數值分析 Team5 Homwork2 v2

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[Theoretical problems]

1. 略

2. Let $(0.1)_{10} = (0.a_1a_2a_3a_4a_5...)_2$

$$\begin{aligned}
(0.2)_{10} &= (a_1.a_2a_3a_4...)_2 \Rightarrow a_1 = 0 \\
(0.4)_{10} &= (a_2.a_3a_4a_5...)_2 \Rightarrow a_2 = 0 \\
(0.8)_{10} &= (a_3.a_4a_5a_6...)_2 \Rightarrow a_3 = 0 \\
(1.6)_{10} &= (a_4.a_5a_6a_7...)_2 \Rightarrow a_4 = 1 \\
(0.6)_{10} &= (0.a_5a_6a_7...)_2 \\
(1.2)_{10} &= (a_5.a_6a_7a_8...)_2 \Rightarrow a_5 = 1 \\
(0.2)_{10} &= (0.a_6a_7a_8...)_2 \\
(0.4)_{10} &= (a_6.a_7a_8a_9...)_2 \Rightarrow a_6 = 0
\end{aligned}$$

we get the relations

$$\begin{aligned}
a_1 &= 0 \\
a_2 &= a_6 = a_{2+4k} = 0 \\
a_3 &= a_7 = a_{3+4k} = 0 \\
a_4 &= a_8 = a_{4k} = 1 \\
a_5 &= a_9 = a_{1+4k} = 1
\end{aligned}$$

therefore

$$\begin{aligned}
(0.1)_{10} &= (0.000110011001100...)_2 \\
&= (-1)^0 * 2^{-4} * (1 + 0.10011001100110011001100...)_2
\end{aligned}$$

$$\begin{aligned}
sign\ s &= 0 \\
exponent\ e &= (-4 + 127) = 123 = (01111011)_2 \\
mantissa\ m &= 10011001100110011001101
\end{aligned}$$

So 0.1's floating-point number is 0 01111011 10011001100110011001101

3. (a) 略

(b)

$$\begin{aligned} b^2 &= 11.1556 \\ 4ac &= 11.1264 \\ b^2 - 4ac &= 0.0292 \end{aligned}$$

(c)

$$\frac{|0.0292 - 0.1|}{0.1} = \frac{0.0708}{0.1} = 0.708$$

[Numerical Problems]

1. The following is our MATLAB code

```
syms epsilon1 epsilon2 e1 e2;
epsilon1 = 1;
e1 = 0;
epsilon2 = 1;
e2 = 0;
while 1+epsilon1 > 1
    epsilon1 = epsilon1/2;
    e1 = e1 -1;
end
fprintf('epsilon1 is 2^%d\n',e1)

while epsilon2 > 0
    epsilon2 = epsilon2/2;
    e2 = e2 - 1;
end
fprintf('epsilon2 is 2^%d\n',e2)
```

$$\epsilon_1 = 2^{-53}, \epsilon_2 = 2^{-1075}$$

These two numbers are not the same , we guess the motion +1 involved in the arithmetic of floating point numbers, makes the accuracy decrease.

2. (a) 略

   (b) 略