MAT0122 ÁLGEBRA LINEAR I

FOLHA DE SOLUÇÃO

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Assinatura

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Exercício: E51 **Data: 19/10/22**

SOLUÇÃO

It is supposed that $H_n^2 = NI_N$ for all n.

Let's prove $v_i\,(1 \leq i \leq N)$ form a basis for $\mathbb{R}^N$.

Let's prove $\text{Span}(v_i) = \mathbb{R}^N$.
If exists $[\alpha_1, \ldots, \alpha_N] \in \mathbb{R}$ such that $[\alpha_1, \ldots, \alpha_N] \cdot [v_1, \ldots, v_N]$ generate the standard vectors of $\mathbb{R}^N$, then $\text{Span}(v_i) = \mathbb{R}^N$.
Let g_i be a standard generator of $\mathbb{R}^N$ with a 1 in position i. For all i such that $1 \leq i \leq N$,

$$H_n \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} = g_i$$

$$H_n H_n \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} = H_n g_i$$

$$NI_N \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} = v_i$$

$$\begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix} = \frac{1}{4} v_i$$

Therefore, all standard generators for $\mathbb{R}^N$ can be obtained through a linear combination of v_i's.

Since the cardinality of set $\{v_1, \ldots, v_N\}$ equals to the cardinality of the set of the standard base and $\text{Span}(v_i) = \mathbb{R}^N$, then $\{v_1, \ldots, v_N\}$ is linearly independent (assumed by the Teorema 6.1.3 from Yoshi's notes).