3. If PM

$\beta_p = \Delta\phi_{max} = max|4\sin(2\pi f_m t)| = 4$

$B_{pm} = 2(\beta_p + 1) f_m = 10 kHz$

4. If f_m is doubled $\beta_p = \Delta\phi_{max}$ still

$B_{pm} = 2(\beta_p + 1) 2f_m = 20 kHz$

$s(t) = A_c \cos(\theta_i(t))$

↓ carrier amplitude

↳ phase / angle changes according to message signal

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$\theta_i(t) = 2\pi f_c t + \underbrace{k_p}_{\text{phase sensitivity}} m(t)$

$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$ For PM

$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$

$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau)$ For FM

$\beta_p = k_p \max(|m(t)|) = \Delta\phi_{max}$

$$\beta_f = \frac{k_f \max(|m(t)|)}{BW} = \frac{\Delta f_{max}}{BW}$$

For sinusoidal message signal $m(t) = A_m \cos(2\pi f_m t)$

$\beta_f = \frac{k_f A_m}{f_m}$ $\qquad \beta_p = k_p A_m$

Recitation 5 (FM - PM)

Q1. $m(t) \Rightarrow BW = 10 kHz$, $\max(m(t)) = 1$

a) $f_d = 10 Hz/V$

$$B_c = 2(\beta + 1) BW = 2\left(\frac{k_f \max(m(t))}{BW} + 1\right) BW$$

$$= 2\left(\frac{(10 Hz/V) \cdot 1V}{10 kHz} + 1\right)(10 kHz) = 2(10^{-3} + 1) 10^4 =$$

b) $f_d = 100 Hz/V$

c) $f_d = 1 kHz/V$ $\qquad f_d$ arttıkça B_c (bandwith requirement artıyor)

2. $m(t) = 10\,\text{sinc}(400t)$ $\qquad$ $\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$

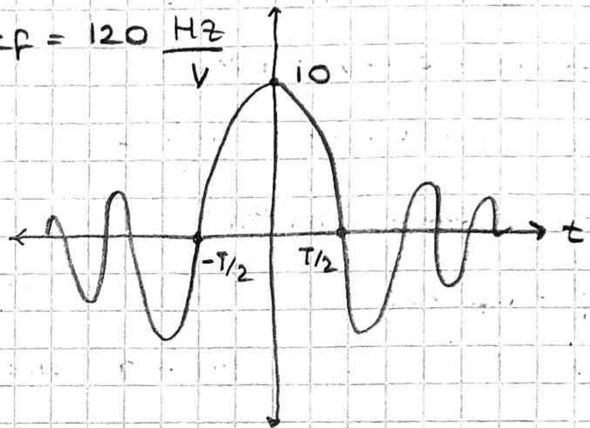
$c(t) = 100\cos(2\pi f_c t)$

mod. index $\beta = 6$

$$u(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right)$$

$$\beta = \frac{k_f \max(m(t))}{BW} = \frac{k_f \cdot 10}{BW} \Rightarrow k_f = 120\ \frac{Hz}{V}$$

$$m(t) = 10\,\frac{\sin(400\pi t)}{400\pi t}$$



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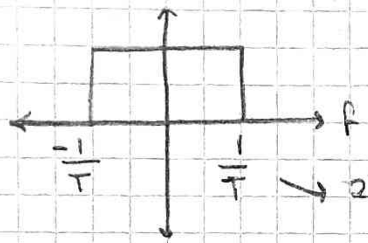


$\frac{1}{T} \rightarrow 200$ Hz

$\sin(400\pi t) = 0$ $\qquad$ $BW = 200$ Hz

$t = \frac{1}{400} = \frac{T}{2}$ $\quad T = \frac{1}{200}$ s

$u(t) =$

b) $\Delta f_{max} = \beta \cdot BW = k_f \max(m(t)) = 120 \cdot 10 = 1200$ Hz

c) $u(t) = \underbrace{100\cos\left(2\pi f_c t + 2\pi\, 120 \int_{-\infty}^{t} 10\,\text{sinc}(400\tau)\,d\tau\right)}_{\text{sinusoidal signal with peak amplitude 100}}$

$$P = \frac{A_p^2}{2} = 5000$$

d) B_c, Carlson's Rule $B_c = 2(\beta+1)BW = 2.8$ kHz

3.

a) $m_1(t) \rightarrow$ FM, $f_c = 10^6$ Hz, $k_f = 5$ Hz/V

$f_i(t) = f_c + k_f m_1(t) \rightarrow \max(f_i(t)) = f_c + k_f \max(m_1(t))$
$= 10^6 + 5 \cdot 10^5 = 1.5$ MHz

b) $m_1(t) \rightarrow$ PM, $k_p = 3$ rad/V

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d\,Q(t)}{dt} = f_c + \frac{k_p}{2\pi}\frac{d\,m_1(t)}{dt}$$

$\max(f_i(t)) = f_c + \frac{k_p}{2\pi} \underbrace{\frac{d\max(m_1(t))}{dt}}_{t \in (0,1)} = 10^6 + \frac{3}{2\pi} 10^5$

$\min(f_i(t)) = 10^6 - \frac{3}{2\pi} 10^5$

c) $m_2(t) \rightarrow$ FM , $k_f = 10^3$ Hz/V

$m_2(t) = \text{sinc}(2.10^4 t)$

$\max(f_i(t)) = 10^6 + 10^3 . 1 = 1.001$ MHz

B_c, $\sin(2.10^4 \pi t) / 2.10^4 \pi t = 0$ $\quad T = \frac{1}{10^4}$, $BW = 10$ kHz

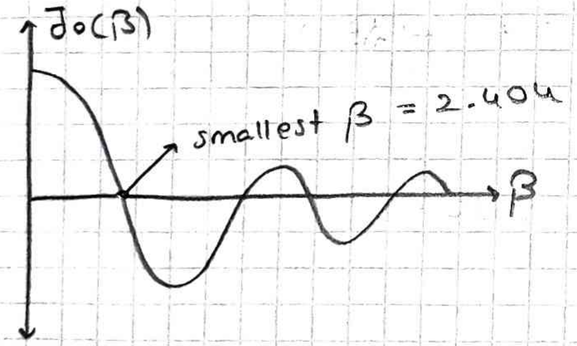
$\beta = k_f \max(m(t)) / BW = 10^3 . 1 / 10^4 = 0.1$

$B_c = 2(0.1 + 1)\, BW = 22$ kHz

4. $u(t) = \sum_{n=-\infty}^{\infty} A_c . J_n(\beta) \cos(2\pi f_c t + 2\pi n f_m t)$

$P_k = \frac{A_c^2}{2} J_n^2(\beta)$ (each sideband power)

$P_c = \frac{A_c^2}{2} J_0^2(\beta)$ (carrier power)



n = sideband sayısı

Recitation 6 (FM - PM)

1. $m(t) = A_m \cos(2\pi f_m t) \rightarrow$ PM with k_p. The unmodulated carrier wave has f_c and A_c. Determine the spectrum of the resulting PM wave, assuming $\Delta\phi_{max}$ $\beta = k_p A_m$ does not exceed 0.3 radian.

$s(t) = A_c \cos[2\pi f_c t + k_p A_m \cos(2\pi f_m t)]$

$\Delta\phi_{max} = k_p \max[m(t)] = k_p A_m \qquad \beta = \Delta\phi_{max}$

$s(t) = A_c \cos[2\pi f_c t + \beta \cos(2\pi f_m t)]$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$$s(t) = A_c \cos(2\pi f_c t)\cos(\beta \cos(2\pi f_m t)) - A_c \sin(2\pi f_c t)\sin(\beta \cos(2\pi f_m t))$$

if $\beta \leq 0.3$, $\cos(\beta \cos(2\pi f_m t)) \approx 1$

$\sin(\beta \cos(2\pi f_m t)) \approx \beta \cos(2\pi f_m t)$

small-angle approximation

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\cos(2\pi f_m t)$$

$$\sin a \sin b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$s(t) \approx A_c \cos(2\pi f_c t) - \frac{1}{2}\beta A_c \sin[2\pi(f_c+f_m)t] - \frac{1}{2}\beta A_c \sin[2\pi(f_c-f_m)t]$$

$$F\{\sin(2\pi A t)\} = \frac{1}{2j}[\delta(f-A) - \delta(f+A)]$$

The spectrum of $s(t)$ is

$$S(f) \approx \frac{1}{2}A_c[\delta(f-f_c) + \delta(f+f_c)] - \frac{1}{4j}\beta A_c[\delta(f-f_c-f_m) - \delta(f+f_c+f_m)] - \frac{1}{4j}\beta A_c[\delta(f-f_c+f_m) - \delta(f+f_c-f_m)]$$

2. $m_1(t) = \begin{cases} a_1 t + a_0, & t \geq 0 \\ 0, & t = 0 \end{cases}$ is applied to FM

$m_2(t) = \begin{cases} b_2 t^2 + b_1 t + b_0, & t \geq 0 \\ 0, & t = 0 \end{cases}$ is applied to PM

which outputs of two angle modulators

$$s_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m_1(\tau)\,d\tau]$$

$$= A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t [a_1 t + a_0]\,d\tau]$$

$$= A_c \cos[2\pi f_c t + 2\pi k_f (\frac{1}{2}a_1 t^2 + a_0 t)]$$

$$s_{PM}(t) = A_c \cos(2\pi f_c t + k_p m_2(t)]$$

$$= A_c \cos(2\pi f_c t + k_p(b_2 t^2 + b_1 t + b_0)], \quad t \geq 0$$

$s_{FM}(t) = s_{PM}(t)$ at $t \geq 0$

i) $\pi k_f a_1 = b_2 k_p$

ii) $2\pi k_f a_0 = k_p b_1$

iii) $2\pi k_f c = k_p b_0$ $\quad c = 0$

3. $c(t) = 100\cos(2\pi f_c t)$ to FM by $m(t) = 5\cos(2000\pi t)$
$f_c = 10^8$ Hz $\Delta f_{peak} = 20$ kHz

1. Determine amp and freq of all signal components that have a power level of at least 10% of power of unmodulated carrier component

2. From Carson's Rule, determine approximated BW of FM signal.

1. $\Delta f_{max} = k_f \max(m(t)) \Rightarrow \beta = \frac{\Delta f_{max}}{f_m}$

$\beta = \frac{20 \times 10^3}{10 \times 10^3} = 2$ wideband FM

$u(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$

$u(t) = 100 \sum J_n(2) \cos[2\pi(10^8 + n.10^4)t]$

power $P = \frac{100^2}{2} = 5000$ %10 = 500

$P_{f_c + nf_m} = \frac{100^2 J_n^2(2)}{2}$ Bessel function table

also $J_n^2(\beta) = J_{-n}^2(\beta)$ four sidebands
$10^8 + 10^4$
$10^8 + 2.10^4$
$10^8 - 10^4$
$10^8 - 2.10^4$

2. $B_c = 2(\beta + 1) BW_{fm}$ for $\beta >> 1 \Rightarrow B_c = 2(1+2)10^4$
$B_c = 2 f_m$ for $\beta << 1$ $= 6 \times 10^4$ Hz

4. $m(t) = 10\cos 16\pi t$, FM

$u(t) = 10\cos[4000\pi t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau]$ $k_f = 10$, $f_c = 2000$ Hz

BPF centered at f_c with BW = 62 Hz

$\beta = \frac{k_f \max(m(t))}{f_m} = \frac{10.10}{8} = 12{,}5$

$u(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau)$

$u(t) = A_c \cos(2\pi 2000 t + 2\pi 10 \int_{-\infty}^{t} 10\cos(2\pi f_m \tau)\, d\tau)$

$= \sum A_c J_n(12{,}5) \cos[2\pi(2000 + 8.n)t]$

BPF $[2000 - 32, 2000 + 32]$ $n = -4$
$n = 4$

The power of the output is then

$$= \frac{10^2}{2} J_0^2(12,5) + 2 \cdot \sum_{n=1}^{4} \frac{10^2}{2} J_n^2(12,5)$$

$= 50 \times 10.2630 = 13.15$

Total power $\Rightarrow P_{Tot} = \frac{10^2}{2} = 50$

The power of the output and BPF is only 26.30% of the trasmitted power.