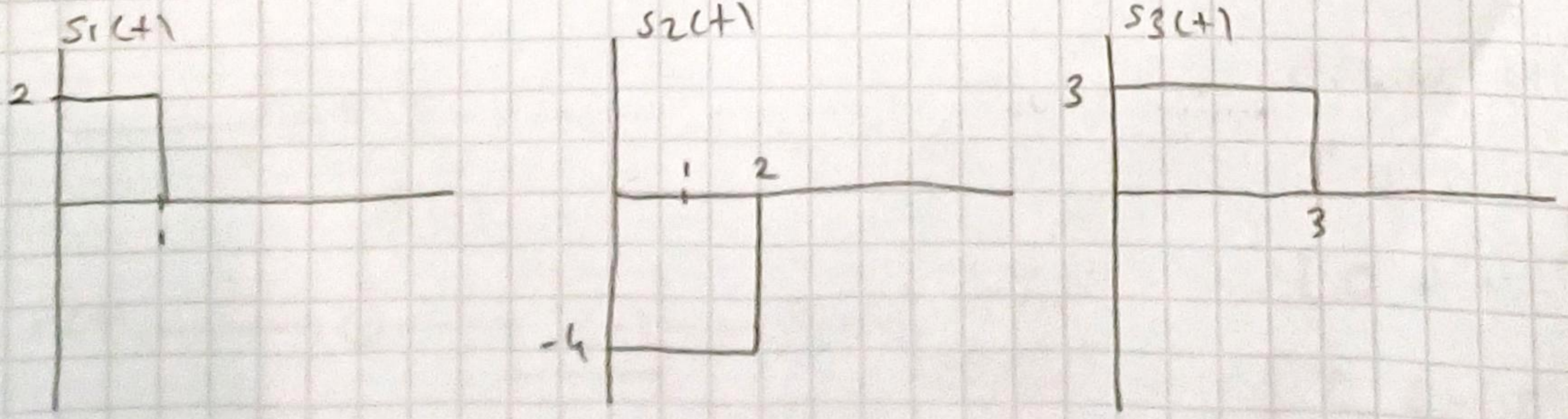
Recitation #1

Q1. a) Using the Gram Schmidt orthogonalization procedure, find a set of orthonormal basis functions to represent three signals ($s_1(t)$, $s_2(t)$, $s_3(t)$)

b) Express each of the signals in terms of basis functions found in part (a)

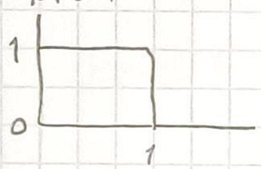


S1. $\mathcal{E}_1 = \int_0^1 (s_1(t))^2\,dt = 4$

$\phi_1(t) = \dfrac{s_1(t)}{\sqrt{\mathcal{E}_1}}$

$c_{21} = \int_{-\infty}^{\infty} s_2(t)\cdot \phi_1^*(t)\,dt = 4$

$\phi_1(t)$

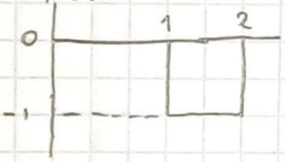


$y_2(t) = s_2(t) - c_{21}\,\phi_1(t)$

$\phi_2(t) = \dfrac{y_2(t)}{\sqrt{\mathcal{E}_{y_2}}}$

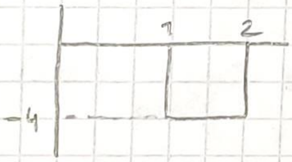
$y_2(t) \longrightarrow s_2(t)$'nin orthogonal parçası

$\phi_2(t)$



while

$y_2(t)$

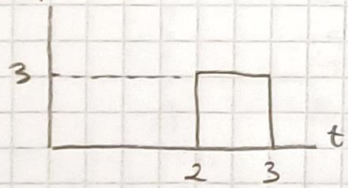


$c_{31} = \int_{-\infty}^{\infty} s_3(t)\cdot \phi_1^*(t)\,dt = 3$

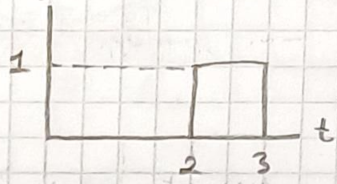
$c_{32} = -3$

$y_3(t) = s_3(t) - c_{31}\,\phi_1(t) - c_{32}\,\phi_2(t)$

$y_3(t)$

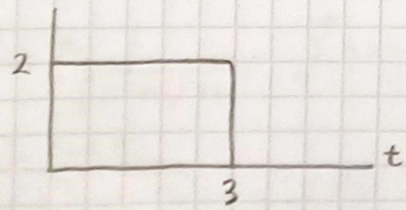


$\phi_3(t) \longrightarrow s_3(t)$'nin orthonormal parçası

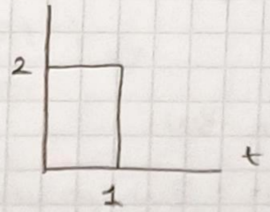


Q.2. Determine a set of orthonormal functions for the signals

$s_1(t)$



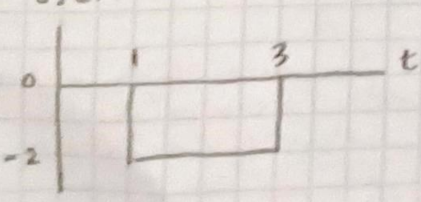
$s_2(t)$



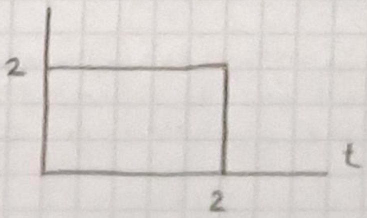
a. Express each signal in terms of found basis func.

b. Determine minimum distance

$s_3(t)$



$s_4(t)$



(ATLANAN) Q.1.b

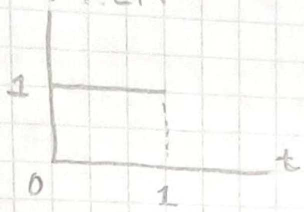
$s_1 = [2\ 0\ 0]$

$s_2 = [-4\ 4\ 0]$

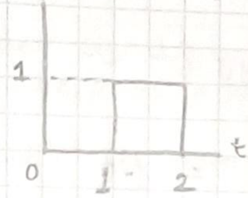
$s_3 = [3\ -3\ 3]$

S.2 Daha kısa süren sinyalleri ilk seçerek soru çözülürse daha kolay olabiliyor. Ama her çözüm kendine özgü

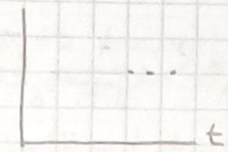
a. $\phi_1(t)$



$\phi_2(t)$



$\phi_3(t)$



$\phi_1(t)$ $s_2(t)$'nin basis karşılığı

$\phi_2(t)$ $s_4(t)$'nin basis karşılığı
(c_{41}: scaler projection of $s_4(t)$ on $\phi_1(t)$)

b. $s_1 = [2 \ 2 \ 2]$

$s_2 = [2 \ 0 \ 0]$

$s_3 = [0 \ -2 \ -2]$

$s_4 = [2 \ 2 \ 0]$

$d_{12} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ (oclidian distance)

$d_{13} = 6$

$d_{14} =$ ⋮

$d_{23} =$

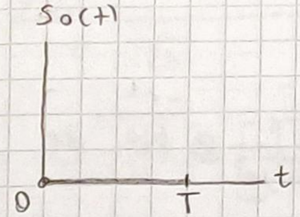
$d_{min} = d_{24} = 2$

Q.3. A binary digital comm system employs the signals
$s_0(t) = 0, \quad 0 \le t \le T$
$s_1(t) = A, \quad 0 \le t \le T$
for transmitting information. This is called on-off signalling. The modulator correlates received signal $r(t)$ with $s_1(t)$ and samples the output of the correlator at $t = T$
a. Determine the optimum detector for an AWGN channel and the optimum threshold, assumming that the signals are equally probable
b. Determine the probability of error as a function of SNR. How does on-off signalling compare with antipodal signalling?

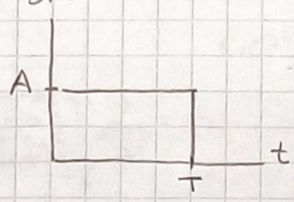
S.3.

$$\varepsilon_1 = \int_{-\infty}^{\infty} (s_1(t))^2 \, dt = A^2 T$$

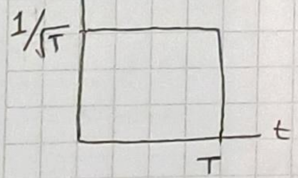
$s_0(t)$

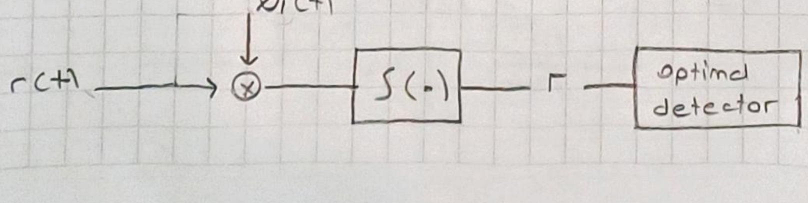


$s_1(t)$



$\phi_1(t)$





$$r(t) = s_m(t) + n(t)$$

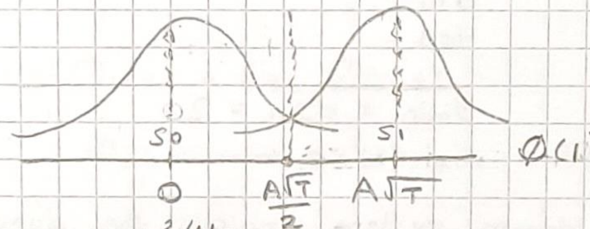
$$r = \int_0^T (s_m(t) + n(t))\, \phi_1(t)\, dt$$

$$= s_m + \frac{1}{\sqrt{T}} \int_0^T (n(t))\, dt = s_m + n$$

$$E\left\{ \frac{1}{\sqrt{T}} \int_0^T n(\tau)\, d\tau \cdot \frac{1}{\sqrt{T}} \int_0^T n(\nu)\, d\nu \right\}$$

$$= \frac{1}{T} \underbrace{\int_0^T \underbrace{\int_0^T \underbrace{E\{ n(\tau)\, n(\nu) \}}_{\frac{N_0}{2}\delta(\tau - \nu)} d\tau}_{N_0/2}\, d\nu}_{\frac{N_0}{2} T} = \frac{N_0}{2}$$

$s_1 = A\sqrt{T}$
$s_0 = 0$



$$f(r \mid s_0) = \frac{1}{\sqrt{\pi N_0}} e^{-r^2/N_0}$$

$$f(r \mid s_1) = \frac{1}{\sqrt{\pi N_0}} e^{-(r - A\sqrt{T})^2/N_0} \qquad e^{\frac{(r - A\sqrt{T})^2 - r^2}{N_0}} \underset{s_0}{\overset{s_1}{\gtrless}} 1$$

$$(r - A\sqrt{T})^2 - r^2 = 0 \qquad r = \frac{1}{2} A\sqrt{T}$$

$$P(e) = P(e \mid s_0)\, P(s_0) + P(e \mid s_1)\, P(s_1)$$

$$\frac{1}{2} \int_{\frac{1}{2}A\sqrt{T}}^{\infty} f(r \mid s_0)\, dr + \frac{1}{2} \int_{-\infty}^{\frac{1}{2}A\sqrt{T}} f(r \mid s_1)\, dr = \frac{1}{2} \int_{\frac{1}{2}A\sqrt{T}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{\frac{-r^2}{N_0}}\, dr$$

standard normal $z = \frac{r - \mu}{\sigma} \Rightarrow \sigma z + \mu = r$

$$\sqrt{\frac{N_0}{2}}\, z = r$$

$$2 \cdot \frac{1}{2} \int_{\sqrt{\frac{N_0}{2}} z = \frac{1}{2} A\sqrt{T}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{\frac{-\frac{N_0 z^2}{2}}{N_0}} \sqrt{\frac{N_0}{2}}\, dz = \int_{\frac{A\sqrt{T}}{\sqrt{2N_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}\, dz$$

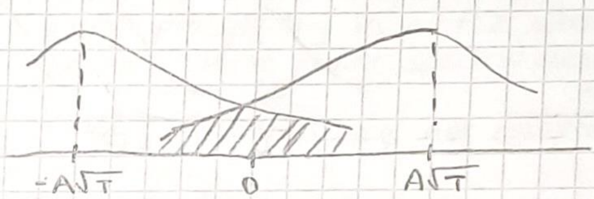
$$Q\left[\frac{A\sqrt{T}}{\sqrt{2N_o}}\right]$$

Q.4. Binary antipodal signals are used to transmit information over an AWGN channel. The prior probabilities for two input symbols (bits) are 1/3 and 2/3

a. Determine the optimum maximum-likelihood decision rule for the detector.

b. Determine the average probability of error as function of E_b/N_o

S4. a. $\dfrac{f(r\mid s_1)}{f(r\mid s_0)} \underset{s_0}{\overset{s_1}{\gtrless}} 1$



$$f(r\mid s_0) = \frac{1}{\sqrt{\pi N_o}} e^{-(r+\sqrt{E_b})/N_o} = \frac{1}{\sqrt{\pi N_o}} e^{-(r-\sqrt{E_b})/N_o}$$

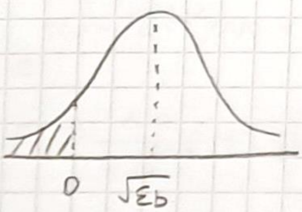
$$r \underset{s_0}{\overset{s_1}{\gtrless}} 0$$

b.

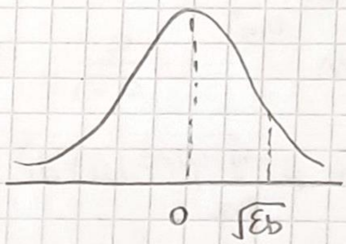
$$P(e) = P(e\mid s_0)P(s_0) + P(e\mid s_1)P(s_1)$$

$$(1-p)\int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_o}} e^{-(r-\sqrt{E_b})^2/N_o}\,dr$$

$$+ (p)\int_{0}^{\infty} \frac{1}{\sqrt{\pi N_o}} e^{-(r+\sqrt{E_b})^2/N_o}\,dr$$



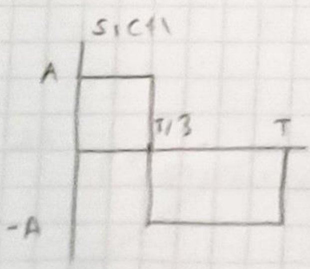
$\Rightarrow$

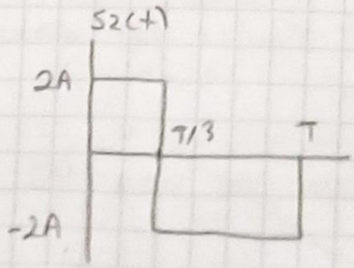


$$\Rightarrow Q\left[\sqrt{\frac{2E_b}{N_o}}\right]$$

$$= (1-p)\,Q\left[\sqrt{\frac{2E_b}{N_o}}\right] + p\,Q\left[\sqrt{\frac{2E_b}{N_o}}\right] = Q\left[\sqrt{\frac{2E_b}{N_o}}\right]$$

Q5. A communication system transmits one of the three messeges m1, m2 and m3 using signals $s_1(t)$, $s_2(t)$ and $s_3(t)$



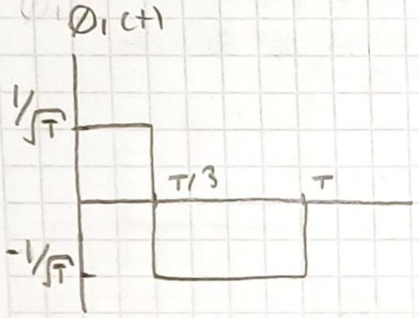


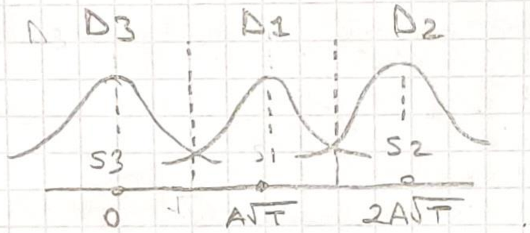
$s_3(t) = 0$

a. Determine an orthonormal basis for the signal set, and depict the signal constellation

b. If the three messages are equiprobable, what are the optimal decision rules for this system

55. a. $\mathcal{E}_1 = \frac{A^2T}{3} + \frac{2A^2T}{3} = A^2T$





b. $D_3 = r < \frac{\sqrt{\mathcal{E}_1}}{2}$

$D_1 = \frac{\sqrt{\mathcal{E}_1}}{2} < r < 3\frac{\sqrt{\mathcal{E}_1}}{2}$

$D_2 = \frac{3\sqrt{\mathcal{E}_1}}{2} < r$

c. $P_e = \sum_{m=1}^{M} p_m \left(\sum_{\substack{1 \le m' \le M \\ m' \neq m}} \int_{D_{m'}} p(r|s_m)\, dr \right)$

$p_1 \int_{D_3} p(r|s_1)\,dr + p_1 \int_{D_2} p(r|s_1)\,dr + p_3 \left(\int_{D_1} p(r|s_3)\,dr + \int_{D_2} p(r|s_2)\,dr \right) + p_2 \left(\int_{D_1} p(r|s_2)\,dr + \int_{D_3} p(r|s_2)\,dr \right)$

$\int_{\sqrt{\frac{N_0}{2}} z = \sqrt{\frac{\mathcal{E}_1}{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz$ $\qquad r = \sigma z + \mu$, $\sigma \to \sqrt{\frac{N_0}{2}}$, $\mu \to 0$

$z = \sqrt{\frac{\mathcal{E}_1}{2N_0}} \Rightarrow Q\left[\sqrt{\frac{\mathcal{E}_1}{2N_0}}\right]$, similar for others

$P_e = \frac{4}{3}\left(Q\left[\sqrt{\frac{\mathcal{E}_1}{2N_0}}\right] \right)$ (D1 için iki kez 1/3 eklemişiz?)

d. $R_s = 3000$
$R_b = R_s \log_2 M = 4755$

Q7. A binary communication system uses two equiprobable messages $s_1(t) = p(t)$, $s_2(t) = -p(t)$. The channel noise is additive WGN with power spectral density $N_0/2$. Assume that we have designed optimal receiver for this channel, and the error probability for the optimal receiver be P_e
a. Find an expression for P_e.
b. If this receiver is used on AWGN channel using the same signals but with the noise power spectral density $N_1 > N_0$, find the resulting error probability P_1 and explain how its value compares with P_e
c. Let P_{e1} denote the error probability in part 2 when an optimal receiver is designed for the new noise power spectral density N_1. Find P_{e1} and compare it with P_1.
d. Answer parts 1 and 2 if the two signals are not equiprobable but have prior probabilities p and $1-p$.

S7. $P_e = Q\left[\sqrt{\dfrac{2E_b}{N_0}}\right]$
a.

b. $P_{e1} = Q\left[\sqrt{\dfrac{2E_b}{N_1}}\right]$

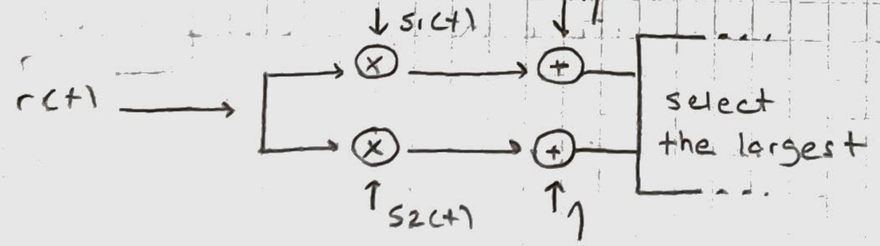
c.

d. $\dfrac{(1-p)}{\sqrt{\pi N_0}} e^{-(r-\sqrt{E_s})^2/N_0} = \dfrac{p}{\sqrt{\pi N_0}} e^{-(r+\sqrt{E_s})^2/N_0}$

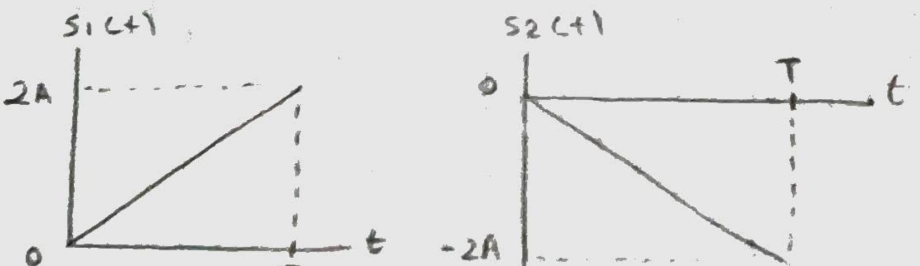
$e^{-(r-\sqrt{E_s})^2/N_0 + (r+\sqrt{E_s})^2/N_0} = \dfrac{p}{1-p}$

$r = \dfrac{N_0}{4\sqrt{E_s}} \ln\left(\dfrac{p}{1-p}\right)$ (N_0 kullanmak daha verimsiz)

S6.



$C(r, s_m) = \int_{-\infty}^{\infty} r(t)\, s_m(t)\, dt - \frac{1}{2}\int_{-\infty}^{\infty} |s_m(t)|^2 dt + \frac{N_0}{2} \ln P(s_m)$



$$\int_{-\infty}^{\infty} r(t)\, s_1(t)\, dt + \frac{No}{2} \ln(P(s_1)) \underset{s_2}{\overset{s_1}{\gtrless}} \int_{-\infty}^{\infty} -r(t)\, s_2(t) + \frac{No}{2} \ln(P(s_2))$$

$$\int_{-\infty}^{\infty} r(t)\, s_1(t)\, dt \underset{s_2}{\overset{s_1}{\gtrless}} \frac{No}{4} \ln\left(\frac{p_2}{p_1}\right)$$ (decision region olarak kullanılabilir.

$$P(e) = \int_{-\infty}^{\infty} (s_1(t) + n(t))\, s_1(t)\, dt = f(e|s_1)$$

$$= \underbrace{\int_0^T (s_1(t))^2 dt}_{\mathcal{E}s} + \underbrace{\int_0^T n(t) \cdot s(t)\, dt}_{n}$$

$$\sigma_n^2 = E\left\{ \int_0^T n(\tau)\, s_1(\tau)\, d\tau + \int_0^T n(v)\, s_1(v)\, dv \right\}$$

$$= \int_0^T \int_0^T s_1(\tau)\, s_1(v)\, \underbrace{E\{n(\tau)\, n(v)\}}\, dv\, d\tau$$

$$= \int_0^T \int_0^T s_1(\tau)\, s_1(v) \quad \frac{No}{2} \delta(\tau - v)$$

$$= \frac{No}{2} \int_0^T (s_1(\tau))^2 d\tau = \frac{No\, \mathcal{E}s_1}{2}$$

$$P(e|s_1) = \int_{-\infty}^{\frac{No}{4}\ln\left(\frac{P_2}{P_1}\right) - \mathcal{E}s} \frac{1}{\sqrt{\pi No \mathcal{E}s}}\, e^{\frac{-x^2}{No\,\mathcal{E}s}}\, dx$$

$$\frac{x-\mu}{\sigma} = z \qquad x = \sqrt{\frac{No\,\mathcal{E}s}{2}}\, z$$

$$Q\left[\frac{\mathcal{E}s\sqrt{2}}{\sqrt{No\,\mathcal{E}s}} - \frac{\sqrt{No}}{2\sqrt{2}} \ln\left(\frac{P_2}{P_1}\right)\right]$$

Q.6 In binary antipodal signalling scheme the signals are given by =

$$s_1(t) = \begin{cases} 2At/T & , \quad 0 \le t \le T/2 \\ 2A(1-t/T) & ; \quad T/2 \le t \le T \\ 0 & , \quad ow \end{cases}$$

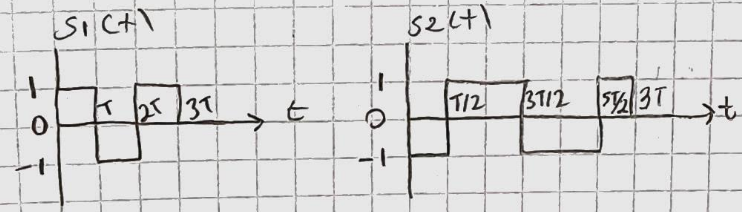
The channel is given AWGN and $S_N(f) = No/2$. The two signals have prior probabilities p_1 and $p_2 = 1 - p_1$

a. Determine the structure of optimal receiver.
b. Determine an expression for the error probability

Q.8 Figure shows a pair of signals $s_1(t)$ and $s_2(t)$ that are orthogonal to each other over the observations interval $0 \le t \le 3T$. The received signal is defined by

$$x(t) = s_k(t) + w(t) \quad 0 \le t \le 3T \quad k = 1,2$$

where $w(t)$ is WGN of zero mean and psd $N_0/2$

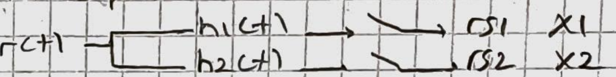


a. Design a receiver that decides in favor of signals $s_1(t)$ or $s_2(t)$ assumming that these two signals are equiprobable.
b. Calculate the average probability of symbol error incurred by this receiver for $E/N_0 = 4$, where E is the signal energy

S.8 a) Assume we use matched filter

$h_1(t) = s_1(T-t)$

$h_2(t) = s_2(T-t)$



since signals are equiprobable & have same energy

$$x_1 \underset{s_2}{\overset{s_1}{\lessgtr}} x_2 \quad \text{where} \quad x_1 = r_{s1}, \quad x_2 = r_{s2}$$

$\hat{m} = \arg\max [\eta_m + r_{sm}]$

where

$\eta_m = \frac{N_0}{2} \ln(P_m) - \frac{1}{2} E_m$

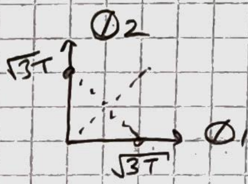
if equiprobable   if equal energy

b) $P(e) = ?$ if $\frac{E}{N_0} = 4$

$$E_2 = E_1 = \int_0^T (1)^2 dt + \int_T^{2T} (-1)^2 dt + \int_{2T}^{3T} (1)^2 dt = 3T = E \text{ (signal energy)}$$

$\phi_1(t) = \frac{s_1(t)}{\sqrt{3T}}$ ✓ $\quad \phi_2(t) = \frac{s_2(t)}{\sqrt{3T}}$

since orthogonal

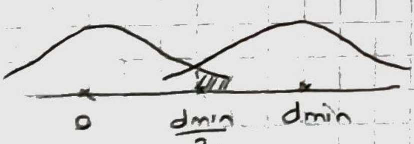


distance $d_{min} = \sqrt{6T}$

since equiprobable

$$P_e = \frac{1}{2} \int_{\frac{d_{min}}{2}}^{\infty} f(r|s_2)\,dr + \frac{1}{2} \int_{-\infty}^{d_{min}/2} f(r|s_2)\,dr$$

same



$$P_e = \int_{d_{min}/2}^{\infty} f(r|s_2)\,dr = \int_{d_{min}/2\sqrt{\frac{N_0}{2}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = Q\left[\frac{d_{min}/2}{\sqrt{N_0/2}}\right] = Q\left[\sqrt{\frac{3T}{N_0}}\right]$$

$$= Q\left[\sqrt{\frac{E}{N_0}}\right]$$

Note: $Q(x)$ can be denoted as $\frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$ in some solutions

Q.9 Consider the optimum detection of the sinusoidal signal in AWGN
a. Determine the correlator output assumming a noiseless input
b. Determine the corresponding matched filter output, assumming that the filter includes a delay T to make it causal
c. Hence show that these two outputs are the same only at time instant $t=T$

$$s(t) = \sin\left(\frac{8\pi t}{T}\right), \quad 0 \le t \le T$$

S.9

a) $y(T) = \int_0^T r(\tau)\, s(\tau)\, d\tau$

$$y(T) = \int_0^T s^2(\tau)\, d\tau = \int_0^T \sin^2\left(\frac{8\pi\tau}{2}\right) d\tau = \int_0^T \frac{1}{2}\left[1-\cos\left(\frac{16\pi\tau}{2}\right)\right]$$

b. $h(t) = s(T-t)$

$$y(t) = \int_{-\infty}^{\infty} r(\lambda)\, h(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} s(\lambda)\, s(T-t+\lambda)\, d\lambda$$

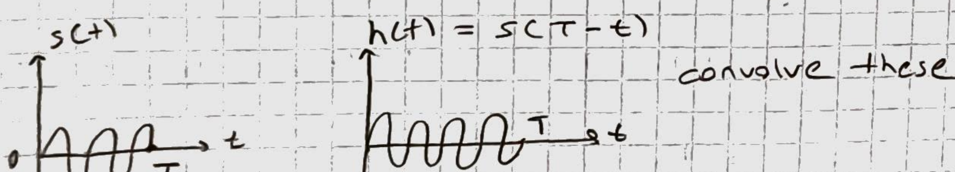
$$= \int_{-\infty}^{\infty} \sin\left(\frac{8\pi\lambda}{T}\right) \sin\left(\frac{8\pi(T-t+\lambda)}{T}\right) d\lambda$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \cos\left[\frac{8\pi(T-t)}{T}\right] d\lambda - \frac{1}{2}\int_{-\infty}^{\infty} \cos\left(\frac{8\pi(T-t+2\lambda)}{T}\right) d\lambda$$

$$= \frac{1}{2}\cos\left(\frac{8\pi(T-t)}{T}\right)\lambda\Big|_0^T - \frac{T}{32\pi}\sin\left(\frac{8\pi(T-t+2\lambda)}{T}\right)\Big|_0^{\lambda=T}$$

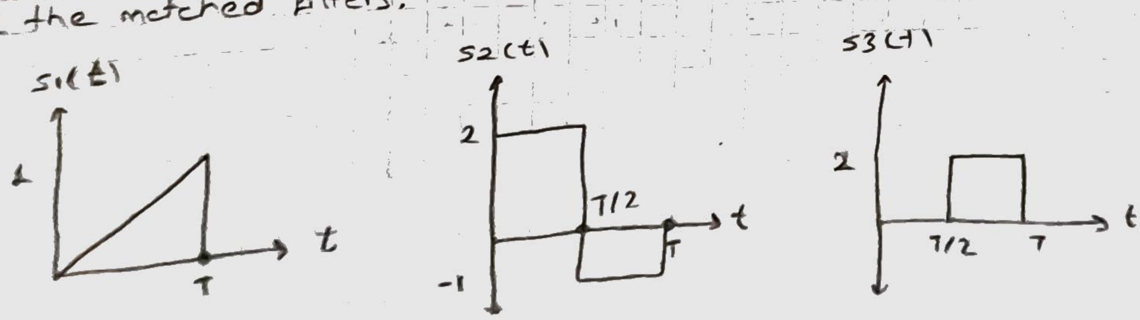
$$= \frac{T}{2}\cos\left(\frac{8\pi(T-t)}{T}\right) - \frac{T}{32\pi}\sin\left(\frac{8\pi(3T-t)}{T}\right) + \frac{T}{32\pi}\sin\left(\frac{8\pi(T-t)}{T}\right)$$

at $T=t$ $\quad y(T) = \frac{T}{2}$

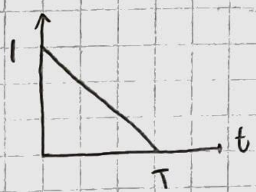


→ Results are the same

Q.10 Sketch the impulse response of filter matched to the pulses shown in the figure. Also determine and sketch the outputs of each of the matched filters.

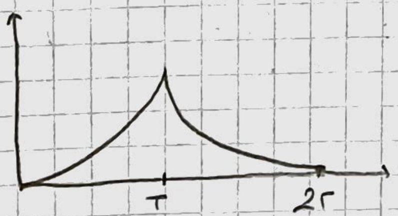


S.10.a $h_1(t)$

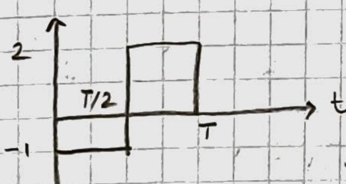


$$y_1(t) = \int_0^{\infty} \frac{\tau}{T}\left(1 - \frac{(t-\tau)}{T}\right) d\tau$$

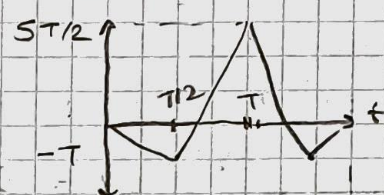
$y_1(t)$



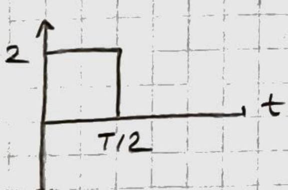
b) $h_2(t)$



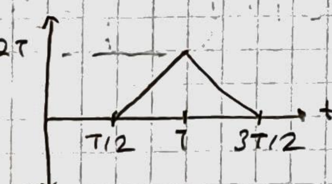
$y_2(t)$



c) $h_3(t)$



$y_3(t)$



Q.11 In an AWGN channel with noise psd $N_0/2$, two equiprobable messages are transmitted by

$$s_1(t) = \begin{cases} At/T, & 0 \le t \le T \\ 0, & \text{ow} \end{cases}$$

$$s_2(t) = \begin{cases} A(1 - t/T), & 0 \le t \le T \\ 0, & \text{ow} \end{cases}$$

a. Determine the structure of the optimal receiver
b. Determine the probability of error

S.11 a)

$$\int_{-\infty}^{\infty} r(t)\, s_1(t)\, dt \underset{s_2}{\overset{s_1}{\gtrless}} \int_{-\infty}^{\infty} r(t)\, s_2(t)\, dt$$

b) $P(e) = ?$ find $P(e|s_1)$

$$\int_{-\infty}^{\infty} s_1(t)\left(s_1(t) - s_2(t)\right) dt + \int_{-\infty}^{\infty} n(t)\left(s_1(t) - s_2(t)\right) dt \underset{s_2}{\overset{s_1}{\gtrless}} 0$$

$$w = \frac{A^2 T}{6} + n$$

$$\sigma_n^2 = E\left\{ \int_{-\infty}^{\infty} n(\tau)\left(s_1(\tau) - s_2(\tau)\right) d\tau \cdot \int_{-\infty}^{\infty} n(\vartheta)\left(s_1(\vartheta) - s_2(\vartheta)\right) d\vartheta \right\}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(s_1(\tau) - s_2(\tau)\right)\left(s_1(\vartheta) - s_2(\vartheta)\right) \frac{N_0}{2} \delta(\vartheta - \tau)\, d\vartheta\, d\tau$$

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} \left(s_1(\tau) - s_2(\tau)\right)^2 d\tau = \int_0^T \left(\frac{2A\tau}{T} - A\right)^2 d\tau$$

$$= \frac{N_0 A^2 T}{6}$$

$P(e|s_1) = \int_{-\infty}^{0} f(w)\, dw$ where w has $\mu = \frac{A^2 T}{6}$ $\sigma^2 = \sqrt{\frac{N_0 A^2 T}{6}}$

$$P(e|s_1) = \int_{-\infty}^{(-A^2T/6)/\sqrt{\frac{N_0 A^2 T}{6}}} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = Q\left[\sqrt{\frac{A^2 T}{6 N_0}}\right] = P(e|s_2)$$

$$P_e = \frac{1}{2} Q\left[\sqrt{\frac{A^2 T}{6 N_0}}\right] + \frac{1}{2} Q\left[\sqrt{\frac{A^2 T}{6 N_0}}\right] = Q\left[\sqrt{\frac{A^2 T}{6 N_0}}\right]$$

Q.12 Consider a signal detector with an input $r = \pm A + n$, where $+A$ and $-A$ occur with equal probability and the noise variable is characterized by Laplacian pdf. Determine the probability of error as function of the parameters A and σ

$$p(n) = \frac{1}{\sqrt{2}\sigma} e^{-|n|\sqrt{2}/\sigma}$$

S.12 $\lambda = \frac{\sqrt{2}}{\sigma}$ $f(n) = \frac{\lambda}{2} e^{-\lambda|n|}$

optimal receiver

$$\frac{p(r|A)}{p(r|-A)} = e^{-\lambda\left[(r-A) - (r+A)\right]} \underset{-A}{\overset{A}{\gtrless}} 1$$

$$-\lambda\left[(r-A) - (r+A)\right] \underset{-A}{\overset{A}{\gtrless}} 0 \qquad r \underset{-A}{\overset{A}{\gtrless}} 0$$

$$P(e) = \frac{1}{2} P(e|A) + \frac{1}{2} P(e|-A)$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} f(r \mid A)\, dr + \frac{1}{2} \int_{-\infty}^{\infty} f(r \mid -A)\, dr$$

$$= \frac{1}{2} \int_{-\infty}^{0} \frac{\lambda}{2} e^{-\lambda |r-A|}\, dr + \frac{1}{2} \int_{0}^{\infty} \frac{\lambda}{2} e^{-\lambda |r+A|}\, dr$$

$$= \frac{\lambda}{2} \int_{A}^{\infty} e^{-\lambda |x|}\, dx = -\frac{1}{2} e^{-\lambda x} \Big|_{A}^{\infty} = \frac{1}{2} e^{-\lambda A}$$