Numerical Analysis - Homework 3

Due on 11/30 , 2016

[Theoretical problems]

1. [20%] Use Gaussian quadrature with $n = 2$ to approximate the following integrals, and compare your results to the exact values of the integrals.

   (a) $\int_1^{1.6} \frac{2x}{x^2-4} dx$.

   (b) $\int_3^{3.5} \frac{x}{\sqrt{x^2-4}} dx$.

2. [20%] Show that the quadrature formula $Q_n(f) = \sum_{i=1}^n w_i f(x_i)$ cannot have a degree of precision greater than $2n-1$ regardless of the choice of w_i and x_i, for $i = 1, \ldots, n$. (Hint: Construct a polynomial with a double root at each x_i, for $i = 1, \ldots, n$)

3. [20%] Calculate the resulting nodes and weights for an open two-point Newton-Cotes quadrature rule on the interval $[a, b]$? What is the degree of precision of the resulting rule?

[Numerical Problems]

1. [40%] Consider the following integral

$$\int_0^1 \frac{4}{1+x^2} dx = \pi.$$

   Implement the midpoint, trapezoid, and Simpson composite quadrature rules seen in class. Test the methods on the above example for step sizes $h = 1, 1/10, 1/100, \ldots, 1/10^6$. Characterize the errors as a function of h for each rule.