MAT0122 ÁLGEBRA LINEAR I

FOLHA DE SOLUÇÃO

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Assinatura

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Exercício: E19 (3.8.10 de PNK) **Data: 13/09/22**

SOLUÇÃO

[1] For $\mathcal{V}$ to be a vector space, it must satisfy three properties:

(1) $\mathcal{V}$ must contain the zero vector.
(2) For every vector v, if $\mathcal{V}$ contains v then it contains αv for every scalar α and is closed under scalar-vector multiplication
(3) For every pair u and v of vectors, if $\mathcal{V}$ contains u and v then it contains $u + v$

1.

$\mathcal{V}$ satisfies property (1). By hypothesis, every vector of $\mathcal{V}$ has an even number of 1's, and 0 is even by definition. Therefore, $\mathcal{V}$ contains the zero vector [0,0,0,0,0].

$\mathcal{V}$ satisfies property (2). Suppose a vector $v \in \mathcal{V}$. There are two scalars regarding GF2: 0 and 1. $0v$ returns the zero vector, included in $\mathcal{V}$ as said previously. $1v$ returns itself, obviously included in $\mathcal{V}$.

$\mathcal{V}$ satisfies property (3). The argument will be divided in cases:

- If u is the zero vector and v is not, then $u + v = v$, contained in $\mathcal{V}$.
- If $u = v$, then $u + v$ equals the zero vector, contained in $\mathcal{V}$.
- If $u \neq v$:
  - If u, v are 2-1's vectors, then:
    - 2 1's overlap (example: $11000, 01100$), then $u + v$ has 2 1's.
    - 0 1's overlap (example: $11000, 00011$), then $u + v$ has 4 1's.
  - If u, v are 4-1's vectors, then at least 3 1's overlap (example: $11110, 01111$). Considering the other 2 1's, they are in the form of $01, 10$, so $u + v$ must have 3 0's and 2 1's.
  - If u is a 2-1's vector and v is a 4-1's vector, then:
    - 2 1's overlap (example: $11110, 11000$), then $u + v$ has 2 1's.
    - 1 1 overlap (example: $11110, 00011$), then $u + v$ has 4 1's.

[1] This portion of text was extracted from the definition of vector space in PNK.

Either way, $u + v$ will have an even number of 1's.
Therefore, $\mathcal{V}$ is a vector space.

2.

$\mathcal{V}$ is not a vector space. By hypothesis, if every vector of $\mathcal{V}$ has an odd number of 1's, every vector must have at least a 1. Therefore, $\mathcal{V}$ does **not** have a zero vector and does not satisfy property (1).