MAT0122 ÁLGEBRA LINEAR I

FOLHA DE SOLUÇÃO

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Assinatura

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Exercício: E14 **Data: 05/09/2022**

SOLUÇÃO

Firstly, we'll prove that $A\Delta B$ is analogous to $\mathbb{1}_A + \mathbb{1}_B$ when operating in $GF(2)^U$.

Proof: Suppose A, B, U sets such that $A \cup B \subset U$; $\mathbb{1}_A, \mathbb{1}_B$ characteristic functions as vectors in $GF(2)^U$. Let $U = \{x_1, \ldots, x_n\}$, $x_i \in U$.

(1) If $x_i \in A$ and $x_i \in B$, $\mathbb{1}_A(x_i) + \mathbb{1}_B(x_i) = 1 + 1 = 0$

(2) If $x_i \in A$ and $x_i \notin B$ (or vice-versa), $\mathbb{1}_A(x_i) + \mathbb{1}_B(x_i) = 1 + 0 = 0 + 1 = 0$

(3) If $x_i \notin A$ and $x_i \notin B$, $\mathbb{1}_A(x_i) + \mathbb{1}_B(x_i) = 0 + 0 = 0$

Note that the sum only returns 1 in case (2). In other words, only if $x_i \in A\Delta B$.

Now that we can treat symmetrical difference as a vector addition, it is possible to conclude that symmetrical difference is associative, since vector addition is associative as well.

$$(\mathbb{1}_A + \mathbb{1}_B) + \mathbb{1}_C = \mathbb{1}_A + (\mathbb{1}_B + \mathbb{1}_C)$$
$$\mathbb{1}_{A\Delta B} + \mathbb{1}_C = \mathbb{1}_A + \mathbb{1}_{B\Delta C}$$

Therefore, $(A\Delta B)\Delta C = A\Delta(B\Delta C)$.