\Delta\|_{\infty-\infty} \leq 1$. Similarly, if the first N channel coefficients are not changing ($\epsilon_0 = \ldots = \epsilon_{N-1} = 0$) but there is uncertainty in the higher order terms, then $\hat{W}(\lambda) = \epsilon\lambda^N$.

We note that this uncertainty formulation is different in nature than what is typically assumed in the stochastic framework (e.g., Chapter 14 in [7]). However, we believe that it captures a number of relevant fading phenomena due to time variations and can be used to design reliable reconstruction algorithms.

The bounds obtained in Theorem 1 can be suitably modified to incorporate channel uncertainty. This can achieved by maximizing the upper bound over set of all admissible Δ. The function Φ will depend on both Q and Δ.

$$\begin{aligned} P_e &= \text{Prob}\{\Phi u(k) \geq g_0\} \\ &\leq \max_{\Delta} C_0 \int_0^t \exp(-t\frac{\|\Phi_1\|_1}{\|\Phi_1\|_\infty}\psi(\frac{t}{\|\Phi_1\|_1}))\frac{\exp\left(-\frac{(1-t)^2}{\|\Phi_2\|_2^2}\right)}{\sqrt{2\pi\|\Phi_2\|_2^2}}dt \\ &\quad + \text{ErrFun}\left(\frac{1}{\|\Phi_2\|_2^2}\right) \end{aligned}$$

for $k \in Z^+$. It turns out that the problem of minimizing over all Q the worst-case P_e over all Δ can be formulated as a robust synthesis problem [1] in ℓ_1. This problem is a topic of ongoing research.

VI. Conclusions

In this paper we presented a control theoretic framework for channel equalization. Traditional methods attempt to convert a bandlimited ISI channel into a memoryless AWGN channel and can lead to performance degradation. Instead, we directly characterize the overall probability of symbol error by means of a Chernoff type bound for a given channel/receiver combination. The main idea is that the randomness of ISI and noise can be exploited to average them out. This results in a new formulation for the channel equalization problem where a receiver is chosen to minimize the exponent in the Chernoff bound. This problem is shown to reduce to a mixed ℓ_1/ℓ_∞ problem whose solution can be completely characterized.

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