$A_m = (2m - 1 - M)\frac{d}{2}$

Question 1 $m = 1, 2, 3, \ldots, M$

Determine the average energy of a set of M PAM signals of the form

$$s_m(t) = s_m \psi(t), \quad m = 1, 2, \ldots, M$$
$$0 \le t \le T$$

where $\mathcal{E}_{av} = \frac{1}{M}\sum_{m=1}^{M} S_m^2$

$$s_m = \sqrt{\mathcal{E}_g} A_m, \quad m = 1, 2, \ldots, M$$

The signals are equally probable with amplitudes that are symmetric about zero and are uniformly spaced with distance d between adjacent amplitudes as shown in Figure 7.11.

Question 3 33721.8 (bits/second) = H(x) (symbol/sec) * fs (bits/symbol)

An information source can be modeled as a bandlimited process with a bandwidth of 6000 Hz. This process is sampled at a rate higher than the Nyquist rate to provide a guard band of 2000 Hz. It is observed that the resulting samples take values in the set $\mathcal{A} = \{-4, -3, -1, 2, 4, 7\}$ with probabilities 0.2, 0.1, 0.15, 0.05, 0.3, 0.2.

$H(x) = -\sum_{i=1}^{6} P_i \log_2(P_i)$ $f_s = 2k + 2.6k$

What is the entropy of the discrete-time source in bits/output (sample)? What is the entropy in bits/sec?

Question 4

Let $Y = g(X)$, where g denotes a deterministic function. Show that, in general, $H(Y) \le H(X)$. When does equality hold?

Question 5

Binary PAM is used to transmit information over an unequalized linear filter channel. When $a = 1$ is transmitted the noise-free output of the demodulator is

$$q_n = \sum_{m=-1}^{1} c_m h_{n-m}$$

$$x_m = \begin{cases} 0.3, & m = 1 \\ 0.9, & m = 0 \\ 0.3, & m = -1 \\ 0, & \text{otherwise} \end{cases}$$

1. Design a three-tap zero-forcing linear equalizer so that the output is

$$q_m = \begin{cases} 1, & m = 0 \\ 0, & m = \pm 1 \end{cases}$$

2. Determine q_m for $m = \pm 2, \pm 3$, by convolving the impulse response of the equalizer with the channel response.

Question 6

The transmission of a signal pulse with a raised cosine spectrum through a channel results in the following (noise-free) sampled output from the demodulator:

$$x_k = \begin{cases} -0.5, & k = -2 \\ 0.1, & k = -1 \\ 1, & k = 0 \\ -0.2, & k = 1 \\ 0.05, & k = 2 \\ 0, & \text{otherwise} \end{cases}$$

1. Determine the tap coefficients of a three-tap linear equalizer based on the zero-forcing criterion.

Question 7 design

$M = 4$ PAM modulation is used for transmitting at a bit rate of 9600 bits/sec on a channel having a frequency response

$$C(f) = \frac{1}{1 + j\frac{f}{2400}}$$

$|f| \leq 2400$, and $C(f) = 0$, otherwise. The additive noise is zero-mean, white Gaussian with power-spectral density $\frac{N_0}{2}$ W/Hz. Determine the (magnitude) frequency response characteristic of the optimum transmitting and receiving filters.

Question 8

Consider a digital communication system that transmits information via QAM over a voice-band telephone channel at a rate 2400 symbols/second. The additive noise is assumed to be white and Gaussian.

1. Determine the $\mathcal{E}_b/N_0$ required to achieve an error probability of 10^{-5} at 4800 bps.
2. Repeat (1) for a bit rate of 9600 bps.
3. Repeat (1) for a bit rate of 19,200 bps.
4. What conclusions do you reach from these results.

Solutions

Question 1

The amplitudes A_m take the values

$$A_m = (2m - 1 - M)\frac{d}{2} \qquad m = 1, 2, \dots M$$

$$\mathcal{E}_{av} = \frac{1}{M}\sum_{m=1}^{M} s_m^2 = \frac{d^2}{4M}\sum_{m=1}^{M}(2m-1-M)^2$$

$$= \frac{d^2}{4M}\mathcal{E}_g \sum_{m=1}^{M}\left[4m^2 + (M+1)^2 - 4m(M+1)\right]$$

$$= \frac{d^2}{4M}\mathcal{E}_g\left(4\sum_{m=1}^{M} m^2 + M(M+1)^2 - 4(M+1)\sum_{m=1}^{M} m\right)$$

$$= \frac{d^2}{4M}\mathcal{E}_g\left(4\frac{M(M+1)(2M+1)}{6} + M(M+1)^2 - 4(M+1)\frac{M(M+1)}{2}\right)$$

$$= \frac{M^2-1}{3}\,\frac{d^2}{4}\,\mathcal{E}_g$$

$$X_2 = \int_0^4 x(t)\,\psi_2(t)\,dt = \frac{1}{2}\int_0^4 x(t)\,dt = 0$$

$$X_3 = \int_0^4 x(t)\,\psi_3(t)\,dt = -\frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_1^2 dt + \frac{1}{2}\int_2^3 dt + \frac{1}{2}\int_3^4 dt = 0$$

~~As it is observed $x(t)$ is orthogonal to the signal waveforms $\psi_n(t)$ $n=1,2,\ldots$ and thus it can not represented as a linear combinations of these functions~~

Question 3

The entropy of the source is

$$H(X) = -\sum_{i=1}^{6} p_i \log_2 p_i = 2{,}4087 \text{ bits/symbol}$$

The sampling rate is

$$f_s = 2000 + 2 \cdot 6000 = 14000 \text{ Hz}$$

③

This means that 14 000 samples are taken per each second and hence the entropy of the source in bits per second is given by

$$H(X) = 2.4087 \times 14\,000 \text{ (bits/symbol)} \times \text{(symbol/sec)} = 33721.8 \text{ bits/second}$$

Question 4

$$H(X,Y) = H(X, g(X)) = H(X) + H(g(X)|X)$$
$$= H(g(X)) + H(X|g(X))$$

But $H(g(X)|X) = 0$ since $g(\cdot)$ is deterministic. Therefore

$$H(X) = H(g(X)) + H(X|g(X))$$

Since each term in the previous equation is non-negative we obtain

$$H(x) \geq H(g(x))$$

Equality holds when $H(x \mid g(x)) = 0$

This means that the values $f(x)$ uniquely determine X, or that $f(\cdot)$ is a one to one mapping.

Question 5

The equivalent discrete time impulse response of the channel is

$$h(t) = \sum_{n=-1}^{1} h_n \delta(t-nT) = 0.3\,\delta(t+T) + 0.9\,\delta(t) + 0.3\,\delta(t-T)$$

If by $\{c_n\}$ we denote the coefficients of the FIR equalizer then the equalized signal is

$$q_m = \sum_{n=-1}^{1} c_n h_{m-n}$$

Which in the matrix notation is written as

$$\begin{pmatrix} 0.9 & 0.3 & 0 \\ 0.3 & 0.9 & 0.3 \\ 0 & 0.3 & 0.9 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} -0.4762 \\ 1.4286 \\ -0.4762 \end{pmatrix}$$

Question 6

$$\underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{q} = \underbrace{\begin{bmatrix} 1 & 0.1 & -0.5 & 0 & 0 \\ -0.2 & 1 & 0.1 & -0.5 & 0 \\ 0.05 & -0.2 & 1 & 0.1 & -0.5 \\ 0 & 0.05 & -0.2 & 1 & 0.1 \\ 0 & 0 & 0.05 & -0.2 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} w_{-2} \\ w_{-1} \\ 0 \\ w_1 \\ w_2 \end{bmatrix}}_{w}$$

$w = C^{-1} q$

Question 7

4 PAM modulation can accomodate $k=2$ bits per transmitted symbol. Thus the symbol interval duration is

$$T = \frac{k}{9600} = \frac{1}{4800} \text{ sec}$$

Since the channel's bandwidth is

$$W = 2400 = \frac{1}{2T}$$

in order to achieve the maximum rate of transmission $R_{max} = \frac{1}{2T}$ the spectrum of the signal pulse should be

$$X(f) = T\,\Pi\left(\frac{f}{2W}\right)$$

Then, the magnitude frequency response of the optimum transmitting and receiving filter is

$$|G_T(f)| = |G_R(f)| = \left[1+\left(\frac{f}{2400}\right)^2\right]^{1/4}$$

$$\Pi\left(\frac{f}{2W}\right) = \begin{cases} \left[1+\left(\frac{f}{2400}\right)^2\right]^{1/4} & |f| < 2400 \\ 0 & \text{o.w.} \end{cases}$$

Question 8

The number of bits per symbol is

$$k = \frac{4800}{R} = \frac{4800}{2400} = 2$$

Thus, a 4 QAM constellation is used for the transmission. The P.E for M-ary QAM system with $M = 2^k$ is

$$P_M = 1 - \left(1 - 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3\mathcal{E}_b}{(M-1)N_0}}\right)\right.$$

with $P_M = 10^{-5}$ $k = 2$

we obtain

$$Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) = 5 \times 10^{-6} \quad \frac{\mathcal{E}_b}{N_0} = 9.7682$$

(2) If bit rate of the [illegible]

is 9600 bps then

$$k = \frac{9600}{2400} = 4$$

In this case a 16-QAM cons is used

and the prob Er is

$$P_M = 1 - \left(1 - 2\left(1 - \frac{1}{4}\right) Q\left(\sqrt{\frac{3 \times 4 \mathcal{E}_b}{15 N_0}}\right)\right)^2$$

Thus

$$Q\left(\sqrt{\frac{3 \times \mathcal{E}_b}{15 N_0}}\right) = \frac{1}{3} \times 10^{-5} \Rightarrow \frac{\mathcal{E}_b}{N_0} = 25.368$$

(3) (6)

If the bitrate of the

frame is 18200

$$k = \frac{18200}{2400} = 8$$

256 QAM

$$P_M = 1 - \left(1 - 2\left(1 - \frac{1}{16}\right) Q\left(\sqrt{\frac{3 \times 8\, \mathcal{E}_b}{255 N_0}}\right)\right)$$

with $P_M = 10^{-5}$ $\frac{\mathcal{E}_b}{N_0} = 658.8822$

(4)

k	2	4	8
SNR ~~dB~~	8.89	14.01	28.18

3dB per addition bit per symbol