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# Channel Equalization Linear,Non linear, Blind and Time Domain and Interference Cancellation

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## **1 LIST OF ABBRIVATIONS**

AWGN - Additive White Gaussian Noise  
BER - Bit Error Rate  
BPSK - Binary phase Shift Keying  
BSS - Blind signal separation  
CCI- Co-channel Interference  
CDMA - Code Division Multiple Access  
CMA - Constant Modulus Algorithm  
CP - Cyclic Prefix  
DFE - Decision Feedback Equalizer  
DMT - Discrete Multi-tone  
DS/SS - Direct Sequence Spread Spectrum  
DSP - Digital Signal Processing  
FIR - Finite Impulse Response  
FISSS - Frequency independent strong signal suppressor  
FLI - Frequency localised interference  
FSBLP - Fractionally Spaced Bilinear Perceptron  
FSDFMLP - Fractionally Spaced Decision Feedback Multilayer Perceptron  
GPC - Gaussian Processes for Classification  
GSA - General Sato Algorithm  
GSM - Global System for Mobile Communication  
GSNR - Geometric Signal- to- Noise Ratio  
IC - Interference Cancellation  
ISI - Intersymbol Interference  
LE - Linear Equalizer  
LE - MSE - Mean Square Error Linear Equalizer  
LE - ZF - Zero Forcing Linear Equalizer  
LMS - Least Mean Square  
LTE - Linear Transversal Equalizer  
MAI - Multiple Access Interference  
MEM - Micro-Electro Mechanical  
MGSNR - Maximum Geometric Signal - to - Noise Ratio  
MIMO - Multiple Input Multiple Output  
MLSE - Maximum Likelihood Sequence Estimation  
MMSE - Minimum Mean Square Error  
MMSE - LE - Minimum Mean Square Error Linear Equalizer  
MSE - Mean Square Error  
MSSNR - Maximum Shortening Signal- to - Noise Ratio  
OFDM - Orthogonal Frequency Division Multiplexing  
PAM - Puls Amplitude Modulation  
PLL - Phase Locked Loop  
QAM - Quadrature Amplitude Modulation  
RF - Radio Frequency  
RLS - Recursive least squares  
SAIC - Single antenna interference cancellation

SAW - Surface acoustic wave  
SCFNN - self-constructing fuzzy neural network  
SGA - Stop and Go Algorithm  
SIR - shortened impulse response  
SRCA - Sign Reduced Constellation Algorithm  
SSNR - Shortening Signal - to - Noise Ratio  
TDMA - Time Division Multiple Access  
TEQ - Time Domain Equalizer  
TFLI - Time-frequency localised interference  
TIR - Target Impulse Response  
TLI - Time localised interference  
Ts - Symbol Time  
ZFE - zero Forcing Equalizer

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### Abstract

This paper presents about different equalization techniques and interference cancellation. Linear Equalizers like zero forcing and minimum mean square error based are discussed. Although linear equalizers are simple to construct, most of practical equalizers are made based on non linear equalizing techniques due to the time varying nature of the channel. Non linear equalization, blind or self-recovering equalization, which doesn't use a training (i.e., known) sequence to characterize the channel, techniques are discussed. To show how these equalizers mitigate ISI, a matlab simulation output of blind equalizers is provided as an example. Then various types of time domain equalizers and interference cancellation methods are covered in this paper.

## 2 INTRODUCTION

In wireless communications systems, efficient use of the available spectrum is one of most critical design issues. Therefore, modern communication systems must evolve to work as close as possible to channel capacity to achieve the demanded binary rates. We need to design digital communication systems that implement novel approaches for both channel equalization and coding and, moreover, we should be able to link them together to optimally detect the transmitted information.

Communication channels introduce linear and nonlinear distortions and, in most cases of interest, they cannot be considered memoryless. Inter-symbol interference (ISI), mainly a consequence of multipath in wireless channels, accounts for the linear distortion. The presence of amplifiers and converters explain the nonlinear nature of communications channels. Communication channels also contaminate the received sequence with random fluctuations, which are typically regarded as additive white Gaussian noise (AWGN) fluctuations.

In the design of digital communication receivers the equalizer precedes the channel decoder. A channel equalizer is an important component of a communication system and is used to mitigate the ISI (inter symbol interference) introduced by the channel. The equalizer depends upon the channel characteristics. In a wireless channel, due to multipath fading, the channel characteristics change with time. Thus it may be necessary for the channel equalizer to track the time varying channel in order to provide reasonable performance. Ideally, the output of an equalizer

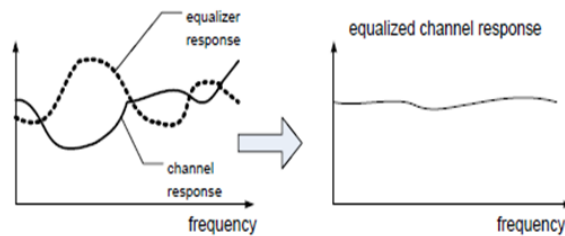


Figure 1: equalized channel response

is a delayed version of the transmitted signal. A fixed equalizer measures the time-invariant channel and compensates the frequency selectivity during the entire transmission of data. An

adaptive equalizer adjusts its coefficients to track a slowly time-varying channel

We can categorize equalizers into two groups. These categories are determined from how the output of an equalizer is used for subsequent control (feedback) of the equalizer. The analog signal  $\hat{d}(t)$  is processed by the decision maker in the receiver. This device determines the value of the digital data being received and applies thresholding operation in order to determine the value of the  $d(t)$ . If  $d(t)$  is not used in the feedback path to adapt the equalizer, the equalization is linear. On the other hand, if  $d(t)$  is fed back to change the subsequent outputs of the equalizer, the equalization is nonlinear. Many filter structures are used to implement linear and nonlinear equalizers. Further, for each structure, there are numerous algorithms used to adapt the equalizer. Figure 2 provides a general categorization of the equalization techniques according to the types, structures, and algorithms used.

The most common equalizer structure is a linear transversal equalizer (LTE). A linear transversal filter is made up of tapped delay lines, with the tapings spaced a symbol period ( $T_s$ ) apart.

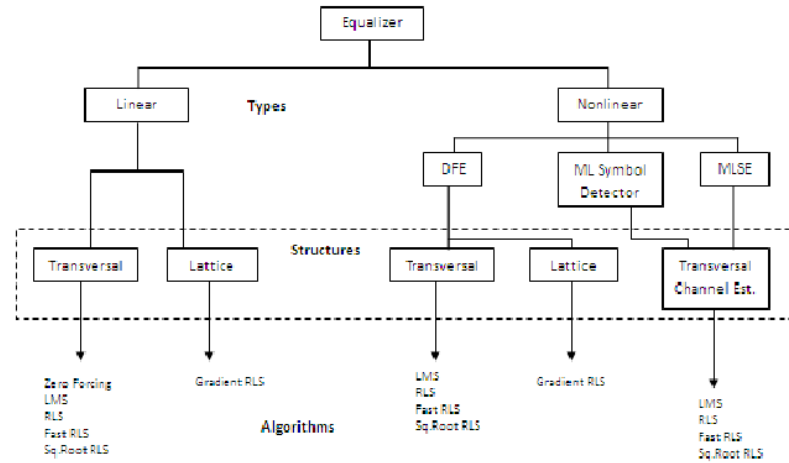


Figure 2: classification of equalizers

### 3 LINEAR EQUALIZATION

Every channel can be characterized by an equivalent discrete-time transfer function. This equivalent channel response, expressed in the time domain, also can be described mathematically by its frequency response  $H(z)$ , where  $z$  is the discrete-time frequency-domain variable (the  $z$ -transform). The zero-ISI condition exists when the transfer function, replicated in the frequency domain at every multiple of the sampling frequency, sums to a flat spectrum. When that is not the case, ISI will occur.

There are two basic approaches to linear equalization, aimed at different optimizing criteria. A linear equalizer can be implemented as an FIR filter. This type of equalizer is the simplest

type available. In such an equalizer, the current and past values of the received signal are linearly weighted by the filter coefficient and summed to produce the output, as shown in figure 3. The output of of this filter before decision making is:

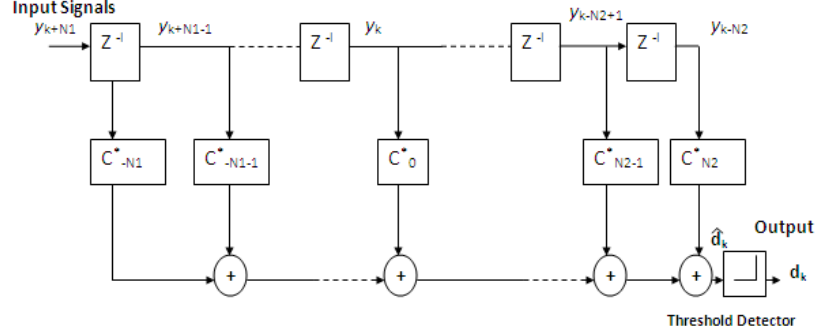


Figure 3: transversal linear equalizers

$$\hat{d}_k = \sum_{n=-N_1}^{N_2} (c_n^*) y_{k-n} \quad (1)$$

where  $c_n^*$  represents the complex filter coefficients or tap weights,  $\hat{d}_k$  is the output at time index  $k$ ,  $y_i$  is the input received signal at time  $t_0 + iT$ ,  $t_0$  is the equalizer starting time, and  $N = N_1 + N_2 + 1$  is the number of taps. The values  $N_1$  and  $N_2$  denote the number of taps used in the forward and reverse portions of the equalizer, respectively.

The linear equalizer can also be implemented as a lattice filter, whose structure is shown in figure 4. The input signal  $y_k$  is transformed into a set of  $N$  intermediate forward and backward error signals,  $f_n(k)$  and  $b_n(k)$  respectively.

### 3.1 ALGORITHMS FOR LINEAR EQUALIZERS

#### 3.1.1 ZERO FORCING (ZF)-EQUALIZER

This section examines the Zero-Forcing Equalizer (ZFE), which is the easiest type of equalizer to analyze and understand, but has inferior performance to some other equalizers to be introduced in later sections.

In zero-forcing equalization, the equalizer attempts to completely inverse the channel by forcing. The equalizer tries to restore the Nyquist Pulse character to the channel. In so doing, the ZFE ignores the noise and shapes the signal  $I_k$  so that it is free of ISI. We observe that the cascade of the discrete-time linear filter model having an impulse response  $f_n$  and an equalizer having an impulse response  $c_n$  can be represented by a single equivalent filter having the impulse response

$$q_n = \sum_{j=-\infty}^{\infty} (c_j) f_{n-j} \quad (2)$$



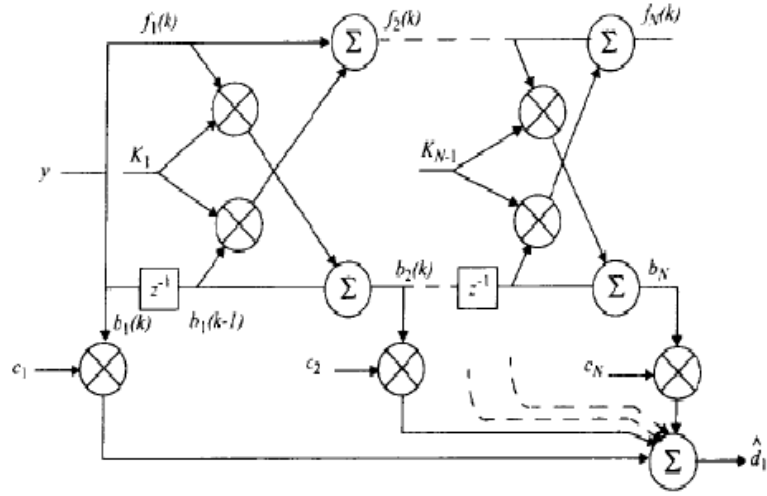


Figure 4: lattice linear equalizers

That is,  $q_n$  is simply the convolution of  $c_n$  and  $f_n$ . The equalizer is assumed to have an infinite number of taps. Its output at the  $k$ th sampling instant can be expressed in the form

$$\hat{I}_k = q_0 I_k + \sum_{n=k}^{\infty} I_n q_{k-n} + \sum_{j=-\infty}^{\infty} c_j \eta_{k-j} \quad (3)$$

The first term in eq 3 represents a scaled version of the desired symbol. For convenience, we normalize  $q_0$  to unity. The second term is the intersymbol interference. The peak value of this interference, which is called the peak distortion, is

$$\begin{aligned} D(c) &= \sum_{n=-\infty; n \neq 0}^{\infty} |q_n| \\ &= \sum_{n=-\infty; n \neq 0}^{\infty} \left| \sum_{j=-\infty}^{\infty} c_j f_{n-j} \right| \end{aligned} \quad (4)$$

Thus,  $D(c)$  is a function of the equalizer tap weights.

With an equalizer having an infinite number of taps, it is possible to select the tap weights so that  $D(c)=0$ , i.e.  $q_n = 0$  for all  $n$  except  $n=0$ . That is the inter symbol interference can be completely eliminated. The values of the tap weights for accomplishing this goal are determined from the condition

$$q_n = \sum_{j=-\infty}^{\infty} c_j f_{n-j} = \begin{cases} 1 & (n = 0), \\ 0 & (n \neq 0) \end{cases} \quad (5)$$

By taking the  $z$  transform of eq. 5, we obtain

$$Q(z) = C(z)F(z) = 1 \quad (6)$$

Or, simply,

$$C(z) = \frac{1}{F(z)} \quad (7)$$

Where  $C(z)$  denotes the  $z$  transform of the  $C_j$ . Note that the equalizer, with transfer function  $C(z)$ , is simply the inverse filter to the linear filter model  $F(z)$ . In other words, complete elimination of the ISI requires the use of an inverse filter to  $F(z)$ . We call such a filter a zero-forcing filter. Figure 5 illustrates in block diagram the equivalent discrete-time channel and equalizer. The cascade of the noise-whitening filter having the transfer function  $\frac{1}{F^*(z^{-1})}$  and the

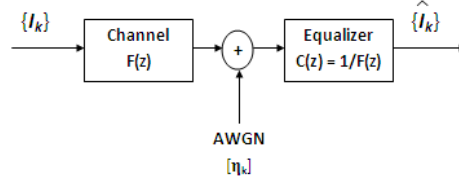


Figure 5: block diagram representation of channel with zero-forcing equalizers

zero-forcing equalizer having the transfer function  $\frac{1}{F(z)}$  results in an equivalent zero-forcing equalizer having the transfer function

$$C(z) = \frac{1}{F(z)F^*(z^{-1})} = \frac{1}{X(z)} \quad (8)$$

As shown in fig. 6. This combined filter has its input the sequence  $Y_k$  of samples from the matched filter. Since the channel characteristics are unknown a priori and, in many cases, the channel

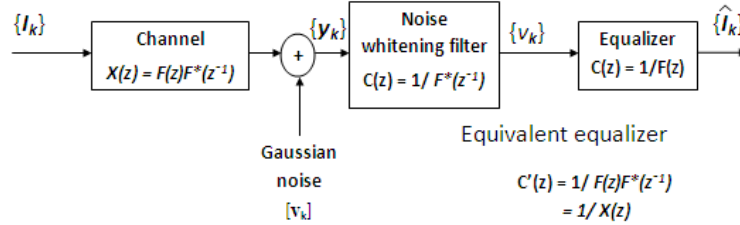


Figure 6: block diagram representation of channel with zero-forcing equalizers and whitening filter

response is time-variant, the equalizers should be designed to be adjustable to the channel response and, to be adaptive to the time variations in the channel response. Now, we present this type of zero-forcing equalizers algorithm.

The zero forcing solution is achieved by forcing the cross-correlation between the error sequence  $e_k = I_k - \hat{I}_k$  and the desired information sequence  $I_k$  to be zero for shifts in the range  $0 \leq n \leq K$ . We have

$$\begin{aligned} E(e_k I_{k-j}^*) &= E[(I_k - \hat{I}_k) I_{k-j}^*] \\ &= E(I_k I_{k-j}^*) - E(\hat{I}_k I_{k-j}^*) \quad j = -K \dots K \end{aligned} \quad (9)$$

We assume that the information symbols are uncorrelated (memory less system), i.e.  $E(I_k I_j) = D_{kj}$ , and that the information sequence  $I_k$  is uncorrelated with the additive noise sequence  $n_k$ .

And we use the expression

$$\hat{I}_k = \sum_{j=-K}^K c_j v_{k-j} \quad (10)$$

We obtain

$$E(e_k I_{k-j}^*) = \delta_{j0} - q_j \quad j = -K, \dots, K \quad (11)$$

Therefore, the conditions

$$E(e_k I_{k-j}^*) = 0 \quad j = -K, \dots, K \quad (12)$$

Are fulfilled when  $q_0 = 1$  and  $q_n = 0$ ,  $1 \leq n \leq K$ . When the channel response is unknown, the cross-correlations given by eq. 9 are also unknown. This difficulty can be circumvented by transmitting a known training sequence  $I_k$  to the receiver, which can be used to estimate the cross-correlation by substituting time averages for the ensemble averages given in eq. 9. After the initial training, which will require the transmission of a training sequence of some predetermined length that equals or exceeds the equalizer length, the equalizer coefficients that satisfy eq. 12 can be determined.

A simple recursive algorithm for adjusting the equalizer coefficients is:

$$c_j^{(k+1)} = c_j^{(k)} + \Delta e_k I_{k-j}^* \quad j = -K, \dots, K \quad (13)$$

Where  $C_j(k)$  is the value of the  $j$ th coefficient at time  $t=KT$ ,  $e_k = I_k - \hat{I}_k$  is the error signal at time  $t=KT$ , and  $\Delta$  is a scale factor that controls the rate of adjustment.

Following the training period, after which the equalizer coefficients have converged to their optimum values, the decisions at the output of the detector are generally sufficiently reliable so that they may be used to continue the coefficient adaptation process. This is called a decision-directed mode of adaptation. In such a case, the cross-correlations in eq. 13 involve the error signal  $\bar{e}_k = \bar{I}_k - \hat{I}_k$  and the detected output sequence  $\bar{I}_{k-j}$ ,  $j=-K, \dots, K$ . Thus in the adaptive mode, eq. 13 becomes

$$c_j^{(k+1)} = c_j^{(k)} + \Delta \bar{e}_k \bar{I}_{k-j}^* \quad j = -K, \dots, K \quad (14)$$

Figure 7 illustrates the zero-forcing equalizer in the training mode and the adaptive mode of operation. Thus, for the LE-ZF, the resulting cascade will again have the net flat response, and zero ISI, because  $C(z)F(z) = 1$ . There is a problem with the ZF approach when spectral nulls exist, since the gain of  $C(z)$  in those regions becomes very high. That results in excessive noise enhancement, since the channel noise will also be passing through the equalizer.

The ZF criterion aims to eliminate ISI completely. But because of the problems cited, it may be more desirable from an error-probability standpoint to allow some ISI if the upshot would be less noise enhancement and a smaller overall mean square error (MSE) at the equalizer output from the exact transmitted symbol. Such an approach is called the MSE criterion, resulting in LE-MSE. Next we will discuss MSE.

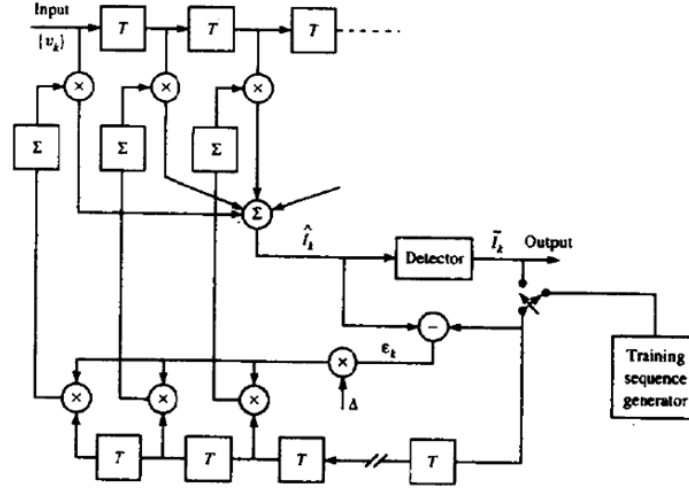


Figure 7: an adaptive zero-forcing equalizer

### 3.1.2 MINIMUM MEAN-SQUARE ERROR (MMSE)-EQUALIZER

The Minimum Mean-Square Error (MMSE) equalizer balances a reduction in ISI with noise enhancement. The MMSE-LE always performs as well as, or better than, the ZFE and is of the same complexity of implementation. Nevertheless, it is slightly more complicated to describe and analyze than is the ZFE. The MMSE-LE uses a linear time-invariant filter  $c_k$ , but the choice of filter impulse response  $c_k$  is different than the ZFE.

The MSE criteria for filter design does not ignore noise enhancement because the optimization of this filter compromises between eliminating ISI and increasing noise power. Instead, the filter output is as close as possible, in the Minimum MSE sense, to the data symbol  $I_k$ .

While the ideal criterion would be minimum detection-error probability, that is not a straightforward figure to determine. A reasonable design goal is to minimize the combined error of additive white Gaussian noise (AWGN) and ISI. In the MSE criterion, the tap weight coefficients  $C_j$  of the equalizer are adjusted to minimize the mean square value of the error.

$$e_k = I_k - \hat{I}_k \quad (15)$$

Where  $I_k$  is the information symbol transmitted in the  $K$ th signaling interval and  $\hat{I}_k$  is the estimate of that symbol at the output of the equalizer, i.e.

$$\hat{I}_k = \sum_{j=-K}^K c_j v_{k-j} \quad (16)$$

The performance index for the MSE criterion, denoted by  $J$ , is defined as:

$$\begin{aligned} J &= E|e_k|^2 \\ &= E|I_k - \hat{I}_k|^2 \end{aligned} \quad (17)$$

Let us consider the case of infinite number of taps. We shall derive the tap weight coefficients that minimize  $J$  when the equalizer has an infinite number of taps. In this case, the estimate  $\hat{I}_k$  is expressed as:

$$\hat{I}_k = \sum_{j=-\infty}^{\infty} c_j v_{k-j} \quad (18)$$

Substitution of eq. 18 into the expression for  $J$  given in eq. 17 and expansion of the result yields a quadratic function of the coefficients  $C_j$ . This function can be easily minimized with respect to the  $C_j$  to yield a set (infinite in number) of linear equations for the  $C_j$ .

Alternatively, the set of linear equations can be obtained by invoking the orthogonality principle in mean square estimation. That is, we select the coefficients  $C_j$  to render the error  $e_k$  orthogonal to the signal sequence  $V_{k-l}^*$  for  $-\infty < l < \infty$ . Thus,

$$E(e_k v_{k-l}^*) = 0, \quad -\infty < l < \infty \quad (19)$$

Substitution for  $e_k$  in eq. 19 yields

$$E[(I_k - \sum_{j=-\infty}^{\infty} c_j v_{k-j}) v_{k-l}^*] = 0$$

or equivalently,

$$\sum_{j=-\infty}^{\infty} c_j E(v_{k-j} v_{k-l}^*) = E(I_k v_{k-l}^*), \quad -\infty < l < \infty \quad (20)$$

To evaluate the moments in eq. 20, we use the expression for  $V_k$ :

$$v_k = \sum_{n=0}^L f_n I_{k-n} + \eta_k \quad (21)$$

where  $\eta_k$  is white gaussian noise,  $f_k$  is a set of tap coefficients of an equivalent discrete-time transversal filter having a transfer function  $F(z)$  and  $I_n$  is the sequence of information symbols. Thus, we obtain

$$\begin{aligned} E(v_{k-j} v_{k-l}^*) &= \sum_{n=0}^L f_n^* f_{n+l-j} + N_0 \delta_{lj} \\ &= \begin{cases} x_{l-j} + N_0 \delta_{lj} & (|l-j| \leq L), \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \quad (22)$$

and

$$E(I_k v_{k-l}^*) = \begin{cases} f_{-l}^* & (-L \leq l \leq 0), \\ 0 & \text{otherwise.} \end{cases} \quad (23)$$

Now, if we substitute eq. 22 and eq. 23 into eq. 20 and take the  $z$  transform of both sides of the resulting equation, we obtain

$$C(z)[F(z)F^*(z^{-1}) + N_0] = F^*(z^{-1}) \quad (24)$$

Therefore, the transfer function of the equalizer based on the MSE criterion is

$$C(z) = \frac{F^*(z^{-1})}{F(z)F^*(z^{-1}) + N_0} \quad (25)$$

When the noise-whitening filter is incorporated into  $C(z)$ , we obtain an equivalent equalizer having the transfer having the transfer function.

$$\begin{aligned} C'(z) &= \frac{1}{F(z)F^*(z^{-1}) + N_0} \\ &= \frac{1}{X(z) + N_0} \end{aligned} \quad (26)$$

We observe that the only difference between this expression for  $C'(z)$  between this expression for  $C(z)$  and the one based on the peak distortion criterion is the noise spectral density factor  $N_0$  that appears in eq. 26. when  $N_0$  is very small in comparison with the signal, the coefficients that minimize the peak distortion  $D(c)$  are approximately equal to the coefficients that minimize the MSE performance index  $J$ . that is in the limit as  $N_0 \rightarrow 0$ , the two criterion yield the same solution for the tap weights. Consequently, when  $N_0=0$ , the minimization of the MSE results in complete elimination of the ISI. In general, when  $N_0 \rightarrow 0$ , there is both residual ISI and additive noise at the output of the equalizer.

A measure of the residual ISI and additive noise is obtained by evaluating the minimum value of  $J$ , denoted by  $J_{min}$ , when the transfer function  $C(z)$  of the equalizer is given by eq. 25. since  $J = E|e_k|^2 = E(e_k I_k^*) - E(e_k \hat{I}_k^*)$ , and since  $E(e_k \hat{I}_k^*) = 0$  by virtue of the orthogonality conditions given in eq. 19, it follows that

$$\begin{aligned} J_{min} &= E(e_k I_k^*) \\ &= E|I_k|^2 - \sum_{j=-\infty}^{\infty} c_j E(v_{k-j} I_k^*) \\ &= 1 - \sum_{j=-\infty}^{\infty} c_j f_{-j} \\ &= \frac{N_0}{N_0 + 1} \end{aligned} \quad (27)$$

Let us now turn our attention to the case in which the transversal equalizer spans a finite time duration. The output of the equalizer in the  $K$ th signaling interval is

$$\hat{I}_k = \sum_{j=-K}^K c_j v_{k-j} \quad (28)$$

The MSE for the equalizer having  $2K+1$  taps, denoted by  $J(K)$ , is

$$J(K) = E|I_k - \hat{I}_k|^2 = E|I_k - \sum_{j=-K}^K c_j v_{k-j}|^2 \quad (29)$$

Minimization of  $J(K)$  with respect to the tap weights  $C_j$  or, equivalently, forcing the error  $e_k = I_k - \hat{I}_k$  to be orthogonal to the signal samples  $V_{j-l}^*$ ,  $|l| \leq K$ , yields the following set of simultaneous equations:

$$\sum_{j=-K}^K c_j \Gamma_{lj} = \xi_l \quad l = -K, \dots, -1, 0, 1, \dots, K \quad (30)$$

Where

$$\Gamma_{lj} = \begin{cases} x_{l-j} + N_0 \delta_{lj} & (|l-j| \leq L), \\ 0 & (\text{otherwise}) \end{cases} \quad (31)$$

And

$$\xi_l = \begin{cases} f_{-l}^* & (-L \leq l \leq 0), \\ 0 & (\text{otherwise}) \end{cases} \quad (32)$$

It is convenient to express the set of linear equations in matrix form. Thus,

$$\Gamma C = \xi \quad (33)$$

where  $C$  denotes the column vector of  $2K + 1$  tap weight coefficients,  $\Gamma$  denotes the  $(2K + 1) \times (2K + 1)$  Hermitian covariance matrix with elements  $\Gamma_{lj}$ , and  $\xi$  is a  $(2K + 1)$  dimensional column vector with elements  $\xi_l$ . The solution of eq. 33 is:

$$C_{opt} = \Gamma^{-1} \xi \quad (34)$$

The solution of  $C_{opt}$  can be efficiently performed by use of the Levinson-Durbin algorithm.

Alternatively, an iterative procedure that avoids the direct matrix inversion may be used to compute  $C_{opt}$ . Probably the simplest iterative procedure is the method of steepest descent, in which one begins by arbitrarily choosing the vector  $C$ . This yields the basic LMS (least-mean-square) algorithm for recursively adjusting the tap weight coefficients of the equalizer first proposed by Widrow and Hoff (1960).

$$\hat{C}_{k+1} = \hat{C}_k + \Delta e_k V_k^* \quad (35)$$

Here it has been assumed that the receiver has knowledge of the transmitted information sequence in forming the error signal between the desired symbol and its estimate. Such knowledge can be made available during a short training period in which a signal with a known information sequence is transmitted to the receiver for initially adjusting the tap weights. The length of this sequence must be at least as long as the length of the equalizer so that the spectrum of the transmitted signal adequately covers the bandwidth of the channel being equalized.

A practical scheme for continuous adjustment of the tap weights may be either a decision-directed mode of operation in which decisions on the information symbols are assumed to be correct and used in place of  $I_k$  in forming the error signal  $e_k$ , or one in which a known pseudo-random probe sequence is inserted in the information-bearing signal either additively or by interleaving in time and the tap weights adjusted by comparing the received probe symbols with the known transmitted probe symbols. In the decision directed mode of operation, the error signal becomes  $\bar{e}_k = \bar{I}_k - \hat{I}_k$ , where  $\bar{I}_k$  is the decision of the receiver based on the estimate  $\hat{I}_k$ . as long as the receiver is operating at low error rates, an occasional error will have a

negligible effect on the convergence of the algorithm.

If the channel response changes, this change is reflected in the coefficients  $f_k$  of the equivalent of the equivalent discrete-time channel model. It is also reflected in the error signal  $e_k$ , since it depends on  $f_k$ . Hence, the tap weights will be changed according to eq. 35 to reflect the change in the channel. A similar change in the tap weights occurs if the statistics of the noise or the information sequence change. Thus, the equalizer is adaptive. Minimum value of  $J(K)$  is:

$$\begin{aligned} J_{min}(K) &= 1 - \sum_{j=-k}^0 c_j f_{-j} \\ &= 1 - \xi^{t*} \Gamma^{-1} \xi \end{aligned} \quad (36)$$

## 4 NON LINEAR EQUALIZERS

Due to the increasing demand for high speed data transmission in communication systems, channels have inevitably become much noisier and more crowded than ever. This has consequently resulted in signal distortion at the receiver end. Generally, equalizers are used at the receiver end to recover distorted signals. Traditionally, the most commonly used strategy for reducing ISI is the linear finite-duration impulse response (FIR) filter based equalizer. However, the performance of linear channel equalizers employing linear filters with an FIR or lattice structure is quite poor, especially when the channel encounters severe nonlinear distortion. In this case, we need to use nonlinear channel equalizers in order to attain a lower bit error rate (BER), lower mean squared error (MSE), and higher convergence rate than is possible with linear equalizers.

Three very effective nonlinear methods have been developed which offer improvements over linear equalization techniques. These are:

1. Decision Feedback Equalization (DFE)
2. Maximum Likelihood Symbol Detection
3. Maximum Likelihood Sequence Estimation (MLSE)

Other Examples of non linear equalizers

- SCFNN (self-constructing fuzzy neural network)
- Gaussian processes for classification (GPC)
- other neural network Based Nonlinear Channel Equalizer

### 4.1 DECISION FEEDBACK EQUALIZATION (DFE)

As the name implies, decision feedback equalization (DFE) uses decisions on the symbol stream to deal with distorted sequences. The idea behind decision feedback equalization is that once information symbols have been detected and decided upon, the ISI that it induces on future symbols can be estimated and subtracted out before detection of subsequent symbols.

The logic behind it is as follows: If you know the correct answer to a sequence of symbols, and you know that the channel sums those past symbols into a symbol currently being detected,



then you just need to subtract out what you know about those symbol values. Also, the information can be used to predict future ISI effects to mitigate degradation from symbols on both sides of the observed symbol.

First, however, you have to have confidence in the detection performance, or the feedback information is in error. Because the DFE is a feedback loop, the "catch" with the DFE is the need to worry about incorrect decisions, which can lead to error propagation. When operating as it is supposed to, the DFE provides significant advantages in terms of noise enhancement for severe ISI. That's because the spectral-null issue exists no longer and because correct decisions result in noise-free feedback.

A decision-feedback equalizer (DFE) is a nonlinear equalizer that employs previous decisions as training sequences. The detected symbols (or the output of the feedback filter) is subtracted from the output of the equalizer for adaptation. While better in performance in general, a major drawback of DFE is its potential catastrophic behavior due to error propagation. In practical, DFE is often combined with training sequence based equalization for robustness. Unfortunately, the derivation of the transfer function of the DFE is more complex than the intuitiveness of the LE described above.

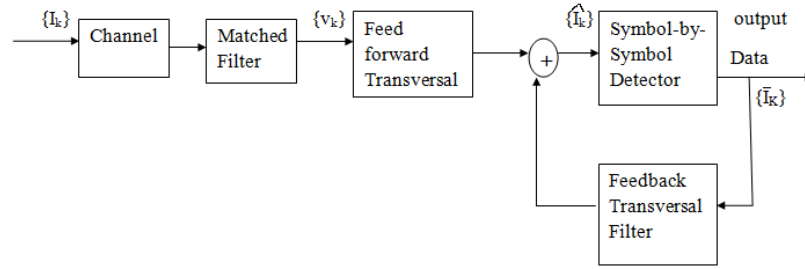


Figure 8: decision feedback equalizers

The decision feedback equalizer consists of two filters, a feedforward filter and a feedback filter. Both have taps spaced at the symbol interval  $T$ . The input to the forward section is the received signal sequence  $V_k$ . In this respect, the feedforward filter is identical to the linear transversal equalizer. The feedback filter has as its input the sequence of decisions on previously detected symbols. Functionally, the feedback filter is used to remove that part of the ISI from the present estimate caused by previously detected symbols. The equalizer output can be expressed as:

$$\hat{I}_K = \sum_{j=-K_1}^0 c_j v_{k-j} + \sum_{j=1}^{K_2} c_j \bar{I}_{k-j} \quad (37)$$

Where  $\hat{I}_k$  is an estimate of the  $K$ th information symbol,  $C_j$  are the tap coefficients of the filter, and  $[\bar{I}_{K-1}, \dots, \bar{I}_{K-K_2}]$  are previously detected symbols. The equalizer is assumed to have  $K_1 + 1$  taps in its feedforward section and  $K_2$  in its feedback section. It should be observed that this equalizer is nonlinear because the feedback filter contains previously detected symbols  $\bar{I}_k$ .

Both the peak distortion criterion and the MSE criterion result in a mathematically tractable optimization of the equalizer coefficients. Since the MSE criterion is more prevalent in practice, we focus our attention on it. Based on the assumption that previously detected symbols in the feedback filter are correct, the minimization of MSE leads to the following set of linear equations for the coefficients of the feedforward filter:

$$\sum_{j=-K_1}^0 \psi_{lj} c_j = f_{-l}^* \quad l = -K_1, \dots, 0 \quad (38)$$

Where

$$\psi_{lj} = \sum_{m=0}^{-l} f_m^* f_{m+l-j} + N_0 \delta_{lj}, \quad l, j = -K_1, \dots, 0 \quad (39)$$

The coefficients of the feedback filter of the equalizer are given in terms of the coefficients of the feedforward section by the following expression:

$$c_k = - \sum_{j=-K_1}^0 c_j f_{k-j}, \quad K = 1, 2, \dots, K_2 \quad (40)$$

As in the case of the linear adaptive equalizer, the coefficients of the feedforward filter and the feedback filter in a decision-feedback equalizer may be adjusted recursively, instead of inverting a matrix as implied by eq. 38. Based on the minimization of the MSE at the output of the DFE, the steepest-descent algorithm takes the form

$$C_{k+1} = C_k + \Delta E(e_k V_k^*) \quad (41)$$

Where  $C_k$  is the vector of equalizer coefficients in the  $K$ th signal interval,  $E(e_k V_k^*)$  is the cross-correlation of the error signal  $e_k = I_k - \hat{I}_k$  with  $V_k$  and  $V_k = [v_{k+K_1}, \dots, v_k, I_{k-1}, \dots, I_{k-K_2}]^t$ , representing the signal values in the feedforward and feedback filters at time  $t=KT$ . The MSE is minimized when the cross-correlation vector  $E(e_k V_k^*) = 0$  as  $k \rightarrow \infty$ .

Since the exact cross-correlation vector is unknown at any time instant, we use as an estimate the vector  $e_k V_k^*$  and average out the noise in the estimate through the recursive equation

$$\hat{C}_{k+1} = \hat{C}_k + \Delta e_k V_k^* \quad (42)$$

This is the LMS algorithm for the DFE. As in the case of a linear equalizer, We may use a training sequence to adjust the coefficients of the DFE initially. Upon convergence to the optimum coefficients (minimum MSE), we may switch to a decision-directed mode where the decisions at the output of the detector are used in forming the error signal  $e_k$  and fed to the feedback filter, this is the adaptive mode of the DFE. In this case, the recursive equation for adjusting the equalizer coefficient is

$$\bar{C}_{k+1} = \bar{C}_k + \Delta \bar{e}_k V_k^* \quad (43)$$

Where  $\bar{e}_k = \bar{I}_k - \hat{I}_k$  and  $V_k = [v_{k+K_1}, \dots, v_k, \bar{I}_{k-1}, \dots, \bar{I}_{k-K_2}]^t$ . If we compare decision feedback equalizer with the linear equalizer, a considerable gain in performance can be

achieved relative to the linear equalizer by the inclusion of the decision feedback section, which eliminates the ISI from previously detected symbols.

The decision feedback equalizer yields a significant improvement in performance relative to the linear equalizer having the same number of taps and there is still a significant degradation in performance of the decision feedback equalizer due to the residual ISI, especially on channels with severe distortion.

## 4.2 MAXIMUM LIKELIHOOD SEQUENCE ESTIMATION

let us assume symbol  $s_k$  is transmitted from a finite set of possible transmit vectors  $\mathbf{s}$ . given a receive vector  $\mathbf{r}$ , which is actually not the same as  $s_k$  due to the time varying channel and the additive white gaussian noise, we ask for the most probable transmit vector  $\hat{\mathbf{s}}$ , that is, the one for which the conditional probability  $P(\mathbf{s}/\mathbf{r})$  that  $\mathbf{s}$  was transmitted given that  $\mathbf{r}$  has been received becomes maximal. the estimate of the symbol is:

$$\hat{\mathbf{s}} = \arg \max_{\mathbf{s}} P(\mathbf{s}/\mathbf{r}) \quad (44)$$

The receiver technique described above, which finds the most likely transmit vector, is called maximum likelihood sequence estimation (MLSE). Maximum likelihood sequence estimation (MLSE) is a nonlinear estimation technique that replaces the equalizing filter with a MLSE estimation.

The MSE based linear equalizers described previously in section 2.1.2 are optimum with respect to the criterion of minimum probability of symbol error when the channel does not introduce any amplitude distortion. Yet this is precisely the condition in which an equalizer is needed for a mobile communications link. this limitation on MSE based equalizers led researchers to investigate optimum or nearly optimum nonlinear structures. These equalizers use various forms of the classical maximum likelihood receiver structure. Using a channel impulse response simulator within the algorithm, the MLSE tests all possible data sequences (rather than decoding each received symbol by itself), and chooses the data sequence with the maximum probability as the output. An MLSE usually has a large computational requirement, especially when the delay spread of the channel is large. Using the MLSE as an equalizer was first proposed by Forney in which he setup a basic MLSE estimator structure and implemented it with the Viterbi algorithm.

The MLSE can be viewed as a problem in estimating the state of a discrete time finite state machine, which in this case happens to be the radio channel with coefficients  $f_k$  and with a channel state which at any instant of time is estimated by the receiver based on the  $L$  most recent input samples. Thus the channel  $M^L$  states, where  $M$  is the size of the symbol alphabet of the modulation. That is, an  $M^L$  trellis is used by the receiver to model the channel over time. The Viterbi algorithm then tracks the state of the channel by the paths through the trellis and gives at stage  $k$  a rank ordering of the  $M^L$  most probable sequences terminating in the most recent  $L$  symbols. The block diagram of a MLSE receiver based on the DFE is shown in figure 9. The MLSE is optimal in the sense that it minimizes the probability of a sequence error. The MLSE requires knowledge of the channel characteristics in order to compute the metrics for making decisions. The MLSE also requires knowledge of the statistical distribution of the noise corrupting the signal. Thus, the probability distribution of the noise determines the

form of the metric for optimum demodulation of the received signal. Notice that the matched filter operates on the continuous time signal, whereas the MLSE and channel estimator rely on discretized (nonlinear) samples.

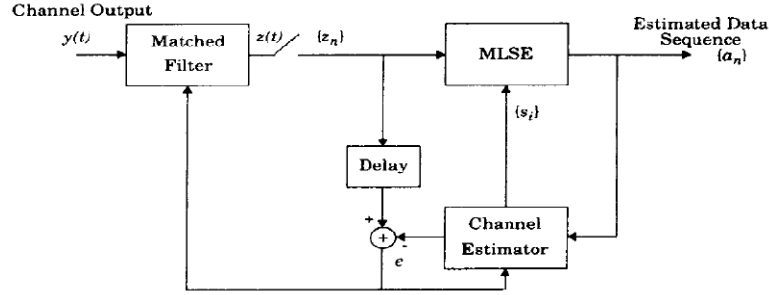


Figure 9: The structure of a maximum likelihood sequence estimator (MLSE) with an adaptive matched filter

## 5 BLIND EQUALIZERS

Identification of nonlinear systems is of considerable practical interest, since many real-life systems exhibit nonlinear characteristics. Examples of such systems are encountered in satellite and microwave channels with nonlinear amplifiers, underwater and magnetic recording channels, and physiological modeling.

In the conventional zero-forcing or minimum MSE equalizers, we assumed that a known training sequence is transmitted to the receiver for the purpose of initially adjusting the equalizer coefficients. However, there are some applications, such as multipoint communication networks, where it is desirable for the receiver to synchronize to the received signal and to adjust the equalizer without having a known training sequence available. Equalization techniques based on initial adjustment of the coefficients without the benefit of a training sequence are said to be blind or self-recovering Equalizers. Blind equalization uses a type of adaptive equalizer that doesn't use a training (i.e., known) sequence to characterize the channel. The term blind is used because the equalizer performs the equalization blindly on the data without a reference signal. Instead the blind equalizer relies on the knowledge of signals structure and its statistics to perform the equalization.

In digital communications, blind equalization approaches are important for the following reasons: No training input and no interruption of the transmission are necessary to equalize the channel. Therefore, for channels exhibiting multipath phenomena, changing characteristics, or high data rates, blind methods are attractive. Blind identification and equalization of channels is potentially useful since no training sequence needs to be transmitted and, hence, there is no reduction in the effective data rate. Identification of nonlinear dynamics is also a subject of interest in biomedical research, since many physiological signals undergo nonlinear transformations. For example, the auditory nervous system includes memoryless nonlinearities and the response of photoreceptors is modeled as a Volterra series expansion. Blind identification of

such systems is attractive in cases where the design of the experiment (input sequence) may be difficult, or the input to the system is not accessible.

Among some algorithms of blind equalizers like CMA (Constant Modulus Algorithm), GSA (General Sato Algorithm), SGA (Stop and Go Algorithm), SRCA (Sign Reduced Constellation Algorithm, SRCA) etc., Stop and Go is one of the most important algorithms. One of the major disadvantage of Blind Equalizers is that all blind equalizers converge very slowly. However, one can reduce the convergence time by employing some other techniques. In this term paper, we simulate the CMA algorithm of Blind Equalization.

A Blind Equalizer, as opposed to a data-trained equalizer, is able to compensate amplitude and delay distortions of a communication channel using only the channel output samples and the knowledge of the basic statistical properties of the data symbols. Sato was the one who first introduced the concept of blind equalizers in 1975, since then blind equalization has attracted significant scientific interest due to its potentials in terms of Overhead reduction and Simplification of point to multipoint communication.

## 5.1 BLIND EQUALIZATIONS ALGORITHMS

Sato was the first who introduced the idea of blind equalization for multilevel pulse amplitude modulation, and after it Godard combined Sato idea with another algorithm and obtain a new blind equalization scheme for QAM data transmission. Sato proposed algorithm which was designed only for real valued signal and PAM. However, its complex valued extension is straightforward which was derived by Godard.

### 5.1.1 SATO ALGORITHM

Satos pioneering contribution in 1975, describe first blind equalization algorithm proposed, which was designed for real-valued signals only. However, its extension to complex-valued signals and QAM is straightforward. Sato algorithm dedicated to real valued signal  $z(n)$ , which uses the following cost function:

$$J(n) = E[(z(n) - \gamma^2)] \quad (45)$$

Where  $\gamma$  is the sato's scaling coefficient and  $E[\cdot]$  represents the expectation over all possible transmitted data sequence, and  $z(n)$  is the equalizer output. It is clear that this cost function is forcing the absolute value of the equalized signal to a fixed value  $\gamma$ . For multilevel constellations the minimization of the Sato cost function of above equation, may not seem to lead to correct update of the equalizer taps at each iteration. Sato's coefficient update equation is given as:

$$C(n+1) = C(n) - \mu y(n) \epsilon^{sato}(n) J(n) = E[(z(n) - \gamma^2)] \quad (46)$$

Where  $y(n)$  is the input to the equalizer,  $C(n)$  is tap coefficient at time  $n$  and  $\epsilon^{sato}(n)$  is the Sato's error defined as:

$$\epsilon^{sato}(n) = z(n) - \gamma \text{sgn}(z(n)) \quad (47)$$

so taps of equalizer are updated according to the equation defined above. As it can be seen from above equation, this algorithm uses only the sign of equalized output values  $z(n)$  in order to

update equalizer coefficients  $C(n)$ . Setting the value of  $\gamma$  in above equation is very important, since it actually directs the signal  $z(n)$  to the point of its convergence that is to the original constellation points. A way of achieving this is by constraining the mean value of the error update term of above equation is zero. Therefore, the optimum value is set for  $\gamma$  in the minimum mean squared error sense. The optimum value of  $\gamma$  set by Sato in order to achieve minimum MSE is given by:

$$\gamma = \frac{E[a(n)^2]}{E[abs(a(n))]} \quad (48)$$

which is only for real valued signals, where  $a(n)$  is the signal to be transmitted.

### 5.1.2 CONSTANT MODULUS ALGORITHM (CMA)

Among the blind channel equalization schemes, constant modulus algorithm (CMA), due to its robustness and easy implementation, is an excellent choice to correct the distortions caused by transmission channels. Godard algorithm which he developed for complex valued signal is the most popular scheme for blind equalization of QAM signals. Figure 10 shows output of an adaptive CMA for 3000 received noisy signal. The CMA attempts to minimize the constant

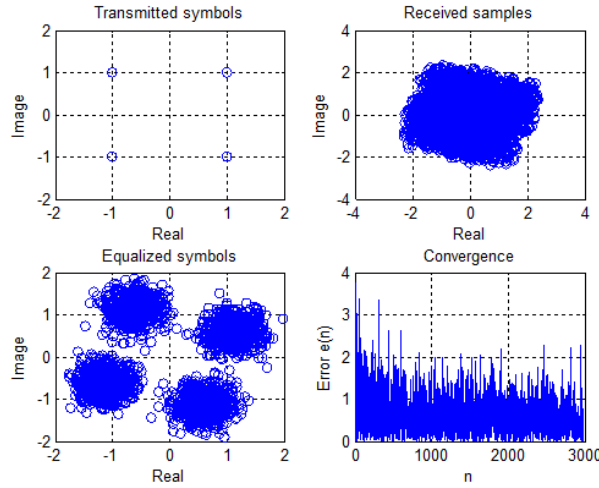


Figure 10: Adaptive CMA output

modulus cost function  $J_{CM}$ . CMA adjusts the taps of equalizer in an attempt to minimize the difference between samples squared magnitude and Godard dispersion constant  $R_2$ , which depends only on input data symbols  $a_k$ . So value of  $J_{CM}$  depends on difference between squared magnitude of received samples  $z(n)$  and the Godard dispersion constant  $\gamma$ .

$$J_{CM} = E[(abs(z(n))^2 - R_2)^2] \quad (49)$$

Where  $z(n)$  is the equalizer output at time  $n$ . The equalizer coefficient update equation in CMA uses a gradient descent to minimize  $J_{CM}$ . The equation is given by Godard as,

$$C(n+1) = C(n) - \mu y(n) z(n) [(abs(z(n))^2 - R_2)] \quad (50)$$

Where in order to find out the value of  $R_2$ , Godard uses the exactly same method as used by Sato, to obtain the value of  $R_2$ ,

$$R_2 = \frac{E[a(n)^4]}{E[abs(a(n))^2]} \quad (51)$$

## 6 TIME DOMAIN EQUALIZERS

DUE TO THE recent interest in discrete multitone (DMT) modulation systems, the design of time-domain equalizers (TEQs) or channel-shortening equalizers has received much attention. One popular method for combating ISI in DMT modulation is to use a guard sequence, called the cyclic prefix (CP). The CP is prepended to each symbol. But due to the long impulse response of typical channels encountered in DMT systems, TEQs are required to shorten the overall channel response to one sample more than the length of the cyclic prefix used. Without a TEQ, unwanted intercarrier interference (ICI) and intersymbol interference (ISI) occur.

Time domain equalizer design methods can be categorized into three major approaches: minimizing Mean Squared Error (MSE), maximizing Shortening SNR (SSNR), and maximizing channel capacity. The Minimum MSE (MMSE) approach is the first application of channel shortening to multicarrier systems. Adaptive MMSE design methods are commonly used in practical systems. Maximizing SSNR is equivalent to minimize the energy of the component of the channel impulse response that cause ISI. Neither the MMSE nor the Maximum SSNR (MSSNR) methods attempt to maximize channel capacity directly. Al-Dhahir and Cioffi propose the Maximum Geometric SNR (MGSNR) method to shorten the channel impulse response while maximizing an approximation to the channel capacity.

### 6.1 MINIMUM MEAN SQUARED ERROR (MMSE) DESIGN

The idea behind the MMSE TEQ design method may be explained by Fig. 10. The structure consists of an FIR equalizer in cascade with the channel and a parallel branch that consists of a delay and an FIR filter with a target impulse response (TIR). The goal in the MMSE design of the vector of TEQ taps  $\mathbf{w}$  is to minimize the mean square of the error between the output of the equalizer and the output of the TIR.

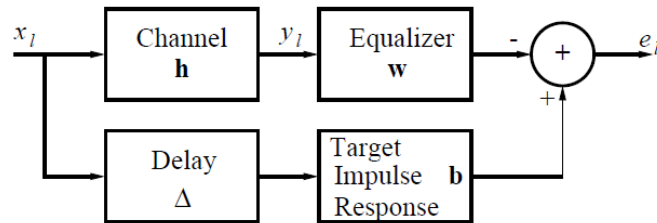


Figure 11: Block diagram of the minimum mean-squared error (MMSE) equalizer. The equalizer is an FIR filter with impulse response  $\mathbf{w}$ . The bottom path does not physically exist, but is part of the design method.

The optimum value of the tap  $w_{opt}$  of the equalizer can be obtained from:

$$b^T R_{xy} = w_{opt}^T R_{yy} \quad (52)$$

where  $R_{xy} = E[x_k y_k^T]$ ,  $R_{yy} = E[y_k y_k^T]$  and

$$b = b_{opt} = \frac{R_{\Delta}^{-1} e_{i_{opt}}}{R_{\Delta}^{-1}(i_{opt}, i_{opt})} \quad (53)$$

where  $e_i$  is the  $i^{th}$  unit vector.

$$R_{\Delta} = S^T R_{x/y} S \quad (54)$$

and

$R_{\Delta}^{-1}(i, i)$  is the  $i_{th}$  element in the diagonal of the  $R_{\Delta}^{-1}$  matrix

and

$$S = [0_{(v+1)x\Delta} I_{(v+1)x(v+1)} 0_{(v+1)x(N_w+L-\Delta-v-1)}]^T \quad (55)$$

where  $0_{m \times n}$  is a  $m \times n$  matrix of zeros,  $I_{n \times n}$  is an  $n \times n$  identity matrix, and  $v+1$  is the number of elements in  $b$ ,  $N_w$  is the number of symbols and  $L$  is the number of taps.

$$i_{opt} = \arg \max_{0 \leq i \leq v} R_{\Delta}^{-1}(i, i) \quad (56)$$

The MMSE design method formulates the square of the difference between the TIR (target impulse response) SIR (shortened impulse response) as the error and minimizes it. The method minimizes the difference between the TIR and SIR both inside and outside the target window. In fact, the difference between the TIR and SIR inside the target window does not cause ISI. Both the TIR and SIR inside the target window have higher amplitudes, which means that difference inside the target window might contribute more to the MSE than the difference outside.

The MMSE design method maximizes the SNR at the TEQ output. The equalizer frequency response tends to be a narrow bandpass filter placed at a center frequency, which has high SNR. The equalizer increases the output SNR by filtering out the low SNR regions of the channel frequency response. This would ensure that only the ISI is minimized instead of the combination of noise and ISI.

Since the MMSE method in general cannot force the error to become exactly zero, some residual ISI will remain. To maximize channel capacity, the residual ISI should be placed in frequency bands with high channel noise. This ensures that the residual ISI would be small compared to the noise and the effect on the SNR would be negligible. The MMSE design method does not have a mechanism to shape the residual ISI in frequency. Therefore, it is not optimal in the sense of maximizing channel capacity.

Another drawback of the MMSE design method is the deep notches in the frequency response of the designed TEQ. The subchannels in which a notch appears cannot be used for data transmission because the gain in the subchannel is too small.



## 6.2 MAXIMUM SHORTENING SNR DESIGN

Seeing the TEQ design problem as a channel shortening problem rather than a equalization problem, Melsa, Younce, and Rohrs propose a different solution. The goal is to find a TEQ that minimizes the energy of the SIR outside the target window, while keeping the energy inside constant. They have a reasonable assumption that the channel impulse response is known.

The MSSNR method minimizes the part of the SIR that causes ISI. If the energy outside the target window were zero, then the channel would be perfectly shortened and ISI would be totally eliminated. The solution which gives zero energy outside the target window is optimum also in the sense of maximum channel capacity since this is the case where ISI is totally canceled. In practice, however, this optimum solution cannot be achieved. For this case, the MSSNR solution is not guaranteed to yield maximum channel capacity solution. The reason is similar to that of the MMSE design method; i.e., the residual ISI power cannot be placed in high noise regions in the frequency domain. The method only minimizes the energy outside the target window and does not care where the residual ISI lies in frequency. A second problem with the MSSNR design approach is the computation complexity due to the eigenvalue and Cholesky decompositions.

## 6.3 MAXIMUM GEOMETRIC SNR DESIGN

In a communication system, the ultimate goal is to reach optimum channel capacity. Al-Dhahir and Cioffi introduced the idea of a TEQ design method to optimize channel capacity.

The maximum geometric SNR (MGSNR) method maximizes a channel capacity cost function that is based on a geometric SNR definition as:

$$GSNR = \Gamma \left( \left[ \prod_{i \in S} \left( 1 + \frac{SNR_i^{EQ}}{\Gamma} \right) \right]^{1/N} - 1 \right) \quad (57)$$

where  $i$  is the subchannel index  $S$  is the set of the indices of the used  $\bar{N}$  subchannels out of the  $N/2 + 1$  subchannels  $\Gamma$  is the SNR gap for achieving Shannon channel capacity and is assumed to be constant over all subchannels, and,

$$SNR_i^{EQ} = \frac{S_{x,i} |B_i|^2}{S_{n,i} |W_i|^2} \quad (58)$$

where  $S_{x,i}$  is signal power  $S_{n,i}$  is the noise power and  $B_i$  and  $W_i$  are the gains of  $b$  and  $w$  in the  $i_{th}$  subchannel respectively.

All of the subchannels act together like  $\bar{N}$  AWGN channels with each channel having an SNR equal to the GSNR. Therefore maximizing the GSNR is equivalent to maximizing the channel capacity. But, MGSNR method is not optimum in the sense of maximizing channel capacity due to many inaccurate approximations and assumptions.

TIR  $w_{opt}$  is found using:

$$w_{opt}^T = b_{opt}^T R_{xy} R_{yy} \quad (59)$$

where  $R_{xy}$  and  $R_{yy}$  are the channel input-output cross-correlation and channel output autocorrelation matrices, respectively.

MGSNR TEQ has drawbacks. These are its derivation is based on a subchannel SNR definition  $SNR_i^{EQ}$  that does not include the effect of ISI and it depends on the parameter  $MSE_{max}$  which has to be tuned for different channels and it assumes that  $b$  and  $w$  are independent.

In conclusion, MMSE and MGSNR methods have more disadvantages compared to MSSNR methods. However, none of the MMSE and MSSNR methods optimize channel capacity and the MGSNR methods optimize only an approximation to the channel capacity.

## 7 INTERFERENCE CANCELLATION

Interference Cancellation techniques are any technique or combination of techniques that allow an existing receiver to operate with higher levels of interference. The motivation of improving a receiver's performance in co-channel interference is to increase the spectrum efficiency of a system usually by allowing a greater geographical re-use of frequencies (although in the case of CDMA systems improved spectrum efficiency usually comes by allowing greater use of the orthogonal code space). It is a general principle that a communication system should be designed to avoid interference in the first place, either through network planning or with effective radio resource management and medium access control. However, increasing use of license exempt spectrum means that interference is unavoidable and so the radio system must not only avoid interference but also mitigate against its presence.

A key point is that the strategies employed to mitigate interference are very dependent on the source of the interference and its relationship to the wanted signal. The use of interference cancellation techniques can also make systems more reliable, either by design or by incorporating additional signal processing into existing systems, where a retrofit is practical. Interference cancellation techniques have long been applied to radio systems in conjunction with adaptive arrays, primarily in military applications, but also in some civil applications. Adaptive arrays exploit the spatial separation of the wanted and interfering signals to spatially filter or cancel the interfering signals. In some applications the use of an antenna array is prohibitive, for example in a mobile radio handset, and modifications to detection techniques have been devised that permit interference cancellation using a single antenna.

Recently so called Single Antenna Interference Cancellation (SAIC) techniques have been introduced into the GSM standard where modified handsets can operate at lower signal to interference ratios than unmodified handsets, permitting greater frequency re-use and hence greater capacity for a fixed amount of spectrum.

### 7.1 SURVEY OF IC TECHNIQUES

This section reviews a diverse variety of IC techniques that have appeared in the literature.

### 7.1.1 TYPES OF INTERFERENCE

*Intersymbol Interference (ISI)*: results from time dispersion in a real channel. The non-flat frequency response of the channel will cause the pulse to spread out or disperse so that the distorted pulse has a greater duration at the receiver than was transmitted. When a stream of such pulses is sent, these are distinct entities (symbols) at the transmitter while at the receiver, each pulse will overlap with others. Equalizers are designed to reverse the effects of ISI. This form of interference was not considered further within this section.

*Co-channel Interference (CCI)*: arises when two or more signals overlap in the frequency domain. This arises in cellular systems where a given frequency band is used in multiple cells according to the frequency reuse scheme.

*Multiple Access Interference (MAI)*: arises in a multiuser system where several different users share the same bandwidth. Early multiple access systems avoided MAI by employing orthogonal signals for different users (time division and frequency division multiplexing). More recent systems take a different approach. In a spread-spectrum multiple access system, all users share the same bandwidth without time division multiplexing. In this type of system, the interaction between different users' signals and the ability of the receiver to recover a given user's data from the signal multiplex are key aspects of the design process. Although the signals belonging to different users may be orthogonal when time aligned, the uplink (mobile transmitter to base station receiver) is typically asynchronous so that radio frequency signals belonging to individual users are non-orthogonal. Further, multipath interference will cause loss of orthogonality.

While it is useful to classify the interference according to its source and relation to the wanted signal, in applying IC techniques the effectiveness will also depend on the interfering signals characteristics, as now described.

The term *Frequency Localized Interference (FLI)* will be used in a relative sense to designate any form of interference whose bandwidth is much less than that of the desired signal. The term narrowband interference is often used in the literature on spread-spectrum communications.

*Time Localized Interference (TLI)* is a type of interference that arises from impulsive sources such as pulse jammers, car ignition systems, packet burst systems and electrical storms. This type of interference can be wideband or even pan-spectral because impulse functions, being of short duration, have a very broad spectrum. The term impulsive interference is a common synonym.

For some systems, the interference can be both Time and Frequency Localised (TFLI), where it traces trajectories in the time-frequency plane and so both dimensions are required to describe its behaviour. Examples include chirped or frequency hopped systems.

### 7.1.2 CLASSIFICATION OF IC TECHNIQUES

A useful way of classifying techniques is by the type of structure involved. This covers both DSP structures and hardware. For the most part, this classification also provides an efficient logical organization because each category is associated with a body of theoretical principles

that can be unified into a coherent entity. This classification is used as the structural basis for the main body of this section: Filter based, Filter based, Transform methods, Joint detection/Multi-user detection, Cyclostationarity, Neural networks, Higher order statistics and source separation, Spatial processing, Analogue techniques.

**FILTER BASED:** Filter based methods synthesise a filter that provides a desired frequency response function. While non-linear filter structures are possible, this section focuses on linear approaches, non-linear methods are discussed elsewhere (for example later in this Section on Neural Networks). The optimum filter, as derived by Wiener, is based on prior knowledge of the spectrum of the wanted and interfering signals. The filter enhances regions of the spectrum with high SNR and suppresses those with low SNR. In many applications the assumption that noise is additive white Gaussian noise (AWGN) is made, and a matched filter is consequently derived. The most common filter architecture is a transversal filter, with the difference between approaches being how the filter coefficients are derived. While the Wiener filter may exist as a theoretical optimum at any given point in time, the covariance functions that define it are not generally known a priori. The task of an adaptive algorithm is to maintain close to optimum filter weights on the basis of the received signal and possibly other information. The best known methods for adaptive linear filters are the least mean square (LMS) algorithm and recursive least squares (RLS) algorithms.

**TRANSFORM METHODS:** It is well known that a signal can be analysed and processed by decomposing it into frequency domain components and, for some operations, processing in the frequency domain can be more efficient. Frequency domain processing is an alternative approach to IC, as will be shown in this section. The Fourier transform as a means to represent

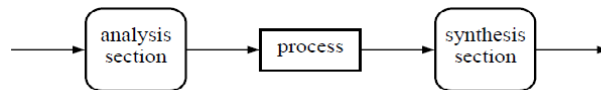


Figure 12: Transform Space Processing

a signal in a different basis is not the only transform that can be used. In fact, a number of other transforms are widely used and each has its own particular merits. The transform appropriate for a given application depends on the demands and constraints presented by that application, such as the signal type and features to be identified, computational complexity and so on. The established signal decomposition techniques fall into a number of categories: block transforms, filter banks and wavelets.

Once in the transform domain, appropriate processing for IC can be applied. Although there is a wide variety of transforms available, the concept underlying transform-based IC is the same, Figure 12. The (analysis) transform should separate the wanted and interfering signals in the new basis such that removal of a subset of the transform components leaves either the wanted or interfering signals, with minimal distortion. The inverse (synthesis) transform will leave either the wanted signal or the interfering signal, which can then be subtracted from the received signal. This process is commonly known as excision, and IC is achieved with a combination of a transformation and excision process. The chosen combination is most effective when it

is chosen using knowledge of the wanted and interfering, such that the signals are maximally separated in the transform domain.

**JOINT DETECTION/MULTI-USER DETECTION:** Where the wanted and interfering signals are of the same type, transform and conventional filter techniques will be ineffective. In this situation alternative distinguishing characteristics are required. It is then possible to jointly detect wanted and interfering signals, which will result in improved performance for the wanted signal. An example is using independence between channel impulse responses, however, such independence cannot always be guaranteed, such as in line of sight conditions. More commonly, a unique signature waveform is assigned to each user to facilitate signal separation. In this way Code Division Multiple Access (CDMA) methods inherently let different users share the same spectrum.

In order to maximise performance, receiver processing that exploits the characteristics of the interfering signals, joint detection, is used. Joint Detection, or Multi-User Detection as it is often referred to, can be applied to single carrier TDMA systems or spread spectrum CDMA systems or OFDM systems.

Multi-user detection is the study of receiver structures for multi-user communications, such as systems based on CDMA in which users are separated in the code domain. This means that each user has a distinct signature waveform, typically defined by a time domain spreading sequence or frequency hopping sequence. The central problem of multi-user detection is the recovery of individual signals from the coded multiplex.

The effectiveness of this processing depends on the orthogonality of the signature sequences at the receiver. Recovery of signals belonging to individual users is straightforward for the case of orthogonal signature waveforms since these have zero cross-correlation. The signature waveforms are designed to have low cross-correlations, irrespective of time alignment. Since these cross-correlations are non-zero, the base station receiver has to extract each user's signal from the non-orthogonal multiplex. Similarly, if the downlink is non-orthogonal, the mobile receiver has to recover its signal from the non-orthogonal multiplex. From the viewpoint of a given user, the part of the signal multiplex arising from other users that is correlated with his signal is a form of interference called multiple access interference (MAI). The conventional receiver applies a despreading process to receive the signal from the other users, but this ignores the characteristics of the other signals and is therefore sub-optimum.

The optimum multi-user detector is exponentially complex in the number of users and therefore impractical except for small user populations. This led to the development of the decorrelating and MMSE detectors. The basic forms of these detectors all require at least the timing and signatures of all users. The MMSE detector can be implemented in blind adaptive form that requires only the desired user's timing and signature. The decorrelator and MMSE detector both have moderate computational complexity. Better performance can be achieved with non-linear detectors, such as parallel or successive IC methods. Further improvements are possible by using iterative detection techniques, which pass soft information between decoding stages.

**CYCLOSTATIONARITY:** By definition, the statistics of a stationary signal are constant over

time. A cyclostationary signal will have statistics that vary periodically. A signal is cyclostationary of order  $n$  (in the wide sense) if and only if we can find some  $n$ -th order nonlinear transformation of the signal that will generate finite strength additive sine wave components, which result in spectral lines. In contrast, for stationary signals, only a spectral line at zero frequency can be generated.

For many man-made signals encountered in communications, radar, sonar and telemetry systems, certain frequency shifted versions of the signal can be highly correlated with the original signal. This spectral coherence can be exploited for signal selection by adding appropriately weighted and frequency shifted versions of the signal. FRESH filters exploit spectral correlation in addition to temporal correlation. These linear time-variant filters can be used to suppress FLI in a DS/SS signal. Another application of FRESH filters is the separation of spectrally overlapping BPSK or QAM signals. Such signals can be separated, even if their carrier frequencies are identical. FRESH filters have moderate to high computational complexity, depending on the number of frequency shifts employed. In some cases a large excess bandwidth is required for these techniques to be effective and so may not be suitable for all systems.

**NEURAL NETWORKS:** Neural networks have a non-linear mapping capability and are therefore able to eliminate types of interference that linear filters and transforms cannot contend with. In particular, the perceptron based equalisers FSDFMLP and FSBLP can suppress co-channel interference in a multipath fading channel. The FSDFMLP has the best performance of the techniques considered while the much simpler FSBLP is nearly as good. Both of these techniques surpass a conventional decision feedback equaliser. While the computational complexity of the FSDFMLP is high, the FSBLP is comparable to transform space techniques in terms of complexity. Neural networks are particularly useful where interference is not just a linear addition process (e.g. multiplicative or nonlinear distortion).

**HIGHER ORDER STATISTICS AND SIGNAL SEPARATION:** Many signal processing algorithms are based on the theory of (wide sense) stationary random processes. This theory involves only means and covariances, i.e. first and second order statistics. By incorporating higher (than second) order statistics, new algorithms can be developed with capabilities that are simply not possible in the framework of stationary random processes.

Higher order statistics have been exploited in several signal separation algorithms. Such algorithms recover the individual signals from a mixture and therefore have interference cancelling and anti-jamming applications. It should be emphasized that the best known algorithms are designed to exploit the spatial diversity of the received signal by employing a sensor array.

Another issue is how well BSS techniques are able to cope with time-variant channels. These algorithms require a large volume of data, and stationarity is difficult to guarantee in time variant channels. Finally, the assumption of a definite number of sources is built into the theoretical framework. Simple IC involves the desired signal, a source of interference and noise. The problem of interference from an unknown number of sources would need to be considered.

The above source separation algorithms require no knowledge other than all signals are statistically independent. In a communication system the signalling format is known and can

therefore be exploited. For example in digital communications a finite alphabet can be assumed. For some modulation methods the constant modulus constraint can be assumed. Such constraints are frequently used in blind equalisation techniques, where the constant modulus blind equaliser adapts the parameters such that the equalised signal matches the expected statistics of the known signal type. In this way even QAM signals can be equalised, even though they are non-constant amplitude.

Amplitude domain processing is a single antenna technique that can separate some classes of signals where the amplitude distributions are distinct and non-Gaussian, such as narrow band interference in a CDMA system. The amplitude distribution from a sequence of the input signal is estimated and an optimal amplitude transform is used to suppress the interference. The performance of time-domain amplitude methods diminishes rapidly as the number of interfering signal sources increases (even if they are at the same frequency) because of the rapid convergence of the combined amplitude distribution towards the thermal noise distribution.

**SPATIAL PROCESSING:** Adaptive antenna arrays have long been used to cancel interference particularly when the interference is spatially separated from the wanted signal. They have been used particularly in military applications where often the interfering signal is a high level jamming signal. In the civilian arena there has been less use of adaptive antenna arrays for IC.

Multiple antenna techniques such as beamforming, diversity or source separation, can be particularly effective where signal source channels are decorrelated or the sources are spatially separated. The price paid is not just in the additional processing, but also the additional antennas and RF processing. Interference is a particular issue for MIMO systems, where full knowledge of all the signals present is required for reliable decoding. When the receiver has an excess of antennas, the extra degrees of freedom can be used for IC.

**ANALOGUE TECHNIQUES:** Analogue techniques are effective against high amplitude interfering signals that would create non-linear operation in the RF front end or conversion subsystems. For dynamic interference environments a degree of tuning is required.

Where the transmitter and receiver are co-located then adaptive echo cancellation type techniques can be employed to remove the interference. Such techniques can also be used to remove the need for a duplexer in a terminal.

The Frequency Independent Strong Signal Suppressor (FISSS) can separate a strong signal from a weak signal and functions almost independently of the type of interference. Moreover, the device can be retro-fitted to existing equipment.

The analogue neural network is the only technique reviewed that employs analogue processing under digital control. The network can be trained to closely approximate a specified frequency response, at least in amplitude. The phase response of the least complex version is unsatisfactory but could probably be improved with more interconnections or a more sophisticated training algorithm. A major requirement to implement this technique is an array of perhaps 40- 200 first or second order filters whose poles are either fixed or can be set at appropriate

log-spaced frequencies. For practicality, these would need to be available on a single chip or a few chips. Micro-electromechanical (MEM) technology may provide such chips in the future.

The SAW exciser exploits the near-ideal characteristics of SAW devices to implement a real-time Fourier transform. This approach can be used to excise FLI from DS/SS. The system in its basic form is equivalent to a fixed notch filter. The PLL exciser locks onto a high-powered sinusoidal jamming signal, and subtracts a synthesized version of the jamming signal from the received signal. The interference cancelling ability is dependent on the match between the synthesized waveform, basically the output of a voltage controlled oscillator, and the jamming signal.

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## 8 APPENDIX

### SAMPLE MATLAB CODE FOR CMA

```

T=3000; % total number of data
dB=25; % SNR in dB value

%%%%%%%%%% Simulate the Received noisy Signal %%%%%%%%%%
N=20; % smoothing length N+1
Lh=5; % channel length = Lh+1
P=round((N+Lh)/2); % equalization delay
h=randn(1,Lh+1)+sqrt(-1)*randn(1,Lh+1); % channel (complex)
j=sqrt(-1);
h=[0.0545+j*0.05 .2832-.1197*j -.7676+.2788*j -.0641-.0576*j ...
.0566-.2275*j .4063-.0739*j];
h=h/norm(h); % normalize
s=round(rand(1,T))*2-1; % QPSK or 4 QAM symbol sequence
s=s+sqrt(-1)*(round(rand(1,T))*2-1);

% generate received noisy signal
x=filter(h,1,s);
vn=randn(1,T)+sqrt(-1)*randn(1,T); % AWGN noise (complex)
vn = vn/norm(vn) * 10(-dB/20) * norm(x); % adjust noise power with SNR dB value
SNR=20*log10(norm(x)/norm(vn)) % Check SNR of the received samples
x=x+vn; % received signal

%%%%%%%%%% adaptive equalizer estimation via CMA
Lp=T-N; % remove several first samples to avoid 0 or negative subscript
X=zeros(N+1,Lp); % sample vectors (each column is a sample vector)
for i=1:Lp
    X(:,i)=x(i+N:-1:i).';
end
e=zeros(1,Lp); % used to save instant error
f=zeros(N+1,1); f(P)=1; % initial condition
R2=2; % constant modulus of QPSK symbols
mu=0.001; % parameter to adjust convergence and steady error
for i=1:Lp
    e(i) = abs(f' * X(:,i))^2 - R2; f=f-mu*2*e(i)*X(:,i)*X(:,i)'; % update equalizer
    f(P)=1;

```

```

 $i_e = [i/10000 \text{abs}(e(i))] \%$  output information
end

sb=f'*X; % estimate symbols (perform equalization)

% calculate SER
H=zeros(N+1,N+Lh+1); for i=1:N+1, H(i,i:Lh)=h; end % channel matrix
fh=f'*H; temp=find(abs(fh)==max(abs(fh)));
sb1=sb/(fh(temp)); % scale the output
sb1=sign(real(sb1))+sqrt(-1)*sign(imag(sb1)); start=6; sb2=sb1-s(start+1:start+length(sb1));
SER=length(find(sb2==0))/length(sb2)
if 1
subplot(221),
plot(s,'o'); grid,title('Transmitted symbols'); xlabel('Real'),ylabel('Image')
axis([-2 2 -2 2])

subplot(222),
plot(x,'o'); grid, title('Received samples'); xlabel('Real'), ylabel('Image')

subplot(223),
plot(sb,'o'); grid, title('Equalized symbols'), xlabel('Real'), ylabel('Image')

subplot(224),
plot(abs(e)); % show the convergence
grid, title('Convergence'), xlabel('n'), ylabel('Error e(n)')
end

```